Place-Based Redistribution

Cecile Gaubert, Patrick Kline, and Danny Yagan
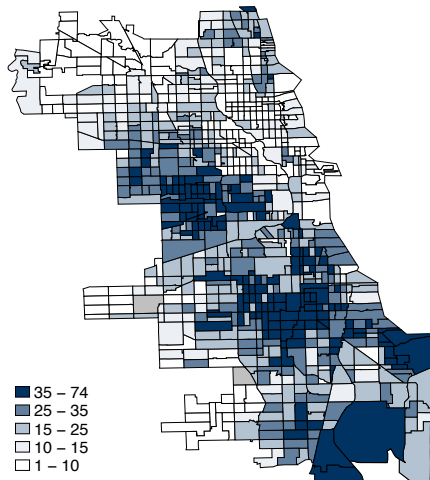
January 15, 2021

Does place-based **redistribution** improve welfare?

- Widespread use of place-based policies: 30% of EU budget, U.S., Canada, UK, France, ...

  1. **Efficiency** [Traditional urban/trade focus]: Internalize agglomeration/congestion externalities

  2. **Equity** [Integrating PF tradeoff]:
     - Places are heterogeneous in income, opportunities, environment
     - A way to transfer resources to the disadvantaged

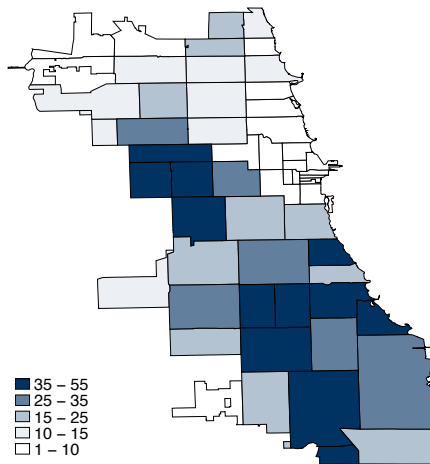# Redistributive motive: Poverty is spatially concentrated

West/South Chicago:
50% Poverty Rates

- Ex: U.S. Empowerment Zones 1993-present
- Cover 1% of pop. $3,000 per full-time worker.

35 – 74
25 – 35
15 – 25
10 – 15
1 – 10

We already redistribute based on income

West/South Chicago:
50% Filers with Negative Income Tax

Legend:
- 35 – 55
- 25 – 35
- 15 – 25
- 10 – 15
- 1 – 10

- Should South Side residents get *extra* transfer?

Appalachia:
50% Poverty Rates

23 – 65
18 – 23
15 – 18
13 – 15
10 – 13
2 – 10

- Should Appalachia residents get *extra* transfer?

Traditional view: No, because of efficiency costs

THE LINDAHL LECTURES

Cities, Agglomeration
and Spatial Equilibrium

EDWARD L. GLAESER

OXFORD

"'Help Poor People, Not Poor Places'...is something of a mantra for many urban and regional economists... [Place-based] aid is inefficient because it increases economic activity in less productive places and decreases economic activity in more productive places." – Glaeser (2008)

- **Theory:** Place-based can usefully complement income-based redistribution
  - **Lower efficiency cost** of equity gains, if limited mobility or limited earnings loss from moving
  - **Unique equity gains** from within-earnings redistribution

- **Quantification:** Optimal transfer to 1% living in poorest tracts $\sim \$3,000 - \$5,500$/household
  - Magnitude depends in particular on which forces drive sorting
  - Comparative advantage constitutes in itself a motive for place-based redistribution

Contributions

- **Urban:** Large literature studying place-based policies [Flatters et al. '74, Glaeser-Gottlieb '08, Albouy '09, Desmet-RossiHansberg '13, Kline-Moretti '14, Neumark-Simpson '15, Ossa '15, Gaubert '18 Austin-Glaeser-Summers '19, Bergman et al. '19, Fagelbaum et al. '19, Hsieh-Moretti '19, Fajgelbaum-Gaubert '20, Slattery-Zidar '20]

    – Main focus: efficiency
    – We characterize optimal *redistribution* in the workhorse urban model

Contributions

- **Urban:** Large literature studying place-based policies [Flatters et al. '74, Glaeser-Gottlieb '08, Albouy '09, Desmet-RossiHansberg '13, Kline-Moretti '14, Neumark-Simpson '15, Ossa '15, Gaubert '18 Austin-Glaeser-Summers '19, Bergman et al. '19, Fagelbaum et al. '19, Hsieh-Moretti '19, Fajgelbaum-Gaubert '20, Slattery-Zidar '20]

    – Main focus: efficiency

    – We characterize optimal *redistribution* in the workhorse urban model

- **Public:** Tagging; commodity taxation [Atkinson-Stiglitz '76, Akerlof '78, Mirrlees '76, Christiansen '84, Diamond-Sheshinski '95, Parsons '96, Cremer-Gahvari '98, Saez '02, Laroque '05, Kaplow '06/'08, Mankiw-Weinzierl '10, Kleven-Kopczuk '11, Rotschild-Scheuer'13, Gordon-Kopczuk '14, Allcott-Lockwood-Taubinsky '19]

    – Tagging: Residential choice is an area where tagging is used. Study its theoretical rationale.

    – Non-linear tax. Productivity differences. Comparative advantage.

Roadmap

1. Equity gains and efficiency costs of place-based redistribution (PBR)

2. Comparison to income-based redistribution

3. Quantification

Model setup

- Model combining key elements from Urban + Public Finance:

    - Heterogeneous skill θ, unobserved
    - Endogenous labor supply $\Rightarrow$ pre-tax income z^*, observed

    - Heterogeneous preferences for locations $\{\varepsilon_j\}$, unobserved
    - Residential choice j^*, observed

- Not in analysis

    - [Market failures (e.g. agglomeration spillovers, local public goods)]
    - [Incidence on landowners (see paper)]

Household preferences

- Unit mass of households $\Theta = (\theta, \varepsilon_0, \varepsilon_1) \sim F(\Theta)$ choose earnings z, consumption of c, h and location j to maximize utility:

$$u_j(\Theta) = U\left(c, h, a_j, \frac{z}{w_j(\theta)}\right) + \varepsilon_j$$

- Budget constraint:

$$c + r_j h = z - T_j(z)$$

- Two locations $j \in \{0, 1\} = \{\textit{Elsewhere}, \textit{Distressed}\}$

  - Amenities: $a_0 \geq a_1$
  - Housing rents r_j: $r_0 \geq r_1$
  - Productivity: $w_0(\theta) \geq w_1(\theta)$

Planner's problem

- Planner maximizes:

$$SWF = \int \omega\left(\Theta\right) v^*\left(\Theta\right) \, dF\left(\Theta\right) = \mathbb{E}\left[\omega v^*\right]$$

  - $\omega\left(\Theta\right)$: Pareto weight on Θ. v^*: Indirect utility.

- Define social marginal welfare weights $\lambda^*\left(\Theta\right)$: **welfare benefit of an extra** $\$1$ **to household** Θ:

$$\lambda^*\left(\Theta\right) \equiv \frac{\omega\left(\Theta\right) \frac{\partial v^*(\Theta)}{\partial I}}{\phi}$$

Redistributive tools

- Income tax $T(z)$, place-blind

- Lump-sum **P**lace-**B**ased **R**edistribution scheme (PBR), indexed by Δ
  - Distressed residents receive lump-sum transfer $\frac{\Delta}{S}$ (S: share of households in Distressed)
  - Elsewhere residents pay lump-sum tax $\frac{\Delta}{1-S}$

Q. What is the first-order welfare effect of a small PBR reform starting from a place-blind system?

Proposition

Implementing a small place-based transfer improves welfare if and only if

$$\frac{dSWF}{d\Delta} = \bar{\lambda}_1 - \bar{\lambda}_0 - \frac{dS}{d\Delta} \cdot \mathbb{E}\Big[T\left(z_0^*\right) - T\left(z_1^*\right)|move\Big] > 0$$

Proposition

Implementing a small place-based transfer improves welfare if and only if

$$\frac{dSWF}{d\Delta} = \bar{\lambda}_1 - \bar{\lambda}_0 - \frac{dS}{d\Delta} \cdot \mathbb{E}\left[T\left(z_0^*\right) - T\left(z_1^*\right) | move\right] > 0$$

- Equity gains depend on average social marginal welfare weights (place as a "tag"):

$$\bar{\lambda}_1 - \bar{\lambda}_0$$

Impact of PBR on social welfare

Proposition

Implementing a small place-based transfer improves welfare if and only if

$$\frac{dSWF}{d\Delta} = \bar{\lambda}_1 - \bar{\lambda}_0 - \frac{dS}{d\Delta} \cdot \mathbb{E}\Big[T(z_0^*) - T(z_1^*)\,|\,move\Big] > 0$$

- Equity gains depend on average social marginal welfare weights (place as a "tag"):

$$\bar{\lambda}_1 - \bar{\lambda}_0$$

- Efficiency cost depends on mobility responses and earnings responses:

$$\underbrace{\frac{dS}{d\Delta}}_{\text{movers}} \cdot \mathbb{E}\Big[\underbrace{T(z_0^*) - T(z_1^*)}_{\text{efficiency cost} > 0}\,|\,\text{move}\Big]$$

1. **Neighborhood Zones**
   PBR between affluent/poor residential neighborhoods with same access to business district:
   - no earnings loss upon moving $\Rightarrow$ no efficiency cost of PBR

When equity gains come at no efficiency cost: Special cases

1 Neighborhood Zones

PBR between affluent/poor residential neighborhoods with same access to business district:

– no earnings loss upon moving $\Rightarrow$ no efficiency cost of PBR

2 Moving costs [Sjaastad '62, Kennan-Walker '10/'11, Bayer-McMillan-Murphy-Timmins '16]

$U(\text{Distressed}) < U(\text{Elsewhere})$, but households stay in Distressed because of high moving costs

– no household wants to pay a moving cost to move to Distressed, even after PBR
– no movers $\Rightarrow$ no efficiency cost of PBR

When equity gains come at no efficiency cost: Special cases

❶ Neighborhood Zones

PBR between affluent/poor residential neighborhoods with same access to business district:

– no earnings loss upon moving ⇒ no efficiency cost of PBR

❷ Moving costs [Sjaastad '62, Kennan-Walker '10/'11, Bayer-McMillan-Murphy-Timmins '16]

U(Distressed) $<$ U(Elsewhere), but households stay in Distressed because of high moving costs

– no household wants to pay a moving cost to move to Distressed, even after PBR
– no movers ⇒ no efficiency cost of PBR

❸ Comp. advantage/Skilled jobs clustering [Moretti '12, DeLaRoca-Puga'17, Autor '19]

High-skilled/high-wage jobs only in Elsewhere; low-skilled jobs in both areas, same low wage.

– high-skill not incentivized to move to Distressed; only low-skill move
– no earnings loss of movers ⇒ no efficiency cost of PBR

Optimal PBR Scheme

- Increase PBR until additional equity gains are outweighed by additional efficiency costs:
  - Efficiency costs include impact of movers on PBR budget

Proposition

The optimal place-based transfer Δ^* obeys:

$$\Delta^* = \frac{\bar{\lambda}_1(\Delta^*) - \bar{\lambda}_0(\Delta^*) - \frac{dS(\Delta^*)}{d\Delta}\mathbb{E}\left[T(z_0^*) - T(z_1^*)\,|\,move\right]}{\frac{dS(\Delta^*)}{d\Delta}/\left[S(\Delta^*)\left(1 - S(\Delta^*)\right)\right]}.$$

2. When does PBR usefully complement income-based redistribution?

- **Compare PBR to an income tax reform** $q\tilde{T}(z)$ that raises same tax at each earnings level

$$\tilde{T}(z) \propto S - s(z)$$

where $s(z)$: share of z-earners who live in Distressed

- Compare effects on social welfare to determine PBR desirability:

$$\frac{dSWF}{d\Delta} \gtrless \frac{dSWF}{dq} = 0$$

Difference in Equity Benefits − Difference in Efficiency Costs $\gtreqless 0$

Difference in equity benefits

- In isolation, PBR's equity gains depend on how $\lambda(\Theta)$ covaries with location choice of households:

$$\mathbb{C}\left(\lambda, j^*\right)$$

- Income tax reform redistributes identically *across* earnings $\Rightarrow$ PBR's net gains are *within* earnings:

$$\mathbb{C}\left(\lambda, j^* | z^*\right)$$

Difference in efficiency costs

- Income tax reform's efficiency cost:

  - Increases marginal tax rates (so long as high earners sort into Elsewhere $s'(z) < 0$)
  - Reduces labor supply of stayers in both locations ($+$ triggers some moves)

- PBR's efficiency costs minus income tax reform's efficiency costs:

$$\underbrace{\left(\frac{dS}{d\Delta} - \frac{dS}{dq} \right) \mathbb{E}\left[T\left(z_0^*\right) - T\left(z_1^*\right) | \text{move} \right]}_{\text{efficiency cost of movers, on net} > 0}$$

$$-\underbrace{\mathbb{E}\left\{ -T'\left(z^*\right) \frac{s'\left(z^*\right)}{S(1-S)} \frac{Z_{1-\tau}}{1 + Z_{1-\tau} T''\left(z^*\right)} \right\}}_{\text{labor supply of stayers distorted by income tax} > 0}$$

PBR desirability on top of optimal income tax is a horserace

Proposition

Place-based redistribution improves welfare in the presence of an optimally chosen income tax iff:

$$\underbrace{\mathbb{E}\left[\mathbb{C}\left(\lambda, j^*|z^*\right)\right]}_{\text{PBR-specific equity benefit}} > \underbrace{\left(\frac{dS}{d\Delta} - \frac{dS}{dq}\right)\mathbb{E}\left[T\left(z_0^*\right) - T\left(z_1^*\right)|move\right]S(1-S)}_{\text{efficiency cost of movers} > 0}$$

$$-\underbrace{\mathbb{E}\left\{-T'\left(z^*\right)s'\left(z^*\right)\frac{Z_{1-\tau}}{1 + Z_{1-\tau}T''\left(z^*\right)}\right\}}_{\text{labor supply of stayers distorted by income tax} > 0}$$

- PBR has low efficiency costs if:
    - Migration rates are limited
    - Earnings losses of movers are limited
    - Labor supply responses are large
    - What drives sorting is key. Return to this in quantification.

PBR desirability on top of optimal income tax is a horserace

Proposition

Place-based redistribution improves welfare in the presence of an optimally chosen income tax iff:

$$\underbrace{\mathbb{E}\left[\mathbb{C}\left(\lambda, j^* | z^*\right)\right]}_{\textit{PBR-specific equity benefit}} > \underbrace{\left(\frac{dS}{d\Delta} - \frac{dS}{dq}\right)\mathbb{E}\left[T\left(z_0^*\right) - T\left(z_1^*\right) | move\right]S(1-S)}_{\textit{efficiency cost of movers} > 0}$$

$$-\underbrace{\mathbb{E}\left\{-T'\left(z^*\right)s'\left(z^*\right)\frac{Z_{1-\tau}}{1 + Z_{1-\tau}T''\left(z^*\right)}\right\}}_{\textit{labor supply of stayers distorted by income tax} > 0}$$

- PBR improves welfare all the more as place-based transfers yield within-earnings equity gains

Rationale for within-earnings redistribution $\lambda_1(z) \geq \lambda_0(z)$

- Consider case where labor supply is separable to isolate key driving forces

$$u_j(\Theta) = \psi\left(g\left(c, h\right), a_j\right) - e\left(\frac{z}{w\left(\theta\right)}\right) + \varepsilon_j$$

  – with $g(c, h)$ homothetic consumption index

1. **Cost-of-living effect**: $P_0 > P_1 \Rightarrow \lambda_1(z) \geq \lambda_0(z)$ if ψ not too concave
   – Households are poorer in real terms in Elsewhere
   – A govt dollar spent in Distressed goes further, as prices are lower
   – Dominates when ψ not too concave.

Rationale for within-earnings redistribution $\lambda_1(z) \geq \lambda_0(z)$

- Consider case where labor supply is separable to isolate key driving forces

$$u_j(\Theta) = \psi\left(g(c,h), a_j\right) - e\left(\frac{z}{w(\theta)}\right) + \varepsilon_j$$

  – with $g(c,h)$ homothetic consumption index

1. **Cost-of-living effect**: $P_0 > P_1 \Rightarrow \lambda_1(z) \geq \lambda_0(z)$ if ψ not too concave
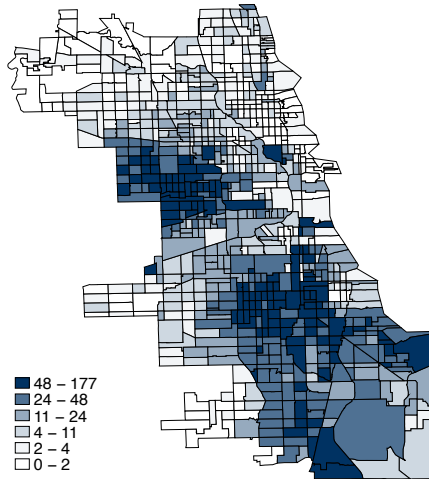   – Households are poorer in real terms in Elsewhere
   – A govt dollar spent in Distressed goes further, as prices are lower
   – Dominates when ψ not too concave.

2. **Amenity effect**: $a_1 < a_0 \Rightarrow \lambda_1(z) \geq \lambda_0(z)$ if amenities - consumption q-substitutes ($\frac{\partial^2 \psi}{\partial g \partial a} < 0$)
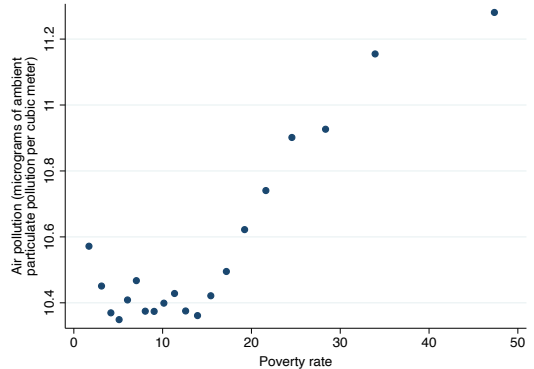   – Disamenities raise the marginal utility of consumption
   – e.g. car rides to avoid crime, healthcare needs and pollution

Disamenities that can raise the marginal utility of consumption

High-Poverty Tracts Have More Murders

High-Poverty Tracts Have Higher Pollution

Rationale for within-earnings redistribution (Why place can be special)

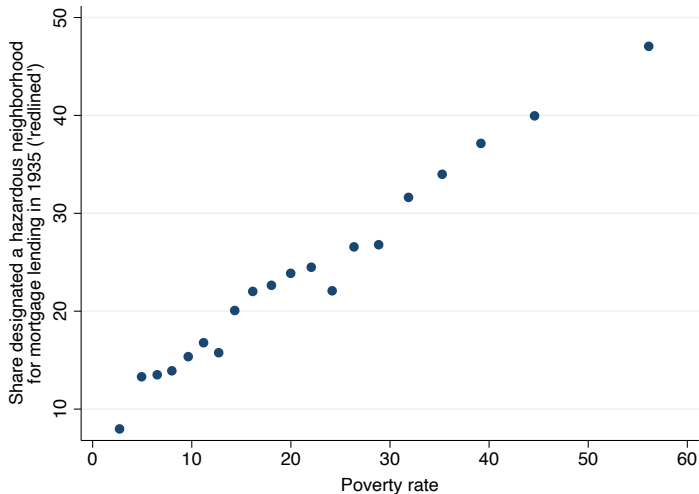- Consider separable case in consumption and/or amenities to isolate key driving forces

$$u_j(\Theta) = \psi \left(g\left(c, h \right), a_j \right) - e\left(\frac{z}{w(\theta)} \right) + \varepsilon_j$$

- with $g(c, h)$ homothetic consumption aggregate

1. **Cost-of-living effect**: $P_0 > P_1 \Rightarrow \lambda_z^1 > \lambda_z^0$ so long as ψ not too concave
   - Dollar spent goes further in buying consumption in low-price location

2. **Amenity effect**: $a_1 < a_0 \Rightarrow \lambda_z^1 > \lambda_z^0$ if amenities and consumption are q-substitutes ($\frac{\partial^2 \psi}{\partial g \partial a} < 0$)
   - Lower amenities in 1 raises marginal utility of consumption, e.g. car rides to avoid crime

3. **Equality and justice**: Residents of Distressed are more deserving [Wilson '87]
   - Suffer from past injustices, unfair treatment
   - Can be folded into high Pareto weights $\omega(\Theta)$ [Saez and Stantcheva '16]

High poverty neighborhoods and past injustices

High-Poverty Tracts Were 5x More Likely Redlined

Rationale for within-earnings redistribution (Why place can be special)

- Consider separable case in consumption and/or amenities to isolate key driving forces

$$U = \psi \left(g \left(c, h \right), a_j \right) - e \left(\frac{z}{w \left(\theta \right)} \right)$$

  – with $g(c, h)$ homothetic consumption aggregate

1. **Cost-of-living effect**: $P_0 > P_1 \Rightarrow \lambda_z^1 > \lambda_z^0$ so long as ψ not too concave
   - dollar spent goes further in buying consumption in low-price location

2. **Amenity effect**: $a_1 < a_0 \Rightarrow \lambda_z^1 > \lambda_z^0$ if amenities and consumption are q-substitutes ($\frac{\partial^2 \psi}{\partial g \partial a} < 0$)
   - lower amenities in 1 raises marginal utility of consumption, e.g. car rides to avoid crime

3. **Equality and justice**: Residents of Distressed are more deserving [Wilson '87]
   - suffer from past injustices, unfair treatment
   - can be folded into high Pareto weights $\omega(\Theta)$ [Saez and Stantcheva '16]

Do people have within-earnings distributive motives across place?

- Non-representative survey of Americans on Amazon MTurk [e.g. Kuziemko-Norton-Saez-Stantcheva '15]

Think about America's cities and towns, which are divided into neighborhoods.

A few neighborhoods are "**distressed**" and have low housing costs but also high poverty, high crime, high pollution, and struggling schools.
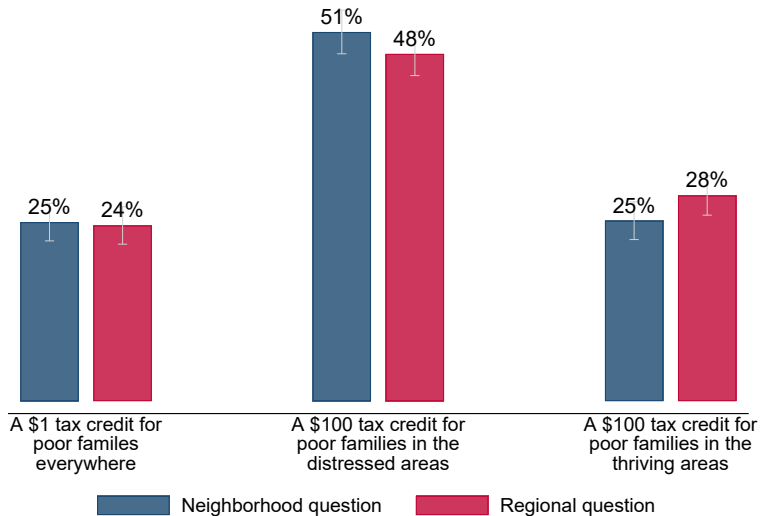
Many other neighborhoods are "**thriving**" and have high housing costs but also low poverty, low crime, low pollution, and great schools.

One percent of poor families live in the many thriving neighborhoods, mostly around rich families. One percent of poor families live in the few distressed neighborhoods, in concentrated poverty.
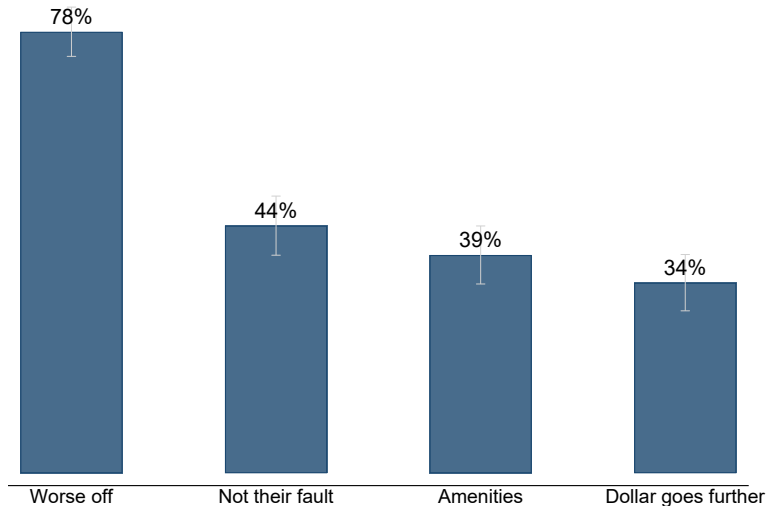
Let's pretend that the government has some extra money for new tax credits. **It wants you to choose the option that would do the most good.** (No one will work less or move as a result of your choice.)

– A \$1 tax credit for poor families everywhere

– A \$100 tax credit for poor families in the distressed neighborhoods

– A \$100 tax credit for poor families in the thriving neighborhoods

Survey results: Marginal dollars should go to distressed areas...

...because of equality, justice, amenity, and dollar-goes-further motives

- Compute optimal transfer scheme to the 1% who live in poorest group of tracts
    - Rank U.S. Census tracts by poverty rates (2013-2017 ACS)
    - Combine into 100 location groups, each with 1% of the population

- Utilitarian planner maximizes $SWF = \mathbb{E}[v^*]$ using three-bracket income tax $T(\cdot)$ and also PBR Δ
    - Baseline SWF features no within-earnings/across place redistributive motive
    - Focus on PBR as a means to reduce efficiency costs

Parametric assumptions

- Baseline utility:
$$u_j(\Theta) = \ln\left(c^{1-\alpha}h^\alpha - \frac{\eta}{1+\eta}\left(\frac{z}{w_j(\theta)}\right)^{\frac{1+\eta}{\eta}}\right) + a_j(\theta) + \frac{1}{\kappa}\varepsilon_j$$

  - Taste shock: $\varepsilon_j \sim$ EV1.
  - Productivity advantage of locations is skill-neutral: $w_j(\theta) = \theta w_j$
  - $\lambda_1(z) = \lambda_0(z)$
  - Skill-specific mean taste for amenities $a_j(\theta)$ drives sorting

- Add income-based sorting:
  - Use Stone-Geary instead of Cobb-Douglas in consumption: $c^{1-\alpha}(h - \underline{h})^\alpha$
  - Housing is a necessity, induces sorting of low-skill into low-rents communities

- Add comparative advantage:
  - Productivity advantage of locations is skill-biased: $w_j(\theta) = w_j\theta^{b_j}$
  - Induces sorting of high-skill into high-wage communities

Calibration

$$u_j(\Theta) = \ln\left(c^{1-\alpha}h^\alpha - \frac{\eta}{1+\eta}\left(\frac{z}{\theta w_j}\right)^{\frac{1+\eta}{\eta}}\right) + a_j(\theta) + \frac{1}{\kappa}\varepsilon_j; \quad \theta \sim \text{log-normal}(\mu_\theta, \sigma_\theta).$$

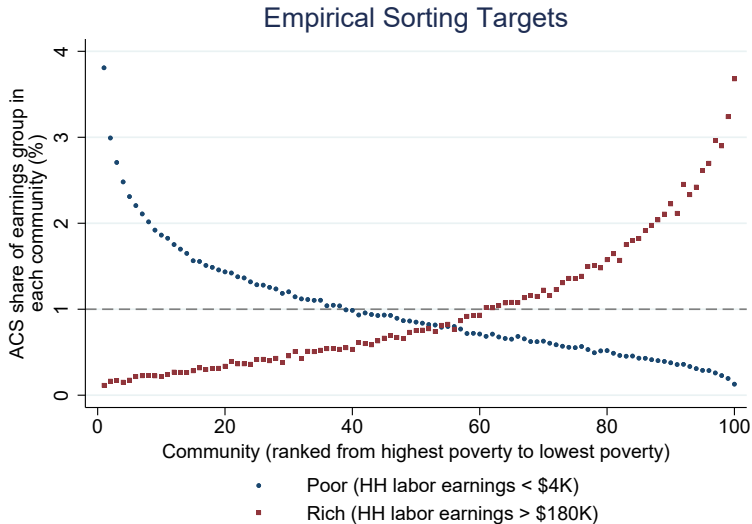- **Baseline Calibration:**
    - Rents $\{r_j\}$: ACS.
    - Wage shifters $\{w_j\}$: from productivity-rent gradient [Hornbeck-Moretti'19]
    - $\kappa = 0.5$: matches population elasticity wrt wage [Kennan-Walker '11]
    - Housing expenditure share $\alpha = .3$. Frisch labor supply elasticity $\eta = .5$ [Chetty et al. '11].
    - Current $T(z)$: \$11K lump-sum transfer w/ brackets 44%, 16%, 27% [Piketty-Saez-Zucman '18]
    - Skill-specific valuation of amenities $\{a_j(\theta)\}$ (and $\mu_\theta, \sigma_\theta$): residual to match distribution of ACS earnings (9 earnings bins) and total population across locations.
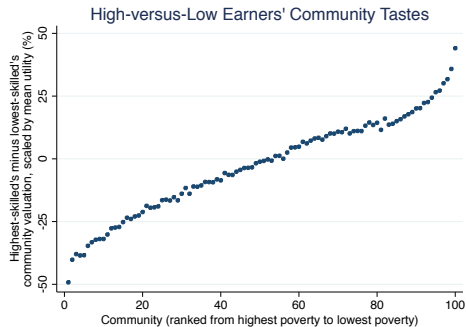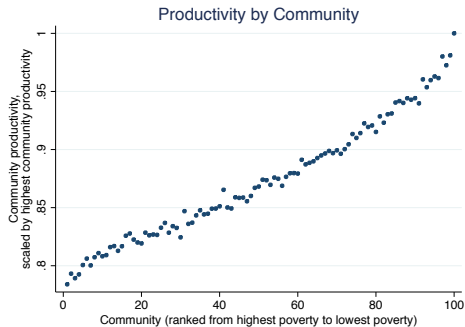
- **Extensions:**
    - Non-homothetic preferences: $(\alpha, \underline{h})$ match housing share between 0.15 and 0.52
    - Comparative advantage: $\{b_j\}$ indexed on $\{w_j\}$ to match estimate in [DeLaRoca-Puga'17]

Empirical Sorting Targets

Poor (HH labor earnings < $4K)
Rich (HH labor earnings > $180K)

Productivity by Community

High-versus-Low Earners' Community Tastes

Optimal PBR: Baseline Results
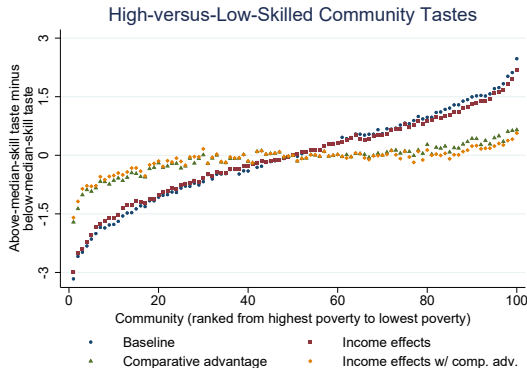
	Optimal level of PBR	Social marginal welfare weight difference narrowed	Increase in Distressed population
	(1)	(2)	(3)
Baseline	$5,500	71%	9%
Capped earnings subsidy	36%	54%	7%
Change top income tax bracket only	$3,600	49%	6%
2x productivity differences	$4,800	63%	8%
2x migration	$4,000	53%	13%

Extensions account for other sorting forces

- Add income-based sorting
- Add comparative advantage of high skill in high-wage cities
- Residual role of skill-specific valuation of amenities is muted compared to baseline

High-versus-Low-Skilled Community Tastes

Optimal PBR with additional sorting forces

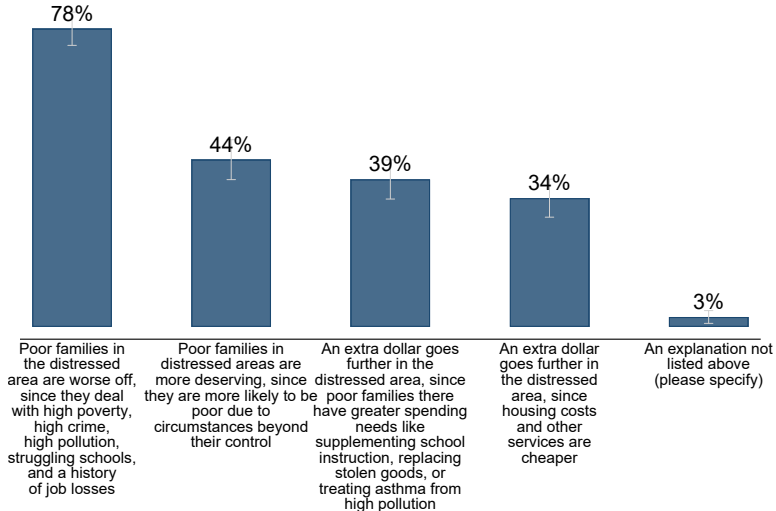| | Optimal Level of PBR | |
| | Calibration | Eliminate skill-taste correlation after calibration |
	(1)	(2)
Baseline	$5,500	$400
Income effects	$3,700	-$400
Comparative advantage	$4,200	$1,600
Income effects + Comparative advantage	$3,100	$700

- Optimal PBR in the range of $3,100-$5,500 depending on sorting forces
- Comparative advantage in isolation provides motive for PBR

Conclusion: No presumption against helping poor places

- Place-based redistribution can deliver unique efficiency and equity benefits

  - Lower efficiency costs: When mobility or productivity differences are low

  - Unique equity benefits: When marginal utilities differ across place, within-earnings

- Urban+Trade: Different rationale for place-based policies, beyond correcting market failures

- PF: Real-world case where tagging on consumption both is used and can be useful

Appendix

Why direct subsidies to the poor to distressed areas?

- 78% — Poor families in the distressed area are worse off, since they deal with high poverty, high crime, high pollution, struggling schools, and a history of job losses
- 44% — Poor families in distressed areas are more deserving, since they are more likely to be poor due to circumstances beyond their control
- 39% — An extra dollar goes further in the distressed area, since poor families there have greater spending needs like supplementing school instruction, replacing stolen goods, or treating asthma from high pollution
- 34% — An extra dollar goes further in the distressed area, since housing costs and other services are cheaper
- 3% — An explanation not listed above (please specify)

- The optimal place-based transfer Δ^* obeys:

$$\Delta^* \approx \frac{\bar{\lambda}_1(0) - \bar{\lambda}_0(0) + \mathbb{E}\left\{\frac{dS(\cdot,0)}{d\Delta}\left[T(z_1^*) - T(z_0^*)\right]\right\}}{\frac{1}{S(1-S)}\left\{\frac{dS}{d\Delta} - \mathbb{C}\left[\frac{dS(\cdot,0)}{d\Delta}, (1-S)\lambda_1(\cdot,0) + S\lambda_0(\cdot,0)\right]\right\} - (\bar{\Lambda}_1(0) + \bar{\Lambda}_0(0)) - \mathbb{E}\left\{\frac{d^2S(\cdot,0)}{d\Delta^2}\left[T(z_1^*) - T(z_0^*)\right]\right\}},$$

  - where: $\Lambda(\Theta) = \frac{\partial \lambda(\Theta)}{\partial I}$ and $\bar{\Lambda}_j = \mathbb{E}\left[\Lambda(\cdot)\,|\,j^* = j\right]$
  - both evaluated at $\Delta = 0$.

back