

A Model of Unconventional Monetary Policy

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Abstract

This paper develops a quantitative monetary DSGE model that allows for financial intermediaries that face endogenous balance sheet constraints. We use the model to simulate a crisis that has some basic features of the current economic downturn. We then use the model to quantitatively assess the effect of direct central bank intermediation of private lending, which is the essence of the unconventional monetary policy that the Federal Reserve has developed to combat the subprime crisis. We show numerically how central bank credit policy might help moderate the simulated crisis. We then compute the optimal degree of central bank credit intervention in this scenario and also compute the welfare gains.

1 Introduction

Over most of the post-war the Federal Reserve has conducted monetary policy by manipulating the Federal Funds rate in order to affect market interest rates.. It has avoided lending directly in private credit markets, other than to supply discount window loans to commercial banks. Even then, it limited discount window activity to loans secured by government Treasury Bills

Since the onset of the subprime crisis in August 2007, the situation has changed dramatically. To address the deterioration in both financial and real activity, the Fed has directly injected credit into private markets. It began in the fall of 2007 by expanding the range of eligible collateral for discount window loans to include agency debt and high grade private debt,. It did so in conjunction with extending the maturity of these types of loans and with extending eligibility to investment banks. Since that time, the Fed has set up a myriad of lending facilities. For example, it has provided backstop funding to help revive the commercial paper market. It has also intervened heavily in mortgage markets by directly purchasing agency debt and mortgage-backed securities. (That mortgage rates have fallen significantly since the onset of this program is perhaps the strongest evidence of a payoff to the Fed's overall new strategy.) Plans are in the works to extend central bank credit to securitized consumer loans.

The Fed's balance sheet provides the most concrete measure of its credit market intervention: Since August 2007 the quantity of private assets it has held has increased from virtually nothing to nearly a trillion and half. Effectively, over this period the Fed has attempted to offset the decline of a considerable fraction of private financial intermediation by expanding central bank intermediation. Now that the short term interest rate is at the zero lower bound, the Fed is unable to stimulate the economy using conventional means. For time being, it must rely exclusively on its new unconventional balance sheet operations.

At the same time, operational models of monetary policy have not kept pace with the dramatic changes in actual practice. There is of a course a lengthy contemporary literature on quantitative modeling of conventional monetary policy, beginning with Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007.) The baseline versions of these models, however, assume frictionless financial markets. They are thus unable to capture financial market disruptions that could motivate the kind of central bank interventions in loan markets that are currently in play. Similarly, models

which do incorporate financial market frictions, such as Bernanke, Gertler and Gilchrist (1999) or Christiano, Motto and Rostagno (2005), have not yet explicitly considered direct central bank intermediation as a tool of monetary policy. Work that has tried to capture this phenomenon has been mainly qualitative as opposed to quantitative (e.g., Kiyotaki and Moore (2008), Adrian and Shin (2008)). Accordingly, the objective of this paper is to try to fill in this gap in the literature: The specific goal is develop a quantitative macroeconomic model where it is possible to analyze the effects of unconventional monetary policy in the same general manner that existing frameworks are able to study conventional monetary policy.

To be clear, we do not attempt to explicitly model the sub-prime crisis. However, we do try to capture the key elements relevant to analyzing the Fed's credit market interventions. In particular, the current crisis has featured a sharp deterioration in the balance sheets of many key financial intermediaries. As many observers argue, the deterioration in the financial positions of these institutions has had the effect of disrupting the flow of funds between lenders and borrowers. Symptomatic of this disruption has been a sharp rise in various key credit spreads as well as a significant tightening of lending standards. This tightening of credit, in turn, has raised the cost of borrowing and thus enhanced the downturn. The story does not end here: The contraction of the real economy has reduced asset values throughout, further weakening intermediary balance sheets, and so on. It is in this kind of climate, that the central bank has embarked on its direct lending programs.

To capture this kind of scenario, accordingly we incorporate financial intermediaries within an otherwise standard macroeconomic framework. To motivate why the condition of intermediary balance sheets influences the overall flow of credit, we introduce a simple agency problem between intermediaries and their respective depositors. The agency problem introduces endogenous constraints on intermediary leverage ratios, which have the effect of tying overall credit flows to the equity capital in the intermediary sector. As in the current crisis, a deterioration of intermediary capital will disrupt lending and borrowing in a way that raises credit costs.

To capture unconventional monetary policy in this environment, we allow the central bank to act as intermediary by borrowing funds from savers and then lending them to investors. Unlike, private intermediaries, the central bank does not face constraints on its leverage ratio. (There is no agency problem between the central bank and its creditors because it can commit

to always honoring its debt.) Thus, in a period of financial distress that has disrupted private intermediation, the central bank can intervene to support credit flows. On the other hand, we allow for the fact that, everything else equal, public intermediation is likely to be less efficient than private intermediation. When we use the model to evaluate these credit interventions, we take into account this trade-off,

Section 2 presents the baseline model. The framework is closely related to financial accelerator model developed by Bernanke, Gertler and Gilchrist (BGG, 1999). That approach emphasized how balance sheet constraints could limit the ability of non-financial firms to obtain investment funds. Firms effectively borrowed directly from households and financial intermediaries were simply a veil. Here financial intermediaries, as we discussed, financial intermediaries are may be subject to endogenously determined balance sheet constraints. In addition, we allow for the central bank to lend directly private credit markets.

Another difference from BGG is that, we use as a baseline framework the conventional monetary business cycle framework developed by Christiano, Eichenbaum and Evans (CEE, 2005), Smets and Wouters (SW, 2007) and others. We adopt this approach because this framework has proven to have reasonable empirical properties. Here we use it to study not only conventional interest policy but also unconventional credit market interventions by the central bank.

Section 3 presents a quantitative analysis of the model. We illustrate how financial factors may amplify and propagate some conventional disturbances. We also consider a disturbance to the underlying quality of intermediary assets and then show how this kind of disturbance could create a contraction in real activity that mirrors some of the basic features of the current crisis. We then illustrate the extent to which central credit interventions could moderate the downturn.

In section 4, we undertake a normative analysis of credit policy. We first solve for the optimal central credit intervention in crisis scenario considered in section 3. We do so under different assumptions about the efficiency costs of central bank intermediation. We then compute for each case the net welfare gains from the optimal credit market intervention. We find that so long as the efficiency costs are quite modest, the gains may be quite significant. As we discuss in the concluding remarks in section 5, this finding suggests a way to think about the central bank's choice between direct credit interventions versus alternatives such as equity injections to private intermediaries.

2 The Baseline Model

The core framework is the monetary DSGE model with nominal rigidities developed by CEE and SW. To this we add financial intermediaries that transfer funds between households and non-financial firms. An agency problem constrains the ability of financial intermediaries to obtain from households. Another new feature is a disturbance to the quality of capital. Absent financial frictions, this shock introduces only a modest decline in output, as the economy works to replenish the effective capital stock. With frictions in the intermediation process, however, the shock creates a significant capital loss in the financial sector, which in turn induces tightening of credit and a significant downturn. As we show, it is in this kind of environment that the is a potential role for central bank credit interventions.

There are five types of agents in the model: Households, financial intermediaries, non-financial goods producers, capital producers, and monopolistically competitive retailers. The latter are in the model only to introduce nominal price rigidities. In addition, there is a central bank that conducts both conventional and unconventional monetary policy. Without financial intermediaries the model is isomorphic to CEE and SW. As we show, though addition of financial intermediaries adds only a modest degree of complexity. It has, however, a substantially on model dynamics and associated policy implications..

We now proceed to characterize the basic ingredients of the model.

2.1 Households

There a continuum of identical households of measure unity. Each household consumes, saves and supplies labor. Households save by lending funds to competitive financial intermediaries and possibly also by lending funds to the government.

Within each household there are two types of members: workers and bankers. Workers supply labor and return the wages they earn to the household. Each banker manages a financial intermediary and similarly transfers any earnings back to household. The household thus effectively owns the intermediaries that its bankers manage. The deposits it holds, however, are in intermediaries that it does not own. Finally, within the family there is perfect consumption insurance. As we make clear in the next section, this simple form of heterogeneity within the family allows us to introduce finan-

cial intermediation in a meaningful way within an otherwise representative agent framework.

At any moment in time the fraction $1 - f$ of the household members are workers and the fraction f are bankers. Over time an individual can switch between the two occupations. In particular, a banker this period stays banker next period with probability θ , which is independent history (i.e., of how long the person has been a banker.) The average survival time for a banker in any given period is thus $\frac{1}{1-\theta}$. As will become clear, we introduce a finite horizon for bankers to insure that over time they do not reach the point where they can fund all investment from their own capital. Thus every period $(1 - \theta)f$ bankers exit and become workers. A similar number of workers randomly become bankers, keeping the relative proportion of each type fixed. Bankers who exit give their retained earnings to their respective household. The household, though, provides its new bankers with some start up funds, as we describe in the next sub-section.

Let C_t be consumption and L_t family labor supply. Then households preferences are given by

$$\max E_t \sum_{i=0}^{\infty} \beta^i [\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi}] \quad (1)$$

with $0 < \beta < 1$, $0 < h < 1$ and $\chi, \varphi > 0$. As in CEE and SW we allow for habit formation to capture consumption dynamics. As in Woodford (2003) we consider the limit of the economy as it become cashless, and thus ignore the convenience yield to household's from real money balances.

Both intermediary deposits and government debt are one period real bonds that pay the gross real return R_t from $t - 1$ to t . In the equilibrium we consider, the instruments are both riskless and are thus perfect substitutes. Thus, we impose this condition from the outset. Thus let B_t be the total quantity of short term debt the household acquires, W_t , be the real wage, Π_t net payouts to the household from ownership of both non-financial and financial firms and, T_t lump sum taxes. Then the household budget constraint is given by

$$C_t = W_t L_t + \Pi_t + T_t + R_t B_t - B_{t+1} \quad (2)$$

Note that Π_t is net the transfer the household gives to its members that enter banking at t .

Let ϱ_t denote the marginal utility of consumption. Then the household's first order conditions for labor supply and consumption/saving are standard:

$$\varrho_t W_t = \chi L_t^\varphi \quad (3)$$

with

$$\varrho_t = C_t - hC_{t-1})^{-1} - \beta h E_t(C_{t+1} - hC_t)^{-1}$$

and

$$E_t \beta \Lambda_{t,t+1} R_{t+1} = 1 \quad (4)$$

with

$$\Lambda_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t}$$

2.2 Financial Intermediaries

Financial intermediaries lend funds obtained from households to non-financial firms. Let N_{jt} be the amount of wealth - or net worth - that a banker/intermediary j has at the end of period t ; B_{jt} the deposits the intermediary obtains from households, S_{jt} the quantity of financial claims on non-financial firms that the intermediary holds and Q_t the relative price of each claim. The intermediary balance sheet is then given by

$$Q_t S_{jt} = N_{jt} + B_{jt} \quad (5)$$

For the time being, we ignore the possibility of the central bank supplying funds to the intermediary.

As we noted earlier, household deposits with the intermediary at time t , pay the non-contingent real gross return R_{t+1} at $t + 1$. Thus B_{jt} may be thought of as the intermediary's debt and N_{jt} as its equity capital. Intermediary assets earn the stochastic return R_{kt+1} over this period. Both R_{kt+1} and R_{t+1} will be determined endogenously.

Over time, the banker's equity capital evolves as the difference between earnings on assets and interest payments on liabilities:

$$N_{jt+1} = R_{kt+1} Q_t S_{jt} - R_{t+1} B_{jt} \quad (6)$$

$$= (R_{kt+1} - R_{t+1}) Q_t S_{jt} + R_{t+1} N_{jt} \quad (7)$$

Any growth in equity above the riskless return depends on the premium $R_{kt+1} - R_{t+1}$ the banker earns on his assets, as well as his total quantity of assets, $Q_t S_{jt}$.

Let $\beta \Lambda_{t,t+i}$ be the stochastic discount the the banker at t applies to earnings at $t+i$. Since the banker will not fund assets with a discounted return less than the discounted cost of borrowing, for the intermediary to operate the following inequality must apply:

$$E_t \beta \Lambda_{t,t+1+i} (R_{kt+1+i} - R_{t+1+i}) \geq 0 \quad \forall i \geq 0$$

With perfect capital markets, the relation always holds with equality: The risk-adjusted premium is zero. With imperfect capital markets, however, the premium may be positive due to limits on the intermediary's ability to obtain funds.

So long as the intermediary can earn a risk adjusted return that is greater than or equal to the return the household can earn on its deposits, it pays for the banker to keep building assets until exiting the industry. Accordingly, the banker's objective is to maximize expected terminal wealth, given by

$$\begin{aligned} V_{jt} &= \max E_t \sum_i (1 - \theta) \theta^i \beta^i \Lambda_{t,t+i} (N_{jt+1+i}) \\ &= \max E_t \sum_i (1 - \theta) \theta^i \beta^i \Lambda_{t,t+i} [(R_{kt+1+i} - R_{t+1+i}) Q_{t+i} S_{jt+i} + R_{t+1+i} N_{jt+i}] \end{aligned} \quad (8)$$

To the extent the discounted risk adjusted premium in any period, $\beta^i \Lambda_{t,t+i} [(R_{kt+1+i} - R_{t+1+i})]$, is positive, the intermediary will want to expand its assets indefinitely by borrowing additional funds from households. To motivate a limit on its ability to do so, we introduce the following moral hazard/costly enforcement problem: At the beginning of the period the banker can choose to divert the fraction λ of available funds from the project and instead transfer them back to the household of which he or she is a member. The cost to the banker is that the depositors can force the intermediary into bankruptcy and recover the remaining fraction $1 - \lambda$ of assets.. However, it is too costly for the depositors recover the fraction of funds that the banker diverted.

Accordingly for lenders to be willing to supply funds to the banker, the following incentive constraint must be satisfied:

$$V_{jt} \geq \lambda Q_t S_{jt} \quad (9)$$

The left side is what the banker would lose by diverting a fraction of assets. The right side is the gain from doing so.

We can express V_{jt} as follows:

$$V_{jt} = v_t \cdot Q_t S_{jt} + \eta_t N_{jt} \quad (10)$$

with

$$\begin{aligned} v_t &= E_t\{(1 - \theta)\beta\Lambda_{t,t+1}(R_{kt+1} - R_{t+1}) + \beta\Lambda_{t,t+1}\theta x_{t,t+1}v_{t+1}\} \\ \eta_t &= E_t\{(1 - \theta) + \beta\Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1}\} \end{aligned} \quad (11)$$

where $x_{t,t+i} \equiv Q_{t+i}S_{jt+i}/Q_tS_{jt}$, is the gross growth rate in assets between t and $t+i$, and $z_{t,t+i} \equiv N_{jt+i}/N_{jt}$ is the gross growth rate of net worth. The variable v_t has the interpretation of the expected discounted marginal gain to the banker of expanding assets Q_tS_{jt} by a unit, holding net worth N_{jt} constant, and while η_t is the expected discounted value of having another unity of N_{jt} , holding S_{jt} constant. With frictionless competitive capital markets, intermediaries will expand borrowing to the point where rates of return will adjust to ensure v_t is zero. The agency problem we have introduced, however, may place limits on the arbitrage. In particular, as we next show, when the incentive constraints is binding, the intermediary's assets are constrained by its equity capital.

Note first that we can express the incentive constraints as

$$\eta_t N_{jt} + v_t Q_t S_{jt} \geq \lambda Q_t S_{jt} \quad (12)$$

If this constraint binds, then the assets the banker can acquire will depend positively on his/her equity capital:

$$\begin{aligned} Q_t S_{jt} &= \frac{\eta_t}{\lambda - v_t} N_{jt} \\ &= \phi_t N_{jt} \end{aligned} \quad (13)$$

where ϕ_t ratio of privately intermediated assets to equity, which we will refer to as the (private) leverage ratio.. Holding constant N_{jt} , expanding S_{jt} raises the bankers' incentive to divert funds. The constraint (13) limits the intermediaries leverage ratio to the point where the banker's incentive to cheat is exactly balanced by the cost. In this respect the agency problem

leads to an endogenous capital constraint on intermediary's ability to acquire assets.

Given $N_{jt} > 0$, the constraint binds only if $0 < v_t < \lambda$. In this instance, it is profitable for the banker to expand assets (since $v_t > 0$). Note that in this circumstance the leverage ratio that depositors will tolerate is increasing in v_t . The larger is v_t , the greater is the cost to the banker from being forced into bankruptcy. If v_t increases above λ , the incentive constraint does not bind: the franchise value of the intermediary always exceed the gain from diverting funds. In the equilibrium we construct below, under reasonable parameter values the constraint always binds.

We can now express the evolution of the banker's net worth as

$$N_{jt+1} = [(R_{kt+1} - R_{t+1})\phi_t + R_{t+1}]N_{jt} \quad (14)$$

Note that the sensitivity of N_{jt+1} to the ex post realization of the excess return $R_{kt+1} - R_{t+1}$ is increasing in the leverage ratio ϕ_t . In addition, it follows that

$$z_{t,t+1} = N_{jt+1}/N_{jt} = (R_{kt+1} - R_{t+1})\phi_t + R_{t+1}$$

$$x_{t,t+1} = Q_{t+1}S_{jt+2}/Q_tS_{t+1} = (\phi_{t+1}/\phi_t)(N_{jt+1}/N_t) = (\phi_{t+1}/\phi_t)z_{t,t+1}$$

Importantly, all the components of ϕ_t do not depend on firm-specific factors. Thus to determine total intermediary demand for assets we can sum across individual demands to obtain:

$$Q_tS_{It} = \phi_t N_t \quad (15)$$

where S_{It} reflects the aggregate quantity of intermediary assets and N_t denotes aggregate intermediary capital. In the general equilibrium of our model, variation in N_t , will induce fluctuations in overall asset demand by intermediaries. Indeed, a crisis will feature a sharp contraction in N_t .

We can derive an equation of motion for N_t , by first recognizing that it is the sum of the net worth of existing banker/intermediaries, N_{et} , and the net worth of entering (or "new") bankers, N_{nt} .

$$N_t = N_{et} + N_{nt} \quad (16)$$

Since the fraction θ of bankers at $t - 1$ survive until t , N_{et} is given by

$$N_{et} = \theta[(R_{kt} - R_t)\phi_t + R_t]N_{t-1} \quad (17)$$

Observe that the main source of variation in N_{et} will be fluctuations in the ex post return on assets R_{kt} . Further, the impact on N_{et} is increasing in the leverage ratio ϕ_t .

As we noted earlier, newly entering bankers receive "start up" funds from their respective households. We suppose that the startup money the household gives its new banker a transfer equal to a small fraction of the value of assets that exiting bankers had intermediated in their final operating period. The rough idea is that how much the household feels that is new bankers need to start, depends on the scale of the assets that the exiting bankers have been intermediating. Given that the exit probability is i.i.d., the total final period assets of exiting bankers at t is $(1 - \theta)Q_tS_{t-1}$. Accordingly we assume that each period the household transfers the fraction $\xi/(1 - \theta)$ of this value to its entering bankers. Accordingly, in the aggregate,

$$N_{nt} = \xi Q_t S_{t-1} \quad (18)$$

Combining (17) and (18) yields the following equation of motion for N_t .

$$N_t = \theta[(R_{kt} - R_t)\phi_t + R_t]N_{t-1} + \xi Q_t S_{t-1}$$

Observe that ξ helps pin down the steady state leverage ratio QS/N . Indeed, in the next section we calibrate ξ to match this evidence. The resulting value, as we show, is quite small.

2.3 Credit Policy

In the previous section we characterized how the total value of privately intermediated assets, Q_tS_{pt} , is determined. We now suppose that the central bank is willing to facilitate lending. Let Q_tS_{gt} be the value of assets intermediated via government assistance and let Q_tS_t be the total value of intermediated assets: i.e.,

$$Q_tS_t = Q_tS_{pt} + Q_tS_{gt} \quad (19)$$

To conduct credit policy, the central bank issues government debt to households that pays the riskless rate R_{t+1} and then lends the funds to non-financial firms at the market lending rate R_{kt+1} . We suppose that government

intermediation involves efficiency costs: In particular, the central bank credit involves an efficiency cost of τ per unit supplied. This deadweight loss could reflect the costs of raising funds via government debt. It might also reflect costs to the central bank of identifying preferred private sector investments. On the other hand, the government always honors its debt: Thus, unlike the case with private financial institutions, is no agency conflict than inhibits the government from obtaining funds from households. Put differently, unlike private financial intermediation, government intermediation is not balance sheet constrained.

An equivalent formulation of credit policy involves having the central bank channel funds to non-financial borrowers via private financial intermediaries, as has occurred in practice. Here we assume that the government has an advantage over private households in enforcing payment of debts by private intermediaries. In particular, it is not possible for an intermediary to walk away from a financial obligation to the federal government, the same way it can a private entity. Unlike private creditors, the federal government has various means to track down and recover debts. It follows that In this instance, the balance sheets constraints that limit intermediaries ability to obtain private credit do not constrain their ability to obtain central bank credit. Accordingly, in this scenario, after obtaining funds from households at the rate R_{t+1} , the central bank lends freely to private financial intermediaries at the rate R_{kt} , which in turn lend to non-financial firms at the same rate. Private intermediaries earn zero profits on this activity: the liabilities to the central bank perfectly offset the value of the claims on non-financial firms, implying that there is no effect on intermediary balance sheets. The behavior of the model is thus exactly same as if the central bank directly lends to non-financial firms. Note that in this instance, the efficiency cost τ is interpretable as the cost of publicly channeling funds to private intermediaries as opposed to directly to non-financial firms.

Accordingly, suppose the central bank is willing to fund the fraction ψ_t of intermediated assets: i.e.,

$$Q_t S_{gt} = \psi_t Q_t S_t \quad (20)$$

It issues amount of government bonds B_{gt} , equal to $\psi_t Q_t S_t$ to funds this activity. It's net earnings from intermediation in any period t thus equal $(R_{kt+1} - R_{t+1})B_{gt}$. These net earnings provide a source of government revenue and must be accounted for in the budget constraint, as we discuss later.

Since privately intermediated funds are constrained by intermediary net worth, we can rewrite equation (19) to obtain

$$\begin{aligned} Q_t S_t &= \phi_t N_t + \psi_t Q_t S_t \\ &= \phi_{ct} N_t \end{aligned}$$

where ϕ_t is the leverage ratio for privately intermediated funds (see equations (13) and (15), and where ϕ_{ct} is the leverage ratio for total intermediated funds, public as well as well private.

$$\phi_{ct} = \frac{1}{1 - \psi_t} \phi_t$$

Observe that ϕ_{ct} depends positively on the intensity of credit policy, as measured by ψ_t . Later how describe how the central might choose ψ_t to combat a financial crisis.

2.4 Intermediate Goods Firms

We next turn to the production and investment side of the model economy. Competitive non-financial firms produce intermediate goods that are eventually sold to retail firms. The timing is as follows: At the end of period t , an intermediate goods producer acquires capital K_{t+1} for use in production in the subsequent period. After production in period $t + 1$, the firm has the option of selling to capital on the open market. There are no adjustment costs at the firm level. Thus, the firm's capital choice problem is always static, as we discuss below.

The firm finances its capital acquisition each period by obtaining funds from intermediaries. To acquire the funds to buy capital, the firm issues S_t claims equal to the number of units of capital acquired K_{t+1} and prices each claim at the price of a unit of capital Q_t . That is, $Q_t K_{t+1}$ is the value of capital acquired and $Q_t S_t$ is the value of claims against this capital. Then by arbitrage:

$$Q_t K_{t+1} = Q_t S_t \tag{21}$$

We assume that there are no frictions in the process of non-financial firms obtaining funding from intermediaries. The intermediary has perfect information about the firm and has no problem enforcing payoffs. This contrasts with the process of the intermediary obtaining funding from households. Thus, within our model, only intermediaries face capital constraints

on obtaining funds. These constraints, however, affect the supply of funds available to non-financial firms and hence the required rate of return on capital these firms must pay. Conditional on this required return, however, the financing process is frictionless for non-financial firms.

At time $t + 1$ the firm produces output Y_{t+1} , using capital and labor L_{t+1} , and by varying the utilization rate of capital, U_{t+1} . Let A_{t+1} denote total factor productivity. Then production is given by:

$$Y_{t+1} = A_{t+1}(U_{t+1}K_{t+1})^\alpha L_{t+1}^{1-\alpha} \quad (22)$$

What the firm earns in $t + 1$ is the value of output plus the value of its capital stock left over net financing and labor costs. Let P_{mt} be the price of intermediate goods output. Then from the vantage of period t , where the firm makes its' capital decision for $t + 1$, its objective is given by:

$$\max E_t \beta \Lambda_{t,t+1} [P_{mt+1} Y_{t+1} + (Q_{t+1} - \delta(U_t)) \xi_{t+1} K_{t+1} - R_{t+1} Q_t K_{t+1} - W_{t+1} L_{t+1}]$$

where as before $\beta \Lambda_{t,t+1}$ is the firm's stochastic discount factor, $\delta(U_t)$ is the capital depreciation rate, which is increasing and convex in U_t , W_{t+1} is the real wage, and R_{t+1} is the state-contingent required return on capital. In addition ψ_{t+1} is an exogenous factor that affects the effective quantity of capital. That is, after production in $t + 1$, the number of units of capital left over is $(1 - \delta(U_t)) \psi_{t+1} K_{t+1}$. Assuming that capital that replacement price of capital that has depreciated is unity, then the value of the capital stock that is left over is given by $(Q_{t+1} - \delta(U_t)) \psi_{t+1} K_{t+1}$. Finally, ψ_{t+1} may be thought of as a measure of the quality of the existing.

Maximizing with respect to K_{t+1} yields

$$E_t \beta \Lambda_{t,t+1} R_{t+1} = E_t \left\{ \beta \Lambda_{t,t+1} \frac{P_{mt+1} \alpha \frac{Y_{t+1}}{K_{t+1}} + (Q_{t+1} - \delta(U_{t+1})) \xi_{t+1}}{Q_t} \right\} \quad (23)$$

At an interior optimum, the discounted cost of capital must equal the discounted return.

At $t + 1$, the firm chooses the utilization rate and labor demand as follows:

$$P_{mt+1} \alpha \frac{Y_{t+1}}{U_{t+1}} = \delta'(U_{t+1}) \quad (24)$$

$$P_{mt+1} \alpha \frac{Y_{t+1}}{L_{t+1}} = W_{t+1} \quad (25)$$

2.5 Capital Producing Firms

At the end of period t , competitive capital producing firms buy capital from intermediate goods producing firms and then repair depreciated capital and builds new capital. They then sell both the new and re-furbished capital. As we noted earlier, the cost of replacing worn out capital is unity. The value of a unit of new capital is Q_t , as is the value of a unit of re-furbished capital. If I_t is total investment by a capital producing firm, then the firms profits at t are given by.

$$Q_t K_{t+1} - (Q_t - \delta(U_t))\xi_t K_t - I_t$$

Let $I_{nt} \equiv I_t - \delta(U_t)\xi_t K_t$, be the firm's objective. Then we can express is maximization problem, as to choose I_{nt} , K_{t+1} and K_t to solve

$$\max Q_t(K_{t+1} - \xi_t K_t) - I_{nt} \quad (26)$$

subject to

$$K_{t+1} - \xi_t K_t = I_{nt} - S\left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}}\right)(I_{nt} + I_{ss}) \quad (27)$$

where I_{ss} is steady state investment, which consists only of replacement investment. As in CEE, we allow for flow adjustment costs of investment, but restrict these costs to modify the net investment flow.

The first order condition for investment gives the follow "Q" relation for net investment:

$$Q_t = [1 - S\left(\frac{I_{nt}^a}{I_{nt-1}^a}\right) - S'\left(\frac{I_{nt}^a}{I_{nt-1}^a}\right)\left(\frac{I_{nt}^a}{I_{nt-1}^a}\right)]^{-1} [1 - \beta E_t \Lambda_{t,t+1} Q_{t+1} S'\left(\frac{I_{nt+1}^a}{I_{nt}^a}\right)\left(\frac{I_{nt+1}^a}{I_{nt}^a}\right)^2] \quad (28)$$

where $I_{nt}^a \equiv I_{nt} + I_{ss}$ may be thought of as "adjusted" net investment.

2.6 Retail Firms

Final output Y_t is a CES composite of a continuum of mass unity of differentiated retail firms, that use intermediate output as the sole input. The final output composite is given by

$$Y_t = [\int_0^1 Y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df]^{\frac{\varepsilon}{\varepsilon-1}} \quad (29)$$

where Y_{ft} is output by retailer f . From cost minimization by users of final output:

$$Y_{ft} = (\frac{P_{ft}}{P_t})^{-\varepsilon} Y_t \quad (30)$$

$$P_t = [\int_0^1 P_{ft}^{\frac{1}{1-\varepsilon}} df]^{1-\varepsilon} \quad (31)$$

Retailers simply re-package intermediate output. It takes one unit of intermediate output to make a unit of retail output. The marginal cost is thus the relative intermediate output price P_{mt} . We introduce nominal rigidities following CEE. In particular, each firm period a firm is able to freely adjusts price with probability $1 - \varsigma$. In between these periods, the firm is able to index its price to the lagged rate of inflation. The retailers pricing problem then is to choose the optimal reset price P_t^* to solve

$$\max \sum_{i=0}^{\infty} \theta^i \beta^i \Lambda_{t,t+i} [\frac{P_t^*}{P_{t+i}} \prod_{k=0}^i (1 + \pi_{t+k-1}) - P_{mt+i}] Y_{ft} \quad (32)$$

where π_t is the rate of inflation from $t - i$ to t . The first order necessary conditions are given by:

$$\sum_{i=0}^{\infty} \theta^i \beta^i \Lambda_{t,t+i} [\frac{P_t^*}{P_{t+i}} \prod_{k=0}^i (1 + \pi_{t+k-1}) - \mu P_{mt+i}] Y_{ft} = 0 \quad (33)$$

with

$$\mu = \frac{1}{1 - 1/\varepsilon}$$

From the law of large numbers, the following relation for the evolution of the price level emerges.

$$P_t = [(1 - \theta)(P_t^*)^{\frac{1}{1-\varepsilon}} + \theta(\Pi_{t-1} P_{t-1})^{\frac{1}{1-\varepsilon}}]^{1-\varepsilon} \quad (34)$$

2.7 Resource Constraint and Government Policy:

Output is divided between consumption, investment, government consumption, G_t and expenditures on government intermediation, $\tau\psi_t Q_t K_{t+1}$. We suppose further that government expenditures are exogenously fixed at the level G . The economy-wide resource constraint is thus given by

$$Y_t = C_t + I_t + G + \tau\psi_t Q_t K_{t+1} \quad (35)$$

Government expenditures, further, are financed by lump sum taxes and government intermediation:

$$G = T_t + (R_{kt} - R_t)B_{gt-1}$$

where government bonds, B_{gt-1} , finance total government intermediated assets, $Q_t\psi_{t-1}S_{t-1}$.

We suppose monetary policy is characterized by a simple Taylor rule with interest-rate smoothing. Let i_t be the net nominal interest rate, i the steady state nominal rate, and Y_t^* the natural (flexible price equilibrium) level of output. Then:

$$i_t = (1 - \rho)[\bar{i} + \iota_\pi \pi_t + \iota_y (\log Y_t^* - \log Y_t) + \rho i_{t-1} + \epsilon_t] \quad (36)$$

where the smoothing parameter ρ lies between zero and unity, and where ϵ_t is an exogenous shock to monetary policy, and where the link between nominal and real interest rates is given by the following Fisher relation

$$1 + i_t = R_{t+1} \frac{P_{t+1}}{P_t} \quad (37)$$

Finally, we also introduce a feedback role for credit policy. We suppose that the central bank injects credit in response to movements in credit spreads, according to the following feedback rule:

$$\psi_t = \psi + \nu[(R_{kt+1} - R_{t+1}) - (R_k - R)] \quad (38)$$

where ψ is the steady state fraction of publicly intermediated assets and $R_k - R$ is the steady state premium. In addition, the feedback parameter exceeds unity. According to this rule, the central bank expands credit as the spread increase relative to its steady state value.

This completes the description of the model.

3 Model Analysis

3.1 Calibration

Table 1 list the choice of parameter values for our baseline model. Overall there are eighteen parameters. Fifteen are conventional. Three (λ, ξ, θ) are specific to our model.

We begin with the conventional parameters. For the discount factor β , the depreciation rate δ , the capital share α , the elasticity of substitution between goods, ε , and the government expenditure share, we choose conventional values. Also, we normalize the steady state utilization rate u at unity. We use estimates from Justinano, Primiceri and Tambalotti (2009) to obtain values for the following preference and technology parameters: the habit parameter h , the elasticity of marginal depreciation with respect to the utilization rate, ζ , and the inverse elasticity of net investment to the price of capital η_i . We pick the relative utility weight on labor χ to fix hours worked at one third of available time. Finally, we choose a Frisch elasticity of labor supply equal to 3. Because, unlike most of the existing quantitative models, we do not allow for wage rigidity, we choose a relatively high labor supply elasticity, though one that is not inconsistent with the evidence if we interpret this elasticity as applying to the extensive margin.

We the price rigidity parameter, γ , to have prices fixed on average for a year. We choose a high value to compensate for not including real price rigidities in the model. Our parametrization leads to a slope coefficient in the Phillips curve that is consistent with the evidence. In addition, we set indexing parameter γ_p at .5, following CEE. Finally, the feedback coefficients in the monetary policy rule, κ_π and κ_y obey a conventional Taylor rule, with a smoothing parameter ρ . This rule is consistent with the evidence for post-1984.

Our choice of the financial sector parameters - the fraction of capital that can be diverted λ , the proportional transfer to entering bankers ξ , and the survival probability θ - is meant to be suggestive. We pick these parameters to hit the following three targets: a steady state interest rate spread of one hundred basis points; a steady state leverage ratio of four; and an average horizon of bankers of a decade. We base the steady state target for the spread on the pre-2007 spreads between mortgage rates and government bonds and between BAA corporate vs. government bonds. The choice of the leverage ratio is a rough guess of a reasonable economy-wide mortgage. For

the mortgage sector, which was about one third of total assets in 2007, this ratio was between twenty and thirty to one. It was obviously much smaller in other sectors.

3.2 Experiments

We begin with several experiments designed to illustrate how the model behaves. We then consider a "crisis" experiment that mimics some of the basic features of the current downturn. We then consider the role of central bank credit policy in moderating the crisis.

Figure 1 shows the response of the model economy to three disturbances: a technology shock, a monetary shock, and shock to intermediary net worth. In each case, the direction of the shock is set to produce a downturn. The figure then shows the responses of three key variables: output, investment and the premium. In each case the solid shows the response of the baseline model. The dotted line gives the response of the same model, but with the financial frictions removed.

The technology shock is a negative one percent innovation in TFP, with a quarterly autoregressive factor of 0.95. The intermediary balance mechanism produces a modest amplification of the decline in output the baseline model relative to the conventional DSGE model. The amplification is mainly the product of substantially enhanced decline investment: on the order of fifty percent relative to the frictionless model. The enhanced response of investment in the balance model is a product of the rise in the premium, plotted in the last panel on the right. The unanticipated decline in investment reduces asset prices, which produces a deterioration in intermediary balance sheets, pushing up the premium. The increase in the cost of capital, further reduces capital demand by non-financial firms, which enhances the downturn in investment and asset prices. In the conventional model without financial frictions, of course, the premium is fixed at zero.

The monetary shock is an unanticipated twenty-five basis point increase in the short term interest rate. The effect on the short term interest rate persists due to interest rate smoothing by the central bank. Financial frictions lead to greater amplification relative to the case of the technology shock. This enhanced amplification is due to the fact that, everything else equal, the monetary policy shock has a relatively large effect on investment and asset prices. The latter triggers the financial accelerator mechanism.

To illustrate how at the core of the amplification mechanism in the first

two experiments is procyclical variation in intermediary balance sheets, we consider a redistribution of wealth from intermediaries to households. In particular, we suppose that intermediary net worth declines by one percent and is transferred to households. In the model with no financial frictions, this redistribution has no effect (it is just a transfer of wealth within the family.) The decline in intermediary in our baseline model, however, produces a rise in the premium, leading to a subsequent decline in output and investment.

We now turn to the crisis experiment. The initiating disturbance is a decline in capital quality. What we are trying to capture, is a shock to the quality of intermediary assets that produce a enhanced decline in the capital of these institutions, due to their high degree of leverage. In this rough way, we capture the broad dynamics of the sub-prime crises. We fix the size of the shock so that the downturn is of broadly similar magnitude to the one we have recently experienced.

We first consider the disturbance to the economy without credit policy and then illustrate the effects of credit policy. The initiating shock is a five percent decline in capital quality, with a quarterly autoregressive factor of 0.66. Absent any changes in investment, the shock produces a roughly ten percent decline in effective capital stock over a two year period. The loss in value of the housing stock relative to the total capital stock was in this neighborhood.

In the model without financial frictions, then shock produces only a modest decline in output. Output falls a bit initially due to the reduced effective capital stock. Because capital is below its steady state, however, investment picks up. Individuals consume less and eventually work more.

By contrast, in the model with frictions in the intermediation process, there is a sharp recession. The deterioration in asset quality produces a magnified decline in intermediary capital. The interest rate spread skyrockets as a consequence, and output tanks. Output initially falls about three percent relative to trend and then decreases to about six percent relative to trend. Though the model does not capture the details of the recession, it does produce an output decline of similar magnitude. Recovery of output to trend does not occur until roughly five years until after the shock. This slow recovery is also in line with current projections. Contributing to the slow recovery is the delayed movement of intermediary capital back to trend. It is mirrored in persistently above trend movement in the spread. Note that over this period the intermediary sector is effectively deleveraging: It is building up equity relative to assets. Thus the model captures formally the informal

notion of how the need for financial institutions to deleverage can slow the recovery of the economy.

We now consider credit interventions by the central bank. Figure 3 considers several different intervention intensities. In the first case, the feedback parameter ν in the policy rule given by equation (38) equals 50. At this value, the credit intervention is roughly of similar magnitude to what has occurred in proactive. The solid line portrays this case. In the second, the feedback parameter is raised to 500, which increases the intensity of the response. The dashed line portrays this case. Finally, for comparison, the dashed and dotted line portrays the case with no credit market intervention.

In each instance, the credit policy significantly moderates the contraction. The prime reason is that central intermediation dampens the rise in the spread, which in turn dampens the investment decline. The moderate intervention ($\nu = 50$) produces an increase in the central bank balance sheet equal to approximately ten percent of the value of the capital stock. This is roughly in accord with the degree of intervention that has occurred in practice. The aggressive intervention further moderates the decline, though the gain relative to the moderated intervention is small. In this case, central bank lending increases to approximately twenty percent of the value of the capital stock.

3.3 Optimal Policy and Welfare

We now consider the welfare gains from central bank credit policy and also compute the optimal degree of intervention. We take as the objective the household's utility function.

We start with the crisis scenario of the previous section. We take as given the Taylor rule for setting interest rates. This rule may be thought of as describing monetary policy in normal times. We suppose that it is credit policy that adjusts to the crisis. We then ask what is the optimal choice of the feedback parameter ν in the wake of the capital quality shock. In doing the experiment, we take into account the efficiency costs of central bank intermediation, as measured by the parameter τ . We consider a range of values for τ .

Following Faia and Monacelli (2007), we begin by writing the household utility function in recursive form:

$$\Omega_t = U(C_t, L_t) + \beta E_t \Omega_{t+1} \quad (39)$$

We then take a second order approximation of this function about the steady state. We next take a second order approximation of the whole model about the steady state and then use this approximation to express the objective as a second order function of the predetermined variables and shocks to the system. In doing this approximation, we take as given the policy-parameters, including the feedback credit policy parameter ν . We then search numerically for the value of ν that optimizes Ω_t as a response to the capital quality shock.

To compute the welfare gain from the optimal credit policy we also compute the value of Ω_t under no credit policy. We then take the difference in Ω_t in the two cases to find out how much welfare increases under the optimal credit policy. To convert to consumption equivalents, we ask how much the individuals consumption would have to increase each period in the no credit policy case to be indifferent with the case under the optimal credit policy. Because we are just analyzing a single crisis and not an on-going sequence, we simply cumulate to the present value of consumption-equivalent benefits and normalize by one year's steady state consumption. Put differently, we suppose the economy is hit with a crisis and then ask what are the consumption-equivalent benefits from credit policy in moderating this single event. Since we are analyzing a single event, it makes sense to us to cumulate up the benefits instead of presenting them as an indefinite annuity flow, where most of the flow is received after the crisis is over.

Figure 4 presents the results for a range of values of the steady state markup and also a range of values of the efficiency cost τ . In the baseline case of with a fifteen percent markup and no efficiency cost ($\tau = 0$), the benefit from credit policy of moderating the recession is worth 6.50 percent of one years recession. At reasonable levels of the efficiency cost (e.g. ten basis points), the gain is on the order of 5.0 percent of steady state consumption. It decreases to zero, as the efficiency costs goes to forty basis points, as fairly large number. Though we do not report the results here, for τ less than forty basis points the optimal credit policy comes closely to fully stabilizing the markup

The welfare loss is increasing in the steady state distortion. If we interpret this distortion from the markup as also capturing distortionary tax effects, then a much high value may be justified. For for example, average effective labor taxes are in the range of thirty to thirty five percent. Assuming the price markup is in the neighborhood of fifteen to twenty percent, then a steady state distortion up to fifty percent is reasonable. Accordingly, we consider two alternatives to our baseline: one involving a thirty-three percent

markup and the other a fifty percent markup. As the figure illustrates, the gains to the optimal credit policy increase substantially, increasing to over fourteen percent of one year's consumption for the τ equals zero case and remaining above ten percent as τ reaches twenty basis points.

One factor moderating the welfare loss is that the marginal disutility of labor declines during the downturn. It could be that the simple preference structure we use to pin down labor supply gives a misleading read on the utility gains from increased leisure during recession. Accordingly, we redo the exercise, this time considering the case of perfectly elastic labor supply. In this case individuals only care about consumption fluctuations. As figure 6 illustrates, in this case the benefits from the optimal credit policy increase roughly twenty percent across the board.

4 Concluding Remarks

We developed a quantitative monetary DSGE model with financial intermediaries that face endogenously determined balance sheet constraints. We then used the model to evaluate the effect of expanding central bank credit intermediation to combat a simulated financial crisis. We find that the welfare benefits may be substantial if the efficiency costs of government intervention are modest.

If we abstract from the issue of efficiency costs, an equivalent type of credit intervention in our model would be direct equity injections into financial intermediaries. Expanding intermediaries equity would of course expand the volume of assets that they can intermediate. In our view, a key factor in choosing between these two policies involves the efficiency costs of the policy action. For certain types of lending, e.g. securitized high grade assets such as mortgaged-backed securities, the costs of central bank intermediation might be relatively low. In this case, direct central bank intermediation may be justified. In other cases, e.g. C&I loans that requires constant monitoring of borrowers, central bank intermediation may be highly inefficient. In this instance, capital injections may be the preferred route. By expanding our model to allow for asset heterogeneity we can address this issue.

Finally, we consider a one time crisis and evaluated the policy response. In subsequent work we plan to model the phenomenon as an infrequently occurring rare disaster in spirit of Barro (2009) and others. In this literature the disaster is taking as a purely exogenous event. Within our framework we

can evaluate the gains from various policy responses, using the same tools as applied in this literature to compute welfare.

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Table 1: Parameter Values for Baseline Model

Households		
β	0.995	Discount rate
h	0.700	Habit parameter
χ	5.584	Relative utility weight of labor
φ	0.333	Inverse Frisch elasticity of labor supply
Financial Intermediaries		
λ	0.383	Fraction of capital that can be diverted
ξ	0.003	Proportional transfer to the entering bankers
θ	0.972	Survival rate of the bankers
Intermediate good firms		
α	0.330	Effective capital share
u	1.000	Steady state utilization rate
$\delta(u)$	0.025	Steady state depreciation rate
ζ	1.000	Elasticity of marginal depreciation with respect to utilization rate
Capital Producing Firms		
η_i	2.500	Inverse elasticity of net investment to the price of capital
Retail firms		
ε	11.000	Elasticity of substitution
γ	0.750	Probability of keeping prices fixed
γ_P	0.500	Measure of price indexation
Government		
κ_π	1.500	Inflation coefficient of the Taylor rule
κ_X	-0.500	Output gap coefficient of the Taylor rule
$\frac{G}{Y}$	0.200	Steady state proportion of government expenditures

Figure 1: Responses to Technology (a) , Monetary (m) and Wealth (w) Shocks

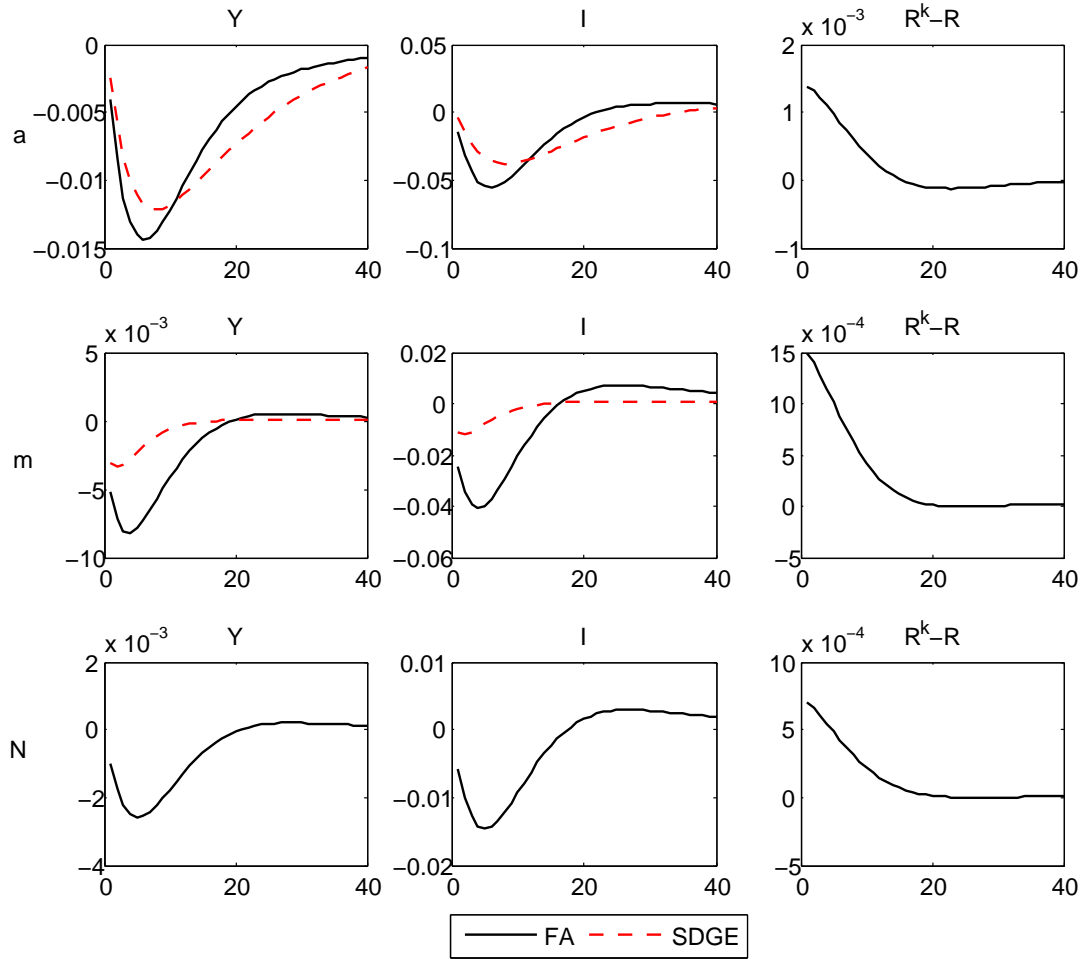


Figure 2: Responses to a Capital Quality Shock

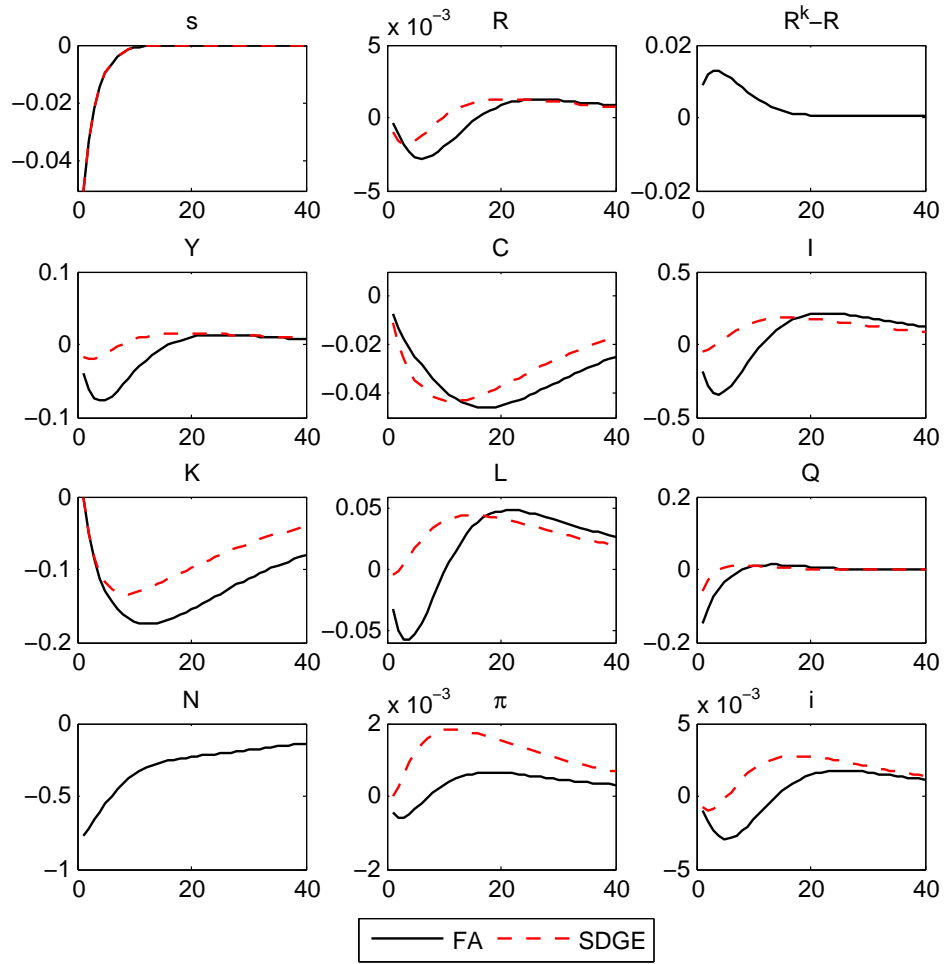


Figure 3: Responses to a Capital Quality Shock with Credit Policy

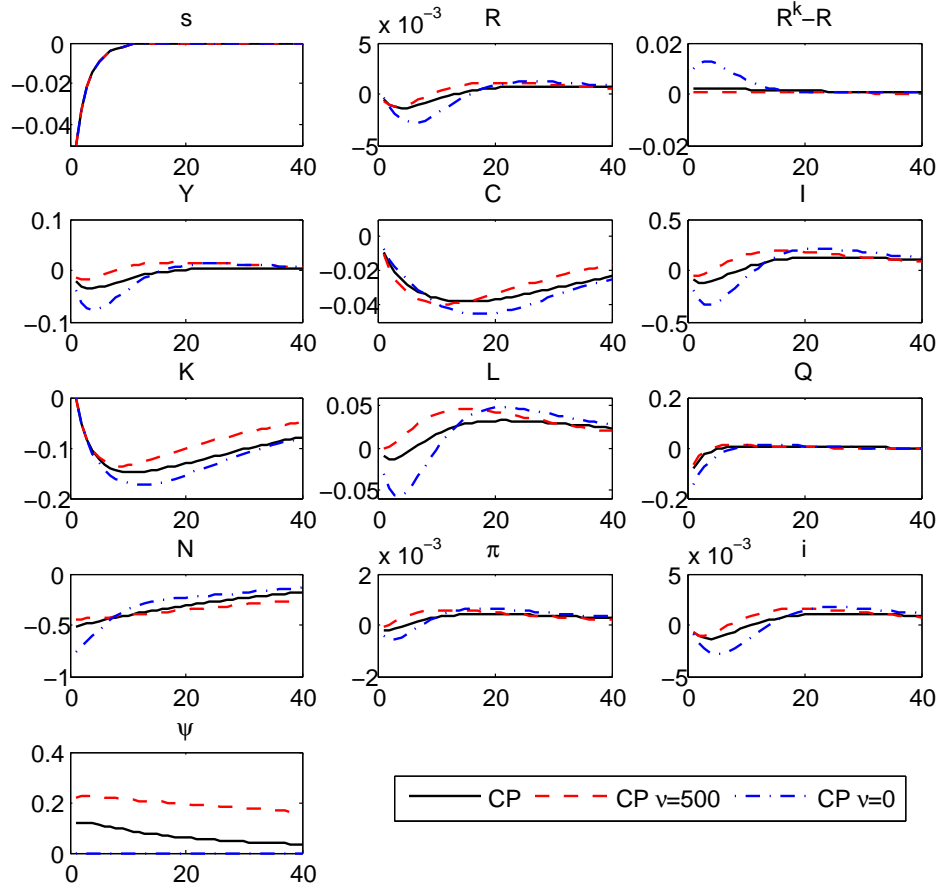


Figure 4: One year consumption equivalent welfare gains from optimal credit policy as a function of efficiency costs τ and steady state markup

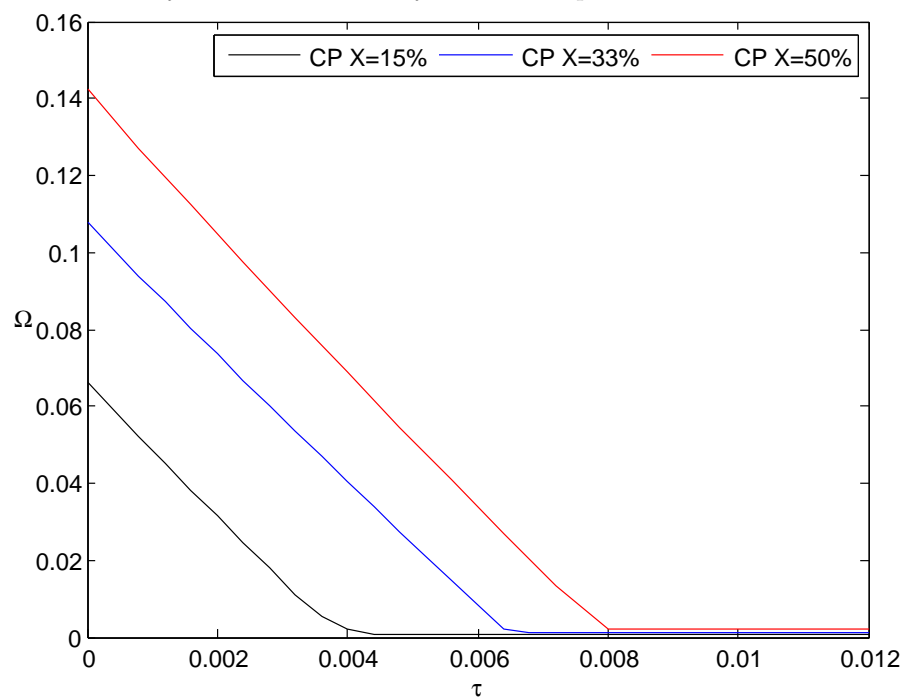


Figure 5: One year consumption equivalent welfare gains from optimal credit policy as a function of efficiency costs τ and steady state markup with high labor supply elasticity

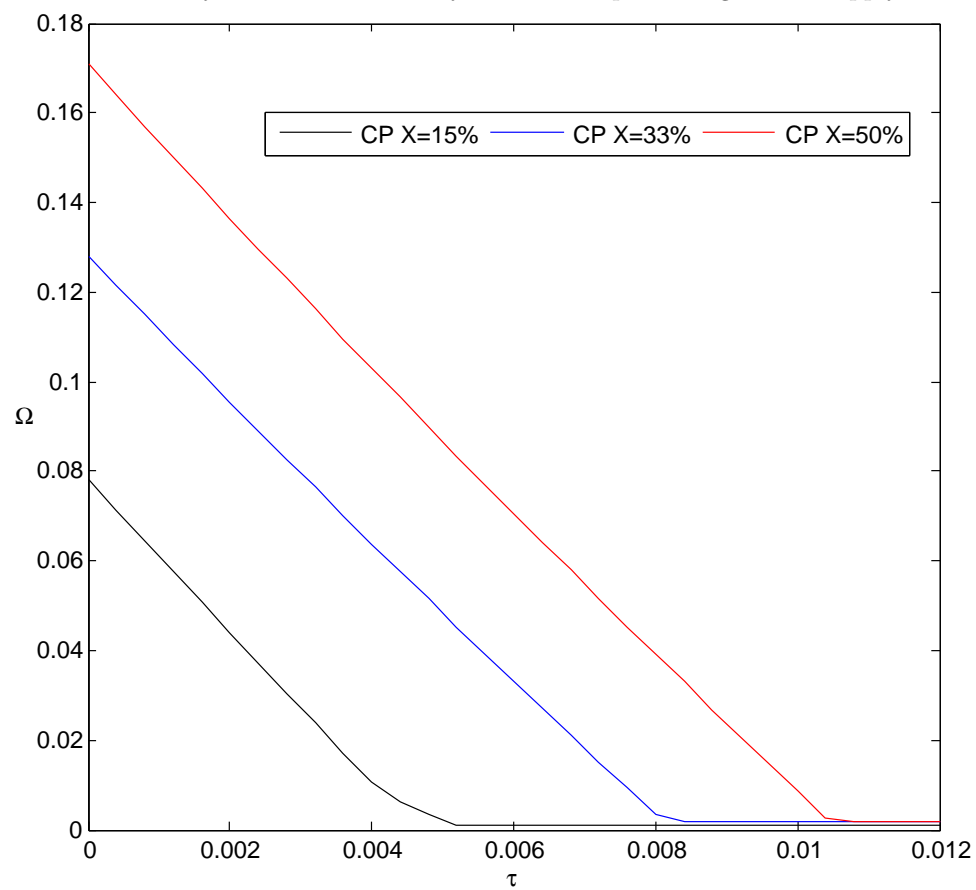


Figure 6: Credit vs. premium policy

