Lecture 4
Olley-Pakes Method

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Outline

• Review static production function results
  – What have we learned?
• Olley-Pakes approach
  – Dealing with dynamics and selection
• Identification
## OLS Production function estimates

**Dep Var=Log(Sales/L) 1986-1995 (T=10)**

<table>
<thead>
<tr>
<th>Est method</th>
<th>Unbalanced (N=1,888)</th>
<th>Balanced (N=681)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Totals</td>
<td>Within</td>
</tr>
<tr>
<td>Log L</td>
<td>.005 (.006)</td>
<td>-.109 (.021)</td>
</tr>
<tr>
<td>Log (K/L)</td>
<td>0.584 (.019)</td>
<td>0.423 (.034)</td>
</tr>
<tr>
<td>s.e.</td>
<td>.452</td>
<td>.232</td>
</tr>
<tr>
<td>R²</td>
<td>.472</td>
<td>.883</td>
</tr>
<tr>
<td>Durbin-Watson</td>
<td>0.273 (.000)</td>
<td>1.159 (.000)</td>
</tr>
<tr>
<td>Hausman Test</td>
<td>37.5 (.000) with 5 d.f.</td>
<td>14.1 (.003) with 3 d.f.</td>
</tr>
</tbody>
</table>

*Std. error estimates are consistent in presence of HS and autocorrelation.*
Static production function summary

- Substantial heterogeneity
  -Exiting firms have higher variance
  -HS-consistent s.e.s are 4-5 times as large

- Dynamics appear to be important
  -Within firm residuals are serially correlated
  -Serial correlation of AR(1) type with large coefficient
    (first diff nearly uncorrelated)

- Exit and entry are pervasive
  -1888 = 681 for 10 yrs + 557 for 5-10 yrs + 750 <5 yrs

- Endogeneity of inputs
  -Chosen by firm as same time as output?
Olley-Pakes overview

• A method for robust estimation of the production function allowing for
  – Endogeneity of some of the inputs
  – Selection (exit)
  – Unobserved (quasi-) permanent differences across firms

• Main requirement (limitation) of their method:
  – There is a monotonic relationship between a firm-level decision variable (investment in this case) and the unobserved firm-level state variable “productivity.”
  – Exit is also conditioned on the unobserved productivity.

• OP Method also useful if you have only one or two of these problems - somewhat more robust than some of the other techniques used in the past
The setting

• Telecomm equipment industry 1972-87
• Major restructuring since late 1960s
  – Lots of tech change – new products (fax, modem, fiber optics, digital switches…)
  – Deregulation and breakup of Bell monopoly
• Under telecomm monopoly until 1980s
  – Essential monopoly in equipment because
    • Procurement practices of Western Electric
    • Requirements for all equipment connected to public network.
The setting (2)

- 1968 - Carterphone decision weakens Bell control over user equipment
- 1975 - program of registration and certification of new equipment – set up by FCC
- 1982 – consent decree
- 1984 – deregulation – 7 RBOCs, none with equipment mfr, immediate impact on equipment manufacturing
- 1996 – Baby bells must open local mkts; ATT spins off equipment and research lab as Lucent
- *Telecomm industry evolution not yet done, thanks to continuous arrival of new technology*……..
The question

• What was the effect of deregulation on productivity?
  – Initial conditions:
    • Heterogeneity among plant
    • Serial correlation in productivity within plant
  – Induced lots of entry and exit
  – Productivity increased
  – Break down productivity increase
    • Average productivity level
    • Due to reallocation of labor
    • Due to reallocation of assets to more productive plants
Model

Derived from a theoretical model of firm-level competition, where firms have idiosyncratic efficiencies but face the same market structure and factor prices.

Profits = f(capital $k_{it}$, efficiency $w_{it}$; factor prices, other firms)

$w_{it}$ evolves as an exogenous Markov process:

\[
E[w_{it} | w_{it-1}, \Omega_{it-1}] = f(w_{it-1})
\]

Simple fixed effects ($\alpha_i$) are a special case of this process.

\[
E[\alpha_i | \Omega_{it-1}] = \alpha_i
\]

*Note that I omit the other state variable considered by Olley-Pakes, firm age, for simplicity.*
Model (continued)

Sequence of moves by a firm:
1. Compare continuation to salvage (liquidation) value. Exit if too low.
2. If firm continues in operation, choose labor and investment (knowing current efficiency $w_{it}$)
3. Investment choice gives capital next period:

$$k_{it+1} = (1 - \delta)k_{i,t} + i_{it}$$
Solution to theoretical model

Olley and Pakes give the Bellman equation; references to other theoretical papers. Solution generates two firm decision rules:

1. Exit if efficiency falls below a level that increases monotonically with capital stock (larger firms are more likely to survive given a particular efficiency draw):
   \[
   \text{Exit if } w_{it} < w_{it}(k_{it})
   \]

2. If no exit, investment is a function of current state variables:
   \[
   i_{it} = f(w_{it}, k_{it})
   \]

Note that variations in market structure and factor prices over time are subsumed in the \( t \) subscript. This means you need industry specific time dummies if you use more than one industry (market).
Estimation

Problem: how to estimate parameters of production function consistently when we have

1. Simultaneity between output and variable inputs.
2. Unobserved heterogeneity in productivity (correlated effects).
3. Exit of inefficient (low productivity) firms (sample selection).

Production function:

\[ y_{it} = \beta_0 + \alpha l_{it} + \beta_0 k_{it} + w_{it} + \eta_{it} \]

\( w_{it} \) is known by firm when it makes the labor and investment decision
\( \eta_{it} \) is the true “error,” not known by the firm or by the econometrician.
Estimation

Consider the following regression:

\[ E[y_{it} | l_{it}, k_{it}, \text{survive to } t] \]
\[ = \beta_0 + \alpha l_{it} + \beta k_{it} + E[w_{it} | l_{it}, k_{it}, \text{survive to } t] \]

The last term is clearly nonzero, since the labor choice depends on the productivity level, and the productivity level is correlated with capital via past productivity and selection.

In addition, we know that firms with larger capital stocks continue in operation at lower productivity levels, so the term is decreasing in \( k_{it} \), leading to a downward bias in \( \beta \).

Pakes (1994) shows that investment (if it is nonzero) is strictly increasing in productivity given \( k_{it} \), so we have

\[ w_{it} = h_t(i_{it}, k_{it}) \]
Output and exit as a function of k(size)

Cutoff output given $w = f(i,k)$
Estimation

This fact (that efficiency is a function of capital and investment) allows us to correct for problem (1), simultaneity, in the following way:

\[ y_{it} = \beta_0 + \alpha l_{it} + \beta k_{it} + h_t(i_{it}, k_{it}) + \eta_{it} \]

\[ = \alpha l_{it} + \phi(i_{it}, k_{it}) + \eta_{it} \]

Regress \( y \) on \( l, k, \) and a polynomial function of \( i \) and \( k \) to obtain a consistent estimate of \( \alpha \) and \( \phi(i_{it}, k_{it}) \), the combined effect of capital and the efficiency level. The estimated labor coefficient should be lower now, since we corrected for the downward bias in capital.
Estimation

Given a consistent $\alpha$, problems 2 (correlated effects) and 3 (selectivity) are solved together. We have

$$E[y_{it} - \alpha l_{it} | k_{it}, \text{survive to } t]$$

$$= \beta_0 + \beta k_{it} + E[w_{it} | w_{it-1}, \text{survive to } t]$$

(because $k_{it}$ is determined once $w_{i,t-1}$ and survival are known)

This regression eq implies the following model:

$$y_{it} - \alpha l_{it} = \beta_0 + \beta k_{it} + E[w_{it} | w_{it-1}, \text{surv.}] + w_{it} - E[w_{it} | w_{it-1}, \text{surv.}] + \eta_{it}$$

$$= \beta_0 + \beta k_{it} + E[w_{it} | w_{it-1}, \text{surv.}] + \xi_{it} + \eta_{it}$$

$\xi$ is the efficiency surprise for surviving firms, which does not affect the investment or exit choice last period.
Estimation

\[ y_{it} - \alpha l_{it} = \beta_0 + \beta k_{it} + E[w_{it} | w_{it-1}, \text{surv.}] + \xi_{it} + \eta_{it} \]

The bias term \( g(w_{i,t-1}, \text{survival}) = E[w_{it} | w_{i,t-1}, \text{survived}] \) is a function of both the unobserved firm productivity (lagged) and the survival probability, so two “instruments” are required for estimation. OP’s idea is to use \( h_{i,t-1} = \phi_{i,t-1} - \beta k_{i,t-1} \) as a proxy for the first conditioning variable and the predicted probability of survival from a probit or semi-parametric estimate of exit \( (P) \) as a proxy for the second.

Because the functional form of \( g \) is unknown, they suggest modeling it as a polynomial in \( h \) and \( P \) or as a kernel regression.
Implementation

1. Regress output $y$ on labor $l$ and a polynomial in investment and capital. Obtain $\gamma - \alpha l$ and the estimated $\varphi$ function.

2. Predict the probability of survival as a function of investment and capital, using a probit or semi-parametric model.

3. Regress $\gamma - \alpha l$ on capital and a polynomial that is joint in $P$ (from step 2) and $\varphi - \beta^* k$ (from step 1, lagged). Note that the $\beta$ here is the same as the capital coefficient, so the regression is nonlinear. 

   \textit{Estimates will be consistent under the assumptions of the model, although not efficient in general.}
Simplifying: selection only

If there are no correlated productivity differences, but there is selectivity, this method reduces to the following:

1. Predict the probability of survival as a function of investment and capital, using a probit or semi-parametric model.

2. Regress $y$ on labor and capital and a polynomial in $P$ (from step 1).

*Note that this is simply a semi-parametric version of the Heckman correction. (It does not make sense if you start with the economic model OP use, but it is useful for understanding what is being done).*
Simplifying: heterogeneity only

If the firms have unobserved productivity differences but do not exit, we have a variation of the F.E. model:

\[ y_{it} - \alpha l_{it} = \beta_0 + \beta k_{it} + E[w_{it} | w_{it-1}] + w_{it} - E[w_{it} | w_{it-1}] + \eta_{it} \]

\[ = \beta_0 + \beta k_{it} + E[w_{it} | w_{it-1}] + \xi_{it} + \eta_{it} \]

1. Regress output \( y \) on labor \( l \) and a polynomial in investment and capital. Obtain \( y - \alpha l \) and the estimated \( \phi \).

2. Regress \( y - \alpha l \) on capital and a polynomial in \( \phi - \beta^* k_{i,t-1} \) (from step 1).

Suppose the polynomial is of order 1. Then

\[ y_{it} - \alpha l_{it} = \beta_0 + \beta k_{it} + \rho(\hat{\phi}_{i,t-1} - \beta k_{i,t-1}) + \xi_{it} + \eta_{it} \]

\[ = \beta_0 + \beta k_{it} + \rho(y_{i,t-1} - \alpha l_{i,t-1} - \eta_{i,t-1} - \beta k_{i,t-1}) + \xi_{it} + \eta_{it} \]

\[ \Rightarrow (y_{it} - \alpha l_{it}) - \rho(y_{i,t-1} - \alpha l_{i,t-1}) = \beta_0 + \beta(k_{it} - \rho k_{i,t-1}) + \xi_{it} + \eta_{it} - \rho\eta_{i,t-1} \]
Simplifying OP – “not-so-fixed” effects

Assume the firm efficiency is a simple AR(1):

\[ w_{it} = \rho w_{i,t-1} + \xi_{it} \]
\[ w_{it} = y_{it} - \beta_0 - \alpha l_{it} - \beta k_{it} - \eta_{it} \]

\[ \Rightarrow y_{it} - \rho y_{i,t-1} = \beta_0 + \alpha(l_{it} - \rho l_{i,t-1}) + \beta(k_{it} - \rho k_{i,t-1}) + \xi_{it} + \eta_{it} - \rho \eta_{it} \]

This model is the same as the one on the previous slide (except for the fact that we removed labor first due to simultaneity).

When the effects are \( w_{it}=\alpha_i \), the model reduces to

\[ y_{it} - y_{i,t-1} = \beta_0 + \alpha(l_{it} - l_{i,t-1}) + \beta(k_{it} - k_{i,t-1}) + \eta_{it} - \eta_{i,t-1} \]

Which is just first-difference estimation.

Conclusion: O-P allows for a considerably more general firm-level effect than the simple “fixed effect” model, but the latter is nested within it.
## Production function example

### Production Function Estimates

**US Manufacturing (Low and Medium Tech)**

Dependent Variable: log(S/L)

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Within</th>
<th>FD</th>
<th>LD (5 yr)</th>
<th>OP</th>
</tr>
</thead>
<tbody>
<tr>
<td>log (L)</td>
<td>.005 (.006)</td>
<td>-.109 (.021)</td>
<td>-.256 (.034)</td>
<td>-.087 (.025)</td>
<td>.170 (.022)</td>
</tr>
<tr>
<td>log (K/L)</td>
<td>.584 (.019)</td>
<td>.423 (.034)</td>
<td>.325 (.034)</td>
<td>.418 (.048)</td>
<td>.612 (.006)</td>
</tr>
<tr>
<td>D-W</td>
<td>0.27</td>
<td>1.16</td>
<td>1.73</td>
<td>0.51</td>
<td>0.5</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.452</td>
<td>0.232</td>
<td>0.247</td>
<td>0.348</td>
<td>1.55</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.472</td>
<td>0.883</td>
<td>0.263</td>
<td>0.293</td>
<td>0.285</td>
</tr>
<tr>
<td># obs.</td>
<td>11822</td>
<td>11822</td>
<td>9935</td>
<td>4406</td>
<td>9935</td>
</tr>
</tbody>
</table>

Survival probability and unobserved productivity estimated with a cubic in \( \log L \) and \( \log K \).

Year dummies included in step 1 and step 2 regressions.
Identification

• Levinsohn-Petrin – use materials instead of investment

• Ackerberg-Caves; Bond-Soderstrom critique:
  – No variation in input prices means flexible inputs a function only of firm productivity, so not identified in its presence
  – True even if some inputs are subject to adj. costs.
  – To get identification:
    • Control function (investment or materials) has no optimization errors; included vars like labor have optimization errors
    • *OR*: all variables have adjustment costs, and they differ in speed across variables in significant ways
Subadditivity

• Single product firm
  – cost function subadditive at $Q$ iff:

$$C(Q) < \sum_{i} C(q_i) \quad \forall \text{ vectors } (q_1, q_2, \ldots) \ni Q = \sum_{i} q_i$$

Same as decreasing average costs until $Q$

• Multiproduct firm
  – same definition of subadditivity but $Q$ is a vector of different types of output
  – economies of scope:

$$C(Q_1, Q_2) < C(Q, 0) + C(0, Q_2)$$

• See Evans and Heckman for example of testing for subadditivity in the Bell system