NBER WORKING PAPER SERIES

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Working Paper No. 2624

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
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June 1988

Harvard University and NBER, Stanford University and NBER, and University of Wisconsin and NBER respectively. We are grateful to Tom Abbott for research assistance. Financial support was provided by NSF-PRA grants 79-13740 and 85-12358 and a Sloan Foundation Dissertation Fellowship to the second author. The research supported here is part of the NBER's research program in Productivity. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

It is known that innovations in the market value of manufacturing firms and their R&D expenditures are related (Pakes (1985) and Mairesse and Siu (1984)). This could be due to shifts in the demand for the output of a particular firm, to shifts in the technological opportunities available to the firm, or to both. In this paper we use innovations in patenting activity as an additional piece of information about technological shifts in order to attempt to identify the relative importance of these two types of shocks. We build a simple two factor model of innovations in sales, investment, R&D investment, patent applications, and the rate of return to holding a share of the firm, and estimate it using a time series-cross section of U.S. manufacturing firms (340 firms from 1973 to 1980).

Except in the pharmaceutical industry, we find little evidence of a second factor which can be clearly identified with technological opportunity, although there is evidence of a long run growth factor linking both types of investment, patenting activity, and the market value of the firm. We then go on to demonstrate that this null result could be caused by our use of patent counts as an indicator of the value of the underlying patents: under reasonable assumptions on the value distribution, the changes in patenting rates can account for only an infinitesimal fraction of the changes in the stock market value of the firm, and hence provide essentially no additional information to the estimation procedure. However, the pharmaceutical industry is an important exception to this: here we find that the technological factor is almost as important as the short run demand factor in explaining movements in the rate of return, although both factors together account for less than five percent of the variance of this variable.

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R&D, Patents, and Market Value Revisited:

Is There Evidence of a Second, Technological Opportunity Related Factor?

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1. Introduction

As part of our ongoing research on the economic content of patent statistics we have been trying to ask the question, "Is there additional information in patent numbers on the rate and output of inventive activity above and beyond what is already contained in R&D expenditures data?" Can one use such data to distinguish between "demand side" and technological "opportunity" factors as they affect the rate of inventive activity? In this paper we use information on additional variables, especially physical investment and the stock market rate of return, to help us disentangle the effects of such factors. We do it first in two different ways: once in the framework of an unobservable two-factors model and then in a "causality testing" type of analysis. When both approaches turn out not to be sensitive enough to yield unequivocal information on the presence of such distinct "factors", we turn to an analysis of the potential information content of patent count statistics and show that, in view of the larger variance in patent values, this "failure" of patent numbers to be very informative is not all that surprising. Nevertheless, the analysis of the relationship between patent numbers and patent values turns out to be quite interesting on its own merits and indicates some new avenues for additional research on this range of questions.

Earlier work (Griliches 1981, Ben-Zion 1984, Mairesse-Siu 1984, and Pakes 1985) has shown that fluctuations in market value and R&D are related. It is not clear however, whether these fluctuations arise largely on the demand side, as Schmookler (1966) has argued, or represent shifts in technological opportunity (cf. Rosenberg, 1974). Most likely both forces are involved but it would be interesting if one could separate them and provide some indication of their relative importance.

It is not obvious whether one can separate "demand" from "supply" factors in this area, even conceptually. Our definition of "demand" factors relates to macro shifts in aggregate demand, population, exchange rates, and relative factor prices which make inventive activity more (or less) profitable at a given level of scientific information, a fixed "innovation possibilities frontier." We would identify changes in technological "opportunity" as those scientific and technological breakthroughs which make additional innovation more profitable or less costly at a fixed aggregate or industry level demand. These distinctions are far from sharp, especially given our inability to measure the contributions of science and technology directly. Moreover, what is a technological opportunity in one industry may spillover as a derived demand effect to another. Nevertheless, there is something distinct in these factors, in their sources of change and dynamics, which motivates our pursuit of this topic. We were led to it, in particular, by our hope that the availability of detailed data on patenting would be helpful distinguishing between them.

Patent data could help here if one were willing to assume that independent, "unanticipated" shifts in the level of patenting by firms, represent shifts in technological opportunities and not responses to current changes in economic conditions (demand forces). That is, the identifying assumption we will make is that the economy impinges on the level of patenting only through the level of R&D expenditures (and slowly changing trends) and that the "news" component in the patent statistics is either error (random fluctuation) or a reflection of technological "news", giving information that a particular line of research has turned out to be more (or less) fruitful or easier (harder) than expected when the decision to invest in it was made originally. That is, what we are hoping to identify here are changes in technological opportunity as reflected in "abnormal," "unexpected," bursts (or declines) in the number of patents applied for.[2]

Several implications of this formulation are immediate. If patent statistics contain additional information about shifts in technological opportunities, then they should be correlated with current changes in market value above and beyond their current relationship with R&D and they should affect R&D levels in the future, even in the presence of the change in market value variable since the latter variable is measured with much error. Patents should "cause" R&D in the sense of Granger (1969), a topic which we shall explore in the third section of this paper.

The available evidence on this point is not too encouraging: While Griliches (1981) found a significant independent effect of patents on the market values of firms, above and beyond their R&D expenditures, Pakes

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2. This and the following paragraph restate the formulation of the problem as given by Pakes (1985).

(1985), who studied this question based on a slightly different two-factor formulation, did not detect a significant influence of lagged patents on R&D in the presence of lagged R&D and the stock market rate of return variables. Nor did Hall, Griliches and Housman (1986) find future R&D affecting current patenting as the "causality" argument might have implied.

Since the first two studies were based on relatively small samples (about 100 firms) and since the Hall, Griliches and Hausman paper did not investigate the relationship of market value to the technological opportunity factor, it seems worthwhile to examine it a bit further, using a slightly more general model, and a longer and more updated data set.

2. R&D, Patents, and Market Value in a Two Factors Model

The model we will consider is very simple: we look at five variables which describe the current levels of the firm's output, stock market, and investment performance: sales, capital expenditures, research and development investment, patent applications, and the one period rate of return to holding a share of the firm. All variables are defined as the logarithm of their values (except for the stock market rate of return) and we divide each observation into an "anticipated" (predictable) and unanticipated or "news" part:

$$
\begin{aligned}
(1)\qquad s &= s^{*} + \hat{s} \\
i &= i^{*} + \hat{i} \\
r &= r^{*} + \hat{r} \\
p &= p^{*} + \hat{p} \\
q &= q^{*} + \hat{q}
\end{aligned}
$$

where s is "real" (deflated) sales, i is tangible investment (buildings and equipment), r is R&D expenditures, p is successful patent applications, and

q is the annual stock market rate of return (defined as $(V_t - V_{t-1} + Div_t)/V_{t-1}$). $\hat{x}$ is the anticipated or predictable (on the basis of past data) part of a variable while x^* is its "news" (innovation) component.

We make this division into "anticipated" versus the "news" part, because it is only the "news" part in the number of patents applied which we wish to identify with changes in technological opportunity. Of course, one could try to develop more explicit investment equations and expectation formation equations, derive the "news" component explicity, and impose all the available cross-equation constraints. Since our approach is exploratory, we condition instead on all the available past data, controlling thereby both for size differences and individual firm effects and the past information on which such expectations would be formed. This allows us to concentrate our modelling on the inovation components of these variables, reducing thereby significantly the dimensionality of the estimation problem.

A simple two orthogonal factor model is given by:

$$
\begin{aligned}
(2) \qquad s^* &= \beta_0 d + \epsilon_1 \\
i^* &= \beta_1 d + \epsilon_2 \\
r^* &= \beta_2 d + \eta_1 t + \epsilon_3 \\
p^* &= \eta_2 t + \epsilon_4 \\
q &= d + t + \epsilon_5
\end{aligned}
$$

where d is a "demand" shift factor which affects sales, investment, and R&D concurrently, while t is a "technology" factor which connects patents and R&D. Both factors affect also market value and are normalized to have a unit impact on it. The ϵ's are specific errors or disturbances and are assumed to be uncorrelated with each other and with d and t.[3] (I.e., it is assumed that the two particular factors are adequate to account for the intercorrelation structure in the 5 x 5 matrix.) The major identifying assumptions are that "t" does not enter the s and i equations (in the same year) and that "d" does not enter the p equation, except possibly via r .

If the innovations in r are allowed to affect p directly, via the γr term, the reduced form equation for p^* is

$$p^* = \gamma\beta_2 d + (\gamma\eta_1+\eta_2)t + \gamma\epsilon_3 + \epsilon_4$$

and the resulting variance-covariance matrix of forecast errors (the x*'s) has the following structure

$$(3)\qquad \begin{array}{ccccc}
\beta_0 d^2+\sigma_1^2 & \beta_0\beta_1 d^2 & \beta_0\beta_2 d^2 & \beta_0\gamma\beta_2 d^2 & \beta_0 d^2 \\
 & \beta_1^2 d^2+\sigma_2^2 & \beta_1\beta_2 d^2 & \beta_1\gamma\beta_2 d^2 & \beta_1 d^2 \\
 & & \beta_2^2 d^2+\eta_1^2 t^2+\sigma_3^2 & \gamma\beta_2^2 d^2+\eta_1(\gamma\eta_1+\eta_2)t^2+\gamma\sigma_3^2 & \beta_2 d^2+\eta_1 t^2 \\
 & & & \gamma^2\beta_2^2 d^2+(\gamma\eta_1+n_2)^2 t^2+(\gamma)^2\sigma_3^2+\sigma_4^2 & \gamma\beta_2 d^2+(\gamma\eta_1+\eta_2)t^2 \\
 & & & & d^2+t^2+\sigma_5^2
\end{array}$$

where $d^2 = \mathrm{Var}\ (d)$, $t^2 = \mathrm{Var}\ (t)$ and $\sigma_k^2 = \mathrm{Var}\ (\epsilon_k)$.

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3. For the purposes of estimation, we assume that the distribution of d, t, and the ϵ's is stationary over time, to allow us to pool the estimates. Allowing for changing variances and covariances would yield slightly different standard errors, but the pooled estimates are consistent even in this case.

That this model is identified can be seen by trying to solve this system recursively. The first row yields estimates of β_1, β_2, and γ. The second row, given an estimate of β_1, yields an estimate of d^2 (and another estimate of β_2). Having an estimate of d^2 allows one also to solve for β_0 and subsequently for σ_1^2 and σ_2^2. Given estimates of β_2 and d^2 yields, in row 3, estimates of $\eta_1 t^2$ and in row 4, $\eta_2 t^2$. Multiplying the main diagonal term of row 3 by γ and subtracting it from the first off-diagonal term yields an estimate of $\eta_1\eta_2 t^2$ and hence also estimates η_1, η_2, and t^2 and the rest of the parameters.

Given our assumptions, this is an overidentified model which can be estimated in two passes. First run a seemingly unrelated regression system of x on all the variables assumed to be in the "information" set and produce the variance-covariance matrix of the residuals ("innovations"). Then fit the model to the estimated residual covariances matrix using a maximum likelihood program such as MOMENTS or LISREL (see the references). Justification for proceeding in two stages can be found in MaCurdy (1981).

3. Basic Results

In this section we present the results of such computations. The sample we used starts out with the NBER R&D data base (Cummins et al 1985), which is based on firms that were in existence in 1976 and had good data for at least three years around that period. We selected only firms which had continuous data on the variables of interest for some period within 1970 to 1980. To minimize the measurement problems associated with the discreteness of the patent variable at low values, the particular sample used here was limited to those firms which averaged more than three successful patent applications per year during the period over which we observed them.[4] We also required that the firm's fiscal year end sometime between October 31 and February 28, since our patent data are available only on a calendar year basis. These restrictions brought us down to about 340 firms per year (for up to eight years after allowing for lags) and a total number of firm-year observations = 2377.[5]

The definition of $\hat{x}$ includes two lags of sales, investment, employment, the firm-specific sales deflator, R&D, and q, three lags of patents, and eight year dummies (reflecting common unanticipated macro events). The resulting variance-covariance matrix of residuals and the associated correlation matrix is shown in Table 1. Before we look at it, it is worth noting that as expected, in the forecast equations not shown here, very little of the stock market rate of return is explainable by past x's,[6]

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4. This still leaves us with some observations in individual years for some firms where the number of patent applications is zero. In order to make use of these observations after the log transformation, we set the number of patents in such years arbitrarily to one third.

5. The main points to be made do not appear to be sensitive to the rather drastic selection. Earlier we had used a somewhat tighter patents requirement of at least six patents per year. Results are also similar when the model is run on a larger sample based on an average of 600+ firms.

6. On theoretical grounds, we have in fact excluded all variables except the year dummies from the prediction equation for q in the results presented here. Except for possible measurement or timing problems due to the non-coincidence of the fiscal year and calendar year, the current stock market rate of return should not be predictable on the basis of the prior year variables. This is confirmed in the data. Although the existence of risk premia for the stocks of individual firms would theoretically introduce some systematic variation in q, we found that including two lags of q in the q equation reduced the variance by only one tenth of one percent, and had no visible effect on the covariances.

and that the lags of the stock market rate of return are significant predictors of both physical investment and R&D. As we have previously observed in these data (Hall, Griliches, and Hausman 1986 and Hall 1986), both employment and R&D are highly persistent with a coefficient on the first own lag of above 0.9, while sales, investment, and patents are less so.

Table 1 already gives the bad news. The correlation between the p and q residuals is .007, implying that there is little room for a separate "t" factor, or at least one whose identification depends on its "loading" on both the q and the patents variable. The same conclusion is reached if one uses the formulae in (3) and the data in Table 1 to try to compute the various coefficients. A direct computation yields either negative or close to zero values for var(t). In Columns 1, 2 and 3 of Table 2 we present maximum likelihood estimates of the parameters of a one factor model (for comparison purposes) and two versions of the two factor model (with and without r^* in the p^* equation). To facilitate comparison with the conventional factor model, we have renormalized the factors so their variance is unity, and d^2 and t^2 are now equal to the square of the factor loadings on q. Both models yield a very small negative variance for the second factor (see the estimate of t^2) and almost no improvement in the fit.

Why do the data yield these "bad" results? After verifying our data and sample, we considered the possibility that the appropriate structural relationship to be looking for is between lagged q and contemporaneous sales, investment, R&D, and patents as in Pakes 1985). Our q is measured between the ends of the previous and current year, so this version emphasizes the role of q as a price influencing investment decisions, while contaminating the news component of the variables with some information that was available during the previous year. While we found that covariances were generally higher using lagged q in place of contemporaneous q (and dropping it from the information set of predetermined variables), the results for the two factor model were worse, if anything: the second factor now has a distinctly negative variance.

Although we focus on patents as an indicator of technological success, the reason for our negative result lies partly in the R&D variable itself. If we assume that one factor links sales, investment, and R&D, then there is clear prediction for the covariance of r and q which is based on the relationship between sales, investment, and q. In the cases of both contemporaneous and lagged q, the difference between the observed covariance between r and q and the covariance due to d^2 is insignificantly different from zero or negative, leaving little room for the technological factor. Either η_1 is extremely small, or t^2 is negligible. Only by allowing r to enter the p equation directly can we justify the fairly large covariance between r and p, and this leaves us with only two very small covariances between r and q and p and q with which to estimate t^2.

It was a maintained assumption of our model that the technological opportunity factor is only "news" for R&D, patents, and q. In an effort to explore further why our firm data fail to reveal such a factor, we perform in Table 2 a conventional factor analysis in order to test for the existence of a second factor of any kind. It turns out that the data will not support two factors with a complete set of loadings for all variables, since we have a so-called "Heywood" case, where the off-diagonal covariances imply negative idiosyncratic variances, but there is a two factor model with zero restrictions which will fit the observed data perfectly ($\chi^2(2)=0.07$): this model is shown in Column 4 of Table 2 and it differs from our original model only in that investment as well as R&D, patents, and q is allowed to load on the second factor. While we cannot label this factor as "pure" technological opportunity, because it may also reflect longer run demand forces it is still an interesting finding in and of itself. However, the contribution of this second factor to the variance of q is infinitesimal, on the order of .02 percent, and its loading on p is not particularly significant either.

We have also estimated some of these models on nine fairly coarse industry groupings (see Table 4 for a listing of them) and found that a one factor model was sufficient in all but three of them: Petroleum-Rubber, Metals-Stone-Clay-Glass, and Drugs. Only in drugs was the second factor model sensible, in the sense that the variance estimates and the loadings on p were all positive. These estimates are shown in Table 2a. The coefficients for this industry (column 2 of Table 2a) imply that d^2 accounts for 2.3 percent of the variance in the rate of return and t^2 accounts for 1.5 percent. Thus in this industry, the patent-linked technological factor is almost as important as the shortrun demand factor; this finding is consistent with other evidence on the importance of patents in the pharmaceutical industry, though again the statistical significance of this finding (the factor loading on the patents variable) is not particularily impressive.

Finally, the interesting finding in the preliminary regressions, which create the "news" (unforecastable), components of these variables, is the strength of the employment variable. In Table 3 we show the first differenced (growth rate) version of these preliminary regressions, which are somewhat easier to interpret than the level regressions.[7] Changes in employment are as good or better than changes in deflated sales in explaining changes in investment and R&D. The most significant variables in the sales equation are the lagged stock market rate of return and lagged employment growth. Neither lagged investment, lagged R&D, or lagged patents are significant although patents lagged twice are marginally so.

In the investment change equation the main important variables are the lagged stock market rate of return, the lagged employment change, and the lagged investment change (negatively). Lagged sales change is insignificant in the presence of the lagged employment change, and its coefficient is negative. Neither lagged R&D or lagged patent changes are significant in the investment equation. In the R&D change equation, the significant variables are the lagged employment change [.13(.05)], lagged stock market rate of return [.07(.01)], lagged sales change [.19(.04)], and the lagged R&D change [-0.07(.02)]. In the patent equation, besides lagged patent change [-0.46(.02)], only lagged R&D [.14(.06)] and lagged employment [.29(.12)] are significant. It appears that the employment change contains more information about the "permanent" changes in the firm's fortunes than most of the other variables. This last finding is consistent with Hall's (1986) finding that there is almost no transitory measurement error in the employment

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7. Except for the q variables, which we do not difference, the growth rate regressions will be identical to level regressions with the first lag coefficient constrained to be unity. Since this coefficient was near unity except for patents, and the coefficients of the other lags tended to be equal and opposite in sign, it is somewhat more parsimonious to look at the regression in first differenced form.

variable.

4. The Time Series Relationship of R&D, Patents, and Market Value

Another way of testing for the presence of a second, patents connected factor is to return to the Pakes (1985) model and retest some of his hypotheses on our larger and more recent data set.

In his model, there is one set of news that is associated with contemporenous movements in q, r, and p which induces also subsequent moves in r and p. In particular, his conclusion that the "news" works almost entirely via R&D and hence a one factor model is adequate, is based on the non-significance of lagged patents in the R&D equation, in the presence of lagged R&D and the stock market rate of return variables. Another implication of the two factor hypothesis is that current q should also appear in the patent equation, reflecting the technological news not fully recoverable from current R&D (due to superimposition of other sources of variation).

Table 4 summarizes various tests of this sort performed separately for different industrial groupings and for the sample as a whole. A log R&D equation is computed containing three lagged values of R&D, three lagged values of patents, and the current and three lagged values of the stock market rate of return. The first test asks whether lagged values of q, beyond the first lag (which was the measure used by Pakes) enter this equation. The second test includes the first lagged q, which enters significantly in more than half the industries. The third test asks whether all the lagged patent variables could be deleted from this equation while the fourth test asks whether one lagged value of R&D is enough. The regressions are based on the same unbalanced panel of firms as Table 1, with slightly fewer observations due to the additional lags of r, p, and q.

Turning to the third test first, the one of most interest to us, lagged patents are barely significant in the overall sample and the relevant coefficient is not economically significant (for Log p_{-2} it is 0.02(.01)). Lagged patents are significant in the Chemicals and Electrical groupings; in both cases it is the second lag rather than the first. In Drugs, where we might have expected to see the strongest effect, because of the economic importance of patents in that industry, all the coefficients are insignificant, as is their sum. This may not be unreasonable per se. It says that a burst of patents does not lead to a permanent increase in R&D above and beyond the level which was forecastable from lagged r and q, reflecting a transient rather than permanent technological opportunity.

Lagged q (the stock market rate of return) is a significant determinant of R&D overall and for most of the industrial grouping separately. Two and three year lagged values of q are also significant overall but only for the non-electrical machinery grouping separately, implying that there may be serious costs in adjusting R&D rapidly to current events. Overall, lagged q, the original Pakes formulation, is a somewhat stronger variable than current q (about twice as large on average with substantial inter-industry variation), implying again that it may take some time to adjust R&D to the market and technological news that are implicit in q.

Table 5 lists similar results for the patent equation where the first test asks whether lagged q's enter the equation above and beyond current and lagged R&D levels; the second test asks whether adding current q contributes to the explanation of the variability in patent applications;

while the last test asks whether there is a significant lag in the effect of R&D on patent applications (it is a follow up on the question explored in Hall, Griliches, and Hausman, 1986). At the pooled all-industries level none of these effects seem important. Neither current or lagged q are either individually or jointly significant and lagged R&D values are only barely so. q is marginally significant only for the firms in Primary Metals, Stone, Clay, and Glass while lagged R&D is of some importance in the Petroleum, Drugs and Metals groupings.

Table 6 gives the estimated coefficients for both R&D and patents equations for the overall pooled sample, indicating the type of result yielded by this approach. Note that lagged R&D expenditures are not statistically significant in the patent equation either. The stock market rate of return does explain movements in R&D but not in patents and R&D is close to a random walk while there is much more evidence of a persistence in a firm's propensity to patent. These findings are effectively the same as in Pakes (1985). A bigger sample and more variables have not really increased our ability distinguish between the various hypotheses about the underlying sources of these fluctuations.

5. Patent Counts and Patent Values.

Why are these results so poor and fragile? The answer to this question lies in the noisiness of patents as an indicator of the value of inventive output. Beginning with Schmookler's critics and possibly even earlier the use of patent statistics as an invention indicator has been questioned on many grounds. The problems cited by critics of the patent count methodology are twofold: first, not all useful innovations are patented; in fact in some industries very few may be. Second, the distribution of the value of individual patents is extremely skewed toward low values. There is informal evidence on this point (see Freeman (1982) and the references there) and there are recent estimates by Schankerman and Pakes (1986) and Pakes (1986) of the distribution of the value of patent rights in several European countries. There are also calculations by Grabowski and Vernon (1983) of the distribution of the value of New Chemical Entities for pharmaceutical companies, which probably corresponds to the upper tail of the patent value distribution. This empirical evidence, although somewhat sparse and imprecise, confirms what most of those knowledgable in the field would say: a few patents are worth a great deal in present value terms while most are (nearly) worthless.

From the point of view of the present attempt to identify a second, more technological opportunity related factor, this skewness in the patent value distribution is bad news. In order to identify this second factor, we need to measure the covariance between the increase in the present value of the firm which is signalled by the new patents and q, the stock market rate of return; the closest we can come to measuring this quantity is to measure the covariance between current patent <u>counts</u> and q, and the quality of this measurement is dependent on the quality of the link between counts and values.

It is possible to derive the quantitative implications of such a skewed distribution of values for the quality of the patent count indicator by combining what we know about patent counts in both the time series and cross section dimension with estimates of the distribution of their values. Following Griliches (1981), assume that at any point in time a firm possesses a stock of knowledge, K, which was produced by the past stream of R&D expenditures. Each year, there are surprises in the additions to this stock of knowledge generated by current and past R&D. We assume that the number of patent applications in a year, n_t, is an indicator of the size of the surprise associated with new inventions that occurred during this period.[8] In addition, there are also surprise revaluations of past "news", of the discoveries of previous years. We now proceed to estimate the relative importance of patents as a component of the variance in the firm's rate of return.

Let us decompose the change in the value of the firm (net of its expected dividend and investment policy) into three components:

$$q_t V_t = w_t + \eta_t + u_t$$

where q_t is the rate of return on stock holding, V_t is the total market-value of the firm's assets, and three components w_t, η_t, and u_t are defined to be orthogonal to each other. w_t corresponds to the change in the value of a firm's R&D "position" (program) arising from the "news" associated with current patent applications. η_t reflects revaluations of previous achievements associated with past patents (above and beyond their correlation with current patents). u_t reflects all other sources of fluctuation in the value of the firm, including also possibly the contribution of not patented R&D. We shall first focus on w_t and the role of patent numbers as an indicator of it. Next, we shall ask about the possible magnitude of the variance of w_t (relative to the variance of $q_t V_t$). That is, how large could the contribution of current patents be to the explanation of fluctuations

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8. This is an oversimplification since the average patenting level of the firm is predictable based on its history. However, given the level of the noise in the value distribution, it is not unreasonable to begin with this assumption and then correct the observed variance for the predictability later in the computations. It will make very little difference to the result.

in market value, even if we had perfect measures of the <u>values</u>? Indirectly, we shall be also asking how big is the haystack (the fluctuations in market value) relative to the needle that we are searching for. Finally, we shall sketch out a procedure which should allow us, in principle, to estimate the variance of η_t, the contribution of the revaluations of past patenting.

In order to decompose the variance of the first component, we make the following stochastic assumptions: 1) The number of patents applied for each year is distributed as a Poisson random variable with a mean which is a distributed lag of past R&D expenditures (see Hausman, Hall, and Griliches 1985). 2) y_i, the underlying value of each patent to the firm is distributed as a log normal random variable with a mean and variance which will be derived from the earlier literature. With these assumptions, the total value of the patent applications of a firm in any one year is

$$W = \sum_{i=1}^{p} y_i$$

where p is Poisson and y is lognormal.

If p is Poisson and y is lognormal, the first two moments of w (under independence) are

$$E[w] = E[py] = \lambda E[y] \qquad \text{where } \lambda = E[p]$$

$$V[w] = V\left(\sum_{i=1}^{n} y_i\right) = \lambda\, V[y] + [Ey]^2\, \lambda$$

Note that in this model, and in Hausman, Hall, and Griliches, λ is taken as a function of R&D. Hence, given an R&D policy, λ is a constant for the firm. Since we shall be allowing for the contribution of R&D changes separately, it is reasonable to make this computation holding R&D constant. We are interested in the value of the news contained in the patent variable above and beyond what is already summarized in the R&D variable. That is, we are looking for patents to measure the "output" value of the R&D process above and beyond, and distinct from, its "input".

The component of the variance of w which could be accounted for by patent <u>numbers</u> corresponds to the last term

$$\mathrm{Var}\ [n\ y] = \lambda[E(y)]^2$$

and its relative size is given by

$$\mathrm{Var}\ [n\ y]\ /\mathrm{Var}\ [w] = 1/(1+V[y]/(E[y]^2) = 1/(1+\tau^2)$$

where τ is the coefficient of variation in the distribution of patent values.

Now we can turn to the literature for some order of magnitude estimates of these various parameters and make a few illustrative calculations. For this purpose we need estimates of Ey, the average increment to the value of the firm associated with an "unanticipated" patent, and Var(y), the variance in this value. It will become clear, shortly, that our conclusions are not particularly sensitive to the precise value of such parameters and hence we will not try to defend them in great detail.

Turning first to the mean value of patents we have estimates of the value of the news associated with patents in the U.S. of between $200,000 (Griliches, 1981) and $800,000 (Pakes, 1985) per patent. There is also some information on this point in our data: using the covariance matrix in Table 1, we can regress q on the news in patents to obtain an approximate estimate of the rate of return to an increase in patenting. This produces an estimate of $98,000 per unexpected patent at the geometric mean of our data (with a very large standard error). The data for the drug industry, where patents are more important, yield a larger and somewhat more precise estimate: an \$821,000 average increase in the value of the firm per unexpected patent. This is in fact very similar to the Pakes (1985) estimate which was based on a smaller sample of larger firms and is therefore more comparable to our drug firm subset.

If we take the upper range of these numbers, \$800,000 per "unexpected" patent, and use $\lambda = 13$, the average (geometric) number of patents received in our sample (per year, per firm), the expected contribution of the variance in patent numbers to the average variance in market value is $13 \cdot (.8)^2\ (\text{mil\$})^2 = \$8.3$ million squared. This is to be compared to the average variance of $q_t V_t$ in our sample. The variance of q is 0.133 which, evaluated at the geometric average value of our firms (\$276 million), yields a variance of market value changes on the order of \$10,000 million squared. Comparing the two variances yields an estimate of the size of the needle as less than one-tenth of one percent (.0008). And this number is already based on the upper range of the available estimates!

The next task is to estimate the potential size of Var(w), the variance of patent values rather than its approximation by patent numbers. For this we need an estimate of Var(y), which we shall try to borrow from Schankerman and Pakes (1986).[9] If we take their numbers for Germany from their table 4 and project the distribution of present values of patent rights back to age zero on the basis of the relationship between their estimates of this

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9. Shankerman and Pakes obtain distributions of the value of the patent right to the since their estimates are based on the decision to patent; while we are assuming that the value of the underlying innovation to the firm is proportional to the patent right value and highly correlated with it.

distribution for age three and age five[10] we get approximate estimates of the mean and variance of the lognormal distribution (9.25 and 2.62 respectively, estimated from the inter-quartile range of this distribution), which imply an arithmetic mean patent value of about \$39,000 and a standard deviation of \$139,000. Of course the average value of the news in a patent for a U. S. manufacturing firm is undoubtedly higher both due to the larger market and the higher value of corporate patents. Multiplying through by 3, to adjust roughly for the relative size of the two countries yields comparable U.S. "estimates" of \$116,000 and \$413,000 respectively.

Since we are looking for upper-bound estimates, we shall take from them the implied estimate of 3.6 for the coefficient of variation in these values. (Similar estimates in Pakes 1986 can be interpreted as implying a coefficient of variation on the order of 3). Applied to our "upper" range estimate of $Ey = \$0.8$ million, it implies a variance in y of $(3.6 \times \$0.8 \text{ million})^2 = \$8.3$ million squared. The total variance of w is now

$$13[(2.88)^2 + (.8)^2] = \$166 \text{ million sq.}$$

or a little over one percent of the total variance in market value. That is, even if we had good estimates of patent values, they would account for very little of the fluctuations in market value. Having numbers instead of values only makes matters worse, reducing this fraction even further. The

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10. The ratio of the arithmetic means in the two years is 1.32, which suggests a depreciation rate of fifteen percent per year for the sample of patents which survive until the fifth year (over 98 percent of all patents granted).

contribution of patent numbers to the variance in their values is only on the order of .7 percent 1/(1+3.6), and their contribution to the explanation of the variance in the unexpected changes in the market values of individual firms is even smaller, an infinitesimal .0008.

For the drug industry, where the variance of market values is smaller (Var q = 0.06) and the average number of patents received is larger (23), the news in patents could account for as much as 2 percent of the observed variance in the market value of firms but patent numbers would pick up less than a tenth of this still rather small fraction.

There are two major problems in using this procedure to estimate the variance of the news in the economic value of patents held by the firm: the first is that the distribution estimated by Schankerman and Pakes is a distribution of the value of patent rights, which may be less than the true economic value of the associated invention to the firm. Thus our estimate may be an underestimate of the variance in the value of inventions. Using the upper bound of the various estimates based on our stock market rate of return data for the drug industry we have tried to correct for this problem. The second problem probably goes in the other direction: some of the change in the firm's patent value this year may not be news, and thus may already be incorporated into the market value at the beginning of the year. But allowing for some predictability of patent numbers would only reduce such fractions further, multiplying them essentially by $1-R^2$ of the prediction equation. Because of these problems we try an alternative approach in the next section.

6. Estimating Variance Components

An alternative approach to this question can be developed from an explicit modelling of the components of variance in stock market value surprises as a function of current and past patenting and R&D activity. In principle, this approach allows us also to estimate the contribution of revisions in the value of past patents to current changes in market value. Unfortunately, the estimates in this case are based essentially on fourth moments of the data and are, therefore, rather imprecise.

The estimating equation, which is derived in greater detail in Appendix A has the form

$$(3) \qquad y_t = \sigma^2_{\epsilon_t} + \mu^{v^2} x_t + \sum_{\tau=0}^{3} \sigma^2_\tau p_{t-\tau} + \xi_t$$

where $y_{it} = (q_{it}V_{it})^2$ is the squared annual change in the stock market value of each firm, and x_{it} is the surprise component in current patent applications

$$x_{it} = (p_{it} - \hat{\mu}^p_{it})^2$$

to be estimated using a patent forcasting equation described in Table A1.

Equation (3) partitions the variance in market value surprises into three components: the contribution of the "News" in current patents -- $(\mu^v)^2 x_t$, the contribution of the revaluations in the past and previously anticipated patents -- $\Sigma\sigma_\tau^2 p_{t-\tau}$ and the contribution of all other sources of market value change--$\sigma^2_{u_t}$. The major assumption made in deriving this equation is that the various individual surprises and revaluations are independent of each other, the common components being captured by time and also possibly firm dummies. The parameters to be estimated are μ^{v^2}, the square of the average patent value, σ^2_0, σ^2_1, σ^2_2, σ^2_3, the variance of the surprises in the values of patents of different vintages, and a set of dummies reflecting common macro events (the $\sigma^2_{u_t}$).

Before we estimate this equation we need to confront the following problem: qV in our data ranges all the way from $7,000 to $12 billion, owing to the large size range of our firms. Since the majority of firms have changes in market value at the lower end of this range, the dependent variable in equation (3) has a very skewed distribution. Consequently we expect that ξ_{it} will have considerable size-related heteroskedasticity.[11] This will not bias the results if we have exactly the correct model for $(q_{it} V_{it})^2$, but it does tend to place more weight on the extreme observations than we might like. If the true model has some nonlinearity, for example, weighted least squares estimates may differ substantially from unweighted ones.

We choose to mitigate this problem in this case by allowing each firm to have its own average level of variance in annual stock market value changes. That is, we write ξ_{it} as

$$\xi_{it} = \alpha_i + v_{it}$$

and we estimate α_i, the firm specific variance of qV, along with the other parameters of the equation. This introduces approximately 400 nuisance parameters into the regression, but, as is well known, the estimates from a linear regression on panel data with fixed effects are still consistent. After introducing these firm effects, we find that the simple Lagrange multiplier test for heteroskedasticity of the residuals now has a value of 51. with 12 degrees of freedom, which is significant but not greatly so.

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11. In fact the TR^2 from a regression of the residuals squared from this regression on the independent variables is 131.2 with 12 degrees of freedom.

The results of estimating equation (3) are shown in Table 7. The F-statistic for the industry effects in column 2 is $F(19, 2468) = 5.65$ and that for the firm effects is column 4 is $F(367, 2120) = 3.18$. Because we believe that not allowing each firm to have its own variance may bias the coefficient estimates (for example, if the general riskiness of the firm is correlated with its patenting activity), we focus on the estimates in columns 3 and 4. From these estimates, we hope to get an approximate measure of μ^v, the expected return to a patent application, and σ_τ^2, $\tau = 0,1,\ldots.$, the variance in the revision of the value of a patent application which is τ years old. The estimates shown are not completely sensible since they imply a negative value of μ^{v^2}, although with a fairly large standard error. Thus, at best, the expected value of a patent application is approximately zero using this methodology (remembering that the variable in question, x_{it}, is imperfectly measured in any case). The estimates of σ_τ^2, on the other hand, imply that the news about patent values is of roughly the same order in the first two years after a patent is applied for, with substantial revisions in value taking place even after three or more years (as is indicated by the large estimate for the P_{-3} coefficient).

We are particularily interested in estimating the part of the variance of the stock market return which is attributable to news about the expected stream of returns from patents, which is

$$(4) \qquad V(v_{q,0} \mid 3_q) = \sum_{\tau=0} \beta^{2\tau} \sigma_\tau^2$$

In Table 7 we display this quantity, under the assumption that $\beta=0.9$. It is quite large, on the order of $500 million squared.

It implies that the variance in the news about the value of patents (current and past) could account for over five percent of the total variance in market value surprises. This is not a negligable amount, given the high market volatility from other sources.

It is also consistent with our previous, "back-of-the-envelope" calculations of the potential contribution of the news value in current patents, which we had estimated at \$166 million squared. If we take the last number in Table 7 (\$538 mill. sq.) to be the variance associated with an average stock of 52 (4 x 13) patents, each with an independent revision in its value, then the implied standard deviation of an individual patent value revision is on the order of \$3.2 million. Given our earlier assumption of a coefficient of variation of 3.6 in patent values, this is in turn fully consistent with the assumed \$800,000 news value per new patent in the previous section.

7. Concluding Comments

In this paper we tried first to use patent and market value data at the firm level to distinguish between "demand pull" and "technological opportunity" push forces affecting inventive activity. Two different ways of looking at it, via an unobservable factors model and via time-series causality testing, yielded interesting but statistically insignificant results. The data were not strong enough to discriminate between the various hypotheses.

We asked ourselves, then, perhaps belatedly, the question: can such effects be estimated at all with the data at hand? What is the potential information content of patent data, especially in the context of using market value changes as the variable to be explained?

Two relatively simple conclusions emerged from this examination: (1) If we are interested in estimating the impact of the value of inventive output using only patent counts as an indicator, we are, at best, likely to capture only about six to ten percent of it, using such measures. (2) Trying to do so in the context of market value change equations makes this task almost impossible, primarily because of the stock market's very high volatility. Our conclusions can be restated in terms of the following orders of magnitude. Fluctuations in the market's evaluation of the patented portion of firm's R&D program could account, perhaps, for about five percent of the total variance in market value surprises, of which about one-fifth might be attributable to the news associated with current patent applications. But, using only the number of current patent applications would account for less then 0.1 percent of the total variance, making tests of any hypotheses associated with these effects rather difficult to perform.

Yet another way of making the same point is to note that the estimated 0.08 patent counts variance component translates itself into a required sample size of 5000 for a t-ratio (significance level) of 2 ($t = 2 = .0008N$, $N = 5000$). We are not that far away from it in the aggregate regressions: Our sample size is about 2500 and we would need "only" to double our sample size to approach "significance." But at the industry level the outlook is not particularly optimistic. Only in the drug industry, where we estimate that patent counts could account for about one percent of the variance in market values, might a modest expansion in sample size be adequate.

We do develop an alternative approach, using a model of the variance in market values, which allows us to estimate the variance component associated with patent count fluctuations, but here too, because the approach relies on fourth moments of the data for identification, larger samples and/or more relevant variance in the independent variables may be a prerequisite for further progress.

The major conclusion of this paper is that one should probably not be looking at data on stock market fluctuations if one is trying to test detailed hypotheses about the information content of patent statistics. Because the variability of stock market values exceeds by an order of magnitude the variability associated with any of the "real" causes that one might estimate, our estimates do not tell us that the returns to inventive activity are small or that the topic we have been pursuing is not interesting, only that we have been looking for our particular needle in a very large haystack and should not be really surprised when we are turned back empty-handed.

Table 1

Innovations in Sales, Investment, R&D, Patents and Market Value

Number of Firms = 340
Number of Observations = 2377
1973-1980

Variances and Covariances

	s	i	r	p	q
s	.014177	.016585	.007351	.00017	.008257
i		.186597	.014282	.003742	.012137
r			.041678	.007567	.005521
p				.270047	.001277
q					.132988

Correlation Coefficients

	s	i	r	p	q
s	1.0	.322	.302	.0028	.190
i		1.0	.162	.017	.077
r			1.0	.071	.074
p				1.0	.007
q					1.0

Residual variances, covariances and correlations from regressions containing the same variables lagged once and twice, lagged values of employment and the sales deflator, and year dummies.

Variables:

- s - deflated sales
- i - investment
- r - R&D
- p - patents
- q - stock market rate of return.

All variables are in logarithms of original values except for q.

Table 2

Estimates of Factor Models

Number of Observations = 2377

	One Factor Model	---------- Two Factor Models			--------------------
			r in p eq.		
			Loadings on Factor 1		
s	.096(.005)	.095(.014)	.095(.014)	.108(.017)	.095(.011)
i	.173(.012)	.174(.020)	.174(.020)	.154(.025)	.156(.027)
r	.077(.006)	.078(.009)	.078(.009)	.068(.011)	.102(.034)
p	.011(.013)	-	-	-	.041(.033)
q	.084(.009)	.085(.010)	.085(.010)	.076(.014)	.070(.014)
d^{2***}	.0070(.0016)	.0073(.0017)	.0073(.0017)	.0058(.0027)	.0048(.0020)
			Loadings on Factor 2		
s	-	-	-	-	.051(.041)
i	-	-	-	.043(.034)	.035(.043)
r	-	**	**	.088(.065)	-.046(.056)
p	-	**	**	.085(.064)	-.073(.063)
q	-	**	**	.005(.023)	.032*
t^{2***}	-	-.00005(.0009)	-.000003(.00006)	.00002(.00022)	.00102*
			Idiosyncratic Variances		
s	.0049(.0010)	.0050(.0010)	.0050(.0010)	.0025(.0036)	.0025(.0036)
i	.1567(.0055)	.1565(.0056)	.1565(.0056)	.1611(.0077)	.1611(.0077)
r	.0358(.0012)	.0374(.0033)	.0385(.0018)	.0292(.0108)	.0292(.0109)
p	.2699(.0078)	.4384(2.78)	.3179(2.17)	.2629(.0131)	.2630(.0130)
q	.1260(.0038)	.1252(.0040)	.1257(.0038)	.1271(.0040)	.1271(.0040)
log L	-1785.4	-1782.5	-1782.5	-1778.2	-1778.2
χ^2	14.4	8.6	8.6	0.07	0.05
DF	5	3	2	2	1

Table 2a

Estimates of the Factor Models
Pharmaceutical Industry

Number of Observations = 240

	One Factor Model	Two Factor Models: r in p eq.	
		Loadings on Factor 1	
s	.061(.007)	.066(.032)	.069(.013)
i	.133(.024)	.127(.060)	.123(.030)
r	.079(.011)	.077(.034)	.074(.015)
p	.011(.037)	-	-
q	.043(.019)	.038(.017)	.036(.019)
d^{2}***	.0018(.0016)	.0014(.0013)	.0013(.0014)
		Loadings on Factor 2	
s	-	-	-
i	-	-	.024(.040)
r	-	.0458(.033)	.050(.037)
p	-	.323(.392)	.295(.216)
q	-	.029(.023)	.033(.027)
t^{2}***	-	.0009(.0014)	.0011(.0018)
		Idiosyncratic Variances	
s	.0024(.0008)	.0018(.0008)	.0014(.0017)
i	.0887(.0089)	.0902(.0080)	.0909(.0093)
r	.0085(.0015)	.0072(.0025)	.0072(.0026)
p	.2411(.0220)	.1368(.1302)	.1539(.1270)
q	.0582(.0054)	.0579(.0049)	.0578(.0054)
log L	243.5	254.1	254.3
χ^2	24.1	3.0	2.6
DF	5	3	2

Notes to Table 2 and Table 2a

*This parameter was fixed at this value since the model with two free factors was not estimable in this data (see the text).

**Because the estimate of the variance of the second factor t^2 is negative these factor loadings are not computable (they are imaginary).

***Since we estimated the factor models with the variance of the factors normalized to be unity, the t^2 and d^2 parameters are just the square of the factor loadings for q, and are derived estimates.

Table 3

Growth Rate Regressions

Dependent Variable: Growth Rate of	s	i	r	p	q
e_{-1}	.35(.03)	.20(.10)	.13(.05)	.29(.12)	-
s_{-1}	-.07(.03)	-.04(.10)	.20(.04)	.01(.11)	-
i_{-1}	.007(.006)	-.30(.02)	.002(.010)	.004(.025)	-
r_{-1}	.006(.011)	.07(.04)	-.07(.02)	.13(.05)	-
p_{-1}	-.004(.005)	.03(.02)	.005(.008)	-.46(.02)	-
p_{-2}	.012(.005)	.01(.02)	.03(.01)	-.17(.02)	-
d_{-1}	-.46(.05)	.43(.19)	.19(.09)	-.03(.23)	-
q_{-1}	.04(.01)	.34(.03)	.07(.01)	.05(.03)	-
q_{-2}	.006(.007)	.11(.02)	.03(.01)	.008(.029)	-
			Year Effects		
1973	.12(.01)	.19(.03)	.01(.01)	-.08(.04)	-.21(.02)
1974	.03(.01)	.23(.03)	-.01(.01)	-.001(.04)	-.23(.02)
1975	-.03(.01)	-.14(.04)	-.05(.02)	-.06(.04)	.54(.02)
1976	.11(.01)	-.20(.04)	-.03(.01)	-.09(.05)	.42(.02)
1977	.05(.01)	-.07(.03)	-.05(.01)	-.07(.04)	.04(.02)
1978	.10(.01)	.06(.03)	.004(.01)	-.08(.03)	.12(.02)
1979	.07(.01)	.15(.03)	.03(.01)	-.13(.04)	.29(.02)
1980	-.01(.01)	.05(.03)	.002(.02)	-.35(.04)	.30(.02)
Std. Error	.123	.418	.206	.535	.366
R^2	.36	.25	.28	.21	.43

These are based on the same dataset as the first stage regressions for Table 1. The variables e and d are employment and the firm specific sales deflator respectively. s, i, and r are deflated variables (see Cummins et al 1986 for details of the deflation).

Table 4

The R&D Equation: Summary of Test Statistics**

Number of Firms = 340
1973-1980

F Statistics for the Hypothesis of Zero Coefficients

Sample Groupings	NT^+ / R^2	Test 1 $q_{-2}q_{-3}$ F(Prob)	Test 2 $q_{-1}q_{-2}q_{-3}$ F(Prob)	Test 3 All p F(Prob)	Test 4 $r_{-2}r_{-3}r_{-4}$ F(Prob)
1) Food & Chemicals	278 .998	.18 (.83)	.20 (.89)	4.41* (.01)	.30 (.74)
2) Petroleum & Rubber	190 .998	1.63 (.20)	2.54 (.06)	1.42 (.24)	.76 (.47)
3) Metals, Stone Clay Glass	173 .993	1.40 (.25)	2.41 (.07)	.90 (.45)	4.96* (.01)
4) Drugs	228 .999	.24 (.79)	1.12 (.34)	1.89 (.13)	6.14* (.00)
5) Machinery, Engines, & Fabric. Metals	460 .984	4.65* (.01)	4.00* (.01)	.83 (.49)	2.14 (.12)
6) Computers & Sci. Instruments	370 .991	.91 (.40)	4.98* (.00)	1.29 (.27)	10.43* (.00)
7) Elec. Mach. & Electronics	290 .996	.22 (.80)	1.67 (.17)	3.23* (.03)	1.26 (.29)
8) Motor Vehicles & Aircraft	181 .997	1.00 (.37)	3.49* (.02)	1.33 (.27)	2.32 (.10)
9) Other	157 .989	.32 (.73)	3.88* (.01)	1.82 (.15)	1.81 (.17)
Total Sample	2154 .995	9.53* (.000)	24.73* (.000)	5.55* (.05)	1.60 (.20)

See the following page for notes to the table.

Notes to Table 4:

**The estimated equation is of the form

$$r = \text{year dummies},\ q, q_{-1}, q_{-2}, q_{-3},\ r_{-1}, r_{-2}, r_{-3},\ p_{-1}, p_{-2}, p_{-3}.$$

where all the lower case variables except q are logarithms of the original variables.

The sample is an unbalanced panel of firms with good R&D and market value data, a fiscal year end between October and February, and an average number of patents over the whole period of at least three per year. The period covered is 1973-80, for between 160 to 340 firms. In a few cases where the number of patents was zero in a particular year, p was set to .33.

+NT is the number of firm-year observations; there are up to eight years per firm.

*Statistically significant rejection of the null hypothesis. The numbers in parenthesis are the "probability levels" corresponding to the particular test statistics.

Table 5

Patent Equation: Summary of Test Statistics

Sample Groupings	NT / R^2	Test 1 lag q F(prob)	Test 2 All q F(prob)	Test 3 lag r F(prob)
		F Statistics for the Hypothesis of Zero Coefficients		
1) Food & Chemicals	278 .990	.97 (.41)	.87 (.48)	.70 (.56)
2) Petroleum & Rubber	190 .989	1.76 (.16)	1.34 (.26)	3.94* (.01)
3) Metals, Stone & Glass	173 .976	.90 (.45)	2.46* (.05)	4.65* (.00)
4) Drugs	228 .981	.98 (.41)	.89 (.47)	5.88* (.00)
5) Machinery, Engines, & Fabric. Metals	460 .952	1.90 (.13)	1.42 (.23)	1.74 (.16)
6) Computers & Sci. Inst.	217 .984	1.52 (.21)	1.21 (.31)	1.65 (.18)
7) Elec. Mach. & Electronics	270 .964	2.18 (.09)	2.25 (.06)	.66 (.58)
8) Motor Vehicles & Aircraft	181 .981	.44 (.73)	.33 (.86)	.15 (.93)
9) Other	157 .960	1.02 (.39)	.78 (.54)	1.44 (.23)
Total Sample	2154 .974	.51 (.68)	.61 (.65)	3.85* (.01)

<u>Notes</u>:
The equation estimated is of the form:

$$p = \text{year dummies},\ q, q_{-1}, q_{-2}, q_{-3},\ r, r_{-1}, r_{-2}, r_{-3},\ p_{-1}, p_{-2}, p_{-3}.$$

See the notes to Table 2 for additional detail.

Table 6

Estimated Log R&D and Log Patents Equations for U.S. Manufacturing Firms 1973-1980

	Dependent Variable	
	Log R&D	Log Patents
Number of Observations	2154	2154
q	.046(.012)	.029(.032)
q_{-1}	.092(.012)	.037(.033)
q_{-2}	.048(.011)	.010(.029)
q_{-3}	.011(.011)	-.007(.030)
Log R&D	- -	.212(.056)
Log R&D$_{-1}$	.961(.021)	.053(.077)
Log R&D$_{-2}$	.018(.028)	-.143(.073)
Log R&D$_{-3}$	.017(.020)	-.021(.052)
Log P$_{-1}$	.012(.009)	.489(.022)
Log P$_{-2}$	.024(.009)	.250(.024)
Log P$_{-3}$	-.026(.009)	.139(.022)
Standard Error	.204	.528

Notes:

This sample is the same as that in Tables 2 and 3. It is "unbalanced," in the sense that the number of available data points varies from year to year. Both equations contain year dummies.

Standard errors are in parentheses.

Table 7

Equation for the Variance in Market Valuation of Patents

2498 Observations
1973-1980

Unweighted Estimates

Model	(1)	(2)	(3)	(4)
$(p_t-\mu_t^p)^2$	-.012 (.006)	-.008 (.006)	-.019 (.008)	-.015 (.008)
P	-2.21 (.68)	-1.96 (.67)	1.32 (.77)	0.22 (.69)
P_{-1}			1.33 (.87)	
P_{-2}			0.46 (.89)	
P_{-3}			5.52 (.79)	
$\sum_{r=1}^{3} P_{-r}$	1.55 (.21)	1.16 (.22)		2.58 (.29)
Industry dummies	no	yes	no	no
Firm dummies	no	no	yes	yes
Estimated V^*	.89 (.30)	.36 (.30)	5.63 (.25)	5.38 (.25)
Estimated μ^V	---	---	---	---

All equations contain year dummies.

The dependent variable is the change in market value of the firm during the year, squared. The coefficients of patents and of the dependent variable are in units of a hundred million dollars squared (10^{16} $\$^2$).

*This estimate is calculated using equation (5) with $\beta = 0.9$.

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Appendix A

Estimating the Distribution of the Value of News in Patenting

In this appendix we present the theoretical justification for the regression equation (3) which allows us to perform a variance decomposition of the variance in the firm's one period rate of return into the fraction due to news in the value of patents (both those taken out this year and in prior years) and all other news.

Let there be some underlying measure space which suffices to define the joint distribution of the sequence of net returns to all possible patents and the number of patents applied for, say $(\Omega, \mathfrak{F}, \mathcal{P})$. The sequence, $(\ldots \mathfrak{F}_t \leq \mathfrak{F}_{t+1} \leq \mathfrak{F}_{t+2}, \ldots \leq \mathfrak{F})$, is a sequence of increasing sub σ-fields ("information sets"). We assume that all moments needed in the subsequent presentation are finite. Any other assumptions are stated explicitly where necessary.

For the generic patent taken out in year q, let

$$\text{(A1)} \qquad v_{q,0} = \sum_j \beta^j r_{q+j} \quad , \qquad 0 \leq \beta \leq 1 \quad ,$$

where $\{r_{q+j}\}$ are the sequence of net returns associated with the patent, and define

$$\text{(A2)} \qquad v_{q,a} = \sum_j \beta^j r_{q+a+j} \quad ,$$

so that $\{v_{q,a}\}$ is the sequence of discounted values of net returns remaining after "a" years $(a \geq 0)$. Then

(A3) $$v_{q,0} = \eta_{q,0} + \sum_{a} \beta^{a} \eta_{q,a}$$

where

$$\eta_{q,a} = (E_a - E_{a-1}) v_{q,a} , \qquad \text{for } a \geq 1$$

$$\eta_{q,0} = E_0 \sum \beta^{j} r_{q+j}$$

and

$$E_t x = \int x \, \mathcal{P}(dx|\mathfrak{I}_t),$$

the conditional expectation of x where we condition on $\mathfrak{I}_t$, the information available at time t. Note that for a > 0

(A4) $$E[\eta_{q,a}|\mathfrak{I}_{q+\tau}] = \begin{cases} 0 & \text{if } \tau < a \\ \eta_{q,a} & \text{if } \tau \geq a \end{cases}$$

That is, the innovation in the returns to a patent taken out in year q which occur in year q+a are in the information set for years q+a and later. We define the initial expected value of the patent as

(A5) $$E[v_{q,0} \,|\mathfrak{I}_{q-1}] = E[\eta_{q,0} \,|\mathfrak{I}_{q-1}] = \mu_q^{v}$$

$\{\eta_{q,j}\}$ is a sequence of mutually uncorrelated random variables by construction.

With these preliminaries out of the way, we can write down the value of the patent stocks held by a firm and partition this value into the predictable (measurable $\mathfrak{I}_{q-1}$) and unpredictable part. We expect only the latter part to be reflected in the current stock market rate of return (at year q). Let W_t be the expected discounted value of net returns associated with the patents owned by the firm in year t. Then

(A6)
$$W_t = \sum_{q=0} W_{q,t-q}$$

where

$$W_{q,t-q} = E\left[\sum_{i=1}^{p(q)} v_{i,q,t-q} \;\middle|\; \mathfrak{I}_t \right] .$$

$p(q)$ denotes the number of successful patent applications taken out in year q and $v_{i,q,t-q}$ is the value of the ith patent taken out in year q remaining in year t. Note that $p(q)$ is known with certainty at any $t \geq q$, but not at $t = q - 1$ (that is, $p(q)$ is measurable w.r.t. $\mathfrak{I}_q$).

Under the assumption that the stock market value of the firm in year t-1 incorporates all the information in $\mathfrak{I}_{q-1}$, the theoretical model for the relationship between news in the value of patents and the stock market value of the firm gives

(A7)
$$\begin{aligned} q_t V_t &= W_t - E_{t-1} W_t + \varepsilon_t \\ &= (W_{t,0} - E_{t-1} W_{t,0}) + (W_{t-1,1} - E_{t-1} W_{t-1,1}) + \cdots + \varepsilon_t \\ &= (E_t - E_{t-1}) \left[\sum_{i=1}^{p(t)} v_{i,t,0} \right] \\ &\quad + \sum_{i=1}^{p(t-1)} \eta_{i,t-1,1} + \sum_{i=1}^{p(t-2)} \eta_{i,t-2,2} + \cdots + \varepsilon_t \end{aligned}$$

To simplify the analysis we assume:

A1: The distribution of $(p_t, v_{1,t,0}, v_{2,t,0}, v_{3,t,0} \cdots)$ conditional on $\mathfrak{I}_{t-1}$ factors into the distribution of p_t conditional on $\mathfrak{I}_{t-1}$ and that of $(v_{1,t,0}, v_{2,t,0}, v_{3,t,0}, \cdots)$. Moreover $(v_{1,t,0}, v_{2,t,0}, \cdots)$ consist of a sequence of exchangeable random variables.

Assumption A1 implies that each member of the sequence $\{v_{1,t,0}\}$ has a common mean which is independent of the number of patents applied for (there is no ex ante relationship between the quantity of patents and their expected values). Given assumption A1, we have that

$$\text{(A8)} \qquad E_{t-1}\left[\sum_{i=1}^{p(t)} v_{i,t,0}\{i \le p(t)\}\right] = \mu_t^v \, \mu_t^p \quad ,$$

where μ_t^v is defined above and $\mu_t^p = E_{t-1}p(t)$. Hence

$$\text{(A9)} \qquad (E_t - E_{t-1})\left[\sum_{i=1}^{p(t)} v_{i,t,0}\{i \le p(t)\}\right] = \sum_{i=1}^{p(t)} \bar{\eta}_{i,t,0} + \mu_t^v\,(p_t - \mu_t^p)$$

where $\bar{\eta}_{i,t,0} = \eta_{i,t,0} - \mu_t^v$. Substituting into equation (0) we have

$$\text{(A10)} \qquad q_t V_t = \mu_t^v(p_t - \mu_t^p) + \sum_{i=1}^{p(t)} \bar{\eta}_{i,t,0} + \sum_{i=1}^{p(t-1} \eta_{i,t-1,1} + \cdots + \varepsilon_t$$

Thus, we have decomposed the one period rate of return to owning the firm into a part consisting of the surprise due to the number of patent applications this year, a part consisting of the news in the value of patents applied for this year, parts which are news about the value of old patents held by the firm which is learned this year, and, finally, an residual ε_t which contains all other news which affects the value of the firm, but is uncorrelated with current or past patents.

The simplest way to see the variance relationships is to assume

A2: The double sequence $\left\{\eta_{i,t-q,q}\right\}_{i=1,q=0}^{p(q)}$ is a sequence of mutually uncorrelated random variables. Moreover, ϵ_t is independent of p_t, p_{t-1}, ..., r_t, r_{t-1}, ...

Then

$$\text{(A11)} \qquad E[(q_t V_t)^2 \mid p_t, p_{t-1}, \ldots, r_t, r_{t-1}, \ldots]$$

$$= \mu_t^{v^2} (p_t - \mu_t^p)^2 + \sigma^2_{t,0}\, p_t + \sigma^2_{t-1,1}\, p_{t-1} + \ldots + \sigma^2_\epsilon$$

This is our regression function. Parameters are σ^2_ϵ, $\mu_t^{v^2}$, $\sigma^2_{t,0}$, $\sigma^2_{t,1}$, ..., "independent" variables are $(p_t - \mu_t^p)^2$, p_t, p_{t-1}, and so forth.

In order to actually estimate the regression, we need to know $(p_t - \mu_t^p)^2$, the squared surprise in patent applications for the ith firm in year t conditional on s_{t-1}. Since this quantity is unobservable, we estimate it by regressing the observed p_{it} on lagged p, lagged r, time dummies, and industry dummies (to control for the differing propensity to patent in different industries). We then form a predicted number of patents in each year for each firm μ^p_{it} using the estimated regression coefficients and the observed p and r of the firm. The estimated log patents equation which we use is shown in Table A1. We found that the fit was improved substantially when we included squared lagged patents and squared lagged R&D ($F(2,2575) = 63.2$). Although the industry dummies are not very significant ($F(19,2555) = 2.0$), we included them because some industries patent substantially less than others and not controlling for this would overestimate the variance of the news in the patent counts. Thus the equation we used for predicting μ^P in the following computation is based on column (3) of Table A1.

Now let $y_t = (q_t V_t)^2$, and $x_t = (p_t - \mu_t^P)^2$. Rewriting equation (A11), we have (under A1 and A2)

$$\text{(A12)} \qquad y_t = \sigma^2_{\varepsilon_t} + \mu_t^{v^2} x_t + \sum_{\tau=0} \sigma^2_{t-\tau,\tau} \, p(t-\tau) + \xi_t$$

where ε is the "error" in the equation and

$$E[\xi_t \mid x_t, p_t, p_{t-1}, \ldots] = 0$$

For the empirical work reported in the paper, we assume in addition

$$\text{A3:} \quad \sigma_{t-\tau,\tau} = \sigma_\tau, \quad \mu_t^v = \mu^v.$$

A3 merely specifies that the distribution from which patent values are drawn does not change during our time period, which is about eight to ten years. Using this assumption, the regression equation becomes

$$\text{(A13)} \qquad y_t = \sigma^2_{\varepsilon_t} + \mu^{v^2} x_t + \sum_{\tau=0} \sigma^2_\tau \, p_{t-\tau} + \xi_t$$

which gives us time-homogeneous coefficients (except the constant term). The subscript i indexes firms and t indexes years. When we estimate equation (A13), we include three lagged values of patent application counts in addition to the current value.

Table A1

Estimates of the Expected Number of Patents

2586 Observations
1973-1980

Model	(1)	(2)	(3)
P_{-1}	.77 (.02)	.75 (.03)	.75 (.03)
P_{-2}	.24 (.03)	.26 (.03)	.26 (.03)
P_{-3}	-.06 (.02)	-.08 (.02)	-.07 (.02)
R_{-1}	.08 (.02)	.12 (.03)	.11 (.03)
R_{-2}	-.17 (.04)	-.14 (.04)	-.14 (.04)
R_{-3}	.07 (.03)	.04 (.03)	.04 (.03)
$P_{-1} \cdot R_{-1}$		$.20\ 10^{-3}$ $(.13\ 10^{3})$	$.19\ 10^{-3}(.13\ 10^{-3})$
P_{-1} Squared		$.11\ 10^{-3}$ $(.22\ 10^{-3})$	$.13\ 10^{-3}(.24\ 10^{-3})$
P_{-2} Cubed		$-.37\ 10^{-6}$ $(.56\ 10^{-6})$	$-.42\ 10^{-6}(.57\ 10^{-6})$
P_{-1} Fourth power		$.50\ 10^{-9}$ $(.37\ 10^{-9})$	$.54\ 10^{-9}(.38\ 10^{-9})$
R_{-1} Squared		$-.14\ 10^{-3}$ $(.18\ 10^{-3})$	$-.10\ 10^{-3}(.08\ 10^{-3})$
R_{-1} Cubed		$.19\ 10^{-6}$ $(.08\ 10^{-6})$	$.16\ 10^{-6}(.08\ 10^{-6})$
R_{-1} Fourth power		$-.07\ 10^{-9}$ $(.02\ 10^{-9})$	$-.07\ 10^{-9}(.02\ 10^{-9})$
R_{-1} Squared $\cdot P_{-1}$		$.01\ 10^{-6}$ $(.07\ 10^{-6})$	$.01\ 10^{-6}(.07\ 10^{-6})$
P_{-1} Squared $\cdot R_{1}$		$.55\ 10^{-6}$ $(.17\ 10^{-2})$	$-.54\ 10^{-6}(.17\ 10^{-6})$
Industry dummies	no	no	yes
Standard error	16.8	16.5	16.6
R-squared	.970	.97	.97

The dependent variable is the number of patent applications taken out by the firm during the year which were later granted.

All equations include year dummies. In column (3), twenty industry dummies (at roughly the two and one half digit level) were included in the regression.