Advanced Microeconomics  
(Economics 104)  
Spring 2011  
Introduction

Topics

– Terminology and notations: 
  functions, 
  preferences, 
  utility representation, and 
  profiles.

– Games and solutions: 
  strategic vs. extensive games, and 
  perfect vs. imperfect information.

– Rationality: 
  a rational agent, and 
  boundedly rational agent.

– Formalities: 
  a strategic game of perfect information.
Terminology and notations (OR 1.7)

Sets

- $\mathbb{R}$ is the set of real numbers.
- $\mathbb{R}_+$ is the set of nonnegative real numbers.
- $\mathbb{R}^n$ is set of vectors on $n$ real numbers.
- $\mathbb{R}_+^n$ is set of vectors of $n$ nonnegative real numbers.

For $x, y \in \mathbb{R}^n$,

$$x \geq y \iff x_i \geq y_i$$

for all $i$.

$$x > y \iff x_i \geq y_i \text{ and } x_j > y_j$$

for all $i$ and some $j$.

$$x >> y \iff x_i > y_i$$

for all $i$. 
Functions

A function $f : \mathbb{R} \to \mathbb{R}$ is

- increasing if $f(x) > f(y)$ whenever $x > y$,
- non decreasing if $f(x) \geq f(y)$ whenever $x > y$, and
- concave if

$$f(\alpha x + (1 - \alpha)x') \geq \alpha f(x) + (1 - \alpha)f(x')$$

$\forall x, x' \in \mathbb{R}$ and $\forall \alpha \in [0, 1]$. 

Let $X$ be a set. The set of maximizers of a function $f : X \to \mathbb{R}$ is given by $\arg \max_{x \in X} f(x)$. 
Preferences

\( \succeq \) - a binary relation on some set of alternatives \( A \subseteq \mathbb{R}^n \). From \( \succeq \) we derive two other relations on \( A \):

- strict performance relation
  \[ a \succ b \iff a \succeq b \text{ and not } b \succeq a \]

- indifference relation \( a \sim b \iff a \succeq b \text{ and } b \succeq a \)

\( \succeq \) is said to be
- complete if
  \[ a \succeq b \text{ or } b \succeq a \]
  \( \forall a, b \in A. \)

- transitive if
  \[ a \succeq b \text{ and } b \succeq c \text{ then } a \succeq c \]
  \( \forall a, b, c \in A. \)
Utility representation

A function \( u : A \to \mathbb{R} \) is a utility function representing \( \succeq \) if for all \( a, b \in A \)

\[
a \succeq b \iff u(a) \geq u(b)
\]

\( \succeq \) can be presented by a utility function only if it is complete and transitive (rational).

\( \succeq \) is said to be

- continuous (preferences cannot jump...) if
  for any sequence of pairs \( \{(a^k, b^k)\}_{k=1}^{\infty} \) with \( a^k \succeq b^k \), and \( a^k \to a \) and \( b^k \to b \), \( a \succeq b \).

- (strictly) quasi-concave if
  for any \( b \in A \) the upper counter set \( \{a \in A : a \succeq b\} \) is (strictly) convex.

These guarantee the existence of continuous well-behaved utility function representation.
Profiles

Let $N$ be a the set of players.

$(x_i)_{i \in N}$ or simply $(x_i)$

- a profile, i.e., a collection of values of some variable, one for each player.

$(x_j)_{j \in N/\{i\}}$ or simply $x_{-i}$

- the list of elements of the profile $x = (x_j)_{j \in N}$ for all players except $i$.

$(x_{-i}, x_i)$

- a list $x_{-i}$ and an element $x_i$, which is the profile $(x_i)_{i \in N}$.
Games and solutions (O 1.1; OR 1.1-1.3)

A game is a model of interactive (multi-person) decision-making. We distinguish between:

- noncooperative and cooperative games - the units of analysis are individuals or (sub) groups,
- strategic (normal) form games and extensive form games - players move simultaneously or precede one another, and
- Games with perfect and imperfect information - players are perfectly or imperfectly informed about characteristics, events and actions.

A solution is a systematic description of outcomes in a family of games.

- Nash equilibrium.
- Subgame perfect equilibrium - extensive games with perfect information.
- Perfect Bayesian equilibrium - games with observable actions.
- Sequential equilibrium (and refinements) - extensive games with imperfect information.

The classic references are von Neumann and Morgenstern (1944), Luca and Raiffa (1957) and Schelling (1960) (see R and OR).
Rational behavior and bounded rationality (O 1.2; OR 1.4, 1.6)

Consider

- a set of actions, $A$
- a set of consequences, $C$
- a consequence function $g : A \to C$, and
- a preference relation \( \succ \) on the set $C$.

Given any set $B \subseteq A$ of actions, a rational agent chooses an action $a^* \in B$ such that

$$g(a^*) \succ g(a)$$

for all $a \in B$.

And when \( \succ \) are specified by a utility function $U : C \to \mathbb{R}$

$$a^* \in \arg \max_{a \in B} U(g(a))$$

With uncertainty about

- the environment,
- events in the game, or
- actions of other players and their reasoning,

A rational agent is assumed to have in mind

- a state space $\Omega$,
- a (subjective) probability measure over $\Omega$, and
- a consequence function $g : A \times \Omega \to C$

A rational agent is an expected ($vNM$) utility $u(g(a, \omega))$ maximizer.
Formalities (O 2.1; OR 2.1)

A strategic game of perfect information:

- a finite set $N$ of players,
- for each player $i \in N$:
  - a non-empty set $A_i$ of actions,
  - a preference relation $\succeq_i$ on the set $A = \times_{j \in N} A_j$ of possible outcomes.

We will denote a strategic game by

$$\langle N, (A_i), (\succeq_i) \rangle$$

or by

$$\langle N, (A_i), (u_i) \rangle$$

when $\succeq_i$ can be represented by a utility function $u_i : A \to \mathbb{R}$.

A two-player finite strategic game can be described conveniently in a bimatrix. For example, consider the $2 \times 2$ game

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