A Theory of Optimal Capital Taxation

Thomas Piketty, Paris School of Economics
Emmanuel Saez, UC Berkeley
November 2011
MOTIVATION: CAPITAL TAX THEORY FAILURE

1) Standard economic theory: (Atkinson-Stiglitz, Chamley-Judd) optimal tax rate $\tau_K = 0\%$ on all forms of capital taxes (stock- or flow-based) ⇒ Elimination of all inheritance, property, corporate, and capital income taxes desirable

2) Practice: European Union 27 countries: tax/GDP = 39% and capital tax/GDP=9%. US: tax/GDP = 27% and capital tax/GDP=8%

(inheritance tax/GDP < 1% but significant top rates)

⇒ No government seems to believe this extreme zero-capital tax result which indeed relies on very strong assumptions

3) Huge gap between theory and practice on optimal capital taxation is a major failure of modern public economics
MOTIVATION AND GOALS

With no inheritance (100% life-cycle wealth as in Atkinson-Stiglitz or infinite life as in Chamley-Judd) and perfect capital markets then $1 + r = \text{relative price of present consumption}$

$\Rightarrow \tau_K$ is not an efficient redistributive tool (relative to $\tau_L$) and case for $\tau_K = 0$ is strong

This Paper develops a realistic, tractable optimal capital tax theory based upon two ingredients:

1) **Inheritance:** life is not infinite and inheritance is a significant source of lifetime inequality

2) **Imperfect capital markets:** with uninsurable risk, lifetime capital tax is a useful addition to inheritance tax
KEY RESULTS

0) We develop a dynamic and tractable model of bequests with heterogeneous savings tastes and work abilities

1) We derive simple formulas for optimal inheritance tax rates expressed in terms of estimable parameters (elasticities, bequest flow, social preferences)

⇒ Our theory can account for the variety of observed top bequest tax rates

2) IN PROGRESS Uninsurable risk in individual rate of return on capital can easily explain why significant portion of inheritance tax is optimally partly shifted to capital income

⇒ Our theory can explain actual mix of inheritance vs. lifetime capital taxation [and why top inheritance and top capital income tax rates tend to be correlated]
Figure 1: Top Inheritance Tax Rates 1900-2011

- **U.S.**
- **U.K.**
- **France**
- **Germany**
Figure 2: Top Income Tax Rates 1900-2011
OUTLINE

0) Empirical Facts on Bequest Flows

1) Links with Previous Work

2) Inheritance Tax Model

(a) Basic Model and Optimal Formulas

(b) Extensions: nonlinear bequest tax, elastic labor supply, closed economy, life-cycle, social discounting

3) From Inheritance Taxation to Capital Taxation
EMPIRICAL FACTS: BEQUEST FLOW

\( b_y = B/Y \) = aggregate annual bequest flow \( B \) to national income \( Y \)

U-shape historical pattern in France (Piketty QJE’11)

a) Very large \( b_y \) \( \approx \) 20 – 25% in 19th century (rentier society)

b) Small \( b_y \) \( \approx \) 5% in post-WWII decades (Modigliani lifecycle)

c) Increasing \( b_y \) \( \approx \) 15% today \( \Rightarrow \) Inheritance matters again

U-shape probably less pronounced in US

Key driver of \( b_y \) is \( r - g \) (rate of return on K minus growth rate)

\( r >> g \) \( \Rightarrow \) inherited wealth capitalizes fast \( \Rightarrow b_y \) large

Optimal \( \tau_B \) is increasing with \( b_y \) (or \( r - g \))
Figure 4: Annual inheritance flow as a fraction of national income, France 1820-2008

- Economic flow (computed from national wealth estimates, mortality tables and observed age-wealth profiles)
- Fiscal flow (computed from observed bequest and gift tax data, inc. tax exempt assets)

LINK WITH PREVIOUS WORK

1) Atkinson-Stiglitz JpubE’76: No capital tax in life-cycle model with homogeneous tastes for savings, consumption-leisure separability, and optimal nonlinear labor income tax

2) Chamley EMA’86-Judd JpubE’85: No capital tax in the long-run in an infinite horizon model with homogenous discount rate

3) New Dynamic Public Finance: Capital tax desirable when uncertainty in future earnings ability affects savings decisions

4) Credit Constraints can restore desirability of capital tax to redistribute from the unconstrained to the constrained

5) Time Inconsistent Governments always want to tax existing capital
ATKINSON-STIGLITZ FAILS WITH INHERITANCES

A-S applies when sole source of lifetime income is labor:
\[ c_1 + c_2 / (1 + r) = \theta l - T(\theta l) \quad (\theta = \text{productivity}, \ l = \text{labor supply}) \]

Bequests provide an additional source of life-income:
\[ c + b(left) / (1 + r) = \theta l - T(\theta l) + b(\text{received}) \]

⇒ conditional on \( \theta l \), high \( b(left) \) is a signal of high \( b(\text{received}) \)
⇒ \( b(left) \) should be taxed even with optimal \( T(\theta l) \)

Two-dim. heterogeneity requires two-dim. tax policy tool

**Extreme example:** no heterogeneity in productivity \( \theta \) but pure heterogeneity in bequests motives ⇒ bequest taxation is desirable for redistribution
Dynastic model (each period is a generation) implies that inheritance tax rate $\tau_K = 0$ in the long-run for 2 reasons:

(1) If social welfare is measured by the discounted utility of the first generation then inheritance tax creates an infinitely growing distortion

Not a good social welfare criterion when each period is a generation and there is heterogeneity in tastes for bequests

(2) If social welfare is measured by long-run steady state utility then $\tau_K = 0$ because supply elasticity $e_B$ of bequests with respect to price is infinite

In our theory, $e_B$ is a free parameter
A GOOD THEORY OF OPTIMAL K TAXATION

Should follow the optimal labor income tax progress and hence needs to capture key trade-off robustly:

1) Welfare effects: people dislike taxes on bequests they leave, or inheritances they receive, but people also dislike labor taxes ⇒ trade-off

2) Behavioral responses: bequest taxes might discourage wealth accumulation (but labor taxes might discourage labor supply)

3) Results should be robust to heterogeneity in tastes and motives for bequests within the population

4) Formulas should be expressed in terms of estimable sufficient statistics
AGENT MODEL: MICRO LEVEL

Agent $i$ in cohort $t$ ($1$ cohort = $1$ period = $H$ years)

Receives bequest $b_{ti} = z_ib_t$ at beginning of period $t$ where $b_t$ average bequest and $z_i$ (normalized) bequest received

At the end of period $t$, individual receives (inelastic) labor income $y_{Lti} = \theta_i y_{Lt}$, consumes $c_{ti}$, and leaves bequest $b_{ti+1}$ to unique child so as to maximize:

$$V^i(c_{ti}, b_{ti+1}, \bar{b}_{ti+1}) \quad \text{s.c.} \quad c_{ti} + b_{ti+1} \leq (1 - \tau_B)b_t z_i e^{rH} + (1 - \tau_L)y_{Lt} \theta_i$$

$\tau_B$ = bequest tax rate, $\tau_L$ = labor income tax rate

$b_{ti+1}$ = end-of-life wealth (wealth loving)

$\bar{b}_{ti+1} = (1 - \tau_B)b_{ti+1} e^{rH}$ = net-of-tax capitalized bequest left (bequest loving)

$V^i$ homogeneous of degree one (to allow for growth)
MODEL: MICRO LEVEL PREFERENCES

1) Special Case Cobb-Douglas preferences:

\[ V^i(c_{ti}, b_{t+1}^i, \bar{b}_{t+1}^i) = c_{ti}^{1-s_i} b_{t+1}^{s_{wi}} \bar{b}_{t+1}^{s_{bi}} \text{ with } s_i = s_{wi} + s_{bi} \]

\[ \Rightarrow b_{t+1}^i = s_i \cdot [(1 - \tau_B)b_t z_i e^{rH} + (1 - \tau_L)y_{Lt} \theta_i] = s_i \cdot \tilde{y}_{ti} \]

2) General preferences \( V^i() \) homogeneous of degree one:

\[ V^i(c_{ti}, b_{t+1}^i, (1-\tau_B)e^{rH}b_{t+1}^i) \Rightarrow \text{ FOC } V^i_c = V^i_b + (1-\tau_B)e^{rH}V^i_b \]

All choices are linear in total life-time income \( \tilde{y}_{ti} \)

\[ \Rightarrow b_{t+1}^i = s_i(e^{rH}(1 - \tau_B)) \cdot [(1 - \tau_B)b_t z_i e^{rH} + (1 - \tau_L)y_{Lt} \theta_i] \]

Define \( s_{bi}(e^{rH}(1 - \tau_B)) = s_i \cdot (1 - \tau_B)e^{rH}V^i_b/V^i_c \)

Same as Cobb-Douglas but \( s_i \) and \( s_{bi} \) now depend on \( 1 - \tau_B \)
Open economy with exogenous return $r$ and growth rate $g$

Inelastic labor income $y_{Lt} = y_{L0}e^{gHt}$

Domestic output $y_t = K_t^\alpha L_t^{1-\alpha}$ so that $y_{Lt} = y_t \cdot (1 - \alpha)$ where $1 - \alpha$ is labor share

Period by Period Government budget constraint:

$$\tau_L y_{Lt} + \tau_B b_t e^{rH} = \tau y_t \quad \text{i.e.,} \quad \tau_L (1 - \alpha) + \tau_B b_{yt} = \tau$$

With $\tau =$ exogenous tax revenue requirement

$$b_{yt} = e^{rH} b_t / y_t = \text{inheritance-output ratio}$$

$\tau_L$ is a function of $\tau_B$ to satisfy the budget constraint
EQUIVALENCE BETWEEN $\tau_K$ and $\tau_B$

In basic model, tax $\tau_B$ in inheritance is equivalent to tax $\tau_K$ on annual return $r$ to capital as:

$$\bar{b}_{ti} = (1 - \tau_B)b_{ti}e^{rH} = b_{ti}e^{r(1-\tau_K)H} \quad \text{i.e.,} \quad \tau_K = -\frac{\log(1 - \tau_B)}{rH}$$

E.g., with $r = 5\%$ and $H = 30$, $\tau_B = 25\% \Leftrightarrow \tau_K = 19\%$, $\tau_B = 50\% \Leftrightarrow \tau_K = 46\%$, $\tau_B = 75\% \Leftrightarrow \tau_K = 92\%$

E.g., with $r = 3\%$ and $H = 30$, $\tau_B = 25\% \Leftrightarrow \tau_K = 32\%$, $\tau_B = 50\% \Leftrightarrow \tau_K = 77\%$, $\tau_B = 75\% \Leftrightarrow \tau_K = 154\%$

This equivalence no longer holds with (a) tax enforcement constraints, or (b) life-cycle savings, or (c) insurable risk in $r$

Optimal mix $\tau_B, \tau_K$ then becomes interesting question (see extensions)
MODEL: NO MEMORY SIMPLIFICATION

$\theta_i, s_i, s_{bi}/s_i$ are i.i.d. across and within periods

$\Rightarrow s = E(s_i|\theta_i, b_{ti}), \ s_b = E(s_{bi}|\theta_i, b_{ti}) \Rightarrow$ simple agg. transition:

$$b_{t+1i} = s_i \cdot [(1 - \tau_B)b_{tz} e^{rH} + (1 - \tau_L)y_{Lt}\theta_i]$$

$\Rightarrow b_{t+1} = s \cdot [(1 - \tau_B)b_t e^{rH} + (1 - \tau_L)y_{Lt}]$

Steady-state convergence $b_{t+1} = b_t e^{gH}$:

$$\Rightarrow b_y = \frac{b_t e^{rH}}{y_t} = \frac{s(1 - \alpha - \tau)e^{(r-g)H}}{1 - se^{(r-g)H}}$$

$b_y$ increases with $r - g$ (capitalization effect, Piketty QJE'11)

$r - g = 3\%, \tau = 10\%, H = 30, \alpha = 30\%, s = 10\% \Rightarrow b_y = 20\%$

$r - g = 1\%, \tau = 30\%, H = 30, \alpha = 30\%, s = 10\% \Rightarrow b_y = 6\%$
MODEL: STEADY STATE CONVERGENCE

With $V^i()$ homogenous of degree one and no memory

**Unique steady-state:** for given $\tau_L, \tau_B$, as $t \to +\infty$, $b_{yt} \to b_y$ and distribution of (normalized) inheritance $z$ converges to $\Psi(z)$

Define:

$$e_B = \frac{1 - \tau_B}{b_y} \frac{db_y}{d(1 - \tau_B)}\bigg|_{\tau}$$

$e_B = \text{elasticity of steady-state bequest flow with respect to net-of-bequest-tax rate } 1 - \tau_B$

Cobb-Douglas preferences $\Rightarrow e_B = 0$

For general preferences, $e_B > 0$ (or $< 0$)

$e_B$ is a free parameter in our model
MODEL: GOVERNMENT OBJECTIVE

Government chooses $\tau_B, \tau_L$ to maximize **steady-state** social welfare

$$SWF = \int \omega^i V^i d\Psi(z) dF(\theta)$$

with $\Psi(z)$ cdf of (normalized) inheritance $z$ and $F(\theta)$ cdf of labor productivity $\theta$

subject to budget balance constraint

$$\tau_L y_{Lt} + \tau_B b_t e^{rH} = \tau y_t$$

Consider small $d\tau_B > 0$, can cut $d\tau_L < 0$ by:

$$-y_{Lt} d\tau_L = d\tau_B b_t e^{rH} \left(1 - e_B \frac{\tau_B}{1 - \tau_B}\right)$$
SIMPLIFICATION ASSUMPTIONS LATER RELAXED

0) No Memory in $\theta_i, s_i, s_{bi}/s_i$ processes

1) Linear inheritance tax

2) Inelastic labor supply

3) No lumpsum demogrant

4) Small open economy with fixed $r$

5) No Life-cycle Saving

6) No government debt and steady-state welfare objective

7) Homogeneous $r$ across individuals
OPTIMAL INHERITANCE TAX RATE

**Meritocratic Rawlsian Optimum:** maximize welfare of those receiving no inheritance

\[
\tau_B = \frac{1 + s_b - (s_b/s)e^{-(r-g)H}}{1 + s_b + e_B}
\]

where \( s \) is average savings taste, \( s_b \) bequests savings tastes

\( \tau_B \downarrow \) with \( e_B \) and \( s_b \) (as \( se^{(r-g)H} < 1 \))

If \( s_b = 0 \) then \( \tau_B = 1/(1 + e_B) \) (revenue maximizing rate)

If \( e_B = \infty \) then \( \tau_B = 0 \) (Chamley-Judd)

Even if \( e_B = 0 \), we have \( \tau_B < 1 \) as long as \( s_b > 0 \)

\( \tau_B \uparrow \) with \( r - g \): Taxing bequests raises \( \tau_B b_t e^{rH} \) from inheritors in my cohort but costs \( \tau_B b_t e^{gH} \) to what I leave to my child
OPTIMAL TAX RATE: NUMERICAL EXAMPLES

\[
\tau_B = \frac{1 + s_b - (s_b/s)e^{-(r-g)H}}{1 + s_b + e_B}
\]

0) Base Case: \( r = 5\%, g = 2\%, H = 30, e^{-(r-g)H} = 40\%, e_B = 0, \ s_b = s = 10\% \Rightarrow \tau_B = 63\% \) (or \( \tau_K = 66\% \))

1) If \( s_b/s = .5 \) (bequests half accidental) \( \Rightarrow \tau_B = 81\% \) (or \( \tau_K = 110\% \))

2) If \( g = 4\% \) (post WWII reconstruction) \( \Rightarrow \tau_B = 33\% \) (or \( \tau_K = 27\% \))

3) If \( e_B = 0.5 \) (high elasticity) \( \Rightarrow \tau_B = 43\% \) (or \( \tau_K = 37\% \))

Optimal \( \tau_B \) independent of \( \tau \) (revenue requirement)
**OPTIMAL INHERITANCE TAX RATE**

Optimal tax formula can be also be expressed using bequest flow $b_y$

$$b_y = \frac{b_t e^{rH}}{y_t} = \frac{s(1 - \alpha - \tau)e^{(r-g)H}}{1 - s e^{(r-g)H}}$$

as

$$\tau_B = \frac{1 + s_b - \frac{sb}{s}e^{-(r-g)H}}{1 + s_b + e_B} = \frac{1 - (1 - \alpha - \tau)s_b/b_y}{1 + e_B + s_b}$$

$\tau_B$ increases with $b_y$ (and decreases with $s_b$): Taxing bequests raises $\tau_B b_y y_t$ from inheritors in my cohort but costs $\tau_B s_b \cdot (1 - \tau_L) y_t = \tau_B s_b \cdot (1 - \alpha - \tau + \tau_B b_y) y_t$ to what I leave to my child

$b_y$ formula easier to calibrate with instantaneous variables than $r - g$ formula (see life-cycle extension)
OPTIMAL TAX DERIVATION (Part 1)

\[
\max_{b_{t+1}} V_i((1 - \tau_L)y_{Lt}\theta_i - b_{t+1}, b_{t+1}, (1 - \tau_B)e^{rH}b_{t+1})
\]

Effect of \(d\tau_B > 0, d\tau_L < 0\) on \(V_i\) using envelope theorem

\[
dV^i = -V_c^i y_{Lt} \theta_i d\tau_L - V_b^i b_{t+1} e^{rH} d\tau_B = V_c^i \left[-y_{Lt} \theta_i d\tau_L - \frac{d\tau_B}{1 - \tau_B} \frac{s_{bi}}{s_i} b_{t+1}\right]
\]

Using budget balance equation and \(b_{t+1} = x_i b_t e^{gH}\), we get:

\[
dV^i = d\tau_B e^{rH} b_t V_c^i \left[\theta_i \left(1 - e_B \frac{\tau_B}{1 - \tau_B}\right) - \frac{(s_{bi}/s_i)x_i e^{-(r-g)H}}{1 - \tau_B}\right]
\]

Using no memory assumption \(x_i \perp (s_{bi}/s_i)\) (and the fact that \(\omega^i V_c^i\) is constant among zero-receivers):

\[
\int_{\text{zero receivers}} \omega^i dV^i = 0 \Rightarrow \tau_B = \frac{1 - e^{-(r-g)H}(s_{b}/s)x_0}{1 + e_B}
\]

where \(x_0 = \text{mean (normalized) bequest left of zero-receivers}\)
OPTIMAL TAX DERIVATION (Part 2)

Under no memory, zero-receivers have same $s$ and $y_{Lti}$ than average so relative bequests they leave is

$$x_0 = \frac{y_{Lt}(1 - \tau_L)}{y_{Lt}(1 - \tau_L) + b_t e^{rH}(1 - \tau_B)}$$

Using

$$b_y = \frac{(1 - \alpha)b_te^{rH}}{y_{Lt}} = \frac{s(1 - \alpha - \tau)e^{(r-g)H}}{1 - se^{(r-g)H}}$$

We get

$$x_0 = 1 - (1 - \tau_B)se^{(r-g)H}$$

hence

$$\tau_B = \frac{1 - e^{-(r-g)H}(s_b/s)x_0}{1 + e_B} \Rightarrow \tau_B = \frac{1 + s_b - (s_b/s)e^{-(r-g)H}}{1 + s_b + e_B}$$
OPTIMAL TAX FOR $z_p$-RECEIVERS

Optimum tax rate for receivers at percentile $p$ (of $z$ distribution) is:

$$\tau_B = \frac{1 + s_b - (s_b/s)e^{-(r-g)H} - (1 + s_b + e_B + e_z)z_p}{(1 + s_b + e_B)(1 - z_p) - z_pe_z}$$

$\tau_B \downarrow$ with $z_p$ as taxing bequests has a direct impact on inheritances received ($e_z$ is elasticity of percentile $z_p$ wrt $1 - \tau_B$)

Large inheritors ($z_p > 1$) want bequest subsidy as large as possible

Model allows double counting as taxing bequests hurts both donors ($s_b$ terms) and inheritors ($z_p$ terms)

Distribution of inheritances highly concentrated: bottom 50% inheritors receive 5% of inheritances $\Rightarrow$ Bottom 50%-receivers optimum close to zero-receivers optimum
OPTIMAL TAX FOR GENERAL SWF

Optimum tax rate is:

$$\tau_B = \frac{1 + s_b - (s_b/s)e^{-(r-g)H} - (1 + s_b + e_B)\bar{z}/\bar{\theta}}{(1 + s_b + e_B)(1 - \bar{z}/\bar{\theta})}$$

where $\bar{\theta}$ is average labor ability $\theta$, $\bar{z}$ is average inheritance $z$ received, all weighted by social marginal welfare weights $g_i = \omega_i V^i_c$

This formula nests all the previous ones but $\bar{\theta}$ and $\bar{z}$ are endogenous to $\tau_B$.

If $\bar{z} << \bar{\theta}$ then close to zero-receivers optimum.

Perceptions about wealth inequality and mobility matter a lot:

If bottom receivers expect to leave large bequests, then they may prefer low bequest tax rates $\Rightarrow$ critical to estimate the right distributional parameters.
EXTENSION: MEMORY

Suppose $s_i, s_{bi}/s_i, \theta_i$ are correlated within and across cohorts.

Steady-state $b_y, \Psi(z, \theta)$ still exists under adequate ergodicity assumptions.

Formula for $b_y$ carries over but $s$ is savings rate weighted by life-time resources:

$$\tilde{y}_{ti} = (1 - \tau_B) b_t z_i e^{rH} + (1 - \tau_L) y_{Lt} \theta_i$$

$$\Rightarrow b_y = \frac{b_t e^{rH}}{y_t} = \frac{s(1 - \alpha - \tau)e^{(r-g)H}}{1 - se(r-g)H} \quad \text{with} \quad s = \frac{\int s_i \cdot \tilde{y}_{ti}}{\int \tilde{y}_{ti}}$$

Optimum tax formula becomes

$$\tau_B = \frac{1 + s_{b0} - (s_{b0}/s)e^{-(r-g)H}}{1 + s_{b0} + e_B}$$

with $s_{b0}$ average of $s_{bi}$ weighted by life-time resources among zero-receivers.
EXTENSION: NONLINEAR BEQUEST TAX

Marginal tax rate $\tau_B$ above $b^*_t = \bar{xb}_t$ (and 0 below)

$$\text{Optimum } \tau_B = \frac{1 - e^{-(r-g)H(s_b/s)((x-\bar{x})^+/(x-\bar{x})^+)}}{1 + a \cdot e^*_B}$$

where $a \simeq 1.5$ is Pareto parameter of bequest distribution

$e^*_B$ is elasticity of taxable bequests with respect to $1 - \tau_B$

**Rentier Society:** $x$ thicker tail than $\theta \Rightarrow$ zero receivers hardly ever leave bequests above $b^*_t = \bar{xb}_t$ then $\tau_B \simeq 1/(1 + a \cdot e^*_B)$ [revenue max. top rate]

**Self-Made Wealth:** zero receivers can build large fortunes (and love bequests) then $\tau_B < 1/(1 + a \cdot e^*_B)$

Note: fully nonlinear schedule is intractable (as local MTR change affects full bequest distribution in ergodic equilibrium)
EXTENSION: ELASTIC LABOR SUPPLY

Utility \( \log V^i(c, b, \bar{b}) - h(l) \) with \( y_{Li} = \theta_i w_t l \)

Aggregate labor supply has elasticity \( e_L \) wrt to \( 1 - \tau_L \)

Tax reform \( d\tau_L, d\tau_B \) with budget balance \( \Rightarrow \):

\[
-y_{Lt} \left( 1 - e_L \frac{\tau_L}{1 - \tau_L} \right) d\tau_L = d\tau_B b_t e^{rH} \left( 1 - e_B \frac{\tau_B}{1 - \tau_B} \right)
\]

Easy to obtain the optimum tax \( \tau_B \):

\[
\tau_B = \frac{1 + \left( 1 - \frac{\tau e_L}{1 - \alpha - \tau} \right) \left( s_b - (s_b/s) e^{-(r-g)H} \right)}{1 + s_b \cdot (1 + e_L) + e_B}
\]

\( \tau_B \uparrow \) with \( e_L \) as labor tax is more costly with \( e_L > 0 \) (if \( \tau \) not too small)

Note that \( e_L, e_B \) are GE elasticities where both \( \tau_B \) and \( \tau_L \) change
EXTENSION: ELASTIC LABOR SUPPLY

\[ \tau_B = \frac{1 + (1 - \frac{\tau e_L}{1 - \alpha - \tau}) \left( s_b - (s_b/s) e^{-(r-g)H} \right)}{1 + s_b \cdot (1 + e_L) + e_B} \]

Race between \( e_L \) and \( e_B \)

0) Base Case: \( r = 5\%, g = 3\%, H = 30, e^{-(r-g)H} = 40\%, \alpha = 30\%, \tau = 30\%, s_b = s = 10\%, b_y = 13\%, e_B = 0, e_L = 0 \)
\( \Rightarrow \tau_B = 63\% \) (or \( \tau_K = 66\% \)), \( \tau_L = 31\% \)

1) If \( e_L = 0.5, e_B = 0 \) \( \Rightarrow \tau_B = 70\% \) (or \( \tau_K = 80\% \)), \( \tau_L = 30\% \)

2) If \( e_L = 0, e_B = 0.5 \) \( \Rightarrow \tau_B = 43\% \) (or \( \tau_K = 37\% \)), \( \tau_L = 35\% \)

3) If \( e_L = 0.5, e_B = 0.5 \) \( \Rightarrow \tau_B = 49\% \) (or \( \tau_K = 45\% \)), \( \tau_L = 34\% \)

Optimal \( \tau_B \) now depends on \( \tau \) (revenue requirement)
EXTENSION: LUMPSUM DEMOGRAUNT

Assume bequest taxes fund a demogrant universal transfer $E_t = E_0 e^{gHt}$ Government budget

$$\tau_B b_t e^{rH} = E_t \quad \text{and} \quad \tau_L y_L t = \tau y_t \quad \text{fixed}$$

Assume that $d\tau_B > 0$ is used to fund $dE > 0$ then zero-receivers optimum (assuming inelastic labor supply) is:

$$\tau_B = \frac{1 + s_b - (s_b/s)e^{-(r-g)H}}{1 + s_b + e_B}$$

Same formula as before as govt does not value redistribution within zero-receivers (for general SWF, just replace $\bar{\theta}$ by 1 in formula)

With elastic labor supply, get a formula that involves labor supply income effects
EXTENSION: CLOSED ECONOMY

Suppose economy is closed and capital stock is supplied by inheritances

Production $F(b_t, L_t) = R b_t + w L_t$ with return $R = F_K$ and wage $w = F_L$ endogenous

After-tax price of factors $1 + \bar{R} = (1 + R)(1 - \tau_B)$ and $\bar{w} = w \cdot (1 - \tau_L)$

$\Rightarrow \tau_B, \tau_L$ allow government to fully control after-tax prices

$\Rightarrow$ Optimal tax formulas continue to apply as in open economy with $e_B, e_L$ being the supply elasticities (keeping $R$ and $w$ fixed) as in the standard Diamond-Mirrlees (1971) model
EXTENSION: LIFE CYCLE

Possible to extend model to continuous overlapping generations with life duration \( D \) and utility \( V(U, b, \bar{b}) \) with

\[
U = \left[ \int_0^D e^{-\delta t} c_t^{1-\gamma} dt \right]^{\frac{1}{1-\gamma}} \Rightarrow V(U, b, \bar{b}) = V(\mu \tilde{c}, b, \bar{b})
\]

with \( \tilde{c} = \) capitalized lifetime consumption (at end of life)

Individual budget: \( \tilde{c} + b_{t+H_i} = (1 - \tau_B)b_t e^{rH} + (1 - \tau_L)\bar{y}_{Lt_i} \)

Govt budget **continuously** balanced: \( \tau_L Y_{Lt} + \tau_B B_t = \tau Y_t \)

Optimal \( \tau_B = \frac{1 + s_b/\lambda - (s_b/(\lambda \cdot s)) e^{-(r-g)H}}{1 + s_b/\lambda + e_B} \)

Replace \( s_b \) by \( s_b/\lambda \) where \( \lambda \) is an exogenous factor correcting for when inheritances are received relative to labor income: \( \lambda = 1 \) if inheritances are **realistically** received in mid-adult life (\( \lambda > 1 \) if before mid-life)
EXTENSION: LIFE CYCLE AND TIMING OF TAXES

Optimal $\tau_B$ in discrete model depends on timing of taxes in govt budget

\begin{align*}
(0) \quad \tau_LyLt + \tau_Bb_t e^{rH} &= \tau y_t \quad \text{vs.} \quad (1) \quad \tau_LyLt + \tau_Bb_{t+1} = \tau y_t
\end{align*}

(0) was our initial model, (1) leads to a formula for $\tau_B$ where $s_b$ is replaced by $s_b e^{(r-g)H}$ (hence $\tau_B$ much lower)

No good way to decide between (0) and (1) in discrete model

Life cycle model with realistic continuous budget balance and empirically realistic $\lambda = 1$ implies that (0) is the correct specification
GOVT. DEBT AND CAPITAL ACCUMULATION

Suppose govt maximizes inter-temporal, infinite-horizon SWF

In closed economy, optimum capital stock should be given by modified Golden Rule:

\[ f'(k) = r^* = \delta + \Gamma g \]

where \( \delta \geq 0 \) is discount rate of government, \( \Gamma \) is curvature of SWF, and \( g \) is growth rate

If govt can use debt, then govt can achieve modified Golden Rule (for any tax structure)

In that case, long-run optimal \( \tau_B \) is given by a formula similar to static one (when \( \delta \to 0 \)): capital accumulation is orthogonal to redistributive bequest taxation

If govt cannot use debt, capital stock may be too large or too small and optimal formula for \( \tau_B \) needs to be corrected
FROM INHERITANCE TAX TO LIFETIME K TAX

1) With perfect K markets, it’s always better to have a big tax $\tau_B$ on bequest, and zero lifetime capital tax $\tau_K = 0$, so as to avoid inter-temporal distortion.

2) However in the real world most people prefer paying a property tax of 1% during 30 years rather than a big bequest tax $\tau_B = 30\%$.

3) Total K taxes = 9% GDP, but bequest tax < 1% GDP.

4) In our view, the collective choice in favor of lifetime K taxes is a rational consequence of K markets imperfections, not necessarily of tax illusion.
FUZZY FRONTIER BT CAPITAL AND LABOR

Tax $\tau_K$ on generation return $R$, net bequest is

$$\bar{b}_{ti} = b_{ti}(1 - \tau_B)(1 + R(1 - \tau_K)) \quad \text{with} \quad R = e^{rH} - 1$$

$\tau_B, \tau_K$ is equivalent to $\bar{\tau}_B, \tau_K = 0$ with

$$\bar{\tau}_B = \tau_B + (1 - \tau_B)\tau_K \frac{R}{1 + R}$$

Simplest imperfection: fuzzy frontier between capital income and labor income flows, can be manipulated by taxpayers (self-employed, top executives, etc.)

With fully fuzzy frontier, then govt has to set $\tau_K = \tau_L$ (capital income tax rate = labor income tax rate)

Adjust $\tau_B$ down to keep total tax $\bar{\tau}_B$ the same as before

Bequest tax $\tau_B > 0$ is optimal iff $\bar{\tau}_B$ sufficiently large $\Rightarrow$ comprehensive income tax + bequest tax = what we observe in many countries
UNINSURABLE UNCERTAINTY IN RETURN $R$

Uninsurable uncertainty about future rate of return:

What matters is $b_{ti} e^{r_{ti} H}$ not $b_{ti}$

but at the time of setting the bequest tax rate $\tau_B$, nobody knows what the rate of return $1 + R_{ti} = e^{r_{ti} H}$ is going to be during the next 30 years (idiosyncratic risk + aggregate uncertainty)

$\Rightarrow$ with uninsurable idiosyncratic shocks on returns $r_{ti}$, more efficient to split the tax burden between one-off transfer taxes and lifetime capital taxes

With no moral hazard on $r_{ti}$, 100% tax on $r_{ti}$ (and corresponding reduction in $\tau_B$) is optimal
MORAL HAZARD IN RATE OF RETURN R

Assume rate of return $R_{ti} = \epsilon_{ti} + e_{ti}$

With: $\epsilon_{ti} = \text{i.i.d. random shock with mean } R_0$

$e_{ti} = \text{effort put into portfolio management (how much time one spends checking stock prices, looking for new investment opportunities, monitoring one’s financial intermediary, etc.)}$

$c(e_{ti}) = \text{convex effort cost proportional to portfolio size}$

Define $e_R = \text{elasticity of aggregate rate of return } R \text{ with respect to net-of-capital-income-tax rate } 1 - \tau_K$

If returns mostly random (effort parameter small as compared to random shock), then $e_R$ close to zero

Conversely if effort matters a lot, then $e_R$ large
MORAL HAZARD IN RATE OF RETURN R

Depending on parameters, optimal capital income tax rate $\tau_K$ can be $>$ or $<$ than labor income tax rate $\tau_L$

If $e_R$ small enough and/or by large enough, then $\tau_K > \tau_L$ (=what we observe in UK and US during the 1970s)

Examples: $\tau = 30\%, \alpha = 30\%, s = s_b = 10\%, r = 4\%, g = 2\%, e_B = e_L = 0$

If $e_R = 0$, then $\tau_K = 100\%, \tau_B = 9\%, \tau_L = 34\%$

If $e_R = 0.1$, then $\tau_K = 78\%, \tau_B = 35\%, \tau_L = 35\%$

If $e_R = 0.5$, then $\tau_K = 17\%, \tau_B = 56\%, \tau_L = 37\%$
CONCLUSION

1) Main contribution: simple, tractable formulas for analyzing optimal tax rates on inheritance and capital

2) Main idea: economists’ emphasis on $1 + r = \text{relative price}$ is excessive (intertemporal consumption distortions exist but are probably second-order)

3) The important point about the rate of return to capital $r$ is that

   a) $r$ is large: $r > g \Rightarrow$ tax inheritance, otherwise society is dominated by rentiers

   b) $r$ is volatile and unpredictable $\Rightarrow$ use lifetime K taxes to implement optimal inheritance tax