Optimal Taxation with Endogenous Default under Incomplete Markets

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Abstract

In a dynamic economy, we characterize the fiscal policy of the government when it levies distortionary taxes and issues defaultable bonds to finance its stochastic expenditure. Households anticipate the possibility of default, generating endogenous debt limits that hinder the government's ability to smooth shocks using debt. Default is followed by temporary financial autarky. The government can only exit this state by paying a fraction of the defaulted debt. Since this payment may not occur immediately, in the meantime, households trade the defaulted debt in secondary markets; this device allows us to price the government debt before and during the default.

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1 Introduction

For many governments, debt and tax policies are conditioned by the possibility of default. For emerging economies, default is a recurrent event and is typically followed by a lengthy debt-restructuring process, in which the government and bondholders engage in renegotiations that conclude with the government paying a fraction of the defaulted debt.[1]

We find that emerging economies exhibit lower levels of indebtedness and higher volatility of government tax revenue than industrialized economies — where, contrary to emerging economies, default is not observed in our dataset.[2] In particular, we find that amongst emerging economies, higher spreads are associated with more volatile tax revenues. Also, emerging economies exhibit higher interest rate spreads, especially for high levels of domestic debt-to-output ratios, than industrialized economies. In fact, interest rate spreads in industrialized economies are low and roughly constant for different levels of domestic debt-to-output ratios. Moreover, in emerging economies the highest interest rate spreads are observed after default and during the debt-restructuring period.[3]

These empirical facts indicate that economies that are more prone to default display different government tax policy, as well as different prices of government debt, before default and during the debt-restructuring period. Therefore, the option to default, and the actual default event, will affect the utility of the economy's residents: indirectly, by affecting the tax policy and debt prices, and also directly, by lowering the payoff of their bond holdings.[4]

[1] See Pitchford and Wright (2008) and Benjamin and Wright (2009).

[2] To measure "indebtedness", we use government domestic debt-to-output ratios, where domestic debt is defined as the debt issued under domestic law (see Panizza (2008)); a similar pattern for indebtedness levels is observed for external debt (see Reinhart et al. (2003)). Domestic debt is used as a proxy of domestically held debt since, as argued by Reinhart and Rogoff (2008), in most countries, over most of their history, the former has been mainly in the hands of local residents, while the majority of foreign debt has been held by foreign investors. As a proxy of tax policy, we are using government revenue-to-output ratio or inflation tax.

[3] Some examples are Argentina 2001, Ecuador 1997, and Russia 1998.

[4] Empirical evidence seems to suggest that government default has a significant direct impact on domestic residents; either because a considerable portion of the foreign debt is in the hands of local investors, or because the government also defaults on domestic debt. For example, for Argentina's default in 2001, about 60 percent of the defaulted debt is estimated to have been in the hands of Argentinean residents; local pension funds alone held almost 20 percent of the total defaulted debt (see Sturzenegger and Zettelmeyer (2006)). For Russia's default in 1998 about 60 percent of the debt was held by residents. For Ukraine's default in 1997-98, residents — Ukrainian banks and the National Bank of Ukraine

Our main objective is to understand how the possibility of default and the actual default event affect tax policy, debt prices — before default and during financial autarky —, and welfare of the economy.[5] For this purpose, we analyze the dynamic taxation problem of a benevolent government with access to distortionary labor taxes and non-state-contingent debt in a closed economy under incomplete markets. We assume, however, that the government cannot commit to pay the debt. In case the government defaults, the economy enters temporary financial autarky wherein it faces exogenous random offers to repay a fraction of the defaulted debt that arrive at an exogenous rate.[6] The government has the option to accept the offer — and thereby exit financial autarky — or to stay in financial autarky awaiting new offers. During temporary financial autarky, the defaulted debt still has some value as the recovery rate is positive; a fraction of it will be eventually repaid in the future. Hence, households can trade the defaulted debt in a secondary market from which the government is excluded; the equilibrium price in this market is used to price the debt during the period of default. Finally, in order to keep the model close to the standard optimal tax literature, e.g. Aiyagari et al. (2002), we assume that the government commits itself to its optimal path of taxes as long as the economy is not in financial autarky.

In the model, the government has three policy instruments: (1) distortionary taxes, (2) government debt, and (3) default decisions that consist of: (a) whether to default on the outstanding debt and (b) whether to accept the offer to exit temporary financial autarky.

In order to finance the stochastic process of expenditures, the government faces a trade-off between levying distortionary taxes and not defaulting, or issuing debt and thereby increasing the exposure to default risk. The option to default introduces some degree of

(NBU) among others — held almost 50 percent of the outstanding stock of T-bills (see Sturzenegger and Zettelmeyer (2006)). See Reinhart and Rogoff (2008) for a discussion and stylized facts on domestic debt defaults.

[5] In this model, financial autarky is understood as the period during which the government is precluded from issuing new debt/savings. Also, throughout this paper, we will also refer to the restructuring period as the financial autarky.

[6] While in our model we allow only for outright default on government bonds, governments could liquidate the real value of the debt and repayments through inflation risk, which could be viewed as a form of partial default. In several economies, however, this second option may not available, either because the country has surrendered the control over its monetary policy (for example, as in the eurozone, Ecuador, and Panama), or a significant portion of the government debt is either foreign-currency denominated, or local-currency denominated but indexed to the CPI or a similar index. We see our environment particularly appropriate for this class of economies.

state contingency on the payoff of the debt since the financial instrument available to the government becomes an option, rather than a non-state-contingent bond. This option, however, does not come free of charge: in equilibrium households anticipate the possibility of default, demanding a compensation for it in the pricing of the bond; this originates a "Laffer curve" type of pattern for the bond proceedings, thereby implying endogenous debt limits. In this sense, our model generates "debt intolerance" endogenously.[7]

The main insight of the paper is that these borrowing limits hinder the government's ability to smooth shocks using debt, thus rendering tax policy more volatile, and implying higher interest rate spreads. In equilibrium, the government may optimally decide not to honor its debt contracts —even though the bondholders are the households whose welfare it cares about— because default would prevent the government from incurring in the future tax distortions that would come along with the service of the debt. We believe this is a novel motive to default on government debt which, to our knowledge, had not been explored before in the literature.

The possibility of default introduces a trade-off between the cost of the lack of commitment to repay the debt, reflected in the price of the debt, and the flexibility that comes from the option to default and partial payments, reflected in the payoff of the bond.

In a benchmark case, with quasi-linear utility and i.i.d. government expenditure but allowing for offers of partial payments to exit financial autarky, we characterize analytically the determinants of the optimal default decision and its effects on the optimal taxes, debt and allocations. In particular, we first show that default is more likely when the government expenditure or debt is higher, and that the government is more likely to accept any given offer to pay a fraction of the defaulted debt when the level of defaulted debt is lower. These theoretical results have implications for haircuts and duration of debt restructuring processes that are aligned with the data. Second, we show that prices — both outside and during financial autarky — are non-increasing on the level of debt, thus implying that spreads are non-decreasing and also implying the existence of endogenous borrowing limits. Third, we show that the law of motion of the optimal government tax policy departs from the standard martingale-type behavior found in Aiyagari et al. (2002) (henceforth, AMSS). In particular, we show that the law of motion of the optimal government tax policy is affected, on the one hand, by the benefit from having more state-contingency on the payoff of the bond, but, on the other hand, by the cost of having the option to default.[8]

[7] A term coined by Reinhart et al. (2003).

[8] See also Farhi (2010) for an extension of Aiyagari et al. (2002) results to an economy with capital.

Finally, we calibrate a more general model; this model is qualitatively consistent with the differences observed in the data between emerging and industrialized economies. In terms of welfare, the numerical simulations suggest a nonlinear relationship between household utility and the probability of receiving an offer of partial payments. In particular, increasing the probability of receiving offers for exiting autarky decreases welfare when this probability is low/medium to begin with, but increases it when the probability is high.

The paper is organized as follows. We first present the related literature. Section 2 presents some stylized facts. Section 3 introduces the model. Section 4 presents the competitive equilibrium, and section 5 presents the government's problem. Section 6 derives analytical results. Section 7 contains some numerical exercises. Section 8 briefly concludes. All proofs are gathered in the appendices.

1.1 Related Literature

The paper builds on and contributes to two main strands in the literature: endogenous default and optimal taxation.

Regarding the first strand, we model the strategic default decision of the government as in Arellano (2008) and Aguiar and Gopinath (2006), which, in turn, are based on the seminal paper by Eaton and Gersovitz (1981). Our model, however, differs from theirs in several ways. First, we consider distortionary taxation; Arellano (2008) and references therein implicitly assume lump-sum taxes. Second, in our model the government must pay at least a positive fraction of the defaulted debt to exit financial autarky through a *debt-restructuring process*; in Arellano (2008) and references therein, the government is exempt from paying the totality of the defaulted debt upon exit of autarky. We consider a simple debt-restructuring process, indexed by two parameters, wherein renegotiation opportunities arrive exogenously, but the government endogenously chooses whether to accept of reject the repayment offers. We make this modeling assumption because we are interested in studying only the consequences of this process on the optimal fiscal policy and welfare.[9] Third, our economy is closed—i.e., "creditors" are the representative household—; Arellano (2008) and references therein assume an open economy with foreign creditors. This difference is key to capture the direct impact of the default event in the residents of the economy. Empirical evidence suggests that when governments renege

[9]See Benjamin and Wright (2009), Pitchford and Wright (2008), Yue (2010) and Bai and Zhang (2012) for ways of modeling the entire deb-restructuring process endogenously.

their debt contracts domestic residents and banks are severely affected, either because the default is on external debt and large fraction of it is held by them, or because it directly involves domestic debt; see footnote 4 for particular examples.[10]

Regarding the second strand, we build our framework on Aiyagari et al. (2002). Their economy is a closed one wherein the government chooses distortionary labor taxes and non-state-contingent *risk-free* debt, taking into account restrictions from the competitive equilibria, to maximize the households' lifetime expected utility.

In their work, by imposing non-state-contingent debt, AMSS reconcile the behavior of optimal taxes and debt observed in the data with the theory developed in the seminal paper of Lucas and Stokey (1983), in which the government has access to state-contingent debt under complete markets. These papers work under the assumption of full commitment on both the tax policy and the debt policy. Our work relaxes this last assumption and, as a consequence, generates endogenous debt limits, reflected in the equilibrium prices. It is worth noting that all these papers (and ours) take market incompleteness as exogenous, since the goal is to study the implications of this assumption. Albeit outside the scope of this paper, it would be interesting to explore ways of endogenizing market incompleteness; the paper by Hopeynhan and Werning (2009) seems a promising avenue for this.

Following the aforementioned literature, we assume that the government can commit itself to a tax policy outside temporary financial autarky. During financial autarky taxes are set mechanically to cover the government expenditure. Finally, when the government regains access to financial markets, we assume that it is able to revise and reset its fiscal policy. This feature is related to Debortoli and Nunes (2010). That work studies the dynamics of debt in the Lucas and Stokey (1983) setting but with the caveat that at each time t, with some given probability, the government can lose its ability to commit to taxes; the authors refer to this as "loose commitment." Thus, our model provides a mechanism that "rationalizes" this probability of "loosing commitment" by allowing for endogenous default, and resetting of fiscal policy when the debt settlement is reached. It is worth noting that in their model the budget constraint during the no-commitment stage remains essentially the same, whereas ours does not.

Finally, in recent independent papers, Doda (2007) and Cuadra et al. (2010) study the procyclicality of fiscal policy in developing countries by solving an optimal fiscal-policy

[10] Although outside the scope of this paper, allowing for both type of lenders could be an interesting avenue for future research. See Broner et al. (2010) for a paper studying this issue in a more stylized setting.

problem. Their work differs from ours in two main aspects. They assume first an open small economy (i.e., foreign lenders) and, second, no secondary markets.[11]

2 Stylized Facts

In this section, we present stylized facts regarding the domestic government debt-to-output ratio and central government revenue-to-output ratio of several countries: industrialized economies (IND), emerging economies (EME) and a subset of these: Latin American (LAC).[12]

In the dataset set, IND do not exhibit default events, whereas EME/LAC (LAC in particular) do exhibit several defaults.[13] Thus, we take the former group as a proxy for economies with access to risk-free debt and the latter group as a proxy for economies without commitment. It is worth to point out that we are not implying that IND economies are a type of economy that will never default; we are just using the fact that in our dataset IND economies do not show default events, to use them as a proxy for the type of economy modeled in AMSS (i.e., one with risk-free debt). There is still the question of which characteristics of an economy will prompt it to behave like IND or EME/LAC economy. One possible explanation is that for IND default could be more costly due to a higher degree of financial integration, affecting more severely the balance sheets of financial intermediaries and, thus, the financing conditions of firms, leading in turn to a sharper drop in productivity for the overall economy.[14] As we will see below, in the model this effect is captured in a "reduced form" by a parameter κ that determines the productivity level during financial autarky.

The main stylized facts that we found are, first, that EME/LAC economies have higher default risk than IND economies and that within the former group, the default risk is much higher for economies with high levels of debt-to-output ratio. Second, EME and LAC economies exhibit tighter debt ceilings than economies that do not default (in

[11] Aguiar et al. (2009) also allow for default in a small open economy with capital where households do not have access to neither financial markets nor capital and provide labor inelastically. The authors' main focus is on capital taxation and the debt "overhang" effect.

[12] For government revenue-to-output ratios, we used the data from Kaminsky et al. (2004), and for the domestic government debt-to-output ratios, we used the data from Panizza (2008). See appendix F for a detailed description of the data.

[13] In our sample for LAC, four countries defaulted, and most notable, Argentina defaulted repeatedly.

[14] In a general equilibrium setup, Mendoza and Yue (2012) endogenize the output loss during sovereign defaults as an outcome resulting from the substitution of imported inputs by less-efficient domestic ones when the financing costs of the former rises due to sovereign default risk.

this dataset, represented by IND). Third, economies with higher default risk exhibit more volatile tax revenues than economies with low default risk, and this fact is particularly notable for the group of EME/LAC economies (where defaults are more pervasive).

As shown below, our theory predicts that endogenous borrowing limits are more active for a high level of indebtedness. That is, when the government debt is high (relative to output), the probability of default is higher, thus implying tighter borrowing limits, higher spreads and higher volatility of taxes. But when this variable is low, default is an unlikely event, thereby implying slacker borrowing limits, lower spreads and lower volatility in the taxes. Hence, implications in the upper tail of the domestic debt-to-output ratio distribution can be different from those in the "central part" of it. Therefore, the mean and even the variance of the distribution are not too informative, as they are affected by the central part of the distribution; quantiles are better suited for recovering the information in the tails of the distribution.[15]

Figure 2.1 plots the percentiles of the domestic government debt-to-output ratio and of a measure of default risk for three groups: IND (black triangle), EME (blue square) and LAC (red circle).[16] The X-axis plots the time series averages of domestic government debt-to-output ratio, and the Y-axis plots the values of the measure of default risk.[17] For each group, the last point on the right correspond to the 95 percentile, the second to last to the 90 percentile and so on; these are comparable between groups as all of them represent a percentile of the corresponding distribution. EME and LAC have lower domestic debt-to-output ratio levels than IND; in fact the domestic debt-to-output ratio value that amounts for the 95 percentile for EME and LAC, only amounts for roughly 85 percentile for IND (which in both cases is only about 50 percent of debt-to-output

[15]We refer the reader to Koenker (2005) for a thorough treatment of quantiles and quantile-based econometric models.

[16]This type of graph is not the conventional QQplot as the axis have the value of the random variable which achieves a certain quantile and not the quantile itself. For our purposes, this representation is more convenient.

[17]The measure of default risk is constructed as the spread using the EMBI+ real index from J.P. Morgan for countries for which it is available and using the 3-7 year real government bond yield for the rest, minus U.S. bond return. Although bond returns are not entirely driven by default risk but also capture other factors related to risk appetite, uncertainty and liquidity, for our purpose they constitute a valid conventional proxy of default risk. Furthermore, our spreads are an imperfect measure of default risk for domestic debt since EMBI+ considers mainly foreign debt. However, it is still informative since domestic default are positively correlated with defaults on sovereign debt, at least for the period of 1950's onwards, see figure 10 in Reinhart and Rogoff (2008).

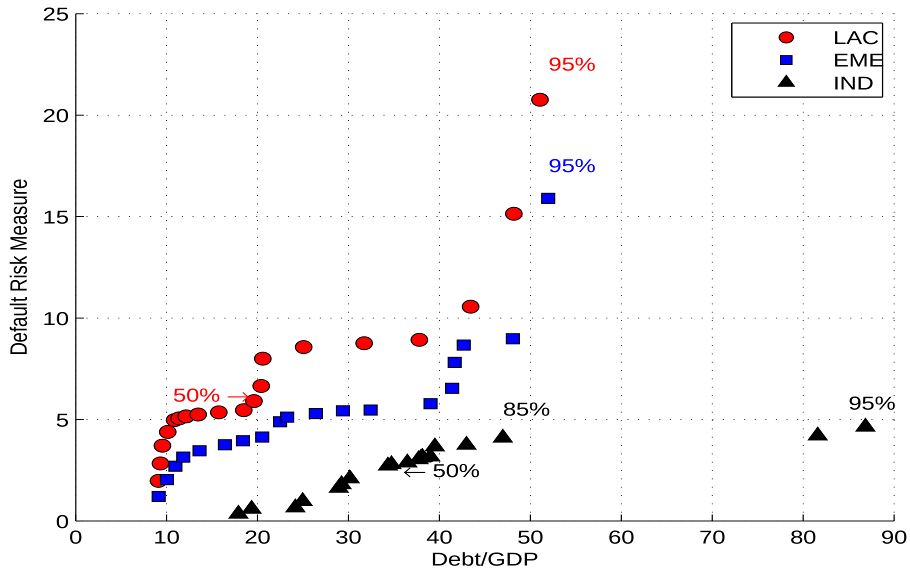

Figure 2.1: The percentiles of the domestic government debt-to-output ratio and of a measure of default risk for three groups: IND (black triangle), EME (blue square) and LAC (red circle)

ratio).[18] *Thus, economies that are prone to default (EME and LAC) exhibit tighter debt ceilings than economies that do not default (in this dataset, represented by IND).*

Figure 2.1 also shows that for the IND group, the default risk measure is low and roughly constant for different levels of debt-to-output ratios. On the other hand, the default risk measure for the EME group is not only higher, but increases substantially for high levels of debt-to-output ratios. We consider this as evidence that, for EME economies, higher default risk is more prevalent for high levels of debt-to-output ratios.

Table 2.1: (A) Measure of default risk for EME and IND groups for different levels of debt-to-output ratio; (B) standard deviation of central government revenue over GDP (%) for EME and IND groups for different levels of default risk.

(A)			(B)		
Debt/GDP	EME	IND	Default Risk	EME	IND
25%	5.4	2.0	25%	0.9	1.4
75%	10.7	2.9	75%	2.5	1.7

[18] We obtain this by projecting the 95 percentile point of the EME and LAC onto the X-axis and comparing with the 85 percentile point of IND.

Table 2(A) compares the measure of default risk between IND and EME matching them across low and high debt-to-output ratio levels. That is, for both groups (IND and EME) we select economies with debt-to-output ratio below the 25th percentile (these are economies with low debt-to-output) and for these economies we compute the average risk measure; we do the same for those economies with debt-to-output ratio above the 75th percentile (these are economies with high debt-to-output). For the case of low debt-to-output ratio, the EME group presents higher (approximately twice as high) default risk than the IND group; however, for high debt-to-output ratio economies, this difference is quadrupled. *Thus, economies that are prone to default (EME and LAC) exhibit higher default risk than economies that do not default (in this dataset, represented by IND), and, moreover, the default risk is much higher for economies in the former group that have high levels of debt-to-output ratio.*

Table 2(B) compares the standard deviation of the central government revenue-to-output ratio between IND and EME matching them across low and high default risk levels. It shows that for IND there is little variation of the volatility across low and high levels of default risk. For EME, however, the standard deviation of the central government revenue-to-output ratio is higher for economies with high default risk.[19] It is worth noting that all the EME with high default risk levels defaulted at least once during our sample. *Thus, economies with higher default risk exhibit more volatile tax revenues than economies with low default risk.* This is particularly notable for the group of EME/LAC economies.

These stylized facts establish a link between (a) default risk/default events, (b) debt ceilings and (c) volatility of tax revenues. In particular, the evidence suggests that economies that show higher default risk, also exhibit lower debt ceilings and more volatile tax revenues. The theory below sheds light upon the forces driving these facts.[20]

3 The Model

In this section we describe the stochastic structure of the model, the timing and policies of the government and present the household's problem.

[19] We looked also at the inflation tax as a proxy for tax policy; results are qualitatively the same.

[20] It is important to note that we are not arguing any type of causality; we are just illustrating co-movements. In fact, in the model below all three features are endogenous outcomes of equilibrium.

3.1 The Setting

Let time be indexed as $t = 0, 1, ...$ Let (g_t, δ_t) be the vector of government expenditure at time t and the fraction of the defaulted debt which is to be repaid when exiting autarky, respectively. If the economy is not in financial autarky, δ_t is equal to one in order to model the option of the government to repay the totality of the debt or to default. These are the exogenous driving random variables of this economy. Let $\omega_t \equiv (g_t, \delta_t) \in \mathbb{G} \times \bar{\Delta}$, where $\mathbb{G} \subset \mathbb{R}$, $\bar{\Delta} \equiv \Delta \cup \{1\} \cup \{\bar{\delta}\}$ and $\Delta \subset [0, 1)$, and in order to avoid technical difficulties, we assume $|\mathbb{G}|$ and $|\Delta|$ are finite.[21] The set Δ models the offers — as fractions of outstanding debt — to repay the defaulted debt; and $\bar{\delta}$ is designed to capture situations where the government does not receive any offer to repay.[22]

For any $t \in \{1, ..., \infty\}$, let $\Omega^t = (\mathbb{G} \times \bar{\Delta})^t$ be the space of histories up to time t, a typical element is $\omega^t = (\omega_0, \omega_1, ..., \omega_t)$.

3.2 The government policies and timing

In this economy, the government finances exogenous government expenditures by levying labor distortionary taxes and trading one-period, discount bonds with households. The government, however, cannot commit to repay and may default on the bonds at any point in time.

Let $\mathbb{B} \subseteq \mathbb{R}$ be compact. Let $B_{t+1} \in \mathbb{B}$ be the quantity of bonds issued at time t to be paid at time $t + 1$; $B_{t+1} > 0$ means that the government is borrowing at time t from households. Let τ_t be the linear labor tax. Also, let d_t be the default decision, which takes value 1 if the government decides to default and 0 otherwise. Finally, let a_t be the decision of accepting an offer to repay the defaulted debt. It takes value 1 if the offer is accepted and 0 otherwise.

The timing for the government is as follows. Following a period with financial access, after observing the current government expenditure, the government has the option to default on 100 percent of the outstanding debt carried from last period, B_t.

As shown in figure 3.2, if the government opts to exercise the option to default at time t, it cannot issue bonds in that period and runs a balanced budget, i.e., tax revenues equal government expenditure. At the beginning of next period, time $t + 1$, with probability

[21] For a given set, $|S|$ is the cardinal of the set.

[22] An alternative way of modeling this situation is to work with $\bar{\Delta} \equiv \Delta \cup \{1\} \cup \{\emptyset\}$ where $\emptyset$ indicates no offer. Another alternative way is to add an additional random variable, $\iota \in \{0, 1\}$ that explicitly indicates if the government received an offer ($\iota = 1$) or not ($\iota = 0$) and let $\bar{\Delta} \equiv \Delta \cup \{1\}$.

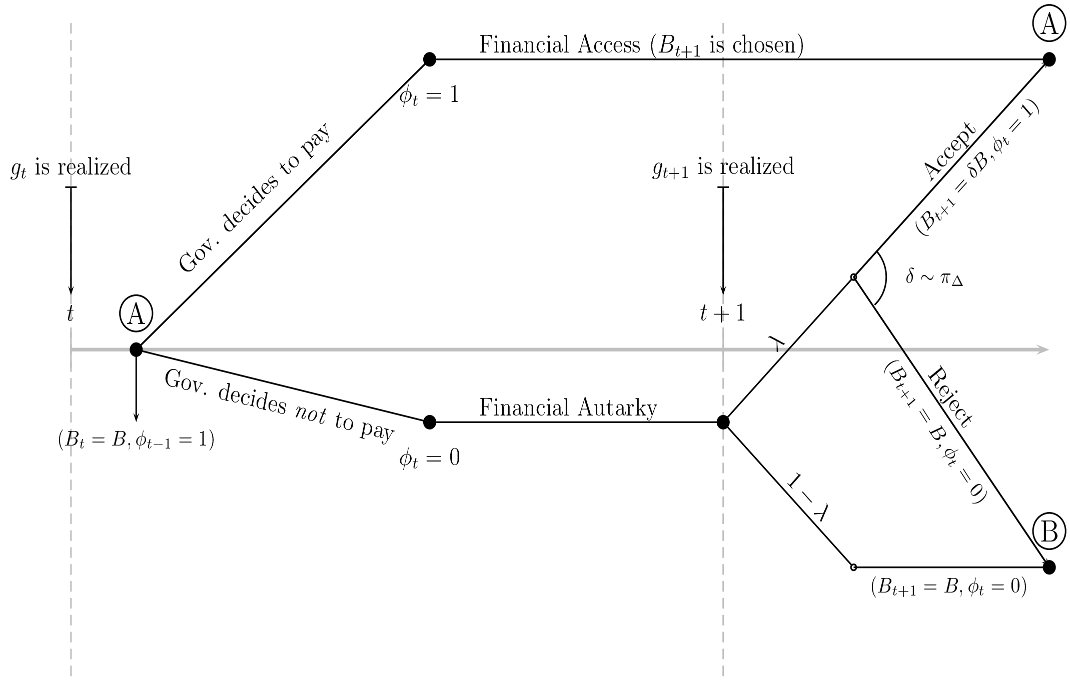

Figure 3.2: Timing of the Model

$1-\lambda$, the government remains in temporary financial autarky for that period (node B). With probability λ, the government receives a random offer to repay a fraction δ of the debt, and has the option to accept or reject it. If the government accepts the offer, it pays the restructured amount (the outstanding defaulted debt times the fraction δ), and it is able to issue new bonds for the following period (node A). If the government rejects the offer, it stays in temporary financial autarky (node B).

Finally, if the government decides not to default, it levies distortionary labor taxes, and allocates discount bonds to the households to cover the expenses g_t and liabilities carried from last period. Next period, it has again the option to default, for the new values of outstanding debt and government expenditure (node A).

As it will become clear later, default on bonds can be seen as a negative lump-sum transfer to households, but a costly one. Default will turn to be costly for two reasons. First, households anticipate the government default strategies and demand higher returns to bear the bond. Second, default is assumed to be followed by temporary financial autarky. During autarky, the government is not only unable to smooth taxes but also could suffer an ad-hoc output cost, as shown later.

We now formalize the probability model mentioned above. Let $\pi_{\mathbb{G}} : \mathbb{G} \to \mathcal{P}(\mathbb{G})$ be the Markov transition probability function for the process of government expenditures and let $\pi_\Delta \in \mathcal{P}(\Delta)$ be the probability measure over the offer space Δ.[23]

Also, for any t, let ϕ_t be the variable that takes value 0 if at time t the government cannot issue bonds during this period, and value 1 if it can. The implied law of motion for ϕ_t is $\phi_t \equiv \phi_{t-1}(1-d_t) + (1-\phi_{t-1})a_t$. That is, if at time $t-1$, the government could issue bonds, then $\phi_t = (1-d_t)$, but if it was in financial autarky, then $\phi_t = a_t$, reflecting the fact that the government regains access to financial markets only if the government decides to renegotiate the defaulted debt.

Assumption 3.1. *For any* (t, ω^t), $\Pr(g_t = g|\omega^{t-1}) = \pi_{\mathbb{G}}(g|g_{t-1})$ *for any* $g \in \mathbb{G}$ *and*

$$\Pr(\delta_t = \delta|g_t, \omega^{t-1}) = \begin{cases} \mathbf{1}_{\{1\}}(\delta) & if\ \phi_{t-1} = 1 \\ (1-\lambda)\mathbf{1}_{\{\bar{\delta}\}}(\delta) + \lambda\pi_\Delta(\delta) & if\ \phi_{t-1} = 0 \end{cases}$$

for any $\delta \in \bar{\Delta}$.[24]

Essentially, this assumption imposes a Markov restriction on the probability distribution over government expenditures and also additional restrictions over the probability of offers. In particular, this assumption implies that in financial autarky with probability $1-\lambda$, $\delta = \bar{\delta}$ (i.e., receiving no offer) and with probability λ, an offer from the offer space is drawn according to π_Δ. Also, if $\phi_{t-1} = 1$ (i.e., the government was not in financial autarky at period $t-1$), then if the government decides *not to default* at time t, it will pay the totality of the outstanding debt and therefore $\delta_t = 1$ with probability one.

Finally, we use Π to denote the probability distribution over Ω^∞ generated by assumption 3.1, and $\Pi(\cdot|\omega^t)$ to denote the conditional probability over Ω, given ω^t.

The next definitions formalize the concepts of government policy, allocation, prices of bonds and the government budget constraint. In particular, it formally introduces the fact that taxes, default decisions and debt depend on histories of past realizations of shocks, and in particular that debt is non-state contingent (i.e., B_{t+1} only depends on the history up to time t, ω^t).

Definition 3.1. *A government policy is a collection of stochastic processes* $\boldsymbol{\sigma} = (B_{t+1}, \tau_t, d_t, a_t)_{t=0}^\infty$, *such that for each* t, $(B_{t+1}, \tau_t, d_t, a_t) \in \mathbb{B} \times [0,1] \times \{0,1\}^2$ *are measurable with respect to* ω^t *and* (B_0, ϕ_{-1}).

Definition 3.2. *An allocation is a collection of stochastic processes* $(g_t, c_t, n_t)_{t=0}^\infty$ *such that for each* t, $(g_t, c_t, n_t) \in \mathbb{G} \times \mathbb{R}_+ \times [0,1]$ *are measurable with respect to* ω^t *and* (B_0, ϕ_{-1}).

[23]For a finite set $\mathbb{X}$, $\mathcal{P}(\mathbb{X})$ is the space of all probability measures defined over $\mathbb{X}$. Also, for any $A \subseteq \mathbb{X}$, the function $\mathbf{1}_A(\cdot)$ takes value 1 over the set A and 0 otherwise.

[24]It is easy to generalize this to a more general formulation such as λ and π_Δ depending on g.

Given a government policy, we say an allocation is *feasible* if for any (t, ω^t)

$$c_t(\omega^t) + g_t = \kappa_t(\omega^t) n_t(\omega^t), \tag{3.1}$$

where $\kappa_t : \Omega^t \to \mathbb{R}_+$ is such that $\kappa_t(\omega^t)$ is the productivity at period t, given history ω^t. For simplicity, we set $\kappa_t(\omega^t) = \phi_t(\omega^t) + \kappa(1 - \phi_t(\omega^t))$ with $\kappa < 1$. The parameter κ represents direct output loss following a default event, associated for example with financial disruption in the banking sector, limited insurance against idiosyncratic risk, among others.

Definition 3.3. *A price process is an stochastic process $(p_t)_{t=0}^{\infty}$ such that for each t, $p_t \in \mathbb{R}_+$ is measurable with respect to ω^t and (B_0, ϕ_{-1}).*

Note that p_t denotes the price of one unit of debt in any state of the world, both with access to financial markets and during autarky, where it represents the price of defaulted debt in secondary markets. Finally, we introduce the government budget constraint.

Definition 3.4 (def:sig-att)**.** *A government policy $\boldsymbol{\sigma}$ is attainable, if for all (t, ω^t),*

$$g_t + \phi_t(\omega^t)\delta_t B_t(\omega^{t-1}) \leq \kappa_t(\omega^t)\tau_t(\omega^t)n_t(\omega^t) + \phi_t(\omega^t)p_t(\omega^t)B_{t+1}(\omega^t), \tag{3.2}$$

and $d_t(\omega^t) = 1$ if $\phi_{t-1}(\omega^{t-1}) = 0$ and $a_t(\omega^t) = 0$ if $\phi_{t-1}(\omega^{t-1}) = 1$ or $\delta_t = \bar{\delta}$. [25]

Observe that in equation 3.2, if the government is in financial autarky ($\phi_t(\omega^t) = 0$), its budget constraint boils down to $g_t \leq \tau_t(\omega^t)n_t(\omega^t)$. On the other hand, if the government has access to financial markets ($\phi_t(\omega^t) = 1$), then it has liabilities to be repaid for $\delta_t B_t$ and can issue new debt.[26] The final restriction on $d_t(\omega^t)$ and $a_t(\omega^t)$ simply states that if last period the government was in financial autarky, then it trivially cannot choose to default at time t, and if $\delta_t = \bar{\delta}$ or if last period the government had access to financial markets $a_t(\omega^t)$ is set to 0.

A few final remarks about the "debt-restructuring process" are in order. This process is parameterized by (λ, π_Δ). These parameters capture the fact that debt restructuring is time-consuming but, generally, at the end a positive fraction of the defaulted debt is honored.[27] This debt-restructuring process intends to capture the fact that after defaults

[25] The inequality in equation 3.2 implies that the government can issue lump-sum transfers to the households. Lump-sum taxes are not permitted.

[26] If the government had access to financial markets at time $t-1$ ($\phi_{t-1} = 1$), then by assumption 3.1, $\delta_t = 1$.

[27] We could also allow for, say, $\pi_\Delta(\cdot | g_t, B_t, d_t, d_{t-1}, ..., d_{t-K})$ some $K > 0$, denoting that possible partial payments depend on the credit history and level of debt. See Reinhart et al. (2003), Reinhart and Rogoff (2008) and Yue (2010) for an intuition behind this structure.

(on domestic or international debt, or both), economies see their access to credit severely hindered.[28]

3.3 The Household's Problem

There is a continuum of identical households, that are price takers and have time-separable preferences for consumption and labor processes. They also make debt/savings decisions by trading government bonds. Formally, we define a household debt process as a stochastic process given by $(b_{t+1})_{t=0}^{\infty}$ where $b_{t+1} : \Omega^t \to [\underline{b}, \overline{b}]$ is the household's savings in government bonds at time $t+1$ for any history ω^t.[29]

For convenience, let q_t denote the price of defaulted debt at time t, i.e., $q_t = p_t$ if $\phi_t = 0$. Given a government policy $\boldsymbol{\sigma}$, for each t, let $\varrho_t : \Omega^t \to \mathbb{R}$ be the payoff of a government bond at period t; i.e.,

$$\varrho_t(\omega^t) = \phi_t(\omega^t)\delta_t + (1 - \phi_t(\omega^t))q_t(\omega^t). \tag{3.3}$$

A few remarks about ϱ are in order. First, since the household takes government actions as given, from the point of view of the households the government debt is an asset with payoff that depends only on the state of the economy, and this dependence clearly illustrates that default decisions add certain degree of state contingency to the government debt. In particular, if $\phi_t(\omega^t) = 1$, then $\varrho_t(\omega^t) = \delta_t$ denoting the fact that the government pays a fraction δ_t. If the government defaults or rejects the repayment option, the household can sell each unit of government debt in the secondary market at a price $\varrho_t(\omega^t) = q_t(\omega^t)$.

The household's problem consists of choosing consumption, labor and debt processes in order to maximize the expected lifetime utility. That is, given (ω_0, b_0) and $\boldsymbol{\sigma}$,

$$\sup_{(c_t, n_t, b_{t+1})_{t=0}^{\infty} \in \mathbb{C}(g_0, b_0; \boldsymbol{\sigma})} E_{\Pi(\cdot|\omega_0)} \left[\sum_{t=0}^{\infty} \beta^t u(c_t(\omega^t), 1 - n_t(\omega^t)) \right]$$

[28] The duration of debt restructurings after sovereign defaults in particular on external debt has received considerable attention in the literature. For instance, for Argentina's default in 2001 the settlement with the majority of the creditors was reached in 2005. In the default episodes of Russia (1998), Ecuador (1999) and Ukraine (1998), the renegotiation process lasted 2.3, 1.7 and 1.4 years, respectively, according to Benjamin and Wright (2009). In general, domestic debt restructuring periods tend to be not as long as in the case of external debt. For example, as documented by Sturzenegger and Zettelmeyer (2006), after the default by Russia in 1998 it took six months to restructure the domestic GKO bonds.

[29] We assume $b_{t+1} \in [\underline{b}, \overline{b}]$ with $[\underline{b}, \overline{b}] \supset \mathbb{B}$ so in equilibrium these restrictions will not be binding.

where $\beta \in (0,1)$ is the discount factor, $E_{\Pi(\cdot|\omega_0)}[\cdot]$ is the expectation using the conditional probability $\Pi(\cdot|\omega_0)$, and $\mathbb{C}(g_0, b_0; \boldsymbol{\sigma})$ is the set of household's allocations and debt process, given government policy $\boldsymbol{\sigma}$, such that for all t and all $\omega^t \in \Omega^t$,

$$c_t(\omega^t) + p_t(\omega^t) b_{t+1}(\omega^t) = (1 - \tau_t(\omega^t))\kappa_t(\omega^t) n_t(\omega^t) + \varrho_t(\omega^t) b_t(\omega^{t-1}) + T_t(\omega^t),$$

where $T_t(\omega^t) \geq 0$ are lump-sum transfers from the government. The previous equation indicates that after-tax labor income, proceedings from bond holdings and government transfers have to be sufficient to cover consumption and new purchases of government bonds.

4 Competitive Equilibrium

We now define a competitive equilibrium for a given government policy and derive the equilibrium taxes and prices.

Definition 4.1. *Given $\omega_0, B_0 = b_0$ and ϕ_{-1}, a competitive equilibrium is a government policy, $\boldsymbol{\sigma}$, an allocation, $(g_t, c_t, n_t)_{t=0}^{\infty}$, a household debt process, $(b_{t+1})_{t=0}^{\infty}$, and a price process $(p_t)_{t=0}^{\infty}$ such that:*

1. *Given the government policy and the price process, the allocation and debt process solve the household's problem.*

2. *The government policy, $\boldsymbol{\sigma}$, is attainable.*

3. *Given $\boldsymbol{\sigma}$, the allocation is feasible.*

4. *For all (t, ω^t), $B_{t+1}(\omega^t) = b_{t+1}(\omega^t)$, and $B_{t+1}(\omega^t) = B_t(\omega^{t-1})$ if $\phi_t(\omega^t) = 0$.*

Observe that the market clearing for debt indicates that $B_{t+1}(\omega^t) = b_{t+1}(\omega^t)$. In addition, if the economy is in financial autarky — where the government cannot issue debt, and thus agents can only trade among themselves —, imposing $B_{t+1}(\omega^t) = B_t(\omega^{t-1})$ implies, since agents are identical, that in equilibrium $b_t(\omega^{t-1}) = b_{t+1}(\omega^t)$, i.e., agents do not change their debt positions.

4.1 Equilibrium Prices and Taxes

In this section we present the expressions for equilibrium taxes and prices of debt. The former quantity is standard (e.g. Aiyagari et al. (2002) and Lucas and Stokey (1983)); the latter quantity, however, incorporates the possibility of default of the government. The following assumption is standard and ensures that u is smooth enough to compute first order conditions.

Assumption 4.1. *$u \in \mathbb{C}^2(\mathbb{R}_+ \times [0,1], \mathbb{R})$ with $u_c > 0$, $u_{cc} < 0$, $u_l > 0$ and $u_{ll} > 0$, and $\lim_{l \to 0} u_l(l) = \infty$.*[30]

Henceforth, for any (t, ω^t), we use $u_c(\omega^t)$ as $u_c(c_t(\omega^t), 1-n_t(\omega^t))$ and proceed similarly for other derivatives and functions.

From the first order conditions of the optimization problem of the households (assuming an interior solution) the following equations hold for any (t, ω^t), [31]

$$\frac{u_l(\omega^t)}{u_c(\omega^t)} = (1 - \tau_t(\omega^t))\kappa_t(\omega^t), \tag{4.4}$$

and

$$\begin{aligned} p_t(\omega^t) =& E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(\omega^{t+1})}{u_c(\omega^t)} \varrho_{t+1}(\omega^{t+1}) \right] \\ =& \beta E_{\Pi(\cdot|\omega^t)} \left[\frac{u_c(\omega^{t+1})}{u_c(\omega^t)} \phi_{t+1}(\omega^{t+1}) \delta_{t+1} \right] + \beta E_{\Pi(\cdot|\omega^t)} \left[\frac{u_c(\omega^{t+1})}{u_c(\omega^t)} (1 - \phi_{t+1}(\omega^{t+1})) q_{t+1}(\omega^{t+1}) \right] \end{aligned} \tag{4.5}$$

Given the definition of ϱ and the restrictions on Π, equation 4.5 implies for $\phi_t(\omega^t) = 1$, [32]

$$\begin{aligned} p_t(\omega^t) =& \beta \int_{\mathbb{G}} \left(\frac{u_c(\omega^t, g', 1)}{u_c(\omega^t)} (1 - d_{t+1}(\omega^t, g', 1)) \right) \pi_{\mathbb{G}}(dg'|g_t) \\ & + \beta \int_{\mathbb{G}} \frac{u_c(\omega^t, g', \bar{\delta})}{u_c(\omega^t)} d_{t+1}(\omega^t, g', \bar{\delta}) q_{t+1}(\omega^t, g', \bar{\delta}) \pi_{\mathbb{G}}(dg'|g_t), \end{aligned} \tag{4.6}$$

and for $\phi_t(\omega^t) = 0$

$$\begin{aligned} q_t(\omega^t) =& \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \left(\frac{u_c(\omega^t, g', \delta')}{u_c(\omega^t)} \delta' a_{t+1}(\omega^t, g', \delta') \right) \pi_{\Delta}(d\delta') \pi_{\mathbb{G}}(dg'|g_t) \\ & + \beta\lambda \int_{\mathbb{G}} \left\{ \int_{\Delta} \left(\frac{u_c(\omega^t, g', \delta')}{u_c(\omega^t)} (1 - a_{t+1}(\omega^t, g', \delta')) \pi_{\Delta}(d\delta') \right) \right\} q_{t+1}(\omega^t, g', \bar{\delta}) \pi_{\mathbb{G}}(dg'|g_t) \\ & + \beta(1-\lambda) \int_{\mathbb{G}} \left(\frac{u_c(\omega^t, g', \bar{\delta})}{u_c(\omega^t)} \right) q_{t+1}(\omega^t, g', \bar{\delta}) \pi_{\mathbb{G}}(dg'|g_t). \end{aligned} \tag{4.7}$$

[30] $\mathbb{C}^2(X, Y)$ is the space of twice continuously differentiable functions from X to Y. The assumption $u_{cc} < 0$ could be relaxed to include $u_{cc} = 0$ (see the section 6 below).

[31] See appendix B for the derivation.

[32] The notation (ω^t, g, δ) denotes the partial history ω^{t+1} where $(g_{t+1}, \delta_{t+1}) = (g, \delta)$.

Equation 4.5 reflects the fact that, in equilibrium, households anticipate the default strategies of the government and demand higher returns to compensate for the default risk. The second line in the Euler equation 4.6 shows that, due to the possibility of partial repayments in the future, defaulted debt has positive value and agents can sell it in a secondary market at price $q_{t+1}(\omega^{t+1})$. Equation 4.7 characterizes this price. Each summand in the right hand side corresponds to a "branch" of the tree depicted in figure 3.2. The first line represents the value of one unit of debt when an offer arrives and the government decides to repay the realized fraction of the defaulted debt next period. The second and third lines capture the value of one unit of debt when either the government decides to reject the repayment offer, or it does not receive one. A final observation is that, as it will become clear later on, when $\phi_{t+1}(\omega^{t+1}) = 0$, $u_c(\omega^{t+1})$ is only a function of g_{t+1} (not the entire past history ω^{t+1}) because in equilibrium the government runs a balanced budget.

To shed some more light on equations 4.6 and 4.7, consider the case where $u_c = 1$, $\lambda = 0$. In this case, for any (t, ω)

$$p_t(\omega) = \beta \int_{\mathbb{G}} (1 - d_{t+1}(\omega^t, g'))\pi_{\mathbb{G}}(dg'|g_t).$$

Here, the bond price is simply the discounted one-period ahead probability of not defaulting. Also observe that, since $\lambda = 0$, it follows that $q_t(\omega^t) = \int_{\mathbb{G}} q_{t+1}(\omega^t, g', \bar{\delta})\pi_{\mathbb{G}}(dg'|g_t)$, which by substituting forward and invoking standard transversality conditions, yields $q_t(\omega^t) = 0$. These pricing equations are analogous to those in Arellano (2008) and Aguiar and Gopinath (2006) and references therein. See also Chatterjee and Eyingungor (2012) for the equilibrium prices in the presence of long-term debt.

The novelty of these pricing equations with respect to the standard sovereign default model is the presence of secondary market prices, q_t. By imposing a positive recovery rate (with some probability), the model is able to deliver a positive price of defaulted debt during the financial autarky period. In sections 6 and 7, we shed some light on the pricing implications of this model and how it relates with the data.

4.2 Characterization of the Competitive Equilibrium

In this environment, the set of competitive equilibria can be characterized by a sequence of non-linear equations which impose restrictions on $(d_t, a_t, B_{t+1}, n_t)_{t=0}^{\infty}$ and are derived from the first order conditions of the household, the budget constraint of the government and the feasibility condition. The next theorem formalizes this claim.

Henceforth, we call $(d_t, a_t, B_{t+1}, n_t)_{t=0}^{\infty}$ an *outcome path* of allocations. We say an outcome path is *consistent with a competitive equilibrium* if the outcome path and $(c_t, p_t, b_{t+1}, \tau_t, g_t)_{t=0}^{\infty}$, derived using the outcome path and market clearing, feasibility and first order conditions, is a competitive equilibrium. Also, let

$$Z_t(\omega^t) \equiv z(\kappa_t(\omega^t), n_t(\omega^t), g_t) = \left(\kappa_t(\omega^t) - \frac{u_l(\omega^t)}{u_c(\omega^t)}\right) n_t(\omega^t) - g_t \tag{4.8}$$

be the primary surplus (if it is negative, it represents a deficit) at time t given history $\omega^t \in \Omega^t$.

Theorem 4.1. *Given $\omega_0, B_0 = b_0$ and ϕ_{-1}, the outcome path $(d_t, a_t, B_{t+1}, n_t)_{t=0}^{\infty}$ is consistent with a competitive equilibrium iff for all $(t, \omega^t) \in \{0, 1, 2, ...\} \times \Omega^t$, the following holds:*

$$\begin{aligned} Z_t(\omega^t)u_c(\omega^t) + \phi_t(\omega^t)\{p_t(\omega^t)u_c(\omega^t)B_{t+1}(\omega^t) - \delta_t u_c(\omega^t)B_t(\omega^{t-1})\} &\geq 0, \qquad (4.9) \\ B_{t+1}(\omega^t) = B_t(\omega^{t-1}) \text{ if } \phi_t(\omega^t) &= 0, \end{aligned}$$

and $c_t(\omega^t) = \kappa_t(\omega^t)n_t(\omega^t) - g_t(\omega^t)$ and equations 4.4 and 4.6 hold.

For any $(\omega, B, \phi) \in (\mathbb{G} \times \overline{\Delta}) \times \mathbb{B} \times \{0, 1\}$, let $CE_\phi(\omega, B)$ denote the set of all outcome paths that are consistent with competitive equilibria, given $\omega_0 = \omega$, $\phi_0(\omega_0) = \phi$ and where B is the outstanding debt of time 0, after any potential debt restructuring in that period. We observe that by setting $\phi_0(\omega_0) = \phi$ we are implicitly imposing restrictions on a_0, d_0, ϕ_{-1} and δ_0.[33,34]

5 The Government Problem

The government is benevolent and maximizes the welfare of the representative household by choosing policies. The government, however, cannot commit to repaying the debt, but commits to previous tax promises until a debt restructuring takes place. That is, as long as the government keeps access to financial markets, it honors past promises of taxes. For autarky states, the government chooses taxes that balance its budget. Once the

[33]For example, if $\phi_0 = 1$ we could arrive to it because $\phi_{-1} = 1$ and $d_0 = 0$, given $(g_0, B_0) = (g, B)$, or because $\phi_{-1} = 0$ with defaulted debt $\widetilde{B}_0 = B_0/\delta_0$ – but offer δ_0 is accepted ($a_0 = 1$) and the renegotiated debt becomes $\widetilde{B}_0$.

[34]Constructing the set $CE_\phi(\omega, B)$ is useful since, in order to make a default/repayment decision, the default authority evaluates alternative utility values both for repayment and for autarky that are sustained by competitive equilibrium allocations.

government accepts an offer to restructure the debt, it regains access to financial markets and starts anew, without any outstanding tax promises, by assumption.[35] Therefore, the government problem can be viewed as a problem involving two types of authorities: a *default authority* and a *fiscal authority*. On the one hand, the default authority can be seen as comprised by a sequence of one-period administrations, where the time-t administration makes the default and repayment decision in period t, taking as given the behavior of all the other agents including the fiscal authority. On the other hand, the *fiscal authority* can be viewed as a sequence of consecutive administrations, each of which stays in office until there is a debt renegotiation. While ruling, a fiscal administration has the ability to commit, and chooses the optimal fiscal and debt processes, taking as given the behavior of the default authority. When debt is renegotiated, the fiscal administration is replaced by a new one, which is not bound by previous tax promises, and is free to reset the fiscal and debt policy.[36]

For any $t \in \{0, 1, ...\}$, let $h_t \equiv (\phi_{t-1}, B_t, \omega_t)$ and $h^t \equiv (h_0, h_1, ..., h_t)$ be the *public history* until time t.[37] We use $\mathbb{H}^t$ to denote the set of all public histories until time t.

A *government strategy* is given by a strategy for the default and fiscal authorities, $\boldsymbol{\gamma} \equiv (\boldsymbol{\gamma}^D, \boldsymbol{\gamma}^F)$. The strategy for the default authority $\boldsymbol{\gamma}^D$ specifies a default and a repayment decision for any period t and any public history $h^t \in \mathbb{H}^t$, i.e., $\boldsymbol{\gamma}^D = (\gamma_t^D(\cdot))_{t=0}^{\infty}$ with $\gamma_t^D(h^t) \equiv (d_t(h^t), a_t(h^t))$ for any $h^t \in \mathbb{H}^t$. The strategy for the fiscal authority, $\boldsymbol{\gamma}^F$, specifies next period's debt level for any public history $h^t \in \mathbb{H}^t$ and any ϕ_t, i.e., $\boldsymbol{\gamma}^F = (\gamma_t^F(\cdot, \cdot))_{t=0}^{\infty}$ with $\gamma_t^F(h^t, \phi_t) \equiv B_{t+1}(h^t, \phi_t)$ for any $(h^t, \phi_t) \in \mathbb{H}^t \times \{0, 1\}$. The fact that $\gamma_t^F(h^t, \phi_t)$ depends on ϕ_t reflects our assumption on the timing protocol by which the default authority moves first in each period. Also, we omit labor taxes (or labor directly) as part of the government strategy because, given (h^t, ϕ_t) and $\gamma_t^F(h^t, \phi_t)$, labor taxes are

[35] A similar feature is present in Debortoli and Nunes (2010), where the government can randomly reoptimize and reset fiscal policies with a given exogenous probability. In our model, however, the *resetting* event, given by the debt restructuring, is an equilibrium outcome that emerges endogenously.

[36] We focus exclusively on symmetric strategies for households, where all of them take the same decisions along the equilibrium path. Similarly, we assume that all default and fiscal administrations choose identical actions conditional on the same state of the economy, to be specified later on, thereby introducing a Markovian structure for optimal strategies.

[37] In our economy an individual household cannot alter prices and faces a (strictly) concave optimization problem. Any deviation from the equilibrium path determined by the Euler equation and the consumption-labor optimality condition, taking prices and policies as given, cannot be profitable from the household's perspective. Hence, there is no need to specify the household's behavior off the equilibrium path as well as to make households' strategies depend on private histories but only on public ones.

obtained by the budget constraint.[38] Finally, note that any strategy $\boldsymbol{\gamma}$ jointly with a stochastic process $(\omega_t)_{t=0}^{\infty}$ generates an *outcome path* of allocations $(d_t, a_t, B_{t+1}, n_t)_{t=0}^{\infty}$. To stress that a particular policy action, say $B_{t+1}(h^t, \phi_t)$, belongs to given strategy we use $B_{t+1}(\boldsymbol{\gamma})(h^t, \phi_t)$.

Let $\boldsymbol{\gamma}|_{(h^t,\phi_t)}$ denote the continuation of strategy $\boldsymbol{\gamma}$ after history $(h^t, \phi_t) \in \mathbb{H}^t \times \{0,1\}$.[39] We say a strategy $\boldsymbol{\gamma}$ is *consistent with a competitive equilibrium*, if after any $(h^t, \phi_t) \in \mathbb{H}^t \times \{0,1\}$, the outcome path generated by $\boldsymbol{\gamma}|_{(h^t,\phi_t)}$ belongs to $CE_{\phi_t}(\omega_t, B)$ with $B = (\delta_t \phi_t(\boldsymbol{\gamma})(h^t) + (1-\phi_t(\boldsymbol{\gamma})(h^t)))B_t(\boldsymbol{\gamma})(h^{t-1}, \phi_{t-1}(\boldsymbol{\gamma})(h^{t-1}))$. For any $h_0 \in \mathbb{H}$ and $\phi_0 \in \{0,1\}$, we denote the set of such strategies as

$$\mathcal{S}(h_0, \phi_0) \equiv \Big\{ \boldsymbol{\gamma} : \; \forall \; (h^t, \phi_t)_{t=1}^{\infty}, \; \boldsymbol{\gamma}|_{(h^t,\phi_t)} \; renders \; (d_\tau(\boldsymbol{\gamma}), a_\tau(\boldsymbol{\gamma}), B_{\tau+1}(\boldsymbol{\gamma}), n_\tau(\boldsymbol{\gamma}))_{\tau=t}^{\infty} \in CE_{\phi_t}(\omega_t, B), \\ with \; B = (\delta_t \phi_t(\boldsymbol{\gamma})(h^t) + (1-\phi_t(\boldsymbol{\gamma})(h^t)))B_t(\boldsymbol{\gamma})(h^{t-1}, \phi_{t-1}(\boldsymbol{\gamma})(h^{t-1})) \Big\}.$$

Henceforth, we only consider strategies that are consistent with competitive equilibrium. Finally, for any public history $h^t \in \mathbb{H}^t$, $\phi \in \{0,1\}$ and $\boldsymbol{\gamma} \in \mathcal{S}(h_0, \phi)$, let

$$V_t(\boldsymbol{\gamma})(h^t, \phi) = E_{\Pi(\cdot|\omega_t)} \left[\sum_{j=0}^{\infty} \beta^j u(\kappa_{t+j}(\omega^{t+j}) n_{t+j}(\boldsymbol{\gamma})(\omega^{t+j}) - g_{t+j}, 1 - n_{t+j}(\boldsymbol{\gamma})(\omega^{t+j})) \right] \tag{5.10}$$

be the expected lifetime utility of the representative household at time t, given strategy $\boldsymbol{\gamma}|_{(h_t,\phi)}$.

5.1 Default and Renegotiation Policies

As mentioned before, the default authority can be viewed as comprised by a sequence of one-period administrations, each of which makes the default and renegotiation decision in its respective period, taking as given the behavior of all the other agents including the other default administrations and the fiscal one. It is easy to see that, for each public history $h^t \in \mathbb{H}^t$, the default authority will optimally choose as follows: if $\phi_{t-1} = 1$

$$d_t^*(\boldsymbol{\gamma})(h^t) = \begin{cases} 0 & if \; V_t(\boldsymbol{\gamma})(h^t, 1) \geq V_t(\boldsymbol{\gamma})(h^t, 0) \\ 1 & if \; V_t(\boldsymbol{\gamma})(h^t, 1) < V_t(\boldsymbol{\gamma})(h^t, 0) \end{cases} \tag{5.11}$$

and if $\phi_{t-1} = 0$

$$a_t^*(\boldsymbol{\gamma})(h^t) = \begin{cases} 1 & if \; V_t(\boldsymbol{\gamma})(h^t, 1) \geq V_t(\boldsymbol{\gamma})(h^t, 0) \\ 0 & if \; V_t(\boldsymbol{\gamma})(h^t, 1) < V_t(\boldsymbol{\gamma})(h^t, 0) \end{cases} \tag{5.12}$$

[38]For this reason we do not include them as part of the public history.

[39]Observe that with a strategy the default authority moves first at $t = 0$, with the continuation strategy, as we defined, the fiscal authority is moving first at t and then the default authority moves at $t + 1$.

Also, recall that by assumption $a^*_t(\boldsymbol{\gamma})(h^t) = 0$ if $\phi_{t-1} = 1$ or $\delta_t = \bar{\delta}$ and $d^*_t(\boldsymbol{\gamma})(h^t) = 1$ if $\phi_{t-1} = 0$. The dependence on $\boldsymbol{\gamma}$ denotes the fact that d^*_t and a^*_t are associated with the strategy of the fiscal authority $\boldsymbol{\gamma}^F$. Indeed, to specify the optimal default and repayment decisions at any history $h^t \in \mathbb{H}^t$ we need to know the value of repayment and the value of default, $V_t(\boldsymbol{\gamma})(h^t, 1)$ and $V_t(\boldsymbol{\gamma})(h^t, 0)$, respectively, which are evidently functions of $\boldsymbol{\gamma}^F$.

In equilibrium, the government will find it sometimes optimal to renege its debt contracts, even though the bondholders are the households whose welfare our (benevolent) government aims to maximize. It will do so because by defaulting it avoids the future tax distortions that would come along with the service of the debt. If this benefit of not repaying exceeds the ad-hoc costs of default (i.e. temporary output loss and financial exclusion), the government will optimally decide not to pay back the bonds. We believe this is a novel motive to default on government debt which, to our knowledge, had not been explored before in the literature.

5.2 Recursive Representation of the Government Problem

Taking as given the optimal decision rules 5.11 and 5.12 for the default authority, we now turn to the optimization problem of the fiscal authority and the recursive representation of the government problem. To do so, we adopt a recursive representation for the competitive equilibria by introducing an adequate state variable. In any competitive equilibrium, as can be seen from the equations in theorem 4.1, all relevant information for households' decision-making in the current period t about future *tax* continuation policy — that is, the policy the government can commit to temporarily— is summarized in $u_c(\omega^t, \omega_{t+1})$ for all ω_{t+1}. By keeping track of the profile of "promised" marginal utilities of consumption, we ensure that the fiscal authority commits to deliver the "promised" marginal utility —as long as the default authority does not restructure the debt— for each realization of g; thereby guaranteeing that the last-period households' Euler equation is satisfied after each possible history.[40] Thus, following Kydland and Prescott (1980) and Chang (1998) among others, it follows that the relevant (co-)state variable is the "promised" marginal utilities of consumption.

We therefore draw our attention to the set of "promised" marginal utilities that can be delivered in a competitive equilibrium. This set differs from the standard set of equilibrium promised marginal utilities in Kydland and Prescott (1980) along some dimensions. In

[40] If the debt is restructured and a new fiscal administration takes power, it sets the current marginal utility at its convenience, which in equilibrium is anticipated by the households.

particular, in an standard Ramsey problem it would suffice to only specify the set of promised marginal utilities, but in our framework with endogenous default decisions we need to also specify continuation values to evaluate alternative courses of action of the default authority. By the same token, we compute this set for any ϕ, even for the value of ϕ not optimally chosen by the default authority through its policy action.

For autarky ($\phi = 0$), the "promised" marginal utilities of consumption are trivially pinned down by the choice of labor that balances the government budget and maximizes the per-period payoff; i.e., for any $g \in \mathbb{G}$, the "promised" marginal utility of consumption equals $m_A(g) \equiv u_c(\mathbf{n}_0^*(g) - g, 1 - \mathbf{n}_0^*(g))$ where[41]

$$\mathbf{n}_0^*(g) = \arg\max_{n\in[0,1]} \{u(\kappa n - g, 1 - n) : z(\kappa, n, g) = 0\}.$$

We now proceed to formally define our object of interest in more generality. For any $h_0 = (\phi_{-1}, g_0, \delta_0, B_0) \in \mathbb{H}$ and $\phi \in \{0, 1\}$, let

$$\begin{aligned}
\Omega(h_0, \phi) = \big\{(\mu, v) \in \mathbb{R}_+ \times \mathbb{R} :& \\
& \exists\, \boldsymbol{\gamma} \in \mathcal{S}(h_0, \phi),\ and\ (V_\tau(h^\tau, 0), V_\tau(h^\tau, 1))_{h^\tau,\tau}\ such\ that : \\
& \mu = m_A(g)\ if\ \phi = 0,\ and\ \mu = u_c(n_0(\boldsymbol{\gamma})(h_0) - g_0, 1 - n_0(\boldsymbol{\gamma})(h_0))\ if\ \phi = 1, \\
& v = V_0(h_0, \phi) \\
& (V_\tau(h^\tau, \phi))_{h^\tau,\tau}\ satisfies\ (5.10)\ for\ any\ \phi \in \{0, 1\}, \\
& \boldsymbol{\gamma}^D|_{h_0,\phi_1(\boldsymbol{\gamma})}\ are\ determined\ by\ (5.11) - (5.12)\Big\},
\end{aligned}$$

For each initial history h_0 and ϕ, the set $\Omega(h_0, \phi)$ assigns the set of all values for marginal utility and lifetime utility values at time zero that can be sustained in a competitive equilibrium, wherein the default authority reacts optimally from next period on. Each pair (μ, v) imposes restrictions on the labor allocation at time 0 (for the case of $\phi = 0$) as well as on the lifetime utility at time 0, given h_0 and ϕ. Finally, note that the set indexed by the value of ϕ optimally chosen by the government contains the promised marginal utilities (and utility values) that can be delivered along the equilibrium path, while its counterpart with the other value of ϕ includes only off-equilibrium marginal utilities.

The correspondence Ω is an equilibrium object, endogenously determined, that can be computed using numerical methods as the largest fixed point of an appropriately constructed correspondence operator, in the spirit of Abreu et al. (1990). Henceforth,

[41]The lemma D.1(1) ensures that $\mathbf{n}_0^*(g)$ exists and is unique for all g.

we proceed to formulate and solve the recursive problem of the fiscal authority as if we already know Ω.

For any $(g, B, \mu) \in \mathbb{G} \times \mathbb{B} \times \mathbb{R}_+$, let $V_1^*(g, B, \mu)$ be the value function of a fiscal authority that had access to financial markets last period and continue to have it this current period (i.e., $\phi_{-1} = \phi = 1$) and that takes as given the optimal behavior of the default and subsequent fiscal authorities, with outstanding debt B and a promised marginal utility of μ and government expenditure g. Similarly, let $V_0^*(g, B)$ be the value function of a fiscal authority that does not have access to financial markets (i.e., $\phi = 0$) and has an outstanding defaulted debt B and government expenditure g. Observe that since in financial autarky the government ought to run a balanced budget, V_0^* does not depend on μ.

Finally, let $\overline{V}_1^*(g, \delta B)$ be the value function of a "new" fiscal authority (i.e., when $\phi_{-1} = 0$ and $\phi = 1$) that takes as given the optimal behavior of the default and subsequent fiscal authorities, when an offer δ is accepted, given government spending g and outstanding defaulted debt B. Note that in this case the fiscal authority does not have any outstanding "promised" marginal utility and thus it sets the current marginal utility at its convenience. By construction of Ω, it follows that

$$\overline{V}_1^*(g, \delta B) = \max\{v | (\mu, v) \in \Omega(0, B, g, \delta, 1)\}, \tag{5.13}$$

as the government maximizes the households' utility without any attached promise of marginal utility to be delivered.[42] Let $\overline{\mu}(g, \delta, B) = \{\mu | (\mu, \overline{V}_1^*(g, \delta B)) \in \Omega(0, B, g, \delta, 1)\}$ be the associated marginal utility.

Given the aforementioned value functions, the optimal policy functions of the default authority in expressions (5.11)-(5.12) become [43] [44]

$$\mathbf{d}^*(g, B, \mu) = \begin{cases} 0 & if\ V_1^*(g, B, \mu) \geq V_0^*(g, B) \\ 1 & if\ V_1^*(g, B, \mu) < V_0^*(g, B) \end{cases} \tag{5.14}$$

and

$$\mathbf{a}^*(g, \delta, B) = \begin{cases} 1 & if\ \overline{V}_1^*(g, \delta B) \geq V_0^*(g, B) \\ 0 & if\ \overline{V}_1^*(g, \delta B) < V_0^*(g, B) \end{cases} \tag{5.15}$$

[42] We are implicitly assuming that the maximum is achieved. This assumption is imposed to ease the exposition and could be relaxed by defining $\overline{V}_1^*$ in terms of a supremum and approximate maximizers.

[43] Implicit in both definition is the refinement that in case of indifference, the government decides to accept/not default on the debt. Without this refinement, the optimal decisions will be correspondences that take any value between 0 and 1 in case of indifference.

[44] As indicated before, by assumption, $\mathbf{d}^*(g, B, \mu) = 1$ if $\phi_{-1} = 0$ and $\mathbf{a}^*(g, \delta, B) = 0$ if $\phi_{-1} = 1$ or $\delta = \bar{\delta}$.

The next theorem presents a recursive formulation for the value functions.

Theorem 5.1. *The value functions V_0^* and V_1^* satisfy the following recursions*

$$V_1^*(g,B,\mu) = \max_{(n,B',\mu'(\cdot))\in\Gamma(g,B,\mu)} \left\{ u(n-g,1-n) + \beta \int_{\mathbb{G}} \max\{V_1^*(g',B',\mu'(g')), V_0^*(g',B')\}\pi_{\mathbb{G}}(dg'|g) \right\}, \tag{5.16}$$

and

$$\begin{aligned} V_0^*(g,B) =& u(\kappa \mathbf{n}_0^*(g) - g, 1 - \mathbf{n}_0^*(g)) + \beta\lambda \int_{\mathbb{G}}\int_{\Delta} \max\{\overline{V}_1^*(g',\delta' B), V_0^*(g',B)\}\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg'|g) \\ &+ \beta(1-\lambda)\int_{\mathbb{G}} V_0^*(g',B)\pi_{\mathbb{G}}(dg'|g) \end{aligned} \tag{5.17}$$

where, for any (g,B,μ),

$$\begin{aligned} \Gamma(g,B,\mu) = \Big\{ & (n,B',\mu'(\cdot)) \in [0,1]\times\mathbb{B}\times\mathbb{R}^{|\mathbb{G}|} : \\ & (B',\mu'(g'),V_1^*(g',B',\mu'(g'))) \in Graph(\Omega(1,\cdot,g',1,1)),\ \forall g' \in \mathbb{G} \\ & \mu = u_c(n-g,1-n) \text{ and } z(1,n,g)\mu + \mathcal{P}_1^*(g,B',\mu'(\cdot))B' - B\mu \geq 0 \Big\} \end{aligned} \tag{5.18}$$

and, for any $(B',\mu'(\cdot))$,

$$\begin{aligned} \mathcal{P}_1^*(g,B',\mu'(\cdot)) =& \beta \int_{\mathbb{G}} \left((1-\mathbf{d}^*(g',B',\mu'(g')))\mu'(g') + \mathbf{d}^*(g',B',\mu'(g'))m_A(g')\mathcal{P}_0^*(g',B')\right)\pi_{\mathbb{G}}(dg'|g) \\ \mathcal{P}_0^*(g,B') =& \beta \int_{\mathbb{G}} \left(\int_\Delta \overline{\mu}(g',\delta',B')\delta' \mathbf{a}^*(g',\delta',B')\pi_\Delta(d\delta') + \pi_A^*(g',B')m_A(g')\mathcal{P}_0^*(g',B') \right) \pi_{\mathbb{G}}(dg'|g) \end{aligned}$$

where $\pi_A^*(g,B) \equiv \{(1-\lambda) + \lambda \int_\Delta (1-\mathbf{a}^*(g,\delta,B))\pi_\Delta(d\delta)\}$ for any (g,B).

Below we present some particular cases of special interest where the recursive representation of the government problem gets simplified.

Example 5.1 (Nondefaultable debt)**.** *Consider an economy with risk-free debt (this is imposed ad-hoc). The value function V_0^* is irrelevant and V_1^* boils down to*

$$V_1^*(g,B,\mu) = \max_{(n,B',\mu'(\cdot))\in\Gamma(g,B,\mu)} \left\{ u(n-g,1-n) + \beta \int_{\mathbb{G}} V_1^*(g',B',\mu'(g'))\pi_{\mathbb{G}}(dg'|g) \right\}$$

where

$$\Gamma(g,B,\mu) = \left\{ \begin{array}{l} (n,B',\mu'(\cdot)) : z(1,n,g)\mu + \beta E_{\pi_{\mathbb{G}}(\cdot|g)}[\mu'(g')]B' - B\mu \geq 0 \\ \text{where } \mu = u_c(n-g,1-n) \end{array} \right\}.$$

In addition, $\bar{V}_1^*$ coincides with the value function for the initial period V_1^* since there is no "re-setting". This case is precisely the type of model studied in Aiyagari et al. (2002).□

Example 5.2 (quasi-linear per-period payoff, $\lambda \geq 0$, and $\pi_\Delta = \mathbf{1}_{\{0\}}$)**.** *Assume that $u(c, 1-n) = c + H(1-n)$ for some function H consistent with assumption 4.1. Under this assumption, μ can be dropped as a state variable since $u_c = 1$ and thus it does not affect the pricing equation. In this case, the value function during financial autarky is given by*

$$V_0^*(g) = \kappa \boldsymbol{n}_0^*(g) - g + H(1 - \boldsymbol{n}_0^*(g)) + \beta \int_{\mathbb{G}} \left(\lambda V_1^*(g', 0) + (1-\lambda) V_0^*(g')\right) \pi_{\mathbb{G}}(dg'|g).$$

This expression follows from the fact that there is no need to keep the debt B as part of the state during financial autarky since none of the defaulted debt is ever repaid, and all the offers of zero repayment are accepted by the government. The value function during financial access is given by

$$V_1^*(g, B) = \max_{(n, B') \in \Gamma(g,B)} \left\{ n - g + H(1-n) + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B'), V_0^*(g')\} \pi_{\mathbb{G}}(dg'|g) \right\},$$

where $\Gamma(g, B) \equiv \{(n, B') \ : \ z(1, n, g) + \beta E_{\pi_{\mathbb{G}}(\cdot|g)}[\mathbf{1}_{\{g : V_1^*(g, B') \geq V_0^*(g)\}}(g')] B' - B \geq 0\}$.

The expression for the price function highlights an important difference between our default model and a model with risk-free debt such as AMSS. Since $u_c = 1$, the market stochastic discount factor is equal to β, and thus in the latter model the government cannot manipulate the return of the discount bond. In our economy with defaultable debt, however, while not being able to influence the risk-free rate, the government is still able to manipulate the return of the discount bond by altering its payoff through the decision of default.

Moreover, assuming H is increasing and strictly concave with $H'(1) < 1$ and $2H''(l) < H'''(l)(1-l)$, we can view the government problem as directly choosing tax revenues R with a per-period payoff given by $W_\kappa(R) = \kappa n_\kappa(R) + H(1 - n_\kappa(R))$ where $n_\kappa(R)$ is the amount of labor needed to collect revenues equal to R, given κ. Under our assumptions, W_κ is non-increasing and concave function. The Bellman equation of the value of repayment is given by

$$V_1^*(g, B) = \max_{(R, B')} \left\{ W_1(R) - g + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B'), V_0^*(g')\} \pi_{\mathbb{G}}(dg'|g) \right\}, \tag{5.19}$$

subject to $R + \beta E_{\pi_{\mathbb{G}}(\cdot|g)}[\mathbf{1}_{\{g : V_1^*(g, B') \geq V_0^*(g)\}}(g')] B' \geq g + B$, and

$$V_0^*(g) = W_\kappa(g) - g + \beta \int_{\mathbb{G}} \left(\lambda V_1^*(g', 0) + (1-\lambda) V_0^*(g')\right) \pi_{\mathbb{G}}(dg'|g). \tag{5.20}$$

The previous equations imply that this government's problem is analogous to that studied in Arellano (2008) and Aguiar and Gopinath (2006) among others where the government chooses how much to "consume", captured by $-R$, given an exogenous process of *"income", $-g$. An important difference, however, is the non-standard per-period payoff which reflects the distortive nature of labor taxes. In particular in our model the per-period payoff has a satiation point at $R = 0$ (i.e., zero distortive taxes).*[45]

We think this last observation is relevant because it allows us to extend some of our results to general sovereign debt models with endogenous default, especially those regarding the impact of the debt restructuring in prices. $\square$

6 Analytical Results

In this section we present analytical results for a benchmark model characterized by quasi-linear per-period utility, i.i.d. government expenditure shocks and debt repayments for exiting financial autarky. The proofs for the results are gathered in appendix E.

Assumption 6.1. *(i) $\kappa = 1$; (ii) $u(c, n) = c + H(1 - n)$ where $H \in \mathbb{C}^2((0, 1), \mathbb{R})$ with $H'(0) = \infty$, $H'(l) > 0$, $H'(1) < 1$, $H''(l) < 0$ and $2H''(l) < H'''(l)(1 - l)$*

Part (i) implies that there are no direct cost of defaults in terms of output. Part (ii) of this assumption imposes that the per-period utility of the households is quasi-linear and it is analogous to assumption in p. 10 in AMSS. As noted above, under this assumption, μ can be dropped as a state variable. This implies that the value functions V_0^*, V_1^* are only functions of (g, B) and the same holds true for the optimal policy functions.

We also assume that government expenditure are i.i.d., formally

Assumption 6.2. *For any $g' \neq g$, $\pi_{\mathbb{G}}(\cdot|g) = \pi_{\mathbb{G}}(\cdot|g')$.*

With a slight abuse of notation and to simplify the exposition we use $\pi_{\mathbb{G}}(\cdot)$ to denote the probability measure of g. Finally, to further simplify the technical details, we assume that $\mathbb{B}$ has only finitely many points, unless stated otherwise.[46]

For the rest of the section, we proceed as if these assumptions 6.1 - 6.2 hold and will not be referenced explicitly.

[45] Another subtle difference with the standard sovereign default literature is that while in our economy government and bondholders share the same preference, in this literature they do not. In particular, the government tends to be more impatient than (foreign) investors, thus bringing about incentives to front-load consumption through borrowing.

[46] This assumption is made for simplicity. It can be relaxed to allow for general compact subsets, but some of the arguments in the proofs will have to be changed slightly. Also, the fact that $\mathbb{B} \equiv \{B_1, ..., B_{|\mathbb{B}|}\}$ is only imposed for the government; the households can still choose from convex sets; only in equilibrium we impose $\{B_1, ..., B_{|\mathbb{B}|}\}$.

6.1 Characterization of Optimal Default Decisions

The next proposition characterizes the optimal decisions to default and to accept offers to repay the defaulted debt as "threshold decisions"; analogously to Arellano (2008) but adapted to this setting. Recall that $\mathbf{d}^*(g, B)$ and $\mathbf{a}^*(g, \delta, B)$ are the optimal decision of default and of renegotiation, respectively, given the state (g, δ, B).

Proposition 6.1. *There exists $\bar{\lambda}$ such that for all $\lambda \in [0, \bar{\lambda}]$, the following holds:*

1. *There exists a $\hat{\delta} : \mathbb{G} \times \mathbb{B} \to \Delta$ such that $\mathbf{a}^*(g, \delta, B) = \mathbf{1}_{\{\delta : \delta \leq \hat{\delta}(g,B)\}}(\delta)$ and $\hat{\delta}$ non-increasing as a function of B.*[47]

2. *There exists a $\bar{g} : \mathbb{B} \to \mathbb{G}$ such that $\mathbf{d}^*(g, B) = \mathbf{1}_{\{g : g \geq \bar{g}(B)\}}(g)$ and $\bar{g}$ non-increasing for all $B > 0$.*

This result shows that for a (non-trivial) range of probabilities of receiving outside offers, $\lambda \in [0, \bar{\lambda}]$, default is more likely to occur for high levels of debt, and so are rejections of offers to exit financial autarky.[48] The latter result implies that the average recovery rate is decreasing in the level of debt, as documented by Yue (2010) in the data.[49] It also follows that other things equal, higher debt levels are on average associated with longer financial autarky periods.[50] Thus, these two results imply a positive co-movement between the (observed) average haircut and the average length of financial autarky.[51]

6.2 Implications for Equilibrium Prices and Taxes

We now study the implications of the above results on equilibrium prices and taxes.

[47] It turns out that the first part of the statement holds for any λ.

[48] In our numerical simulations for the benchmark calibration with $\lambda = 0.2$, all theoretical results from proposition 6.1 hold, therefore implying that $\bar{\lambda} \geq 0.2$.

[49] According to the proposition, the average recovery rate equals $E_{\pi_{\mathbb{G}}}[\int_{\delta' \in \Delta} \delta' \mathbf{1}_{\{\delta \,:\, \delta \leq \hat{\delta}(g,B)\}}(\delta') \pi_{\Delta}(d\delta')]$.

[50] The expected length of autarky — given a defaulted debt of B — is given by $\frac{1}{E_{\pi_{\mathbb{G}}}[F_{\pi_{\Delta}}(\hat{\delta}(g,B))]}$ where $F_{\pi_{\Delta}}$ is the cdf corresponding to π_{Δ}.

[51] This last fact seems to be consistent with the data; see fact 3 in Benjamin and Wright (2009). Cruces and Trebesch (2013) found a similar relationship for 180 sovereign debt restructuring cases of 68 countries between 1970 and 2010. It is important to note, however, that we derived the implications by looking at *exogenous* variations of the debt level; in the data this quantity is endogenous and, in particular, varies with g. This endogeneity issue should be taken into account if one would like to perform a more thorough test of the aforementioned implications. We further explore this issue in the numerical simulations.

Equilibrium prices and endogenous debt limits. Under assumption 6.2 equilibrium prices do not depend on g, i.e., $\mathcal{P}^*_\phi(\cdot) \equiv \mathcal{P}^*_\phi(g,\cdot)$ for any $g \in \mathbb{B}$. By proposition 6.1 it follows that, for any $B' \in \mathbb{B}$,

$$\mathcal{P}^*_1(B') = \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' \leq \bar{g}(B')\}}(g')\pi_{\mathbb{G}}(dg') + \left(\beta \int_{\mathbb{G}} \mathbf{1}_{\{g' > \bar{g}(B')\}}(g')\pi_{\mathbb{G}}(dg')\right) \mathcal{P}^*_0(B') \tag{6.21}$$

and [52]

$$\mathcal{P}^*_0(B) = \frac{\beta\lambda \int_\Delta \left(\int_{\mathbb{G}} \mathbf{1}_{\{\delta:\delta\leq\hat{\delta}(g',B)\}}(\delta)\pi_{\mathbb{G}}(dg')\right)\delta\pi_\Delta(d\delta)}{1-\beta+\beta\lambda\int_\Delta\int_{\mathbb{G}} \mathbf{1}_{\{\delta:\delta\leq\hat{\delta}(g',B)\}}(\delta)\pi_{\mathbb{G}}(dg')\pi_\Delta(d\delta)}. \tag{6.22}$$

A key feature of endogenous default models is the existence of endogenous borrowing limits. A necessary condition for this result is that, due to the possibility of default, equilibrium prices are non-increasing as a function of debt; thus implying a "Laffer-type curve" for the revenues coming from selling bonds. In an economy without debt repayment (e.g., $\pi_\Delta = \mathbf{1}_{\{0\}}$), it follows that $\mathcal{P}^*_0 = 0$ and $\mathcal{P}^*_1(B') = \beta \int_{\mathbb{G}} \mathbf{1}_{\{g:g\leq\bar{g}(B')\}}(g')\pi_{\mathbb{G}}(dg')$ which is non-increasing in B' by proposition 6.1. Moreover, it takes value zero for sufficiently high B'. Therefore, there exists an endogenous debt limit, i.e., finite value of B' that maximize the debt revenue $\mathcal{P}^*_1(B')B'$.

In an economy where we allow for debt repayments, by inspection of equation 6.21 and the fact that $\mathcal{P}^*_0 \geq 0$, it is easy to see that, other things equal, the previous result is attenuated by the presence of (potential) defaulted debt payments and secondary markets. Although, for a general π_Δ is hard to further characterize $\mathcal{P}^*$ analytically, the next proposition shows that when repayment offers exist but are non-random, the price is non-increasing on the level of debt and there are endogenous borrowing limits.

Proposition 6.2. *Suppose $\pi_\Delta(\cdot) = 1_{\delta_0}(\cdot)$ for some $\delta_0 \in [0,1]$. Then there exists a $\bar{\lambda} > 0$, such that for all $\lambda \in [0,\bar{\lambda}]$, $\mathcal{P}^*_i(\cdot)$ is non-increasing for $B > 0$ and for all $i = 0, 1$.*

This proposition shows that high levels of debt are associated with higher return on debt, both before and during financial autarky. This result is consistent with the evidence regarding debt-to-output levels and default risk measures presented in section 2. Moreover, this result in conjunction with the implications derived from proposition 6.1, implies that high levels of debt are associated with higher return on debt, lower (observed) average recovery rate, and, on average, longer financial autarky spells. In particular, it implies that (on average) longer periods in financial autarky are associated with higher spreads during this period.

[52] See lemma E.5(3) in the appendix for the derivation.

In particular, the existence of endogenous borrowing limits implies that the ability to roll over high levels of debt is hindered. This in turn implies not only that labor is higher but due to the concavity of $z(1, \cdot, g)$ labor is more "sensitive" to fluctuations in government expenditure.

Default risk and the law of motion of equilibrium taxes. In order to analyze the ex-ante effect of default risk on the law of motion of taxes, we consider the case $\lambda = 0$ (i.e., autarky is an absorbing state) to simplify the analysis. We also strengthen assumption 6.1 by requiring that $H''(l) < H'''(l)(1-l)$. By proposition 6.1, the default decision is a threshold decision, so for each history $\omega^\infty \in \Omega^\infty$ we can define $T(\omega^\infty) = \inf\{t : g_t \geq \bar{g}(B_t(\omega^{t-1}))\}$ (it could be infinity) as the first time the economy enters in default. For all $t \leq T(\omega^\infty)$ the economy is not in financial autarky, and the implementability constraint is given by

$$B_t(\omega^{t-1}) + g_t \leq \Big(1 - H'(1 - n_t(\omega^t))\Big)\, n_t(\omega^t) + \mathcal{P}_1^*(B_{t+1}(\omega^t))B_{t+1}(\omega^t),$$

where $\mathcal{P}_1^*(g_t, B_{t+1}(\omega^t)) \equiv E_{\pi_{\mathbb{G}}}[1 - \mathbf{d}^*(g', B_{t+1}(\omega^t))]$. Let $\nu_t(\omega^t)$ be the Lagrange multiplier associated to this restriction in the optimization problem of the government, given $\omega^t \in \Omega^t$. In appendix E.2 we derive the FONC of the government and provide a closed form expression for $\nu_t(\omega^t)$ as a *decreasing* nonlinear function of $n_t(\omega^t)$; see equation E.87.[53] Hence, as noted by AMSS, by studying the law of motion of ν_t we can shed light on the law of motion of labor and taxes.

From the FONC of the government it follows (see appendix E.2 for the derivation) [54]

$$\nu_t(\omega^t)\left(1 + \frac{d\mathcal{P}_1^*(B_{t+1}(\omega^t))}{dB_{t+1}} \frac{B_{t+1}(\omega^t)}{\mathcal{P}_1^*(B_{t+1}(\omega^t))}\right) = \int_{\mathbb{G}} \nu_{t+1}(\omega^t, g') \frac{1\{g' \leq \bar{g}(B_{t+1}(\omega^t))\}}{\int_{\mathbb{G}} 1\{g' \leq \bar{g}(B_{t+1}(\omega^t))\}\pi_{\mathbb{G}}(dg')} \pi_{\mathbb{G}}(dg'). \tag{6.23}$$

The Lagrange multiplier associated with the implementability condition is constant in Lucas and Stokey (1983) and, thus, trivially a martingale. In Aiyagari et al. (2002) the Lagrange multiplier associated with the implementability condition is a martingale with respect to the probability measure $\pi_{\mathbb{G}}$.[55] Equation 6.23 implies that the law of

[53] The strengthening of assumption 6.1 is only needed to show that $\nu_t(\omega^t)$ is an decreasing function of $n_t(\omega^t)$; the proof is in appendix E.2.

[54] This derivation assumes that $\mathbb{B}$ is a convex set and $\pi_{\mathbb{G}}$ has a density with respect to the Lesbegue measure, so as to make sense of differentiation. It, also, assumes differentiability of V^*.

[55] The martingale property is also preserved if capital is added to the economy; see Farhi (2010). If we allow for ad-hoc borrowing/savings limits, the equality has to be replaced by the corresponding inequality; see Aiyagari et al. (2002).

motion of the Lagrange multiplier differs in two important aspects. First, the expectation is computed under the so-called *default-adjusted probability measure*, given by $\frac{1\{\cdot \leq \bar{g}(B_{t+1}(\omega^t))\}}{\int_{\mathbb{G}} 1\{g' \leq \bar{g}(B_{t+1}(\omega^t))\} \pi_{\mathbb{G}}(dg')} \pi_{\mathbb{G}}(\cdot)$. The wedge between $\pi_{\mathbb{G}}$ and this new probability stems from the fact that the option to default adds "some" degree of state-contingency to the payoff of the government debt making it lower for high values of government expenditure, and implies that the default-adjusted probability measure is first order dominated by $\pi_{\mathbb{G}}$. In fact, it implies that only the states tomorrow in which there is repayment, are relevant for the law of motion of ν_t.[56] It follows that, in the case that $\nu_{t+1}(\omega^t, \cdot)$ is increasing, the presence of the default-adjusted probability lowers $\nu_t(\omega^t)$ and, consequently, lowers also the tax distortions. Second, $\nu_t(\omega^t)$ in the left-hand side is multiplied by $\left(1 + \frac{d\mathcal{P}_1^*(B_{t+1}(\omega^t))}{dB_{t+1}} \frac{B_{t+1}(\omega^t)}{\mathcal{P}_1^*(B_{t+1}(\omega^t))}\right)$, which can be interpreted as the "markup" that the government has to pay for having the option to default; this effect increases $\nu_t(\omega^t)$ and the tax distortions. These two forces act in opposite directions, and is not clear which one will prevail. In order to shed more light on this issue, in section 7 we explore quantitatively this trade-off by plotting the impulse responses for $\nu_t(\omega^t)$ delivered by our model and a version of Aiyagari et al. (2002).

7 Numerical Results

Throughout this section, we run a battery of numerical exercises in order to assess the performance of the model. We compare our findings with an economy in which the option to default is not present—precisely the model considered in Aiyagari et al. (2002). We denote the variables associated with this model with a (sub)superscript "AMSS"; variables associated to our economy are denoted with a (sub)superscript "ED" (short for Economy with Default).

In the dataset IND economies are proxies of the AMSS model and EME/LAC are proxies of our model. As discussed before, IND do not exhibit default events in the dataset. There is the question of what characteristics of the economy will prompt it to behave like AMSS- or ED-type economies. One possible explanation is that by factors extraneous to the model, such as political instability, ED presents lower discount factor from the government and thus are more prone to default. An alternative explanation, in line with our model, is that for AMSS-type/IND economies, default is more costly because they are financially more integrated, and the financial autarky following a default could

[56]If default occurs, the link between multipliers today and tomorrow is severed and tax rates stop exhibiting persistence, as they are set to balance the budget inheriting the i.i.d. properties of the stochastic process of the government spending.

have a larger impact on financing of the firms, thus lowering their productivity (in our model represented by a lower κ).[57]

For all the simulations the utility function is given by $u(c, 1-n) = c + C_1 \frac{(1-n)^{1-\sigma}}{1-\sigma}$, where $-\sigma$ is the inverse Frisch elasticity of labor supply. In this parametrization, we assume that $|\mathbb{G}|$ consists of five values, evenly spaced between $[0.207, 0.277]$; $\pi_{\mathbb{G}}$ is the uniform distribution over $\mathbb{G}$; the debt state space is given by $\mathbb{B} = [0, 0.1]$, with $|\mathbb{B}| = 1,000$. Finally, we rule out negative lump-sum transfers.

We choose the parameters of the model as follows. We set $\beta = 0.984$, $\sigma = 4$, $\kappa = 1$ and $C_1 = 0.04$. For the benchmark parametrization (Table 7.2) we choose $\lambda = 0.2$, and $\Delta = \{0.3, 0.7\}$ where the probability π_Δ assigns probability of 0.75 and 0.25, respectively.[58]

Table 7.2: Parameter Values

		Parameter	Value
Preferences			
	Preference Parameter	σ	4
	Constant in Preference	χ	0.04
	Time discount factor	β	0.984
Debt Restructuring			
	Probability of receiving offer	λ	0.20
	Offer 1	d_1	0.30
	Offer 2	d_2	0.70
	Probability of offer 1	$\pi(d_1)$	0.75
	Probability of offer 2	$\pi(d_2)$	0.25

To compute the statistics, we perform 5,000 Monte Carlo (MC) iterations, each consisting of sample paths of 2,500 observations for which the first 500 observations were disregarded in order to eliminate the effect of the initial conditions. We then compute the mean statistics across MC simulations. [59]

The results for the whole sample for our model and the risk-free debt model (AMSS)

[57] In our calibration, $\kappa = 0.96$, is sufficiently low to prevent the economy from defaulting.

[58] These values are taken from the 70- percent and the 30-percent haircuts in the debt restructuring following the default by Argentina in 2001 and by Ecuador in 1997, respectively.

[59] The unconditional default frequency is computed as the sample mean of the number of default events in the simulations. The model is solved numerically using value function iterations with a discrete state space and an "outer" loop that iterates on prices until convergence.

Table 7.3: MC Statistics for the whole sample for our model and the risk-free debt model (AMSS).

Statistic	ED	AMSS
Mean(debt/y)(%)	2.5	7.0
Mean(τ)	0.530	0.528
Std.dev.(τ)	0.057	0.038
Autocor.(τ)	0.373	0.630
Mean(y)	0.459	0.460
Mean(c)	0.216	0.218
Std.dev.(y)	0.017	0.011
Std.dev.(c)	0.040	0.033
Mean($r - r^f$) (%)	105.39	0
Mean(default spell)	6.28	NA
Mean(recovery rate) (%)	38.3	NA
Failed reneg. freq. (%)	4.56	NA
Default frequency (%)	1.248	0
20th-percentile(τ)	0.472	0.506
80th-percentile(τ)	0.573	0.553

Table 7.4: MC Statistics for the “financial autarky” sample and “financial access” sample.

	Financial Access		Financial Autarky	
Statistic	ED	AMSS	ED	AMSS
Mean(debt/y)(%)	2.4	6.9	12.7	13.7
Mean(τ)	0.530	0.528	0.557	0.570
Std.dev.(τ)	0.056	0.038	0.093	0.053
Autocor.(τ)	NA	NA	NA	NA
Mean(y)	0.459	0.460	0.448	0.446
Mean(c)	0.217	0.218	0.201	0.200
std.dev.(y)	0.017	0.011	0.031	0.019
std.dev.(c)	0.040	0.033	0.054	0.039
Mean($r - r^f$) (%)	3.40	0	6266.72	0
cor(y, $r - r^f$)	-0.214	0	0	0
cor(τ, $r - r^f$)	0.181	0	0	0
20th-percentile(τ)	0.473	0.506	0.467	0.531
80th-percentile(τ)	0.572	0.552	0.663	0.608

are reported in Table 7.3. In our economy default occurs with an annual default frequency of around 1.25 percent. Bond holders anticipate the default strategies in equilibrium and charge higher bond returns to bear the bond. Facing higher borrowing costs, the government responds by issuing less bonds. Consequently, the average level of indebtedness is significantly lower in our environment than in AMSS model. It is 2.5 percent of output in the former, while 7 percent in the latter economy.[60] Thus, our model is able to generate considerable levels of "debt intolerance", a fact observed to be present in economies prone to default; see section 2.

The presence of endogenous borrowing limits, arising from the possibility of default, hinders the government's ability to smooth taxes. As a result, taxes are higher but particularly more volatile: the standard deviation of tax rates is 50 percent higher in our model. Our model can therefore replicate the corresponding empirical fact documented in section 2. Also, note that the 20th-80th percentile interval for the tax distribution in the economy with defaultable bonds contains its counterpart in the AMSS model, which reflects the fact that the latter distribution is relatively more spread out. Not surprisingly, taxes are less persistent in our environment.[61] The lower autocorrelation is attributed to two main factors. First, a lower persistence of taxes in borrowing states due to the incidence of endogenous default, manifested in the law of motion of the Lagrange multiplier of the implementability constraint; see subsection 6.2.[62] Second, the fact that if default occurs, the economy switches to autarky and the tax rate inherits the stochastic properties of the government spending, which is assumed to be i.i.d. Higher, volatile taxes in our model lead to lower, more volatile labor supply.

Our model also generates a frequency of renegotiation failures of roughly 5 percent, an average recovery rate of almost 40 percent, and a mean autarky spell of 6-7 periods.

While in table 7.3 we focus on the entire sample, in table 7.4 we show the results for two subsamples: "financial autarky" and "financial access". To construct these subsamples, we split each MC simulation into the periods in which the ED economy is in autarky and those in which it is not.

In this environment, the one-period gross risk-free $1+r^f$ rate is equal to the reciprocal of the households' discount factor β. Bond spreads are computed as the differential between annualized bond returns and the risk-free rate. The (one-period) gross return of

[60] For comparison, in the data for Argentina (1990-2005) this ratio is approximately 23 percent. During the default period (2001-2005), it increased to around 45 percent.

[61] Since the "financial autarky" and "financial access" subsamples may contain nonconsecutive periods by construction, the autocorrelation is not computed for them.

[62] As discussed in subsection 6.2, the Lagrange multiplier in AMSS follows a martingale process.

the government bond is given by $1/p_t$, both with borrowing and in financial autarky.[63] As expected, our model is able to generate higher spreads in financial autarky than in financial access. Bond spreads are on average 3.4 percent with borrowing but jump to over $6,000$ percent in autarky. The particular high value of the spread in financial autarky reflects the fact that although the probability of accepting an offer is around 95 percent, less than 40 percent of the defaulted debt is honored. In the data we also observe such high levels of bond yields during autarky: the average sovereign spreads measured by the EMBI+ index reached $5,757$ percent over 2002, the year after the default episode in Argentina, and remained around $5,485$ percent between 2002 and 2004. Also, our model delivers countercyclical bond spreads, a feature well-documented in the data for emerging economies.[64] As indicated by the positive correlation between tax rates and spreads, taxes tend to be higher precisely when borrowing is more expensive. Labor supply is optimally lower in those states leading to a negative commovement between bond returns and output.

During financial autarky, the average debt-to-output ratio is actually the *defaulted* debt-to-output ratio and is around 13 percent. The fact that it is roughly five times larger than in financial access provides additional evidence of endogenous borrowing limits being "active" for high debt levels and is consistent with the stylized facts presented in section 2.

The findings for taxes in financial access and financial autarky echo the results in Table 7.3. When there is borrowing, taxes are relatively more volatile in the economy with the defaultable bond due to the endogenous credit limits. Furthermore, when our economy is in financial autarky, the government is precluded from issuing debt, rendering taxes more volatile that in financial access. The 20th-80th percentile intervals for taxes are consistent with the same patterns.

Both the average output and consumption are similar in both economies, although in both cases these quantities are higher in the financial access subsample. Financial autarky

[63]This calculation of the bond return during autarky is reminiscent to the methodology applied by J.P. Morgan to obtain sovereign bond yields and spreads for the construction of the EMBI+ index. While a country is in default before any debt settlement is reached, bond yields and spreads are calculated assuming future cash flows stay the same as dictated in the original instrument structure. In this situation, future cash flows are not determined based on any expected haircuts until after a restructuring event officially takes place. For more details, see J. P. Morgan (2004).

[64]For both output and tax rates, the correlations with bond spreads are zero in autarky since government spending is i.i.d. —hence prices only depend on the debt level, as shown in expressions 6.21 and 6.22— and bond holdings remain unchanged until the debt is restructured.

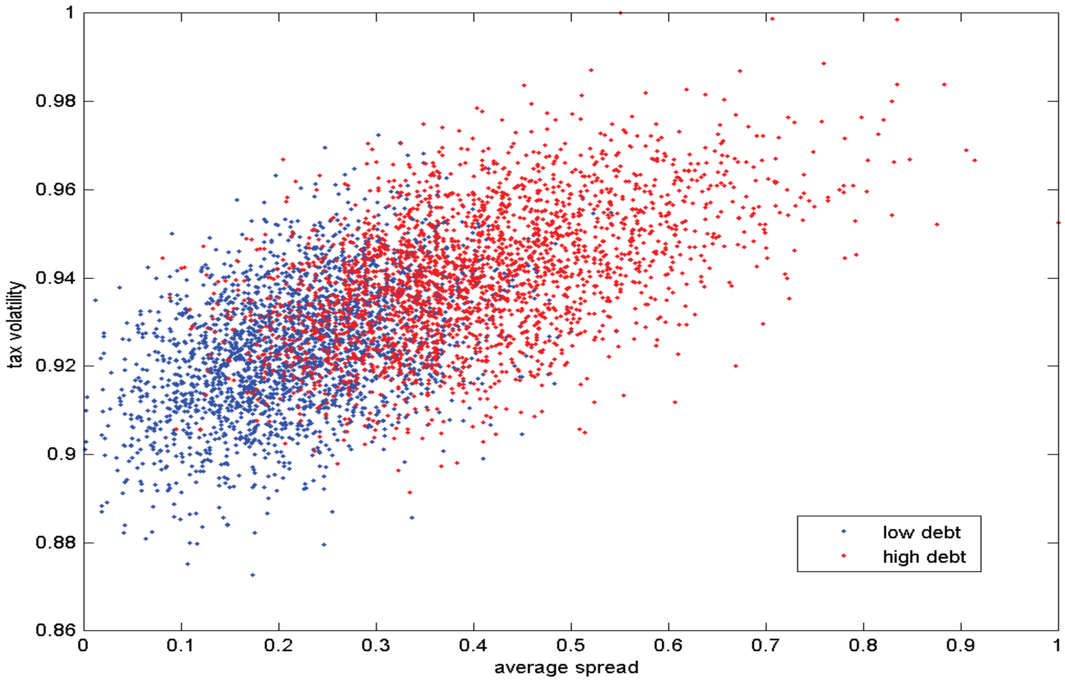

Figure 7.3: Standard deviation of tax rates and mean bond spreads in financial access for low debt (blue) and high debt (red).

is characterized by on average higher indebtness levels and also slightly higher government expenditure (not reported), that is, higher expenditure overall. Given that the primary surplus function z is decreasing in n, it follows that output (and consumption) are lower during financial autarky. This fact shows that our model is able to endogenously generate a drop in output (and consumption) during financial autarky, even even though no ad-hoc output loss is assumed in this calibration.

In addition, in both economies output and consumption are more volatile in the financial autarky subsample. The fact that the primary surplus function is also concave implies a higher sensitivity of these variables to changes in government spending and thus higher volatility during financial autarky. Finally, both output and consumption are more volatile in our economy than in AMSS, portraying the differences in the dynamics of their taxes.

Behavior of Taxes. As mentioned in section 2, our dataset suggests that default risk and tax volatility are positively correlated and that both are higher for high levels of debt-to-output ratio. Figure 7.3 shows that our model is able to generate this pattern. The blue (red) dots show the standard deviation of taxes and spreads for low (high) levels of debt, respectively. Each dot corresponds to the financial access subsample in a MC

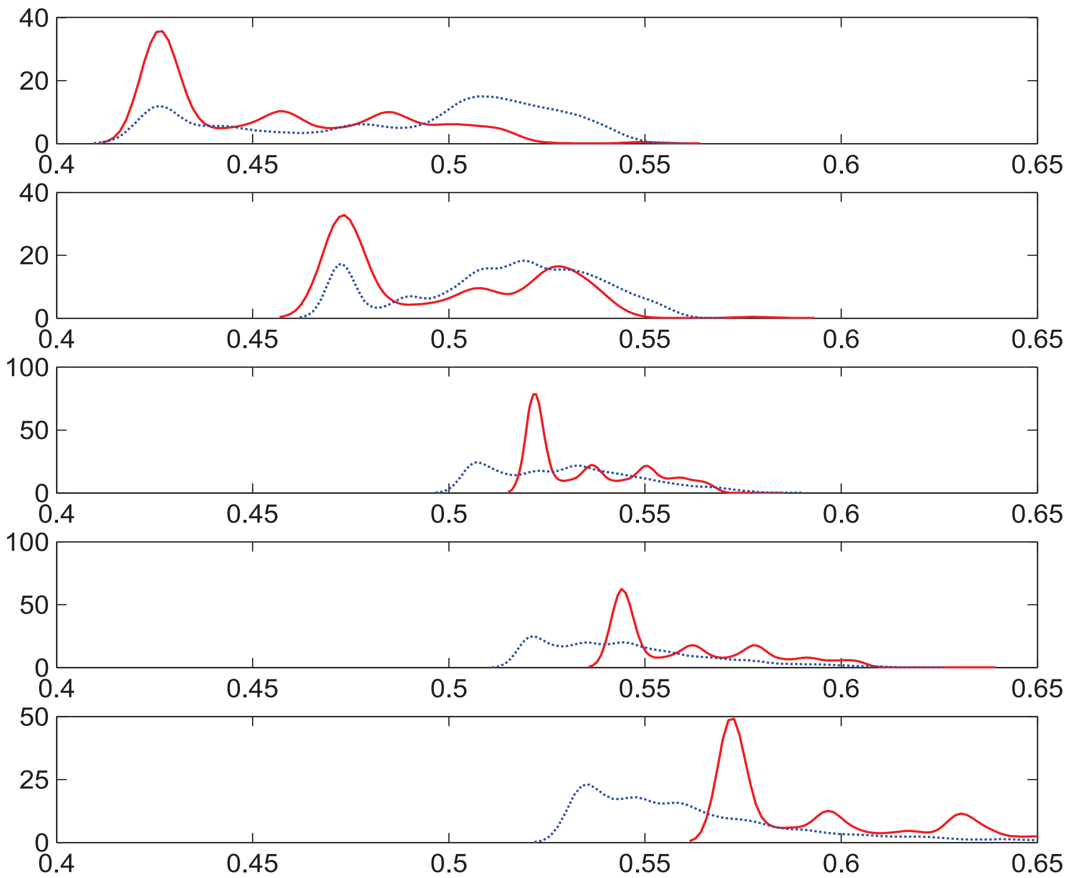

Figure 7.4: Histograms of tax rates in financial access for our model (solid red) and AMSS (dotted blue) conditioned on current g realization, from lowest value (top panel) to highest value (bottom panel).

simulation. For low (high) debt we consider debt-to-output ratios below (over) the median of its distribution. For both cases we can see a positive relationship between spreads and tax volatility. This result follows from the fact that higher spreads are caused by higher risk of default which in turn limits the ability of the government to use debt to smooth taxes when financing government shocks. A second noteworthy observation is that the red cloud is shifted to the upper right corner of the graph with respect to the blue cloud, thus indicating that both spreads and tax volatility are higher for higher level of debts.

In order to shed more light regarding the behavior of taxes when there is risk of default, in figure 7.4 we compare, for different values of government expenditure, the histograms (which were smoothed using kernel methods) of taxes in our model with that in AMSS.

First, as observed in the bottom panels, for high values of g the distribution of taxes in our model is shifted to the right compared to that in AMSS model. This difference between the two models arises from the fact that due to default risk debt is too costly for our government to finance high government expenditure. In contrast, as noted in the top panels, when the g realization is low the situation is reversed and now the distribution of taxes in AMSS model is shifted to the right relative to our model. During those states, the government repays the outstanding debt, which typically is higher in AMSS than in our credit-constrained economy. A final noteworthy observation is that for our model taxes are more concentrated around a single peak (which shifts to the right with the level of government expenditure), indicating more limited borrowing. In contrast, in AMSS the distribution of taxes is more spread-out for each g realization but at the same time more "stable" regarding changes in g, a clear reflection of more tax smoothing.

Table 7.5: MC Statistics for debt renegotiation for different values of λ.

λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Avg. offer accepted	0.40	0.38	0.37	0.35	0.35	0.34	0.33	0.33	0.32	0.32
Avg. duration | High debt	10.9	6.30	4.71	5.18	4.67	4.61	3.99	3.77	3.56	3.34
Avg. duration | Low debt	10.9	6.30	4.70	3.62	3.15	2.90	2.87	2.77	2.70	2.68

Debt Renegotiation. In table 7.5 we present some statistics regarding the debt renegotiation process for different values of λ. From the first row we see that as the probability of receiving an offer increases, the average offer accepted decreases. This fact and the fact that the frequency of rejected offers increases monotonically with λ (bottom panel in figure 7.7), follows because as λ increases the option value of staying in financial autarky increases and thus the government becomes more selective regarding which offers it accepts. Furthermore, it is also the case that as λ increases, the average number of periods in financial autarky declines (as shown in the top panel in figure 7.7), implying a positive relationship between the average haircuts and the average length of the debt restructuring process, in line with the findings of Benjamin and Wright (2009) and Cruces and Trebesch (2013). The last two rows in the table shows the average duration, conditioning on the fact that the defaulted debt is "high" (second row) and "low" (third row).[65] We can see that for both cases, it decreases as the probability of receiving an offer increases, but more importantly it shows that for "high" levels of debt we have, on average, longer financial autarky spells; in fact, the difference can be as large as 50 percent higher for intermediate values of λ. This fact coincides with the implications of proposition 6.1 and, at least for our benchmark case, the numerical simulations indicate that the differences in the duration are non-negligible.

Impulse responses. Figure 7.5 plots the impulse response for debt and taxes for our model and AMSS. The path of government expenditure is plotted in the first panel. We consider two values of initial debt, one "low" ($B_0 = 0$) and one "high" ($B_0 = 0.05$). To finance high government expenditure during the first periods, the government makes use of both instruments: bond issuance and taxes. While in both economies the government accumulates debt in those periods, in our model it does so to a lesser extent due to the presence of endogenous borrowing limits. From $t = 6$ onwards, when government expenditure becomes low, the level of debt decreases, eventually reaching zero. Taxes behave analogously.

In our economy taxes are higher than AMSS during the periods of high government expenditure since borrowing is more limited, but they decrease more rapidly when the realization of government expenditure becomes lower (see the third panel). Overall, not surprisingly, one can see a smoother behavior for taxes in AMSS than in our economy. The last panel plots the behavior of the Lagrange multiplier ν_t studied in subsection 6.2. The fact that ours is above the one of AMSS for periods of high government expenditure reflects the "mark-up" effect mentioned in subsection 6.2. It also follows that the Lagrange multiplier increases during these periods, reflecting the fact that the marginal cost of debt is increasing in the level of debt. From $t = 6$ onwards, as debt decreases, the Lagrange multiplier in our model falls and eventually becomes lower than the one of AMSS. This last feature stems from the facts that the accumulated debt is relatively lower in our

[65] As low (high) defaulted debt we consider debt-to-output ratios in the default episodes below (above) the unconditional median.

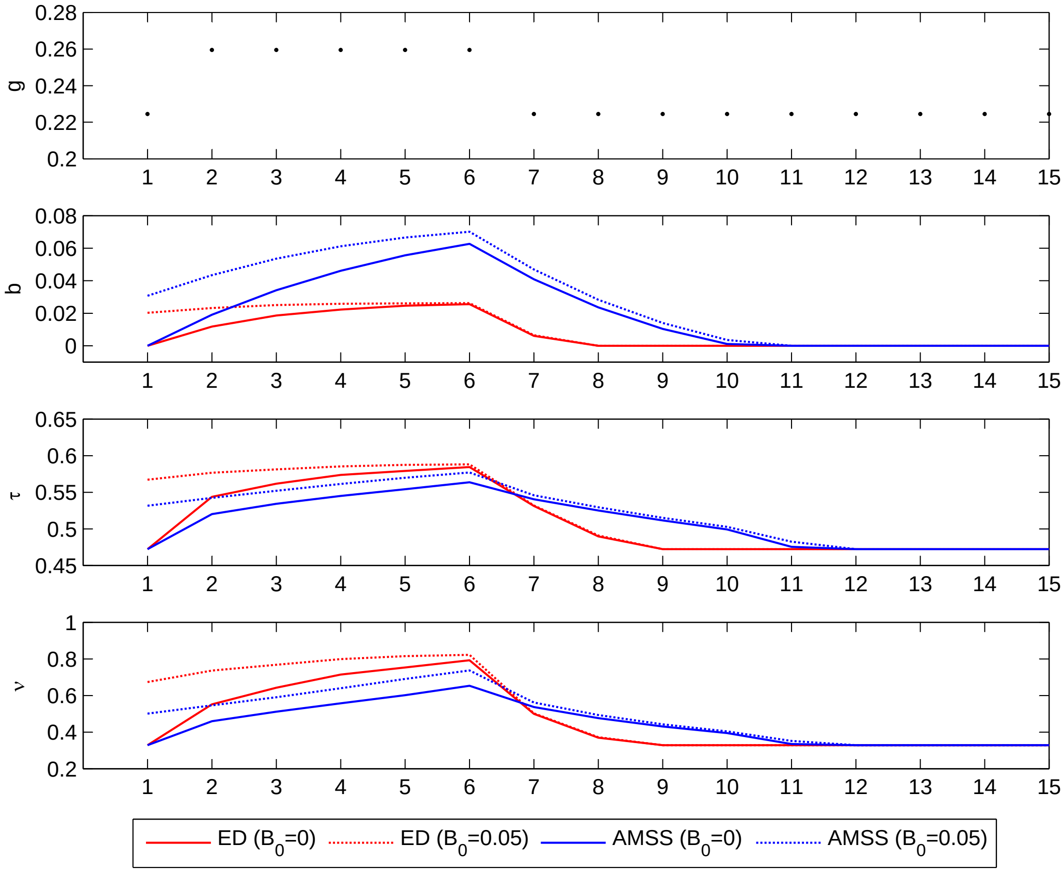

Figure 7.5: Impulse responses for our model (red) and AMSS (blue). Realization of government expenditure (first panel); debt path (second panel); tax path (third panel); and Lagrange multiplier path (fourth path).

model and that, as the level of debt decreases, the "mark-up" effects vanish.

Welfare Analysis. As a measure of welfare we use the compensation in terms of initial consumption that would make the household indifferent between our economy and AMSS. Formally, this compensation denoted by W is computed as

$$W \equiv \frac{\int V^{AMSS}(g,B)\mu_{AMSS}(dg,dB) - \int V_{\phi}^{*}(g,B)\mu(dg,dB,d\phi)}{\int(\mathbf{n}_{\phi}^{*}(g,B)-g)\mu(dg,dB,d\phi)}$$

where μ_{AMSS} and μ are the ergodic distributions generated by the AMSS and our model respectively and V^{AMSS} is the value function corresponding to the AMSS economy. That is, W measures the increase in initial consumption that would make the household indifferent between our economy and AMSS, under the ergodic distribution.[66]

[66]The ergodic distribution is constructed by collecting the last observation from each of the 5,000 MC simulated paths.

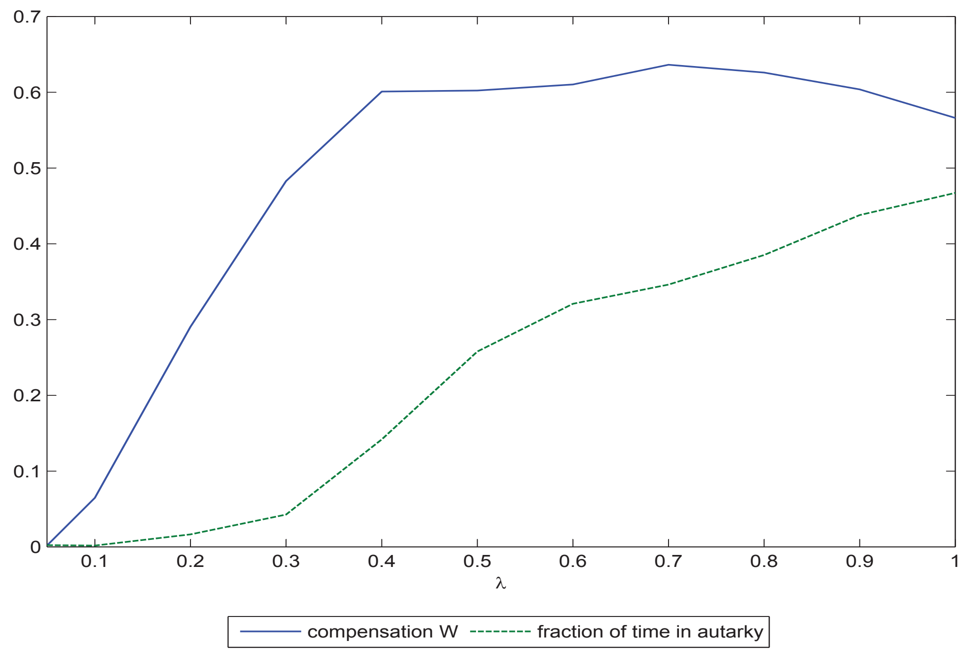

Figure 7.6: Consumption compensation W (solid blue) and fraction of time in autarky under ergodic distribution (dashed green) for different values of arrival probability of offers λ.

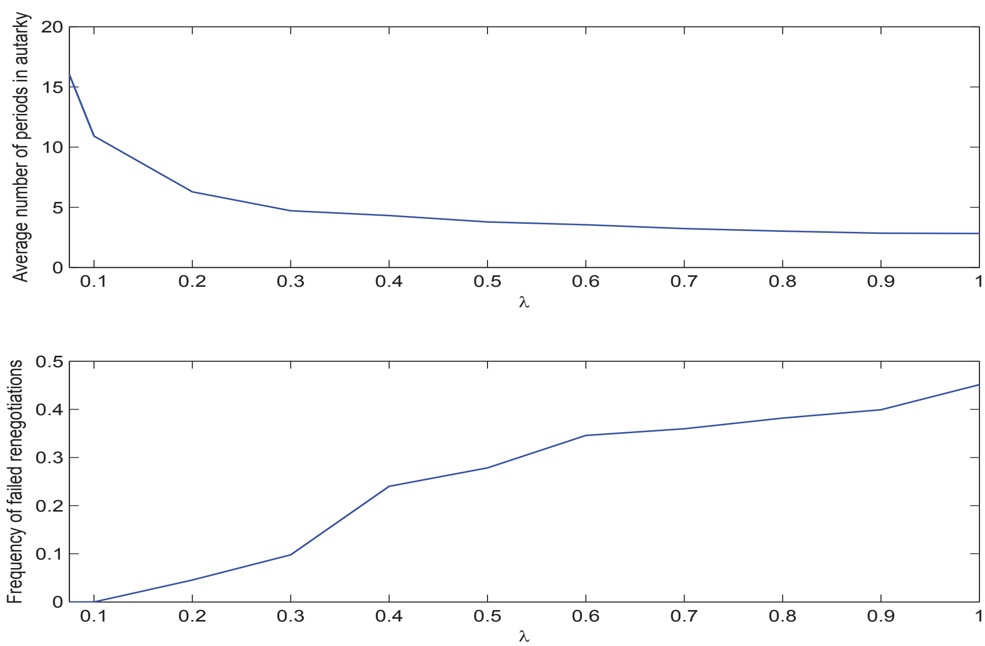

Figure 7.7: Average number of periods in autarky (top panel) and frequency of rejected offers (bottom panel) for different values of arrival probability of offers λ.

Figure 7.6 plots this measure for different values of the arrival probability of offers λ with our, otherwise, benchmark parametrization. As we can see, for low values of λ the compensation is almost zero, which is consistent with the fact that in these cases autarky is very costly pushing down the default frequency and thereby implying that our economy and AMSS are very similar. As λ increases, default occurs more often and welfare decreases in our economy relative to AMSS. In particular, for $\lambda = 0.7$ the compensation is as high as 63.64 percent of the initial consumption. Welfare, however, does not decrease monotonically with λ. In fact, for λ ranging from 0.7 to 1, welfare *increases* in our economy. A key effect for understanding this non-monotonicity is the behavior of the government during financial autarky. The average time the economy spends in financial autarky decreases monotonically, especially for low values of λ (top panel in figure 7.7), and perhaps more importantly the frequency of rejected offers increases monotonically with λ (bottom panel in figure 7.7) as well. These facts imply that, for high values of λ, the government is frequently confronted with the *option* to reject a repayment offer and stay in financial autarky, whereas for low values of λ, this option is presented more infrequently. Thus, roughly speaking, for high values of λ, the time the economy spends in financial autarky is not only low but is driven by choice, whereas for low values of λ it is not. This last observation explains the non-monotonic behavior of welfare with respect to λ and also presents a nuanced view for evaluating how costly renegotiations periods are.

Robustness check: Persistent process for government expenditure. We finally explore the quantitative implications of our model under the assumption of persistence on the stochastic process for government expenditure. To do so, we consider the same parametrization as in our benchmark calibration, except that $|\mathbb{G}| = 3$ and the transition probability for g is

$$\pi_{\mathbb{G}} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.5 & 0.5 \end{bmatrix}.$$

We interpret this state space as a "recession", "normal times" and "booms". We assume that normal times are more persistent than the other two states, and for simplicity we treat these two states symmetrically. For comparison, we also present the results for the case with uniformly distributed g. Table 7.6 reports the MC statistics for our economy and AMSS. When the government expenditure is highly persistent (and less volatile), default tends to occur more often. The annual default frequency is 5.67 percent with persistent g while it only reaches 0.8 percent in the i.i.d. case. The government decides to default usually after a sufficiently long sequence of high government expenditure after increasing substantially its debt level. Because the defaulted debt is significantly higher with persistent g (not reported), the government is less willing to accept high repayment offers, as indicated by a frequency of failed renegotiations of roughly 27 percent. This, in turn, translates into more lengthy debt restructuring processes: the default spell is almost two periods longer with persistent government expenditure. Also, in spite of the higher default frequency, taxes are more persistent, and less volatile than in the i.i.d. case, which is consistent with the stochastic properties of their respective government expenditure processes.

Table 7.6: MC Statistics for the "persistence case" and "i.i.d. case" both for our model and AMSS.

	Persistence case		i.i.d. case	
Statistic	ED	AMSS	ED	AMSS
Mean(debt/y)(%)	1.4	3.6	3.7	7.5
Mean(τ)	0.531	0.529	0.531	0.529
Std.dev.(τ)	0.059	0.046	0.060	0.045
Autocor.(τ)	0.570	0.683	0.448	0.631
Mean(y)	0.458	0.459	0.458	0.459
Mean(c)	0.216	0.217	0.216	0.217
Mean(g)	0.242	0.242	0.242	0.242
Std.dev.(y)	0.018	0.014	0.018	0.013
Std.dev.(c)	0.036	0.031	0.044	0.039
Std.dev.(g)	0.019	0.019	0.029	0.029
Mean($r - r^f$) (%)	1868.66	0	52.72	0
Mean(default spell)	7.88	NA	5.96	NA
Mean(recovery rate) (%)	0.30	NA	0.40	NA
Failed reneg. freq. (%)	26.84	NA	0	NA
Default frequency (%)	5.37	0	0.80	0

8 Conclusion

In this paper we study a government problem in a closed economy, that consists of choosing distortionary taxes with only non-state-contingent government debt, but allowing for partial defaults on the debt.

First, we provide an explanation for the lower debt-to-output ratios and more volatile tax policies observed in emerging economies, vis-*à*-vis industrialized economies. This stems from the fact that the holders of government debt forecast the possibility of default, imposing endogenous debt limits. These limits restrict the ability of the government to smooth shocks using debt, resulting in higher tax variability.

Second, we propose a device to price the debt during temporary financial autarky. Our results show that the spread during the default period is higher than for the rest of the sample; this characteristic is consistent with data for defaulters—e.g., Argentina, Ecuador and Russia.

Third, and last, the numerical simulations suggest that increasing the probability of receiving offers for exiting autarky decreases welfare when this probability is low/medium to begin with, but increases it when the probability is high.

Although this model does a good job of explaining qualitatively several empirical regularities, it could be enriched in several dimensions so as to better fit the data of particular economies. In particular, we think a promising line of future research could be to extend the production side of this economy to allow for physical capital accumulation and productivity shocks. Our model also provides a novel device that allow us to study asset prices of government debt both during periods of financial access and autarky; further research could fully explore the pricing implications of this device for general sovereign debt held by uncertainty or risk averse creditors.[67]

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[67]See Aguiar and Gopinath (2006) and Mendoza and Yue (2012), and Pouzo and Presno (2013) exploring some of these issues.

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A Notation and Stochastic Structure of the Model

Throughout the appendix for a generic mapping f from a set S to T, we use $s \mapsto f(s)$ or $f : S \to T$ to denote it. For the case that a mapping depends on many variables, the notation $s_1 \mapsto f(s_1, s_2)$ is used to denote the function f *only* as a function of s_1, keeping s_2 fixed. Also, for a generic set A, $|A|$ denots the cardinality of A.

B Optimization Problem for the Households

The Lagrangian associated to the household's problem is given by

$$
\begin{aligned}
&\mathcal{L}(\{c_t, n_t, b_{t+1}, \nu_t, \mu_t, \psi_t\}_{t=0}^{\infty}) \equiv \\
&\sum_{t=0}^{\infty} \beta^t E_{\Pi(\cdot|\omega_0)} \Big[\Big\{ u(c_t(\omega^t), 1 - n_t(\omega^t)) - \nu_t(\omega^t) \{ c_t(\omega^t) - (1 - \tau_t(\omega^t))\kappa_t(\omega^t) n_t(\omega^t) + p_t(\omega^t) b_{t+1}(\omega^t) - \varrho_t(\omega^t) b_t(\omega^{t-1}) \} \\
&+ \Psi_t(\omega^t) c_t(\omega^t) + \psi_{1t}(b_{t+1}(\omega^t) - \underline{b}) + \psi_{2t}(\overline{b} - b_{t+1}(\omega^t)) \Big\} \Big],
\end{aligned}
$$

where ν_t and Ψ_t are the Lagrange multipliers associated to the budge constraint and to the restrictions that non-negative bound on consumption, and ψ_{it} $i = 1, 2$ are the Lagrange multipliers associated to the debt limits.

Assuming interiority of the solutions, the first order conditions (FONC) are given by:

$$
\begin{aligned}
c_t(\omega^t)&: u_c(c_t(\omega^t), 1 - n_t(\omega^t)) - \nu_t(\omega^t) = 0 \\
n_t(\omega^t)&: -u_l(c_t(\omega^t), 1 - n_t(\omega^t)) + \nu_t(\omega^t)(1 - \tau_t(\omega^t))\kappa_t(\omega^t) = 0 \\
b_{t+1}(\omega^t)&: p_t(\omega^t)\nu_t(\omega^t) - E_{\Pi(\cdot|\omega^t)}[\beta \nu_{t+1}(\omega^{t+1}) \varrho_{t+1}(\omega^{t+1})] = 0.
\end{aligned}
$$

Then, using $u_j(\omega^t)$ for $u_j(c_t(\omega), 1 - n_t(\omega))$ with $j \in \{c, l\}$, it follows

$$
\frac{u_l(\omega^t)}{u_c(\omega^t)} = (1 - \tau_t(\omega^t))\kappa_t(\omega^t), \tag{B.24}
$$

and

$$
p_t(\omega^t) = E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(\omega^{t+1})}{u_c(\omega^t)} \varrho_{t+1}(\omega^{t+1}) \right]. \tag{B.25}
$$

From the definition of ϱ, equation B.25 implies, for $d_t = 0$ and $a_t = 1$,

$$
p_t(\omega^t) = E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(\omega^{t+1})}{u_c(\omega^t)} (1 - d_{t+1}(\omega)) \right] + E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(\omega^{t+1})}{u_c(\omega^t)} d_{t+1}(\omega^{t+1}) q_{t+1}(\omega^{t+1}) \right].
$$

For $d_t = 1$ and $a_t = 0$, (where in this case recall that $p_t = q_t$)

$$
\begin{aligned}
p_t(\omega^t) = {}& \lambda E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(\omega^{t+1})}{u_c(\omega^t)} a_{t+1}(\omega^{t+1}) \delta_{t+1} \right] \\
&+ E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(\omega^{t+1})}{u_c(\omega^t)} \{1 - \lambda + \lambda(1 - a_{t+1}(\omega^{t+1}))\} q_{t+1}(\omega^{t+1}) \right].
\end{aligned}
$$

C Proofs for section 4.2

The next lemma characterizes the set of competitive equilibria as a sequence of restrictions involving FONC and budget constraints. The proof is relegated to the end of the section.

Lemma C.1. *Suppose assumption 4.1 holds. The tuple* $(c_t, g_t, n_t, b_{t+1}, p_t)_{t=0}^{\infty}$ *and* $\boldsymbol{\sigma}$ *is a competitive equilibrium iff given a* $B_0 = b_0$*, for all* $\omega^t \in \Omega^t$*, for all* t*,*

$$c_t(\omega^t) = \kappa_t(\omega^t)n_t(\omega^t) - g_t, \text{ and } B_{t+1}(\omega^t) = b_{t+1}(\omega^t), \tag{C.26}$$

$$\kappa_t(\omega^t)\tau_t(\omega^t) = \left(\kappa_t(\omega^t) - \frac{u_l(\omega^t)}{u_c(\omega^t)}\right); \tag{C.27}$$

and

$$Z_t(\omega^t) + \phi_t(\omega^t)\{p_t(\omega^t)B_{t+1}(\omega^t) - \delta_t B_t(\omega^t)\} \geq 0, \tag{C.28}$$

$$\text{and } \; if \;\; \phi_t(\omega^t) = 0, \;\; B_{t+1}(\omega^t) = B_t(\omega^{t-1}) \tag{C.29}$$

where

$$p_t(\omega^t) = E_{\Pi(\cdot|\omega^t)}\left[\beta\frac{u_c(\omega^{t+1})}{u_c(\omega^t)}\varrho_{t+1}(\omega^{t+1})\right]. \tag{C.30}$$

Proof of Theorem 4.1. We now show the "$\Rightarrow$" direction. Consider an outcome path $(d_t, a_t, B_{t+1}, n_t)_{t=0}^{\infty}$ that is consistent. This means by lemma C.1 that the tuple $(c_t, g_t, n_t, b_{t+1}, p_t)_{t=0}^{\infty}$ and $\boldsymbol{\sigma}$ is a competitive equilibrium iff given a $B_0 = b_0$, for all $\omega^t \in \Omega^t$, for all t,

$$c_t(\omega^t) = \kappa_t(\omega^t)n_t(\omega^t) - g_t, \text{ and } B_{t+1}(\omega^t) = b_{t+1}(\omega^t), \tag{C.31}$$

$$\kappa_t(\omega^t)\tau_t(\omega^t) = \left(\kappa_t(\omega^t) - \frac{u_l(\omega^t)}{u_c(\omega^t)}\right); \tag{C.32}$$

and

$$Z_t(\omega^t) + \phi_t(\omega^t)\{p_t(\omega^t)B_{t+1}(\omega^t) - \delta_t B_t(\omega^{t-1})\} \geq 0, \tag{C.33}$$

$$\text{and } \; if \;\; \phi_t(\omega^t) = 0, \;\; B_{t+1}(\omega^t) = B_t(\omega^{t-1})$$

where

$$p_t(\omega^t) = E_{\Pi(\cdot|\omega^t)}\left[\beta\frac{u_c(\omega^{t+1})}{u_c(\omega^t)}\varrho_{t+1}(\omega^{t+1})\right]. \tag{C.34}$$

It is easy to see that equations C.32 and C.34 imply equations 4.4 and 4.5. Equations C.32, C.34 and C.33 imply equation 4.9.

We now show the "$\Leftarrow$" direction. Suppose now that the outcome path satisfies that for all $\omega^t \in \Omega^t$, the following equations holds: 4.4, 4.5, 4.9 and

$$c_t(\omega^t) = \kappa_t(\omega^t) n_t(\omega^t) - g_t. \tag{C.35}$$

By using $B_{t+1}(\omega^t) = b_{t+1}(\omega^t)$, equations 4.4, 4.5 and the feasibility condition, we can enlarge the outcome path by $(c_t, p_t, b_{t+1}, \tau_t, g_t)_{t=0}^{\infty}$. Clearly, restrictions C.31, C.32 and C.34 hold. By replacing equations 4.4 and 4.5 on 4.9, it is easy to see that equation C.33 holds too. □

C.1 Proofs of supplementary lemmas

For the proof of Lemma C.1 we need the following lemma (the proof is relegated to the end of the section).

Lemma C.2. *Suppose assumption 4.1 holds. Then first order conditions 4.4 and 4.5 are also sufficient.*

Proof of Lemma C.1. Take $\boldsymbol{\sigma}$ and $(c_t, g_t, n_t, b_{t+1})_{t=0}^{\infty}$, and a price schedule $(p_t)_t$ that satisfy the equations. It is easy to see that feasibility and market clearing holds (conditions 3 and 4). Also, by lemma C.2 optimality of the households is also satisfied.

To check attainability of the government policy (condition 2). Observe that by equations C.26 - C.28 imply for all $\omega^t \in \Omega^t$,

$$g_t + \phi_t(\omega^t)\delta_t B_t(\omega^{t-1}) - \phi_t(\omega^t) p_t(\omega^t) B_{t+1}(\omega^t) \leq \kappa_t(\omega^t)\tau_t(\omega^t) n_t(\omega^t).$$

Finally, we check optimality of the households. We first check that the sequences satisfy the budget constraint. Observe that by equations C.26 - C.28

$$-c_t(\omega^t) + \kappa_t(\omega^t) n_t(\omega^t) + \phi_t(\omega^t)\{\delta_t B_t(\omega^{t-1}) - p_t(\omega^t) B_{t+1}(\omega^t)\} \leq \kappa_t(\omega^t)\tau_t(\omega^t) n_t(\omega^t).$$

If $\phi_t(\omega^t) = 1$, then equation C.28 implies that $b_{t+1}(\omega^t) = B_{t+1}(\omega^t)$ for all t (and for b_0 we assume it is equal to B_0) and thus

$$-c_t(\omega^t) + \kappa_t(\omega^t) n_t(\omega^t) + \delta_t b_t(\omega^{t-1}) - p_t(\omega^t) b_{t+1}(\omega^t) \leq \kappa_t(\omega^t)\tau_t(\omega^t) n_t(\omega^t).$$

This coincides with the budget constraint of the household.

If $d_t(\omega^t) = 1$, but $a_t(\omega^t) = 0$, then equations C.26 and C.28 imply that $b_t(\omega^{t-1}) = b_{t+1}(\omega^t) = 0$ for all t, so

$$-c_t(\omega^t) + \kappa_t(\omega^t) n_t(\omega^t) = \kappa_t(\omega^t)\tau_t(\omega^t) n_t(\omega^t).$$

This coincides with the budget constraint of the household.

Take $\boldsymbol{\sigma}$ and $(c_t, g_t, n_t, b_{t+1}, p_t)_{t=0}^{\infty}$ being a competitive equilibrium. Then it is easy to see that it satisfies the equations. □

Proof of Lemma C.2. Under assumption 4.1 the objective function of the household optimization problem is strictly concave. The budget constraints and debt constraint form a convex set of constraints. Thus, if the transversality condition holds, the FONC are sufficient; this follows from a simple adaptation of the results in Stokey et al. (1989) Ch. 4.5.

In order to verify the transversality condition, it suffices to show that for any $\zeta_t(\omega^t)$ such that $b_t(\omega^t) + \zeta_t(\omega^t) \in \mathbb{B}$,

$$\lim_{T\to\infty} \beta^T E_\Pi[u_c(\kappa_T(\omega^T)n_T(\omega^T) - g_T, 1 - n_T(\omega^T))\varrho_T(\omega^T)\zeta_T(\omega^T)] = 0.$$

Since, by assumption, debt is constrained, this condition follows from Magill and Quinzii (1994) Theorem 5.2. $\square$

D Proofs for section 5

The next lemma characterizes the government surplus function, the proof is relegated to the end of this section.

Lemma D.1. *Let* $(\kappa, n, g) \mapsto z(\kappa, n, g) = \left(\kappa - \frac{u_l(n-g,1-n)}{u_c(n-g,1-n)}\right) n - g$*. Then:*

1. $\arg\max_{n\in[0,1]}\{u(\kappa n - g, 1 - n) \ : \ z(\kappa, n, g) = 0\}$ *exists and is unique.*
2. *Suppose assumption 6.1 holds and let* $\bar{n}(g) = \arg\max_{n\in[0,1]} z(1, n, g)$*. Then,* $n \mapsto z(1, n, g)$ *is decreasing and strictly concave for all* $n \in [\bar{n}(g), 1]$

To show theorem 5.1 we need the following lemma whose proof is relegated to the end of this section.

Lemma D.2 (lem:rec-S)**.** *If, for any* $h_0 = (1, B, g, \delta) \in \mathbb{H}$ *and* $\phi_0 \in \{0, 1\}$*,* $\boldsymbol{\gamma} \in \mathcal{S}(h_0, \phi_0)$*, then* $\boldsymbol{\gamma}|_{h^t,\phi} \in \mathcal{S}(h_t, \phi)$ *for any* $h^t \in \mathbb{H}^t$ *and* $\phi \in \{0, 1\}$*. Moreover,*

$$z(\kappa_{\phi_0}, n_0(\boldsymbol{\gamma})(h_0), g)\mu_0(\boldsymbol{\gamma})(h_0) + \phi_0\{\mathcal{P}_{\phi_0}(g, B_1(\boldsymbol{\gamma})(h_0, \phi_0), \mu_1(\boldsymbol{\gamma})(h_0, h_1(\cdot)))B_1(\boldsymbol{\gamma})(h_0, \phi_0) - \delta\mu_0(\boldsymbol{\gamma})(h_0)B\} \geq 0$$

where $\kappa_\phi = \kappa(1 - \phi) + \phi$ *and* $h_1(\cdot) \equiv (1, B_1(\boldsymbol{\gamma})(h_0, 1), \cdot, 1)$ *and for* $t = 0, 1$

$$\mu_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g')) = u_c(n_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g')) - g', 1 - n_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g')))$$

Proof of Theorem 5.1. By definition of V_1^*, V_0^* and $\overline{V}_1^*$, it follows that:

with $h_0 = (1, B, g, \bar{\delta})$

$$V_0^*(g, B) = \sup_{\boldsymbol{\gamma}} V_0(\boldsymbol{\gamma})(h_0, 0) \tag{D.36}$$

$$\textit{subject to } \boldsymbol{\gamma} = (\boldsymbol{\gamma}^F, \boldsymbol{\gamma}^D) \in \mathcal{S}(h_0, 0) \tag{D.37}$$

$$\boldsymbol{\gamma}^D|_{h_0,\phi=0} \textit{ are determined by } (5.12) - (5.11) \tag{D.38}$$

$$u_c(\kappa n_0(\boldsymbol{\gamma})(h_0) - g, 1 - n_0(\boldsymbol{\gamma})(h_0)) = m_A(g) \tag{D.39}$$

and similarly, with $h_0 = (1, B, g, \delta)$

$$V_1^*(g, \delta B, \mu) = \sup_{\boldsymbol{\gamma}} V_0(\boldsymbol{\gamma})(h_0, 1) \tag{D.40}$$

$$\textit{subject to } \boldsymbol{\gamma} = (\boldsymbol{\gamma}^F, \boldsymbol{\gamma}^D) \in \mathcal{S}(h_0, 1) \tag{D.41}$$

$$\boldsymbol{\gamma}^D|_{h_0,\phi=1} \textit{ are determined by } (5.12) - (5.11) \tag{D.42}$$

$$u_c(n_0(\boldsymbol{\gamma})(h_0) - g, 1 - n_0(\boldsymbol{\gamma})(h_0)) = \mu. \tag{D.43}$$

finally, with $h_0 = (0, B, g, \delta)$

$$\overline{V}_1^*(g, \delta B) = \sup_{\boldsymbol{\gamma}} V_0(\boldsymbol{\gamma})(h_0, 1) \tag{D.44}$$

$$\textit{subject to } \boldsymbol{\gamma} = (\boldsymbol{\gamma}^F, \boldsymbol{\gamma}^D) \in \mathcal{S}(h_0, 1) \tag{D.45}$$

$$\boldsymbol{\gamma}^D|_{h_0,\phi=1} \textit{ are determined by } (5.12) - (5.11). \tag{D.46}$$

The first (sequential) problem consists of selecting $\boldsymbol{\gamma}$, consistent with competitive equilibrium and optimality for the default authority from $t = 1$ on, to maximize the lifetime utility of households, conditional on $h_0 = (1, B, g, \bar{\delta})$ and $\phi = 0$. The solution is given by $V_0^*(g, B)$, which does not depend on δ nor μ. Condition D.39 ensures that the current marginal utility is equal to the autarkic value defined before.

Problem D.40 is analogous to Problem D.36 with $\phi = 1$ instead. In this case, we impose through condition D.43 that the current marginal utility is μ.

Henceforth, we refer to strategies that satisfy the restrictions on the above programs as *admissible*. We also assume that the suprema are achieved; this assumption is to ease the exposition, if this were not the case the proof still goes through by exploiting the definition of the supremum.

By definition, $\overline{V}_1^*(g, \delta B) \geq V_0(\boldsymbol{\gamma})(0, B, g, \delta, 1)$ for all $\boldsymbol{\gamma} \in \mathcal{S}(0, B, g, \delta, 1)$ and $\boldsymbol{\gamma}^D|_{h_0,\phi=1}$ are determined by (5.12)-(5.11). By definition of $\Omega(0, B, g, \delta, 1)$, this implies that for all $(\mu, v) \in \Omega(0, B, g, \delta, 1)$, $\overline{V}_1^*(g, \delta B) \geq v$. On the other hand, assuming that the there exists a strategy $\boldsymbol{\gamma}$ that achieves the supremum, it has to be true that there exists a μ such that $(\mu, \overline{V}_1^*(g, \delta B)) \in \Omega(0, B, g, \delta, 1)$. Therefore,

$$\overline{V}_1^*(g, \delta B) = \max\{v | (\mu, v) \in \Omega(0, B, g, \delta, 1)\}. \tag{D.47}$$

It is easy to see that the same result applies for any time t and any history $(h^t, 0, B, g, \delta)$ (not just $t = 0$ and $h_0 = (0, B, g, \delta)$).

Let $h_0 \equiv (1, B, g, \overline{\delta})$. Suppose that there exists a strategy $\hat{\boldsymbol{\gamma}}$ that achieves the supremum in program D.36. Then [68]

$$\begin{aligned} V_0^*(g, B) =& u(\kappa n_0(\hat{\boldsymbol{\gamma}})(g) - g, 1 - n_0(\hat{\boldsymbol{\gamma}})(g)) \\ &+ \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \max\{V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \delta'), 1), V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \delta'), 0)\} \pi_\Delta(d\delta') \pi_{\mathbb{G}}(dg'|g) \\ &+ \beta(1-\lambda) \int_{\mathbb{G}} V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \overline{\delta}), 0) \pi_{\mathbb{G}}(dg'|g). \end{aligned}$$

[68] Henceforth we abuse notation and use $n_0(\boldsymbol{\gamma})(g)$ instead of $n_0(\boldsymbol{\gamma})(h_0)$.

Observe that, for any $g' \in \mathbb{G}$, $V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \delta'), 0)$ is constant with respect to δ'. Also, note that $\hat{\boldsymbol{\gamma}}|_{h^1,\phi}$ is admissible by lemma D.2. Hence, these observations and definition of V_0^*, imply that $V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \bar{\delta}), 0) \leq V_0^*(g', B)$. It also follows that $V_1(\boldsymbol{\gamma})(h_0, 0, B, (g', \delta'), 0) = V_0(\boldsymbol{\gamma})(0, B, (g', \delta'), 0)$ for any strategy $\boldsymbol{\gamma}$ and any (h_0, g', δ'). Thus

$$V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \bar{\delta}), 0) = V_0^*(g', B), \ \forall g' \in \mathbb{G}. \tag{D.48}$$

Therefore,

$$\begin{aligned}
V_0^*(g, B) =& u(\kappa n_0(\hat{\boldsymbol{\gamma}})(g) - g, 1 - n_0(\hat{\boldsymbol{\gamma}})(g)) && \text{(D.49)}\\
&+ \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \max\{V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \delta'), 1), V_0^*(g', B)\} \pi_\Delta(d\delta') \pi_{\mathbb{G}}(dg'|g) && \text{(D.50)}\\
&+ \beta(1-\lambda) \int_{\mathbb{G}} V_0^*(g', B) \pi_{\mathbb{G}}(dg'|g). && \text{(D.51)}
\end{aligned}$$

By construction, $n_0(\hat{\boldsymbol{\gamma}})(g) = \mathbf{n}_0^*(g)$ and thus,

$$\begin{aligned}
V_0^*(g, B) =& u(\kappa \mathbf{n}_0^*(g) - g, 1 - \mathbf{n}_0^*(g)) \\
&+ \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \max\{V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, (g', \delta'), 1), V_0^*(g', B)\} \pi_\Delta(d\delta') \pi_{\mathbb{G}}(dg'|g) \\
&+ \beta(1-\lambda) \int_{\mathbb{G}} V_0^*(g', B) \pi_{\mathbb{G}}(dg'|g).
\end{aligned}$$

Observe that at $(h_0, \phi_0 = 0, B, g', \delta', \phi_1 = 1)$ a "new" fiscal authority beings at time $t = 1$. By construction, this fiscal authority starts without binding promises regarding the marginal utility of consumption. Since $\hat{\boldsymbol{\gamma}}$ is optimal, it follows that $V_1(\hat{\boldsymbol{\gamma}})(h_0, 0, B, g', \delta', 1) = \overline{V}_1^*(g', \delta' B)$. Therefore,

$$\begin{aligned}
V_0^*(g, B) =& u(\kappa \mathbf{n}_0^*(g) - g, 1 - \mathbf{n}_0^*(g)) + \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \max\{\overline{V}_1^*(g', \delta' B), V_0^*(g', B)\} \pi_\Delta(d\delta') \pi_{\mathbb{G}}(dg'|g) \\
&+ \beta(1-\lambda) \int_{\mathbb{G}} V_0^*(g', B) \pi_{\mathbb{G}}(dg'|g).
\end{aligned} \tag{D.52}$$

We now consider program D.40. With an slight abuse of notation, let $\hat{\boldsymbol{\gamma}}$ be the strategy that achieves the supremum in program D.40. Then,

$$\begin{aligned}
V_1^*(g, \delta B, \mu) =& u(n_0(\hat{\boldsymbol{\gamma}})(g) - g, 1 - n_0(\hat{\boldsymbol{\gamma}})(g)) \\
&+ \beta \int_{\mathbb{G}} \max\{V_1(\hat{\boldsymbol{\gamma}})(h_0, 1, B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), (g', 1), 1), V_1(\hat{\boldsymbol{\gamma}})(h_0, 1, B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), (g', 1), 0)\} \pi_{\mathbb{G}}(dg'|g) \\
\geq& u(n_0(\boldsymbol{\gamma})(g) - g, 1 - n_0(\boldsymbol{\gamma})(g)) \\
&+ \beta \int_{\mathbb{G}} \max\{V_1(\boldsymbol{\gamma})(h_0, 1, B_1(\boldsymbol{\gamma})(h_0, 1), (g', 1), 1), V_1(\boldsymbol{\gamma})(h_0, 1, B_1(\boldsymbol{\gamma})(h_0, 1), (g', 1), 0)\} \pi_{\mathbb{G}}(dg'|g)
\end{aligned} \tag{D.53}$$

where $h_0 = (1, B, g, \delta)$ and the second line holds for any $\boldsymbol{\gamma}$ admissible.

Henceforth, let $\mu_t(\boldsymbol{\gamma})(h^t) \equiv u_c(n_t(\boldsymbol{\gamma})(h^t) - g_t, 1 - n_t(\boldsymbol{\gamma})(h^t))$ for any strategy $\boldsymbol{\gamma}$ and history $h^t \in \mathbb{H}^t$. Observe that, for $h_1 = (1, B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), (g', 1))$, due to lemma D.2, $\hat{\boldsymbol{\gamma}}|_{h^1,\phi}$ is admissible (taking μ as $\mu_1(\hat{\boldsymbol{\gamma}})(h^1)$), because $\hat{\boldsymbol{\gamma}}|_{h^1,\phi} \in \mathcal{S}(h_1, \phi)$, and also $\boldsymbol{\gamma}^D|_{h^1,\phi=1}$ are determined by (5.12)-(5.11). Thus

$$V_1(\hat{\boldsymbol{\gamma}})(h_0, h_1, 1) \leq V_1^*(g', B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\hat{\boldsymbol{\gamma}})(h^1))$$
$$\textit{and } V_1(\hat{\boldsymbol{\gamma}})(h_0, h_1, 0) \leq V_0^*(g', B_1(\hat{\boldsymbol{\gamma}})(h_0, 1)).$$

Therefore, letting $h^1(g') = (1, B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), (g', 1)))$,

$$\begin{aligned} V_1^*(g, \delta B, \mu) \leq & u(n_0(\hat{\boldsymbol{\gamma}})(g) - g, 1 - n_0(\hat{\boldsymbol{\gamma}})(g)) \\ & + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\hat{\boldsymbol{\gamma}})(h^1(g'))), V_0^*(g', B_1(\hat{\boldsymbol{\gamma}})(h_0, 1))\} \pi_{\mathbb{G}}(dg'|g). \end{aligned}$$

By lemma D.2, $(n_0(\hat{\boldsymbol{\gamma}})(g), B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\hat{\boldsymbol{\gamma}})(h^1(\cdot))))$ are such that $u_c(n_0(\hat{\boldsymbol{\gamma}})(g) - g, 1 - n_0(\hat{\boldsymbol{\gamma}})(g)) = \mu$ and

$$z(1, n_0(\hat{\boldsymbol{\gamma}})(g), g)\mu + \mathcal{P}_1^*(g, B_1(\hat{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\hat{\boldsymbol{\gamma}})(h^1(\cdot)) B_1(\hat{\boldsymbol{\gamma}})(h_0, 1) \geq \delta B \mu.$$

Therefore,

$$V_1^*(g, \delta B, \mu) \leq \max_{(n', B', \mu'(\cdot)) \in \Gamma(g, \delta B, \mu)} u(n - g, 1 - n) + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B', \mu'(g')), V_0^*(g', B')\} \pi_{\mathbb{G}}(dg'|g).$$

We now show that the reversed inequality holds. For this we construct the following strategy $\tilde{\boldsymbol{\gamma}}$: (1) $\tilde{\boldsymbol{\gamma}}^D$ are determined by (5.12)-(5.11); (2) for any ϕ and h_1, $\tilde{\boldsymbol{\gamma}}^F(h_0, \phi) = B_1(\tilde{\boldsymbol{\gamma}})(h_0, \phi)$ and $\mu_1(\tilde{\boldsymbol{\gamma}})(h^1)$ are such that

$$z(1, n_0(\tilde{\boldsymbol{\gamma}})(g), g)\mu + \phi\{\mathcal{P}_1^*(g, B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\tilde{\boldsymbol{\gamma}})(h_0, \circ)) B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1) - \delta B \mu\} \geq 0, \tag{D.54}$$

where $\circ$ stands for $(1, B_1(\tilde{\boldsymbol{\gamma}})(h_0, \phi), (\cdot, \phi))$, and $B_1(\tilde{\boldsymbol{\gamma}})(h_0, \phi) = B$ and $\mu_1(\tilde{\boldsymbol{\gamma}})(h^1) = m_A(g')$ if $\phi = 0$; (3) the remainder components of the strategy $\tilde{\boldsymbol{\gamma}}^F$ agree with $\hat{\boldsymbol{\gamma}}^F$, i.e., $\tilde{\boldsymbol{\gamma}}^F|_{h^1,\phi} = \hat{\boldsymbol{\gamma}}^F|_{h^1,\phi}$ for all history $h^1 \in \mathbb{H}^1$ and $\phi \in \{0, 1\}$.

We now verify that $\tilde{\boldsymbol{\gamma}}$ is admissible, which boils down to proving that $\tilde{\boldsymbol{\gamma}} \in \mathcal{S}(h_0, 1)$. Observe that by our construction $(n_0(\tilde{\boldsymbol{\gamma}})(g), B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1))$ satisfy the implementability constraint (equation D.54) at time $t = 0$ for a price given by $\mathcal{P}_1^*(g, B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\tilde{\boldsymbol{\gamma}})(h_0, \circ))$ and it satisfies that $B_1(\tilde{\boldsymbol{\gamma}})(h_0, 0) = B$. Additionally, from lemma D.2, $\hat{\boldsymbol{\gamma}}|_{h^1,\phi} \in \mathcal{S}(h_1, \phi)$, so these two results imply that $\tilde{\boldsymbol{\gamma}} \in \mathcal{S}(h_0, 1)$.

Also, since $\hat{\boldsymbol{\gamma}}|_{h^1,\phi} \in \mathcal{S}(h_1, \phi)$, it follows that $V_1(\hat{\boldsymbol{\gamma}})(h^1, 1) = V_1^*(g', \delta' B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\tilde{\boldsymbol{\gamma}})(h^1))$ and $V_1(\hat{\boldsymbol{\gamma}})(h^1, 0) = V_0^*(g', B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1))$, otherwise there would be an admissible strategy that achieves a higher value for $V_0(\cdot)(h_0, \phi)$ than $\hat{\boldsymbol{\gamma}}$.

Hence, evaluating display D.53 at $\tilde{\boldsymbol{\gamma}}$, it follows that

$$\begin{aligned} V_1^*(g, \delta B, \mu) \geq & u(n_0(\tilde{\boldsymbol{\gamma}})(g) - g, 1 - n_0(\tilde{\boldsymbol{\gamma}})(g)) \\ & + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\tilde{\boldsymbol{\gamma}})(h_0, h_1(g'))), V_0^*(g', B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1))\} \pi_{\mathbb{G}}(dg'|g) \end{aligned}$$

where $h_1(g')$ stands for $(1, B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1), (g', 1))$. Since $(n_0(\tilde{\boldsymbol{\gamma}})(h_0), B_1(\tilde{\boldsymbol{\gamma}})(h_0, 1), \mu_1(\tilde{\boldsymbol{\gamma}})(h^1))$ are arbitrary (other than the fact that they belong to $\Gamma(g, \delta B, \mu)$), it follows that

$$V_1^*(g, \delta B, \mu) \geq \max_{(n, B', \mu'(\cdot) \in \Gamma(g, \delta B, \mu)} u(n - g, 1 - n) + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B', \mu'(g')), V_0^*(g', B')\}\pi_{\mathbb{G}}(dg'|g).$$

$\square$

D.1 Proofs of Supplementary lemmas

Proof of Lemma D.1. (1) Under assumption assumption 4.1, $n \mapsto u'(\kappa n - g, 1 - n) = u_c(\kappa n - g) - u_l(1 - n) = 1 - (1 - \tau)\kappa$ and since $\kappa < 1$ and $\tau \in [0, 1]$ it implies that $u'(\kappa n - g, 1 - n) > 0$. Also, $n \mapsto u(\kappa n - g, 1 - n)$ is continuous. Moreover, $\{n : z(\kappa, n, g) = 0\} = \{n : \kappa(u_c(n - g, 1 - n) - u_l(n - g, 1 - n))n - u_c(n - g, 1 - n)g = 0\}$. Under assumption 4.1, u_c and u_l are continuous, and thus this set is closed (and bounded). Therefore compact. By the theorem of the maximum $\arg\max_{n \in [0,1]}\{u(\kappa n - g, 1 - n) : z(\kappa, n, g) = 0\}$ exists. Uniqueness follows from the fact that $n \mapsto u(\kappa n - g, 1 - n)$ is increasing.

(2) First observe that $n \mapsto z(1, n, g) = (1 - H'(1 - n))n - g$ (with $u_c = 1$) is continuous and thus $\bar{n}(g)$ exists for all $g \in \mathbb{G}$ ($\mathbb{G}$ is such that for all $g \in \mathbb{G}$, $\max_{n \in [0,1]} z(1, n, g) \geq g$). Observe that $n \mapsto z'(1, n, g) = (1 - H'(1 - n)) + H''(1 - n)n$ and $n \mapsto z''(1, n, g) = 2H''(1 - n) - H'''(1 - n)n$. By assumption 6.1, $z''(1, n, g) < 0$ and thus is strictly concave. We now show that z is decreasing. If $\bar{n}(g) = 1$ then the statement is vacuous, so consider $\bar{n}(g) < 1$. Since $\bar{n}(g)$ is the "argmax", $z'(1, \bar{n}(g), g) \leq 0$. Since z is strictly concave, z' is a decreasing, hence $z'(1, n, g) < z'(1, \bar{n}(g), g) \leq 0$ for all $n > \bar{n}(g)$, and the result follows. $\square$

Proof of Lemma D.2. If $\boldsymbol{\gamma} \in \mathcal{S}(h_0, \phi_0)$ it follows that, for any public history h^t with $h_t = (\phi_{t-1}, B_t, \omega_t = (g_t, \delta_t))$ with $B_t = B_t(\boldsymbol{\gamma})(h^{t-1}, \phi)$ and any $\phi \in \{0, 1\}$,

$$z(\kappa_\phi, n_t(\boldsymbol{\gamma})(h^t), g_t)u_c(\omega^t) + \phi\{p_t(\omega^t)u_c(\omega^t)B_{t+1}(\boldsymbol{\gamma})(h^t, \phi) - \delta_t u_c(\omega^t)B_t\} \geq 0$$

and $B_{t+1}(\boldsymbol{\gamma})(h^t, 0) = B_t$,

$$\begin{aligned} p_t(\omega^t)u_c(\omega^t) =& \beta \int_{\mathbb{G}} d_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g'))\mu_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g'))\pi_{\mathbb{G}}(dg'|g_t) \\ &+ \beta \int_{\mathbb{G}} (1 - d_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g')))m_A(g')q_{t+1}(\omega^t, \bar{\delta}, g')\pi_{\mathbb{G}}(dg'|g_t) \end{aligned} \tag{D.55}$$

where $h_{t+1}(g') \equiv (1, B_{t+1}(\boldsymbol{\gamma})(h^t, 1), g', 1)$ and

$$\mu_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g')) = u_c(n_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g')) - g', 1 - n_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g')))$$

and q_t is the "secondary market" price at time t, i.e.,

$$\begin{aligned} q_{t+1}(\omega^{t+1}, \bar{\delta}, g) \equiv& \beta\lambda \int_{\mathbb{G}} \int_{\Delta} a_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g', \delta'))\mu_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g', \delta'))\delta'\pi_{\Delta}(d\delta')\pi_{\mathbb{G}}(dg'|g) \\ &+ \beta \int_{\mathbb{G}} \left(1 - \lambda + \lambda \int_{\Delta} (1 - a_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(g', \delta')))\pi_{\Delta}(d\delta')\right) m_A(g')q_{t+2}(\omega^{t+1}, \bar{\delta}, g')\pi_{\mathbb{G}}(dg'|g) \end{aligned} \tag{D.56}$$

with $h_{t+1}(g',\delta') = (0,\delta' B_{t+1}(\boldsymbol{\gamma})(h^t,1),g',\delta')$.

From equation D.55 it follows that $p_t(\omega^t)u_c(\omega^t) = \mathcal{P}_1(g_t, B_{t+1}(\boldsymbol{\gamma})(h^t,1), \mu_{t+1}(\boldsymbol{\gamma})(h^t, h_{t+1}(\cdot)))$ and from equation D.56 $q_{t+1}(\omega^t,\bar{\delta},g') = \mathcal{P}_0(g', B_{t+1}(\boldsymbol{\gamma})(h^t,1))$. Also, from these equations and the first display it is clear that if $\boldsymbol{\gamma} \in \mathcal{S}(h_0,\phi_0)$, then $\boldsymbol{\gamma}|_{h^t,\phi} \in \mathcal{S}(\phi_{t-1}, B_t, \omega_t, \phi)$. □

E Proofs for section 6

In order to show proposition 6.1, we need the following lemmas (whose proofs are relegated to the end of this section). Throughout this section we assume that assumption 6.1 holds.

Throughout this section, let

$$\Gamma_\phi(g,B) = \{(n,B') : z(\kappa_\phi, n, g) + \phi(\mathcal{P}^*_\phi(g,B')B' - B) \geq 0 \textit{ and } B' = B \textit{ if } \phi = 0\}$$

with $\kappa_\phi \equiv \phi + \kappa(1-\phi)$.

Lemma E.1. *There exists a constant $\infty > C > 0$, such that $|V^*_\phi(g,B)| \leq C$ for all (ϕ,g,B) such that $\Gamma_\phi(g,B) \neq \{\emptyset\}$.*

Lemma E.2. *$B \mapsto V^*_1(g,B)$ is non-increasing for all $g \in \mathbb{G}$.*[69]

Lemma E.3.

$$\max_{g'\in\mathbb{G}} \max_{B_1,B_2\in\mathbb{B}^2} |V^*_0(g',B_1) - V^*_0(g',B_2)| \leq \lambda \frac{\beta C}{1-\beta}.$$

The previous lemma implies that, for any $\epsilon > 0$, there exists a $\lambda(\epsilon) > 0$ such that, for any $\lambda \in [0,\lambda(\epsilon)]$

$$\max_{g'\in\mathbb{G}} \max_{B_1,B_2\in\mathbb{B}^2} |V^*_0(g',B_1) - V^*_0(g',B_2)| \leq \epsilon. \tag{E.57}$$

Lemma E.4. *There exists $\bar{\lambda} > 0$ such that for all $\lambda \in [0,\bar{\lambda}]$, the following holds: For all (g,B) such that $B > 0$ and $\mathbf{d}^*(g,B) = 1$, $\mathcal{P}^*_1(g,B')B' \leq B$ for all $B' \in \mathbb{B}$.*

We observe that for each $B \in \mathbb{B}$, $\mathcal{P}^*_0$ is the fixed point of the following mapping

$$\begin{aligned} q \mapsto & T^*_B[q](\cdot) \\ =& \lambda\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g',\delta',B)\delta'\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg'|\cdot) + \beta \int_{\mathbb{G}} \left((1-\lambda) + \lambda \int_\Delta (1-\mathbf{a}^*(g',\delta',B))\pi_\Delta(d\delta') \right) q(g')\pi_{\mathbb{G}}(dg'|\cdot) \\ =& \lambda\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g',\delta',B)\delta'\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg'|\cdot) + \beta \int_{\mathbb{G}} \left(1-\lambda \int_\Delta \mathbf{a}^*(g',\delta',B)\pi_\Delta(d\delta') \right) q(g')\pi_{\mathbb{G}}(dg'|\cdot) \end{aligned}$$

for any $B \in \mathbb{B}$, and $q \in \{f : \mathbb{G} \to \mathbb{R} \mid f \textit{ uniformly bounded}\}$. We use this insight to derive properties of $\mathcal{P}^*_0$.

[69] This result clearly implies that $\delta \mapsto V^*_1(g,\delta B)$ is non-decreasing for all $g \in \mathbb{G}$ and $B > 0$.

Lemma E.5 (lem:T-q)**.** *Suppose assumption 6.1 holds. Then:*

1. *For each* $B \in \mathbb{B}$*,* T^*_B *is a contraction.*
2. *For any* $(g, B) \in \mathbb{G} \times \mathbb{B}$*,* $\mathcal{P}^*_0(g, B) \in \left[0, \lambda \frac{\beta}{1-\beta} E_{\pi_\Delta}[\delta]\right]$*.*
3. *If* g *is iid (distributed according to* $\pi_{\mathbb{G}}(\cdot)$*), then* $\mathcal{P}^*_0(g, B)$ *is constant in* g *and given by*

$$\mathcal{P}^*_0(g, B) = \frac{\lambda\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g', \delta', B)\delta'\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg')}{1 - \beta + \beta\lambda \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g', \delta', B)\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg')}$$

   and in this case $|\mathcal{P}^*_0(g, B)| \leq \frac{\beta\lambda}{1-\beta+\beta\lambda} < 1$*.*

Proof of Proposition 6.1. **Part (1).** By lemma E.2, $\delta \mapsto V^*_1(g, \delta B)$ is non-increasing, provided $B > 0$ (but this is the only case it matters since the government will never default on savings $B < 0$). On the other hand $V^*_0(g, B)$ is constant with respect to δ. Therefore if for some $\delta \in \Delta$, $\mathbf{a}^*(g, \delta, B) = 1$, then for all $\delta_1 \leq \delta$ the same must hold. Thus, there exists a $\bar{\delta} : \mathbb{G} \times \mathbb{B} \to [0, 1]$ such that

$$\mathbf{a}^*(g, \delta, B) = \mathbf{1}_{\{\delta:\delta\leq\hat{\delta}(g,B)\}}(\delta). \tag{E.58}$$

We now show that $B \mapsto \hat{\delta}(g, B)$ is non-increasing, for all $g \in \mathbb{G}$. It suffices to show that for any δ such that $\delta > \hat{\delta}(g, B_1)$ then $\delta > \hat{\delta}(g, B_2)$ for any $B_1 < B_2$.

Since $\delta > \hat{\delta}(g, B_1)$, it follows that $V^*_1(g, \delta B_1) < V^*_0(g, B_1)$. Let $\epsilon(g, B_1, \delta) \equiv V^*_0(g, B_1) - V^*_1(g, \delta B_1)$. It is easy to see that $\epsilon(g, B_1, \delta) > 0$ for any (g, B_1, δ) such that $\delta > \hat{\delta}(g, B_1)$. Moreover, since g, B_1 and δ belong to discrete sets, there exists a $\epsilon > 0$ such that $\epsilon \leq \epsilon(g, B_1, \delta)$ for all (g, B_1, δ) such that $\delta > \hat{\delta}(g, B_1)$.

Since $B \mapsto V^*_1(g, B)$ is non-increasing (see lemma E.2) for any $g \in \mathbb{G}$, it follows that $V^*_1(g, \delta B_2) \leq V^*_1(g, \delta B_1)$ for all $(g, \delta) \in \mathbb{G} \times \Delta$ (observe that $\delta > 0$ always). Therefore,

$$\begin{aligned} V^*_1(g, \delta B_2) - V^*_0(g, B_2) \leq & V^*_1(g, \delta B_1) - V^*_0(g, B_2) \\ \leq & V^*_1(g, \delta B_1) - V^*_0(g, B_1) + \{V^*_0(g, B_1) - V^*_0(g, B_2)\}, \end{aligned}$$

for all (g, B_1, B_2, δ) such that $\delta > \hat{\delta}(g, B_1)$.

Hence, if $|V^*_0(g, B_1) - V^*_0(g, B_2)| < \epsilon$ for any (g, B_1, B_2), the previous display implies that $V^*_1(g, \delta B_2) - V^*_0(g, B_2) < 0$ and the desired result follows. We now show that $|V^*_0(g, B_1) - V^*_0(g, B_2)| < \epsilon$ for any (g, B_1, B_2). By lemma E.3,

$$|V^*_0(g, B_1) - V^*_0(g, B_2)| < \lambda \frac{\beta C}{1 - \beta}.$$

Thus for any $\varepsilon > 0$, there exists a $\lambda(\varepsilon)$, such that

$$|V^*_0(g, B_1) - V^*_0(g, B_1)| < \varepsilon$$

for all $\lambda \in [0, \lambda(\varepsilon)]$. By setting $\varepsilon = \epsilon$ and $\bar{\lambda} = \lambda(\epsilon)$, the desired result follows.

Part (2). Following Arellano (2008) we show the result in two steps. Throughout the proof $\mathbf{n}^*_\phi$ and $\mathbf{B}^*$ are the optimal policy functions for labor and debt.

Step 1. We show that for any $B_1 < B_2$, $\mathbb{S}(B_1) \subseteq \mathbb{S}(B_2)$ where $\mathbb{S}(B) = \{g : \mathbf{d}^*(g, B) = 1\}$. If $\mathbb{S}(B_1) = \{\emptyset\}$ the proof is trivial, so we proceed with the case in which this does not hold and let $\bar{g} \in \mathbb{S}(B_1)$. If B_2 is not feasible, in the sense that there does not exist any B' such that $B_2 - \mathcal{P}^*_1(g; B')B' - \max_{n\in[0,1]} z(1, n, \bar{g}) \leq 0$, then $\mathbb{S}(B_2) = \mathbb{G}$. And the result holds trivially, so we proceed with the case that B_2 is feasible, given $\bar{g}$.

It follows (since we assume that under indifference, the government chooses not to default) $V^*_1(\bar{g}, B_1) < V^*_0(\bar{g}, B_1)$. Since $B \mapsto V^*_1(\bar{g}, B)$ is non-increasing (see lemma E.2), it follows that

$$V^*_1(\bar{g}, B_2) \leq V^*_1(\bar{g}, B_1), \; for \; all \; B_1 < B_2.$$

Therefore,

$$\begin{aligned} V^*_1(\bar{g}, B_2) - V^*_0(\bar{g}, B_2) \leq & V^*_1(\bar{g}, B_1) - V^*_0(\bar{g}, B_2) \\ \leq & V^*_1(\bar{g}, B_1) - V^*_0(\bar{g}, B_1) + \{V^*_0(\bar{g}, B_1) - V^*_0(\bar{g}, B_2)\}. \end{aligned}$$

Let $\epsilon(\bar{g}, B_1) \equiv -\{V^*_1(\bar{g}, B_1) - V^*_0(\bar{g}, B_1)\}$, observe that $\epsilon(\bar{g}, B_1) > 0$ for any $(B_1, \bar{g}) \in Graph\{\mathbb{S}\}$. Thus, if $V^*_0(\bar{g}, B_1) - V^*_0(\bar{g}, B_2) < \epsilon(\bar{g}, B_1)$, then $V^*_1(\bar{g}, B_2) < V^*_0(\bar{g}, B_2)$ and the desired result follows.

Observe that $|\mathbb{B} \times Graph(\mathbb{S})| < \infty$, so there exists $\epsilon > 0$ such that $\epsilon \leq \epsilon(\bar{g}, B_1)$ for any $\bar{g}$ and B_1 in $Graph(\mathbb{S})$. By lemma E.3 and our derivations in part (1), there exists a $\lambda(\epsilon) > 0$ such that

$$|V^*_0(g, B_1) - V^*_0(g, B_2)| < \epsilon, \; \forall \lambda \in [0, \lambda(\epsilon)] \; and \; (g, B_1, B_2) \in \mathbb{G} \times \mathbb{B}^2.$$

Hence, $V^*_1(\bar{g}, B_2) - V^*_0(\bar{g}, B_2) < 0$, thereby implying that $\bar{g} \in \mathbb{S}(B_2)$.

Step 2. We show that, for any $B \in \mathbb{B}$ and any $g_1 < g_2$ in $\mathbb{G}$, if $\mathbf{d}^*(g_1, B) = 1$, then $\mathbf{d}^*(g_2, B) = 1$. That is, we want to show that $V^*_1(g_2, B) < V^*_0(g_2, B)$. Since default occurs for g_1, it suffices to show that

$$V^*_1(g_2, B) - V^*_0(g_2, B) < V^*_1(g_1, B) - V^*_0(g_1, B) \tag{E.59}$$

or equivalently, $V^*_1(g_2, B) - V^*_1(g_1, B) < V^*_0(g_2, B) - V^*_0(g_1, B)$. Observe that

$$V^*_0(g_2, B) - V^*_0(g_1, B) = r(\mathbf{n}^*_0(g_2)) - r(\mathbf{n}^*_0(g_1)) - (g_2 - g_1). \tag{E.60}$$

where $n \mapsto r(n) = n + H(1 - n)$. And now take $\tilde{n}$ such that

$$z(1, \tilde{n}, g_1) = B - \mathcal{P}^*_1(\mathbf{B}^*(g_2, B))\mathbf{B}^*(g_2, B);$$

i.e., $\tilde{n}$ is such that $(\tilde{n}, \mathbf{B}^*(g_2, B))$ are feasible choices given the state (g_1, B), and recall $z(1, n, g) \equiv (1 - H'(1-n))n - g$ and $(g, B) \mapsto \mathbf{B}^*(g, B)$ is the optimal policy function for debt, when the government has access to financial markets. Observe that if no such choice exists, then trivially $\mathbf{d}^*(g_2, B) = 1$. Also, $\mathcal{P}_1^*$ does not depend on g because of the i.i.d. assumption. Given this construction,

$$\begin{aligned}
& V_1^*(g_2, B) - V_1^*(g_1, B) \\
\leq & r(\mathbf{n}_1^*(g_2, B)) - g_2 + \beta \int_{\mathbb{G}} \mathbb{V}^*(g', \mathbf{B}^*(g_2, B))\pi_{\mathbb{G}}(dg') - \left\{ r(\tilde{n}) - g_1 + \beta \int_{\mathbb{G}} \mathbb{V}^*(g', \mathbf{B}^*(g_2, B))\pi_{\mathbb{G}}(dg') \right\} \\
= & r(\mathbf{n}_1^*(g_2, B)) - r(\tilde{n}) - (g_2 - g_1)
\end{aligned}$$

where $(g, B) \mapsto \mathbb{V}^*(g, B) \equiv \max\{V_1^*(g, B), V_0^*(g, B)\}$. Given this display and E.60, it suffices to show that

$$r(\mathbf{n}_1^*(g_2, B)) - r(\tilde{n}) \leq r(\mathbf{n}_0^*(g_2)) - r(\mathbf{n}_0^*(g_1)). \tag{E.61}$$

We now show this inequality. By construction of $\tilde{n}$,

$$z(1, \tilde{n}, g_1) = z(1, \mathbf{n}_1^*(g_2, B), g_2) \tag{E.62}$$

where $(g, B) \mapsto \mathbf{n}_1^*(g, B)$ is the optimal policy function for labor, when the government has access to financial markets. Since $n \mapsto z(1, n, g)$ is non-increasing in the relevant domain (by relevant domain we mean the interval of n which are in "correct side of the Laffer curve"; see lemma D.1(2)) and $g_1 < g_2$, $\tilde{n} \geq \mathbf{n}_1^*(g_2, B)$. By analogous arguments, it follows that $\mathbf{n}_0^*(g_1) > \mathbf{n}_0^*(g_2)$.

Also, note that

$$z(1, \tilde{n}, g_1) - z(1, \mathbf{n}_0^*(g_1), g_1) = \mathcal{P}_1^*(\mathbf{B}^*(g_2, B))\mathbf{B}^*(g_2, B) = z(1, \mathbf{n}_1^*(g_2, B), g_2) - z(1, \mathbf{n}_0^*(g_2), g_2), \tag{E.63}$$

or equivalently, with $n \mapsto \rho(n) = (1 - H'(1-n))n$

$$\rho(\tilde{n}) - \rho(\mathbf{n}_0^*(g_1)) = \rho(\mathbf{n}_1^*(g_2, B)) - \rho(\mathbf{n}_0^*(g_2)). \tag{E.64}$$

Since $n \mapsto z(1, n, g)$ (and thus ρ) is concave and non-increasing (see lemma D.1(2)), it follows $\tilde{n} > (<)\mathbf{n}_0^*(g_1)$ iff $\mathbf{n}_1^*(g_2, B) > (<)\mathbf{n}_0^*(g_2)$.

Putting all these observations together, we have the following possible orders

$$\begin{aligned}
(I): &\ \mathbf{n}_0^*(g_1) \geq \tilde{n} \geq \mathbf{n}_0^*(g_2) \geq \mathbf{n}_1^*(g_2, B) \\
(II): &\ \mathbf{n}_0^*(g_1) \geq \mathbf{n}_0^*(g_2) \geq \tilde{n} \geq \mathbf{n}_1^*(g_2, B) \\
(III): &\ \tilde{n} \geq \mathbf{n}_0^*(g_1) \geq \mathbf{n}_1^*(g_2, B) \geq \mathbf{n}_0^*(g_2) \\
(IV): &\ \tilde{n} \geq \mathbf{n}_1^*(g_2, B) \geq \mathbf{n}_0^*(g_1) \geq \mathbf{n}_0^*(g_2).
\end{aligned}$$

Moreover, since in (g_1, B) the government defaults, it follows from the proof of lemma E.4 that $B - \mathcal{P}_1^*(B')B' \geq 0$ for any $B' \in \mathbb{B}$, in particular for $B' = \mathbf{B}^*(g_2, B)$. Therefore, $z(1, \tilde{n}, g_1) > z(1, \mathbf{n}_0^*(g_1), g_1)$, and thus $\tilde{n} \leq \mathbf{n}_0^*(g_1)$, and consequently $\mathbf{n}_1^*(g_2, B) \leq \mathbf{n}_0^*(g_2)$. Hence, cases (III) and (IV) are ruled out.

We now study cases (I) and (II). Since $n \mapsto z(1, n, g)$ is strictly concave and non-increasing (see lemma D.1), equation E.64 and (I) and (II) imply

$$\mathbf{n}_0^*(g_1) - \tilde{n} \leq \mathbf{n}_0^*(g_2) - \mathbf{n}_1^*(g_2, B). \tag{E.65}$$

Since $n \mapsto r(n) \equiv n + H(1-n)$ is concave and increasing under our assumptions, the previous inequality implies that

$$r(\mathbf{n}_0^*(g_1)) - r(\mathbf{n}_0^*(g_2)) \leq r(\tilde{n}) - r(\mathbf{n}_1^*(g_2, B)) \tag{E.66}$$

for both case (I) and (II), or equivalently

$$r(\mathbf{n}_1^*(g_2, B)) - r(\tilde{n}) \leq r(\mathbf{n}_0^*(g_2)) - (\mathbf{n}_0^*(g_1)).$$

Which is precisely equation E.61.

Hence, step 2 establishes that $\mathbf{d}^*$ is of the threshold type, since it shows that, for any B, if $\mathbf{d}^*(g, B) = 1$, the same is true for any $g' > g$. That is $\{g : \mathbf{d}^*(g, B) = 1\}$ is of the form $\{g : g \geq \bar{g}(B)\}$. Step 1 shows that the $\bar{g}$ ought to be non-increasing. $\square$

Proof of Proposition 6.2. We first establish the result for $i = 0$. From lemma E.5(3), observe that

$$\mathcal{P}_0^*(B) = \frac{\beta\lambda \int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_{\Delta}(d\delta)\pi_{\mathbb{G}}(dg')}{1 - \beta + \beta\lambda \int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_{\Delta}(d\delta)\pi_{\mathbb{G}}(dg')} \frac{\int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\delta\pi_{\Delta}(d\delta)\pi_{\mathbb{G}}(dg')}{\int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_{\Delta}(d\delta)\pi_{\mathbb{G}}(dg')}.$$

The first term in the RHS is an increasing function (namely $x \mapsto \frac{x}{1-\beta+x}$) of $\beta\lambda \int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_{\Delta}(d\delta)\pi_{\mathbb{G}}(dg')$ Since $B \mapsto \hat{\delta}(g, B)$ is non-increasing (proposition 6.1), it follows that $B \mapsto \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_{\Delta}(d\delta)$ is also non-increasing, this in turn implies that the first term in the RHS is also non-increasing as a function of B.

By our assumption $\pi_{\Delta}(\cdot) = 1_{\delta_0}(\cdot)$, the second term in the RHS is given by

$$\frac{\int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\delta\pi_{\Delta}(d\delta)\pi_{\mathbb{G}}(dg')}{\int_{\mathbb{G}} \int_{\Delta} \mathbf{1}_{\{\delta \leq \hat{\delta}(g',B)\}}(\delta)\pi_{\Delta}(d\delta)\pi_{\mathbb{G}}(dg')} = \delta_0 \frac{\int_{\mathbb{G}} \mathbf{1}_{\{\delta_0 \leq \hat{\delta}(g',B)\}}(\delta)\pi_{\mathbb{G}}(dg')}{\int_{\mathbb{G}} \mathbf{1}_{\{\delta_0 \leq \hat{\delta}(g',B)\}}(\delta)\pi_{\mathbb{G}}(dg')} = \delta_0$$

and thus constant. Hence, $B \mapsto \mathcal{P}_0^*(B)$ is non-increasing.

For $i = 1$, observe that for any $B_1 \leq B_2$,

$$\begin{aligned}
\mathcal{P}_1^*(B_1) =& \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' \leq \bar{g}(B_1)\}}(g')\pi_{\mathbb{G}}(dg') + \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' > \bar{g}(B_1)\}}(g')\pi_{\mathbb{G}}(dg')\mathcal{P}_0^*(B_1) && \text{(E.67)}\\
\geq& \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' \leq \bar{g}(B_2)\}}(g')\pi_{\mathbb{G}}(dg') + \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' > \bar{g}(B_2)\}}(g')\pi_{\mathbb{G}}(dg')\mathcal{P}_0^*(B_1) && \text{(E.68)}\\
\geq& \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' \leq \bar{g}(B_2)\}}(g')\pi_{\mathbb{G}}(dg') + \beta \int_{\mathbb{G}} \mathbf{1}_{\{g' > \bar{g}(B_2)\}}(g')\pi_{\mathbb{G}}(dg')\mathcal{P}_0^*(B_2) && \text{(E.69)}\\
=& \mathcal{P}_1^*(B_2) && \text{(E.70)}
\end{aligned}$$

where the first inequality follows from the fact that $B \mapsto \bar{g}(B)$ is non-increasing (proposition 6.1) and $\mathcal{P}_0^*(B) < 1$ for any $B \in \mathbb{B}$ (see lemma E.5(3)) ; the second inequality follows from the fact that $\mathcal{P}_0^*$ is non-increasing. $\square$

E.1 Proofs of supplementary lemmas

Proof of Lemma E.1. For any $(\phi_-, g, \delta, B) \in \{0,1\} \times \mathbb{G} \times \Delta \times \mathbb{B}$, and any function $(\phi_-, g, \delta, B) \mapsto F(\phi_-, g, \delta, B)$ we define the following operator

$$T[F](\phi_-, g, \delta, B) = \max_{(a,d)\in D(\phi_-,\delta)} T_1[F](\phi_-(1-d) + a(1-\phi_-), g, \delta, \varphi(B, \delta, a, d)) \tag{E.71}$$

with $D(0, \delta) = \{0,1\} \times \{1\}$ if $\delta \neq \bar{\delta}$ and $D(0, \bar{\delta}) = \{0\} \times \{1\}$, also $D(1, \delta) = \{1\} \times \{0,1\}$; $\varphi(B, \delta, a, 0) = B$ and $\varphi(B, \delta, 1, d) = \delta B$ and $\varphi(B, \delta, 0, d) = B$; and

$$T_1[F](\phi, g, \delta, B) = \max_{(n,B')\in\Gamma_\phi(g,B)} \left\{ \kappa_\phi n - g + H(1-n) + \beta \int_{\mathbb{G}} \int_{\bar{\Delta}} F(\phi, g', \delta', B') \pi_{\bar{\Delta}}(d\delta'|\phi) \pi_{\mathbb{G}}(dg') \right\}, \tag{E.72}$$

where $\pi_{\bar{\Delta}}(\cdot|\phi) = \mathbf{1}_{\{1\}}(\cdot)$ if $\phi = 1$ and $\pi_{\bar{\Delta}}(\cdot|\phi) = (1-\lambda)\mathbf{1}_{\{\bar{\delta}\}}(\cdot) + \lambda \pi_\Delta(\cdot)$ if $\phi = 0$.

A fix point of the T operator, is given by

$$\mathbb{V}^*(\phi_-, g, \delta, B) = \max_{(a,d)\in D(\phi_-,\delta)} V^*_{\phi_-(1-d)+a(1-\phi_-)}(g, \varphi(B, \delta, a, d)) \tag{E.73}$$

and for any $\phi \in \{0,1\}$

$$V^*_\phi(g, B) = \max_{(n,B')\in\Gamma_\phi(g,B)} \left\{ \kappa_\phi n - g + H(1-n) + \beta \int_{\mathbb{G}} \int_{\bar{\Delta}} \mathbb{V}^*(\phi, g', \delta', B') \pi_{\bar{\Delta}}(d\delta'|\phi) \pi_{\mathbb{G}}(dg') \right\}. \tag{E.74}$$

In order to verify equation E.74, observe that if $\phi = 0$, $B' = B$ by the restrictions imposed on Γ_0, $\kappa_0 = \kappa$ and

$$\begin{aligned}
\int_{\bar{\Delta}} \mathbb{V}^*(0, g', \delta', B') \pi_{\bar{\Delta}}(d\delta'|0) =& \lambda \int_{\Delta} \mathbb{V}^*(0, g', \delta', B) \pi_{\bar{\Delta}}(d\delta') + (1-\lambda)\mathbb{V}^*(0, g', \bar{\delta}, B) \\
=& \lambda \int_{\Delta} \max_{a\in\{0,1\}} V^*_a(g', B(\delta a + (1-a))) \pi_{\bar{\Delta}}(d\delta') + (1-\lambda)\mathbb{V}^*(0, g', \bar{\delta}, B) \\
=& \lambda \int_{\Delta} \max\{V^*_1(g', B\delta), V^*_0(g', B)\} \pi_{\bar{\Delta}}(d\delta') + (1-\lambda) V^*_0(g', B)
\end{aligned}$$

where the last line follows from the fact that $D(0, \bar{\delta}) = \{0\} \times \{1\}$. If $\phi = 1$, then

$$\begin{aligned}
\int_{\bar{\Delta}} \mathbb{V}^*(1, g', \delta', B') \pi_{\bar{\Delta}}(d\delta'|1) =& \mathbb{V}^*(1, g', 1, B') \\
=& \max_{d\in\{0,1\}} V^*_{(1-d)}(g', \varphi(B', 1, 1, d)) \\
=& \max\{V^*_1(g', \varphi(B', 1, 1, 0)), V^*_0(g', \varphi(B', 1, 1, 1))\} \\
=& \max\{V^*_1(g', B'), V^*_0(g', B')\}.
\end{aligned}$$

Observe that from this fixed point we can derive the functions V^* by using equation E.74.

We now show that the operator T maps bounded functions onto bounded functions. Take F such that $|F(\phi_-, g, \delta, B)| \leq C$ for all (ϕ_-, g, δ, B) and for some finite constant $C > 0$. Then

$$|T[F](\phi_-, g, \delta, B)| = |\max_{(a,d)\in D(\phi_-,\delta)} T_1[F](\phi_-(1-d) + a(1-\phi_-), g, \delta, \varphi(B, \delta, a, d))|.$$

If (g, δ, B) are such that $\Gamma_1(g, \delta B) = \{\emptyset\}$, then by convention, $\phi_-(1-d)+a(1-\phi_-) = 0$ (i.e., there is default/no repayment) and thus $\max_{(a,d)\in D(\phi_-,\delta)} T_1[F](\phi_-(1-d)+a(1-\phi_-), g, \delta, \varphi(B, \delta, a, d)) = F(0, g, \delta, \varphi(B, \delta, 0, 1)) = F(0, g, \delta, B)$ and since by our assumptions over $\mathbb{G}$, $\Gamma_0(g, B) \neq \{\emptyset\}$ for any (g, B), there exists a finite $c' > 0$ such that $|\max_{n\in\Gamma_0(g,B)} \kappa n - g + H(1-n)| \leq c'$. This implies that in this case $|T[F](\phi_-, g, \delta, B)| \leq c' + \beta C$.

Similarly, if (g, δ, B) are such that $\Gamma_1(g, \delta B) \neq \{\emptyset\}$ then $|\max_{n\in\Gamma_1(g,\delta B)} n - g + H(1-n)| \leq c'$ and it follows that $|T[F](\phi_-, g, \delta, B)| \leq c' + \beta C$. Hence, by letting $C = \frac{c'}{1-\beta}$ we showed that T maps bounded functions onto bounded functions.

The fix point $\mathbb{V}^*$ inherits this property, i,.e., $|\mathbb{V}^*(\phi_-, g, \delta, B)| \leq C$ for all (ϕ_-, g, δ, B). This result, the fact that $|\max_{n\in\Gamma_0(g,B)} \kappa n - g + H(1-n)| \leq c'$ and equation E.74 implies that there exists a finite constant $C'' > 0$, such that $|V_0^*(g, B)| \leq C''$. An analogous result holds for $V_1^*(g, B)$ provided that (g, B) are such that $\Gamma_1(g, B) \neq \{\emptyset\}$. □

Proof of Lemma E.2. It is easy to see that $\Gamma_1(g, B_1) \subseteq \Gamma_1(g, B_2)$ for any $B_1 \geq B_2$ and this immediately implies that

$$\begin{aligned}
V_1^*(g, B_1) = &\max_{(n,B')\in\Gamma_1(g,B_1)} \{n - g + H(1-n) + \beta \int_{\mathbb{G}} \max\{V_0^*(g', B'), V_1^*(g', B')\}\pi_{\mathbb{G}}(dg')\\
\leq &\max_{(n,B')\in\Gamma_1(g,B_2)} \{n - g + H(1-n) + \beta \int_{\mathbb{G}} \max\{V_0^*(g', B'), V_1^*(g', B')\}\pi_{\mathbb{G}}(dg')\\
= &V_1^*(g, B_2)
\end{aligned}$$

and the result follows for V_1^*. □

Proof of Lemma E.3. Observe that, for any $(g, B_1, B_2) \in \mathbb{G} \times \mathbb{B}^2$,

$$\begin{aligned}
|V_0^*(g, B_1) - V_0^*(g, B_2)| \leq &\lambda\beta \int_{\mathbb{G}} \int_{\Delta} \mathbf{a}^*(g, \delta, B)|V_1^*(g', \delta B_1) - V_1^*(g', \delta B_2)|\pi_{\Delta}(d\delta)\pi_{\mathbb{G}}(dg'|g)\\
&+ \beta \int_{\mathbb{G}} \{(1-\lambda) + \lambda \int_{\Delta} (1 - \mathbf{a}^*(g, \delta, B))\pi_{\Delta}(d\delta)\}|V_0^*(g', B_1) - V_0^*(g', B_2)|\pi_{\mathbb{G}}(dg'|g)\\
\leq &\lambda\beta \int_{\mathbb{G}} \int_{\Delta} \mathbf{a}^*(g, \delta, B)|V_1^*(g', \delta B_1) - V_1^*(g', \delta B_2)|\pi_{\Delta}(d\delta)\pi_{\mathbb{G}}(dg'|g)\\
&+ \beta \max_{g'\in\mathbb{G}} |V_0^*(g', B_1) - V_0^*(g', B_2)|\\
\leq &\lambda\beta C + \beta \max_{g'\in\mathbb{G}} |V_0^*(g', B_1) - V_0^*(g', B_2)|
\end{aligned}$$

where the last line follows from lemma E.1 and the fact that if (g, δ, B) are such that $\Gamma_1(g, \delta B) = \{\emptyset\}$ then $\mathbf{a}^*(g, \delta, B) = 0$. Therefore,

$$\max_{g'\in\mathbb{G}} \max_{B_1,B_2\in\mathbb{B}^2} |V_0^*(g', B_1) - V_0^*(g', B_2)| \leq \lambda \frac{\beta C}{1-\beta}.$$

□

Proof of Lemma E.4. Suppose not. That is, for any λ, there exists a (g, B) with $B > 0$ such that $\mathbf{d}^*(g, B) = 1$ but there exists a B' such that $\mathcal{P}_1^*(g, B')B' > B$.

First observe that for any (g, B, B') such that $\mathcal{P}_1^*(g, B')B' > B$,

$$z(1, \mathbf{n}(g, B, B'), g) < z(1, \mathbf{n}_0^*(g), g)$$

where $\mathbf{n}(g, B, B')$ is the level of labor that solves $z(1, n, g) + \mathcal{P}_1^*(g, B')B' = B$. Since $n \mapsto z(1, n, g)$ is non-increasing in the relevant domain (see lemma D.1(2)), it follows that $\mathbf{n}(g, B, B') > \mathbf{n}_0^*(g)$, thereby implying that the per-period payoff is greater under no default, i.e.,

$$r(\mathbf{n}(g, B, B')) - g - \{r(\mathbf{n}_0^*(g)) - g\} > 0 \tag{E.75}$$

where $n \mapsto r(n) = n + H(1 - n)$ is increasing our assumptions. Let $U \equiv \{(g, B, B') \in \mathbb{G} \times \mathbb{B}^2 : \textit{equation E.75 holds}\}$. Under our assumptions $|U| < \infty$, so there exists a $\epsilon' > 0$ such that $r(\mathbf{n}(g, B, B')) - g - \{r(\mathbf{n}_0^*(g)) - g\} \geq \epsilon'$ for all $(g, B, B') \in U$.

Consider any $\lambda \in [0, \lambda(0.5\epsilon')]$ where $\epsilon \mapsto \lambda(\epsilon)$ is such that

$$\lambda(\epsilon) | \int_{\mathbb{G}} \{ \int_{\Delta} \max\{V_1^*(g', \delta B') - V_0^*(g', B'), 0\} \pi_{\Delta}(d\delta) \} \pi_{\mathbb{G}}(dg'|g) | \leq \epsilon; \tag{E.76}$$

such λ exists by lemma E.1.

By our hypothesis, there exists a (g, B, B') with $B > 0$ such that $\mathbf{d}^*(g, B) = 1$ and $\mathcal{P}_1^*(g, B')B' > B$. And thus $(g, B, B') \in U$.

By our choice of λ,

$$\int_{\mathbb{G}} V_0^*(g', B') \pi_{\mathbb{G}}(dg'|g) + 0.5\epsilon' \geq \int_{\mathbb{G}} \{ \lambda \int_{\Delta} \max\{V_1^*(g', \delta B'), V_0^*(g', B')\} \pi_{\Delta}(d\delta) + (1 - \lambda) V_0^*(g', B') \} \pi_{\mathbb{G}}(dg'|g). \tag{E.77}$$

By definition of ϵ' and the fact that $(g, B, B') \in U$, it follows that

$$r(\mathbf{n}(g, B, B')) - g + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B'), V_0^*(g', B')\} \pi_{\mathbb{G}}(dg'|g) \tag{E.78}$$

$$> r(\mathbf{n}_0^*(g)) - g + 0.5\epsilon' + \beta \int_{\mathbb{G}} \max\{V_1^*(g', B'), V_0^*(g', B')\} \pi_{\mathbb{G}}(dg'|g) \tag{E.79}$$

$$\geq r(\mathbf{n}_0^*(g)) - g + 0.5\epsilon' + \beta \int_{\mathbb{G}} V_0^*(g', B') \pi_{\mathbb{G}}(dg'|g) \tag{E.80}$$

$$\geq r(\mathbf{n}_0^*(g)) - g + \beta \{ \int_{\mathbb{G}} V_0^*(g', B') \pi_{\mathbb{G}}(dg'|g) + 0.5\epsilon' \} \tag{E.81}$$

$$\geq V_0^*(g, B). \tag{E.82}$$

Since $V_1^*(g, B)$ is larger or equal than the LHS, we conclude that for (g, B) the government decides not to default, but this is a contradiction to the fact that $\mathbf{d}^*(g, B) = 1$. □

Proof of Lemma E.5. **Part 1.** To show part 1 we show that for each $B \in \mathbb{B}$, T^*_B satisfies the Blackwell sufficient conditions. Henceforth, consider $B \in \mathbb{B}$ given, observe that T^*_B is of the form

$$T^*_B[q](g) = A_B(g) + \beta \int_{\mathbb{G}} C_B(g')q(g')\pi_{\mathbb{G}}(dg'|g) \tag{E.83}$$

where $A_B(\cdot) \equiv \lambda\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g',\delta',B)\delta'\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg'|\cdot)$, and $C_B(g) \equiv ((1-\lambda) + \lambda \int_\Delta (1-\mathbf{a}^*(g',\delta',B))\pi_\Delta(d\delta'))$ is non-negative and less than one. Hence for any $g \in \mathbb{G}$ and for any $q \leq q'$, $T^*_B[q](g) \leq T^*_B[q'](g)$ and $T^*[q+a](g) = A_B(g) + \beta \int_{\mathbb{G}} C_B(g')q(g')\pi_{\mathbb{G}}(dg'|g) + \beta \int_{\mathbb{G}} C_B(g')q(g')\pi_{\mathbb{G}}(dg'|g)a \leq A_B(g) + \beta \int_{\mathbb{G}} C_B(g')q(g')\pi_{\mathbb{G}}(dg'|g) + \beta a = T^*_B[q](g) + \beta a$. Therefore T^*_B is a contraction by Blackwell sufficient conditions, see Stokey et al. (1989), moreover its modulus is given by β which does not depend on B.

Part 2. Consider $C \equiv \beta\lambda \frac{E_{\pi_\Delta}[\delta]}{1-\beta}$ such that $|q(g)| \leq C$, then

$$|T^*_B[q](g)| \leq |A_B(g)| + \beta C \leq \beta\lambda E_{\pi_\Delta}[\delta] + \beta C = \beta\lambda E_{\pi_\Delta}[\delta]\{1 + \frac{\beta}{1-\beta}\} = \beta\lambda E_{\pi_\Delta}[\delta]\frac{1}{1-\beta}, \tag{E.84}$$

so in fact T^*_B maps functions bounded by C into themselves; and this holds for any $B \in \mathbb{B}$. Thus the fixed point of T^*_B also satisfies the inequality.

Part 3. Since $\pi_{\mathbb{G}}(\cdot|g)$ are constant with respect to g it follows that

$$\mathcal{P}^*_0(g,B) = \lambda\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g',\delta',B)\delta'\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg') + \beta \int_{\mathbb{G}} \left(1 - \lambda \int_\Delta \mathbf{a}^*(g',\delta',B)\pi_\Delta(d\delta')\right) \mathcal{P}^*_0(g',B)\pi_{\mathbb{G}}(dg')$$

and thus $\mathcal{P}^*_0(g,B)$ is constant with respect to g, abusing notation we denote it as $\mathcal{P}^*_0(B)$. From the display above it follows that

$$\begin{aligned}
\mathcal{P}^*_0(B) =& \frac{\lambda\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g',\delta',B)\delta'\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg')}{1 - \beta \int_{\mathbb{G}} (1 - \lambda \int_\Delta \mathbf{a}^*(g',\delta',B))\pi_\Delta(d\delta'))\,\pi_{\mathbb{G}}(dg')} \\
=& \frac{\lambda\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g',\delta',B)\delta'\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg')}{1 - \beta\,(1 - \lambda \int_{\mathbb{G}} \int_\Delta \mathbf{a}^*(g',\delta',B)\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg'))} \\
=& \frac{\lambda\beta \int_{\mathbb{G}\times\Delta} \mathbf{a}^*(g',\delta',B)\delta'\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg')}{1 - \beta + \beta\lambda\,(\int_{\mathbb{G}} \int_\Delta \mathbf{a}^*(g',\delta',B)\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg'))}.
\end{aligned}$$

Since $\delta \in \Delta$ is such that $\delta \leq 1$, it is easy to see that $|\mathcal{P}^*_0(B)| \leq \frac{\lambda\beta\left(\int_{\mathbb{G}}\int_\Delta \mathbf{a}^*(g',\delta',B)\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg')\right)}{1-\beta+\beta\lambda\left(\int_{\mathbb{G}}\int_\Delta \mathbf{a}^*(g',\delta',B)\pi_\Delta(d\delta')\pi_{\mathbb{G}}(dg')\right)} \leq \frac{\beta\lambda}{1-\beta+\beta\lambda} < 1$.

□

E.2 Derivation of Equation 6.23

By our characterization of the default rule. In this setting, to default or not, boils down to choosing a T (contingent on ω^∞) such that for all $t < T(\omega^\infty)$ there is no default and for $t \geq T(\omega^\infty)$ there is financial autarky. Recall that under our assumptions $u(c,l) = c + H(l)$ and $g_t \sim iid\pi_{\mathbb{G}}$, also $\pi_{\mathbb{G}}$ has a density with respect to Lesbegue, which we denote as $f_{\pi_{\mathbb{G}}}$.

For any $\omega^t \in \Omega^t$ and $t \leq T(\omega^\infty)$,

$$V_1^*(g_t, B_t(\omega^{t-1})) = \max_{(n,B')\in\Gamma(g_t,B_t(\omega^{t-1}),1)} n - g + H(1-n) + \beta \int_{\{g':g'\leq\bar{g}(B')\}} \{V_1^*(g', B') - V_0^*(g')\}\pi_{\mathbb{G}}(dg') \tag{E.85}$$

$$+ \beta \int V_0^*(g')\pi_{\mathbb{G}}(dg') \tag{E.86}$$

and let $\nu_t(\omega^t)$ is the lagrange multiplier of the restriction, $z(1,n,g_t)+\mathcal{P}_1^*(B')B'-B_t(\omega^{t-1}) \geq 0$. By assumption, the solution of B' is in the interior. So the optimal choice $((n_t(\omega^t))_{t=0}^\infty, (B_{t+1}(\omega^t))_{t=0}^\infty)$ satisfy

$$1 - H'(1-n_t(\omega^t)) + \nu_t(\omega^t)\left(\frac{dz(1,n_t(\omega^t),g_t)}{dn}\right) = 0$$

or equivalently

$$\nu_t(\omega^t) \equiv \boldsymbol{\nu}(n_t(\omega^t)) = -\frac{1 - H'(1-n_t(\omega^t))}{1 - H'(1-n_t(\omega^t)) + H''(1-n_t(\omega^t))n_t(\omega^t)}, \tag{E.87}$$

and

$$\begin{aligned}
&\nu_t(\omega^t)\left\{\mathcal{P}_1^*(B_{t+1}(\omega^t)) + \frac{d\mathcal{P}_1^*(B_{t+1}(\omega^t))}{dB_{t+1}}B_{t+1}(\omega^t)\right\} \\
=&\beta\frac{d\int_{\{g':g'\leq\bar{g}(B_{t+1}(\omega^t))\}}\{V_1^*(g', B_{t+1}(\omega^t)) - V_0^*(g')\}\pi_{\mathbb{G}}(dg')}{dB_{t+1}} \\
=&\beta\int_{\{g':g'\leq\bar{g}(B_{t+1}(\omega^t))\}}\frac{dV_1^*(g', B_{t+1}(\omega^t))}{dB_{t+1}}\pi_{\mathbb{G}}(dg') \\
&+\beta\{V_1^*(\bar{g}(B_{t+1}(\omega^t)), B_{t+1}(\omega^t)) - V_0^*(\bar{g}(B_{t+1}(\omega^t)))\}f_{\pi_{\mathbb{G}}}(\bar{g}(B_{t+1}(\omega^t)))\frac{d\bar{g}(B_{t+1}(\omega^t))}{dB_{t+1}}.
\end{aligned}$$

Since $V_1^*(\bar{g}(B_{t+1}(\omega^t)), B_{t+1}(\omega^t)) - V_0^*(\bar{g}(B_{t+1}(\omega^t))) = 0$, the last term in the RHS is naught. Also, $\frac{dV_1^*(g_t,B_t(\omega^{t-1}))}{dB_t} = \nu_t(\omega^t)$ and thus

$$\nu_t(\omega^t)\left\{\mathcal{P}_1^*(B_{t+1}(\omega^t)) + \frac{d\mathcal{P}_1^*(B_{t+1}(\omega^t))}{dB_{t+1}}B_{t+1}(\omega^t)\right\} = \beta\int_{\{g':g'\leq\bar{g}(B_{t+1}(\omega^t))\}}\nu_{t+1}(\omega^t, g')\pi_{\mathbb{G}}(dg') \tag{E.88}$$

We now show that $\boldsymbol{\nu}$ is decreasing. For this it is easier to first establish that $\boldsymbol{\nu}^- \equiv 1/\boldsymbol{\nu}$ is increasing. Observe that

$$\boldsymbol{\nu}^-(n) = -1 - \frac{H''(1-n)n}{1 - H'(1-n)}$$

and thus

$$\frac{d\boldsymbol{\nu}^{-}(n)}{dn} = -\frac{-H'''(1-n)n + H''(1-n)}{1-H'(1-n)} - \frac{(H''(1-n))^2 n}{(1-H'(1-n))^2}.$$

Since $-H'''(1-n)n + H''(1-n) < 0$ by assumption and $1 - H'(1-n) = \tau > 0$, then the first term in the RHS is negative; the second term in the RHS is also negative. Hence $\boldsymbol{\nu}^{-}$ is increasing, which readily implies that $\boldsymbol{\nu}$ is decreasing.

Supplementary Online Material

F Description of the Data

In this section we describe how we constructed the figures presented in section 2.

The industrialized economies group consists of AUSTRALIA (1990-1999), AUSTRIA (1990-1999), BELGIUM (1990-2001), CANADA (1990-2003), DENMARK (1990-2003), FINLAND (1994-1998), FRANCE (1990-2003), GERMANY (1990-1998), GREECE (1990-2001), IRELAND (1995-2003), ITALY (1990-2003), JAPAN (1990-1993), NETHERLANDS (1990-2001), NEW ZEALAND (1990-2003), NORWAY (1990-2003), PORTUGAL (1990-2001), SPAIN (1990-2003), SWEDEN (1990-2003), SWITZERLAND (1990-2003), UNITED KINGDOM (1990-2003) and UNITED STATES (1990-2003).

The emerging economies group consists of ARGENTINA[1] (1998-2003), BOLIVIA[1] (2001-2003), BRAZIL[1] (1997-2003), CHILE[1] (1993-2003), COLOMBIA[1] (1999-2003), ECUADOR[1] (1998-2003), EL SALVADOR[1] (2000-2003), HONDURAS[1] (1990-2003), JAMAICA[1] (1990-2003), MEXICO[1] (1990-2003), PANAMA[1] (1997-2003), PERU[1] (1998-2003), VENEZUELA[1] (1997-2003), ALBANIA (1995-2003), BULGARIA (1991-2003), CYPRUS (1990-2003), CZECH REPUBLIC (1993-2003), HUNGARY (1991-2003), LATVIA (1990-2003), POLAND (1990-2003), RUSSIA (1993-2003), TURKEY (1998-2003), ALGERIA (1990-2003), CHINA (1997-2003), EGYPT (1993-2003), JORDAN (1990-2003), KOREA (1990-2003), MALAYSIA (1990-2003), MAURITIUS (1990-2003), MOROCCO (1997-2003), PAKISTAN (1990-2003), PHILIPPINES (1997-2003), SOUTH AFRICA (1990-2003), THAILAND (1999-2003) and TUNISIA (1994-2003). The LAC group is conformed by the countries with "[1]".

For section 2 we constructed the data as follows. First, for each country, we computed time average, or time standard deviations or any quantity of interest (in parenthesis is the number of observations use to construct these). Second, once we computed these averages, we group the countries in IND, EME and LAC. We do this procedure for (a) central government domestic debt (as % of output) ; (b) central government expenditure (as % of output) ; (c) central government revenue (as % of output) , and (d) Real Risk Measure. The data for (a) is taken from Panizza (2008) ; the data for (b-c) is taken from Kaminsky et al. (2004) ; finally the data for (d) is taken from www.globalfinancialdata.com.[70] [71] [72]

[70]For Greece and Portugal we use central government public debt because central government domestic debt was not available. For Sweden, Ecuador and Thailand we use general government expenditure because central government expenditure was not available. For Albania, Bulgaria, Cyprus, Czech Rep., Hungary, Latvia, Poland and Russia no measure of government expenditure was available and thus were excluded from the sample for the calculations of this variable. The same caveats apply to the central government revenue sample.

[71]We gratefully acknowledge that Kaminsky et al. (2004) and Panizza (2008) kindly shared the dataset used in their respective papers (see references).

[72]For Argentina, Brazil, Colombia, Ecuador, Egypt, Mexico, Morocco, Panama, Peru, Philippines, Poland, Russia, Turkey and Venezuela we used the real EMBI+ as a measure of real risk. For the rest of the countries I used government note yields of 1-5 years maturity, depending on availability.