1. Suppose that there are three firms. The market demand is \( p(q_1, q_2, q_3) = 120 - q_1 - q_2 - q_3 \). Firm \( i \)'s profit is
\[
 u_i(q_1, q_2, q_3) = q_i p(q_1, q_2, q_3) - q_i^2.
\] Compute the Nash equilibrium quantity for each firm.

2. There are 1, \ldots, \, N players with values \( v_1 > v_2 > \cdots > v_N > 0 \) for some object. Each submits a bid for the object in a second-price auction. The player \( i \) who submits the highest bid wins the object and pays the highest bid among the other \( (j \neq i) \) players. In the case of a tie, the lowest-indexed player, i.e. the one with the higher value, wins the object. This can be formalized as the following game:

- \( \mathcal{N} = \{1, \ldots, N\} \).
- \( \mathcal{A} = \mathbb{R}_+ \).
- \( u_i(a_i) = \begin{cases} v_i - \max_{j \neq i} a_j & \text{if } i = \min\left\{ j : a_j \geq a_k, \forall k \in \mathcal{N} \right\} \\ 0 & \text{otherwise} \end{cases} \)

(a) Prove that truth-telling, i.e., the action profile where \( a_i = v_i \) for each player \( i \), is a Nash equilibrium.

(b) Prove that, for every player \( i \), there exists a Nash equilibrium where player \( i \) wins the prize.

(c) Prove that in any equilibrium the payment must be weakly less than \( v_1 \).

**Discussion question**

Aug 30 Three roommates are sharing a three-bedroom apartment with a $2000 monthly rent. The rooms are of different quality and desirability to the different roommates. How should the rooms be assigned and the rent be divided?