Expected Interest Rate Parity

Aggregate Economics Project
Presentations October 17
Notation

\begin{align*}
S_t & \quad \text{spot exchange rate, price of foreign currency (\$/yen)} \\
F_t & \quad \text{forward rate (\$/yen for t+1) contracted today} \\
1+I_t & \quad \text{gross interest rate} \\
1+I^*_t & \quad \text{gross foreign interest rate} \\
s_t & \quad \ln(S_t) \\
f & \quad \ln(F) \\
i & \quad \ln(1+I) \\
i^* & \quad \ln(1+I^*)
\end{align*}
Covered Interest Rate Parity

\[ 1 + I_t = \frac{1}{S_t} \left( 1 + I^*_t \right) F_t, \text{ or in logs} \]

\[ i_t - i^*_t = f_t - s_t \]
Theory: Expected Interest rate parity

\[ i_t - i_t^* = E_t s_{t+1} - s_t \]

=>

\[ f_t = E_t s_{t+1} \]
Example

\[ i_t - i_t^* = f_t - s_t \]
\[ -5\% = \ln(.95) - \ln(1) \]
where \( s \equiv \#$/\text{Yen} \)
suppose \( E_s_{t+1} = \ln(1) > \ln(.95) \)

\[ \text{how to get to equilibrium?} \]
Same mechanism as Random Walk Stock Model

- $E_t s_{t+1}$ given
- Spot price adjusts to clear market
- $f < E_t s_{t+1}$; buy yen $\Rightarrow$ $s_t \uparrow$
- Buy $i^*$ $\Rightarrow$ $i^* \downarrow$
- How much do prices have to change?
- Until $f = E_t s_{t+1} = \ln(1)$
- No arbitrage links: $i - i^* = f - s$
Test the Hypothesis

• Substitute definition in model

\[ i_t - i_t^* = E_t s_{t+1} - s_t \]

\[ s_{t+1} \equiv E_t s_{t+1} + e_{t+1} \Rightarrow \]

\[ s_{t+1} - s_t = a + b(i_t - i_t^*) + e_{t+1}, \text{ or} \]

\[ s_{t+1} - s_t = a + b(f_t - s_t) + e_{t+1} \]

\[ H_0 : \ a = 0 \text{ and } b = 1 \]
Australia \( (s=\ln(\#\text{AU}\\$/$)) \)

Dependent Variable: DLNS  
Method: Least Squares  
Date: 09/29/03  Time: 17:15  
Sample(adjusted): 1993:09 2003:08  
Included observations: 120 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000202</td>
<td>0.002672</td>
<td>0.075411</td>
<td>0.9400</td>
</tr>
<tr>
<td>F_S</td>
<td>-0.288133</td>
<td>0.422369</td>
<td>-0.682182</td>
<td>0.4965</td>
</tr>
</tbody>
</table>

R-squared | 0.003928 | Mean dependent var | -0.000247 |
Adjusted R-squared | -0.004513 | S.D. dependent var | 0.028310 |
S.E. of regression | 0.028374 | Akaike info criterion | -4.270195 |
Sum squared resid | 0.094997 | Schwarz criterion | -4.223737 |
Log likelihood | 258.2117 | F-statistic | 0.465372 |
Durbin-Watson stat | 2.007813 | Prob(F-statistic) | 0.496461 |
Theory: \( s_{t+1} - s_t = f_t - s_t + e_{t+1} \)
Q: Can one make money betting against the theory?

- Use the “carry trade” rules (Cavallo)
  
  \[ i > i^* \text{ lend in home country @ } i \text{ and borrow in foreign @ } i^* \]
  
  \[ i < i^* \text{ borrow @ home and lend in foreign} \]

\[
xr = \begin{cases} 
(i - i^*) - (s_{t+1} - s_t) & ; i > i^* \\
-[(i - i^*) - (s_{t+1} - s_t)] & ; i < i^* 
\end{cases}
\]
Or with the forward rate,

- $f < s$ buy forward $\Rightarrow i < i^*$,
- $f > s$ sell forward

$$x_{rf} = \begin{cases} s_{t+1} - f & \text{buy} \\ -(s_{t+1} - f) & \text{sell} \end{cases} = xr, \text{ eg for buy strategy,}$$

$$x_{rf} = s_{t+1} - f = -[(i - i^*) - (s - s_{t+1})] = xr$$
Results: annual average return
4.2%
Is it risky?

- Sharpe ratios (average excess return/standard deviation)
  - Carry trade  12%
  - VWNYSEretd-risk free 12%

- Is the Sharpe ratio the correct measure?
- How would one reduce risk for the “carry trade” strategy?
How Can these Excess Average Profits Persist?

- Transactions costs absorb most of the profit
## Transactions Cost

### Median Bid-Ask Spreads

<table>
<thead>
<tr>
<th>Country</th>
<th>Spot 100 x Ln(Ask/Bid)</th>
<th>1 month Forward</th>
<th>3 month Forward</th>
<th>Spot Forward</th>
<th>1 month Forward</th>
<th>3 month Forward</th>
<th>Units</th>
<th>Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>0.159</td>
<td>0.253</td>
<td>0.291</td>
<td>10.00</td>
<td>15.93</td>
<td>20.00 Centimes</td>
<td>76:01-98:12</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>0.053</td>
<td>0.096</td>
<td>0.111</td>
<td>0.10</td>
<td>0.20</td>
<td>0.23 Cents</td>
<td>76:01-05:12</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>0.100</td>
<td>0.151</td>
<td>0.176</td>
<td>1.00</td>
<td>1.50</td>
<td>1.88 Centimes</td>
<td>76:01-98:12</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>0.213</td>
<td>0.311</td>
<td>0.319</td>
<td>1.00</td>
<td>1.12</td>
<td>1.13 Pfennig</td>
<td>76:01-98:12</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>0.063</td>
<td>0.171</td>
<td>0.208</td>
<td>1.00</td>
<td>4.00</td>
<td>5.00 Lire</td>
<td>76:01-98:12</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>0.216</td>
<td>0.272</td>
<td>0.280</td>
<td>1.00</td>
<td>1.08</td>
<td>1.13 Yen</td>
<td>78:06-05:12</td>
<td></td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.234</td>
<td>0.344</td>
<td>0.359</td>
<td>1.00</td>
<td>1.25</td>
<td>1.25 Cents</td>
<td>76:01-98:12</td>
<td></td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.255</td>
<td>0.412</td>
<td>0.456</td>
<td>1.00</td>
<td>1.13</td>
<td>1.13 Centimes</td>
<td>76:01-05:12</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>0.055</td>
<td>0.074</td>
<td>0.082</td>
<td>0.10</td>
<td>0.12</td>
<td>0.13 Cents</td>
<td>76:01-05:12</td>
<td></td>
</tr>
<tr>
<td>Euro*</td>
<td>0.043</td>
<td>0.060</td>
<td>0.070</td>
<td>0.04</td>
<td>0.06</td>
<td>0.07 Cents</td>
<td>99:01-05:12</td>
<td></td>
</tr>
</tbody>
</table>

*Note: Euro* likely refers to the Eurozone or a specific currency in the Euro zone.*
### Returns

<table>
<thead>
<tr>
<th>Country</th>
<th>No Transactions Costs</th>
<th>With Transactions Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Belgium*</td>
<td>0.0044</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>(0.0019)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Canada</td>
<td>0.0053</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>(0.0018)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>France*</td>
<td>0.0054</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>(0.0016)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Germany*</td>
<td>0.0011</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>(0.0018)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Italy*</td>
<td>0.0029</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>(0.0017)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Japan†</td>
<td>0.0022</td>
<td>0.036</td>
</tr>
<tr>
<td></td>
<td>(0.0022)</td>
<td>(0.003)</td>
</tr>
</tbody>
</table>
How does one reduce risk?

• Diversify
• Diversify
• Diversify
• The variance of the average is less than the average of the variances
How does one reduce risk?

Sharpe Ratios of the Portfolio Strategies with Transactions Costs Over a Common Sample (79:10-05:12)

<table>
<thead>
<tr>
<th></th>
<th>Equally Weighted</th>
<th>Optimally Weighted</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carry trade</td>
<td>0.154 (0.060)</td>
<td>0.197 (0.057)</td>
<td>0.043 (0.037)</td>
</tr>
<tr>
<td>BGT</td>
<td>0.096 (0.062)</td>
<td>0.137 (0.063)</td>
<td>0.041 (0.032)</td>
</tr>
<tr>
<td>Difference</td>
<td>0.058 (0.069)</td>
<td>0.060 (0.065)</td>
<td></td>
</tr>
</tbody>
</table>
Question

• Do you buy their story that the transactions costs wedge is so large the “carry trade” excess expected profits are an illusion?
Forward Discount Project

- Get monthly data for the US, Japan, and another country. Try Global Financial Data and Datastream.
- Test the model, ie, run the regressions,
  \[ s_{t+1} - s_t = a + b(f_t - s_t) + e_{t+1} \]
- If you find a violation (which you should), then
- Calculate the profits and Sharpe ratios for a “carry trade” strategy
- Explain the tests and results in 3-5 pages