The Forward Discount Premium

Covered Interest Rate Parity says,

\[
\ln(1 + i) = \ln(1 + i^*) + \ln(F_{t+1} / S) \]

\[i - i^* \equiv f_{t+1} - s\]

the forward discount equals the interest rate differential\(^1\). If covered interest rate parity doesn’t hold, then arbitrage profits exist. Accept the covered interest parity as a fact.

Expected Interest Rate Parity\(^2\) is a theory that implies that \(E_{t} s_{t+1} = f_{t+1}\). A test of the theory is the regression,

\[
\Delta s_{t+1} = a + b(i-i^*) + u_{t+1}, \text{ or} \\
\Delta s_{t+1} = a + b(f_{t+1} - s_t) + u_{t+1} \\
since no arbitrage implies, \\
i - i^* = f - s \tag{1.1}
\]

Under the null: \(a = 0, b =1\), and the error e or u is unpredictable.

Profit

The empirical results generally show that expected interest rate parity is not a good approximation to the data. On average the exchange rate does not depreciate enough to compensate for the interest differential. Predictable expected excess returns exist.

How could one make money with this knowledge? A really simple rule is: Invest in the country with the higher rate, ie,

if \((i-i^*) \geq 0\),
then, borrow abroad and invest at home, and
if \((i-i^*) < 0\),
then, borrow at home and invest abroad.

---

\(^1\) I use the notation from the project assignment description.

\(^2\) This assumes that the exchange rate is distributed log-normally.
The realized profit from this rule is,

\[ p^+ = (1 + i) - (1 + i^*) \frac{S_{t+1}}{S_t}; \quad i-i^* \geq 0 \]

(1.2)

\[ p^- = -(1 + i) - (1 + i^*) \frac{S_{t+1}}{S_t}; \quad i-i^* < 0. \]

If the interest differential is greater than the realized exchange rate depreciation then, the profit is positive.

**Empirical Evidence**

**Data**

All the data come from Datastream. The data are monthly (measured on the 26th day of the month) for the exchange rate and the one-month forward rate (as collected by BBI). The data go from 9/26/93 to 9/26/03.

I used the forward discount \((f-s)\) as a proxy for the interest differential, \((i-i^*)\). And I used the log approximation to the profit calculation in equation (1.2), eg,

\[ p^+ \cong (i - i^*) - \Delta s_{t+1} \]

**Australia**

The regression results do not support expected interest rate parity,

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.000202</td>
<td>0.002672</td>
<td>0.075411</td>
<td>0.9400</td>
</tr>
<tr>
<td>F, S</td>
<td>-0.288133</td>
<td>0.422369</td>
<td>-0.682182</td>
<td>0.4965</td>
</tr>
</tbody>
</table>

R-squared: 0.003928
Adjusted R-squared: -0.004513
S.E. of regression: 0.028374
Akaike info criterion: -4.270195
Schwarz criterion: -4.223737
Log likelihood: 258.2117
Durbin-Watson stat: 2.007813

The \( b \) coefficient is significantly less than one (p value of 1.5%).
Visual econometrics in a graph of the data confirm a weak relationship, between the log change in the exchange rate and forward discount.

**Profit**

Can one make a profit betting against the theory?
Yes, on average.

Is it risky? The Sharpe ratio,

\[ \zeta \equiv \frac{\text{mean excess return}}{\text{std}} = 12\% \]

is 12%. The Sharpe ratio for the VWNYSE monthly return with distributions for 1993-2003 is also about 12%.

(Is the Sharpe ratio the correct measure of risk? And how would one reduce risk in the currency speculation exercise?)
Japan

A look at the raw data

shows the level of the exchange rate and the forward rate move closely together.
Test the theory

Dependent Variable: DLNS
Method: Least Squares
Date: 09/29/03   Time: 21:46
Sample(adjusted): 1993:09 2003:08
Included observations: 120 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.003516</td>
<td>0.003826</td>
<td>-0.919084</td>
<td>0.3599</td>
</tr>
<tr>
<td>F_S</td>
<td>-1.008828</td>
<td>0.634076</td>
<td>-1.591020</td>
<td>0.1143</td>
</tr>
</tbody>
</table>

R-squared 0.021002  Mean dependent var 0.000450
Adjusted R-squared 0.012705  S.D. dependent var 0.031993
S.E. of regression 0.031789  Akaike info criterion -4.042865
Sum squared resid 0.119244  Schwarz criterion -3.996407
Log likelihood 244.5719  F-statistic 2.531343
Durbin-Watson stat 1.504952  Prob(F-statistic) 0.114281

The data fail to confirm the theory. The $b$ coefficient is far from one (p value < 1%)

Log changes in the exchange rate are very noisy relative to the forward discount.

Expected interest rate parity predicts a noisy relationship, since the forward rate is the expected future spot rate, $s_{t+1} = E_{t+1} + e_{t+1} = f_t + e_{t+1}$. But the data reveal noise and no systematic relationship.
Profit: Can one make money betting against the theory?

Looks like it!

Sure can! The Sharpe ratio is 0.12.

Is it risky? LTCM made this bet and lost in 1998:8 and 1998:9. Was it a dumb bet, or LTCM unlucky? Here are the numbers.
In August 1998 the monthly interest rate in the US was 0.1% higher than in Japan\(^3\). So invest in the US. Bad move, the dollar depreciated by 7% (yen appreciated 7%) and LTCM lost 6.4% (a 2 std event) on the bet. And September was even worse. The interest differential was 0.4% in favor of the US, but the dollar depreciated by 13% (yen appreciated 13%), and LTCM lost 12%, (a 3.5 std outlier, and the minimum profit in the sample).

---

\(^3\) My exchange forward rate data and in \#yen/$. So I treat Japan as the home country.
Data Warnings

<table>
<thead>
<tr>
<th>Series ID:</th>
<th>EXJPUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source:</td>
<td>Board of Governors of the Federal Reserve System</td>
</tr>
<tr>
<td>Release:</td>
<td>G.5 Foreign Exchange Rates</td>
</tr>
<tr>
<td>Seasonal Adjustment:</td>
<td>Not Applicable</td>
</tr>
<tr>
<td>Frequency:</td>
<td>Monthly</td>
</tr>
<tr>
<td>Units:</td>
<td>Japanese Yen to One U.S. Dollar</td>
</tr>
<tr>
<td>Date Range:</td>
<td>1971-01-01 to 2003-08-01</td>
</tr>
<tr>
<td>Last Updated:</td>
<td>2003-09-02</td>
</tr>
<tr>
<td>Notes:</td>
<td>Averages of daily figures. Noon buying rates in New York City for cable transfers payable in foreign currencies.</td>
</tr>
</tbody>
</table>

Latest Observations:

This is a very nice description and picture. But notice that the monthly data are the average of the daily data. Actual trades take place on a day and profits are realized one month later. Daily movements during the month don’t matter. Averaged data is not appropriate for testing most models.