1. Jive Record sells Britney Spears new album In the Zone, over two periods. The aggregate demand for the album is \( Q(P) = 25 - P \), where \( Q \) is measured in millions of CDs. This means one million people willing to pay as much as $25 for the album, while two million would pay $24, and so on down to 25 million willing to pay $1. Marginal cost of producing and distributing a CD is $5.

   a) What price will the record label set in the first period if it sets the monopoly price? Draw the demand, marginal revenue, marginal costs and the monopoly price and quantity for the first period.

   \[
   MR = 25 - 2Q_1 \quad MC = 5
   \]

   \[
   MR = MC \\
   25 - 2Q_1 = 5 \\
   2Q_1 = 20 \\
   Q_1 = 10, \quad P_1^* = 15
   \]

   b) Given your price in part a), what is the demand the record label faces in the second period? [Hint: which type of consumers did not purchase the CD in the first period?]

   In the first period, only people with valuations greater than $15 would have purchased the CD. The demand in the second period is then just the demand in the first period below 15:

   \[
   Q_2(P) = 15 - P
   \]

   c) Show that the monopoly price it sets in the second period is $10.

   \[
   MR_2 = 15 - 2Q_2 \quad MC_2 = 5
   \]

   \[
   MR = MC \\
   15 - 2Q_2 = 5 \\
   2Q_2 = 10 \\
   Q_2 = 5 \quad \text{and} \quad P_2^* = 10
   \]
d) If consumers have zero costs to waiting, will the record label be able to charge the $P = 15$ in period 1? Explain.

If the record label sets a price of $15 in the first period, then consumers can expect to pay $10 in the second period. If there is no cost to waiting, however, consumers in the first period would not be willing to pay $15 given they can pay $10 in the second period.

e) What would happen if the record label tried to set a price of $10 in period 1?

Again, setting a price of $10 in the first period makes the demand in the second period $Q = 10 - P$. Given this, the firm’s best price in the second period would be $7.50. But the same problem persists, consumers can simply wait until the second period for the lower price.

f) Qualitatively, what do you think allows a producer of a durable good to be able to make a profit even if consumers know the producer will lower the price in the future?

Clearly waiting costs are important. If some consumers value having the album now, the record label can set high price today and a lower price next month, these consumers will purchase as long as the price drop is smaller than the cost of waiting.

2. Suppose that American beer production is a perfectly competitive industry. Market demand for these beers (in millions of barrels per year) is given by:

$$D(p) = 221 - p$$

where $p$ is the price per barrel. Every brewer has the same cost function which is given by:

$$C(q) = 100 + q + q^2.$$ 

In the short-run, there are 10 firms in the industry.

a) Graph the marginal cost and average cost curves for a brewer.

$$AC = 100/q + 1 + q, MC = 1 + 2q$$

b) Does beer making exhibit scale economies, and if so, over what range of production?

Scale economies exist up to the point where $AC = MC$:

$$100/q + 1 + q = 1 + 2q$$

$$100 + q + q^2 = q + 2q^2$$

$$q^2 = 100$$

$$q_{\text{max}} = 10$$

Therefore, $AC$ are falling up until $q = 10$.

c) Derive the expression for the supply curve for an individual brewer (for the 10 firms in the industry, you may assume that the fixed cost has been sunk). Identify the supply curve on the graph from (a).

Since the fixed cost portion of costs are sunk, firms only care about marginal costs above average variable costs, which is true for $q > 0$. The supply curve for an individual firm is where $P = MC$:

$$P = 1 + 2q_i$$

for $q > 0$.

d) Give the expression for the short-run industry supply curve.

Since there are 10 firms, horizontally sum the quantities.

$$q_i = \frac{1}{2}P - \frac{1}{2}$$

$$Q_{\text{total}} = \sum q_i = 10 \times q_i = 5P - 5$$

Inverse demand is:

$$P = 1 + 1/5Q$$ (note that the intercept is the same and the slope is shallower).

e) What is the short-run equilibrium price and quantity.

Where $Q_d = Q_s$

$$5P - 5 = 221 - P$$

$$6P = 226$$

$$P = 226/6 = 38$$

$$Q = 5(38) - 5 = 185$$

$$q_i = 185/10 = 18.5$$

f) Is the industry in its long-run equilibrium? If so, explain why. If not, what are the long-run equilibrium price, quantity, and number of brewers?

No, since $q_i > 10 = q_{\text{max}}$ the industry cannot be in the long run equilibrium.

In the long run:

i) $q_i = 10$ and $P = MC = 21$

ii) $Q_d(P = 21) = 221 - 21 = 200$
iii) If the long run industry quantity is 200, then the long run number of firms will be 200/10 = 20.

Discuss:

1. Perfect competition assumes firms are price takers (that is, an individual firm does not have the ability to raise prices). Is this a fair assumption for the beer industry today? Explain. Even the mass produced beer spend large amounts on advertising trying to differentiate their product and give the brand the ability to raise prices. For the major brands in the mass beer market (Coors, Miller and Bud), however, costs are the major driving factor. Microbreweries attempt (and succeed) at created differentiated products in which the price taking assumption may not hold.

2. Over the last 50 years, concentration in the beer industry has risen. Has this increase in concentration been bad for consumers? Give reasons for and against.

Bad for consumers: The increase in concentration may mean greater ability to raise prices. Good for consumers: Clearly, even with additional market power, the minimum efficient scale as well as the lower marginal costs has made the industry more efficient.

3. The demand you face in Market X is given by \( P = 100 - Q \). You have constant marginal costs of 20 and fixed cost of $180.

   a) What is the optimal price and quantity to set?

   Given this demand, set the monopoly price
   \[
   \text{MR} = 100 - 2Q = \text{MC} = 20
   \]
   
   \[\begin{align*}
   2Q &= 80 \\
   Q &= 40, \ P = 60
   \end{align*}\]

   Profits
   \[
   (P - c)Q - F = (60-20)40 - 180 = 1600 - 180 = 1420
   \]

   b) Calculate your percentage markup \( (P- mc)/P \). Separately, calculate the elasticity of demand at the price and quantity from part a. Does \( (P- mc)/P = 1/\eta \)? (hint: it should).

   \[
   \eta = -(dQ/dP)(P/Q), \text{ since } dQ/dP = -1
   \]
   \[
   = -(1)60/40 = 3/2 \Rightarrow 1/\eta = 2/3
   \]

   c) Actually, your demand in Market X is \( P = 120 - Q - 20N \), where \( N \) is the number of firms in the market (including your firm). Suppose that \( N = 2 \), what happens to the price you set, your profits and the markup?

   At \( N = 2 \):
   \[
   P = 120 - Q - 40 = 80 - Q
   \]

   MR = 80 - 2Q

   Set equal to MC
   
   \[\begin{align*}
   80 - 2Q &= 20 \\
   Q &= 30 \text{ and } P = 50
   \end{align*}\]

   Profits
   
   \[= (50 - 20)30 - 180 = 900 - 180 = 720\]

   Clearly, the mark-up has gone down.

   d) In period 1, you are the only firm to have entered Market X. If other firms face the same demand as you do \( (P = 120 - Q - 20N) \), would you expect another firm to enter? (hint: look at your answer in part c).

   Since at \( N = 2 \), profits for each firm are greater than zero, you would expect another firm to enter to take advantage of the profits.

   e) Note that this game has elements of monopoly and perfect competition. You set a monopoly price based off your residual demand, but entry erodes profits. In the long run, how many firms should there be in this industry?

   At \( N = 3 \), the demand each firm faces is \( P = 60 - Q \). \( P^M = 40 \). Profits \( (40-20)20 = 200 - 180 = 20 \). Since at \( N = 4 \), each firms makes negative profits. We’d expect 3 firms to be in the market.

4. Firms 1 and 2 each produce a single product, also called 1 and 2, which have the following (inverse) demand curves:

   \[
P(q_1, q_2) = 120 - q_1 - q_2
   \]

   Each firm faces zero marginal costs and has fixed costs of \( F = 900 \).

   a) Show that firm 1’s best response curve will be: \( r_1(q_2) = 60 - \frac{1}{2}q_2 \).

   Maximize firm 1’s profits for a given \( q_2 \):

   

Econ 121-Fall 2004 Page 3 Problem Set 1 Answers
\[
\pi = P(q_1, q_2) \times q_1 - 900 \\
= (120 - q_1 - q_2)q_1 - 900 \\
= 120q_1 - q_1^2 - q_2q_1 - 900 \\
\frac{d\pi}{dq_1} = 120 - 2q_1 - q_2 = 0 \\
2q_1 = 120 - q_2 \\
r_1(q_2) = q_1^* \\
= 60 - \frac{1}{2}q_2
\]

b) Draw the best response curves for both firms with \( q_1 \) on the x-axis and \( q_2 \) on the y-axis.

(\text{graph drawn below})

c) Solve for the Cournot equilibrium. Show this point on your graph.

\[
q_2 = r_2(q_1) = 60 - \frac{1}{2}q_1 \\
= 60 - \frac{1}{2}(60 - \frac{1}{2}q_2) \\
= 60 - 30 + \frac{1}{4}q_2 \\
(1-\frac{1}{4})q_2 = 60 - 30 \]

\[
q_2 = (60 - 30b) / (1-\frac{1}{4}) \]

\[
q_2 = 30/(3/4) = 40.
\]

Now consider a slightly different form of the same question: Firms 1 and 2 each produce a single product, also called 1 and 2, which have the following (inverse) demand curves:

\[
P_1(q_1, q_2) = 120 - q_1 - bq_2 \\
P_2(q_2, q_1) = 120 - q_2 - bq_1
\]

where \( 0 < b < 2 \). Each firm faces zero marginal costs and has fixed costs of \( F = 900 \).

d) Explain how you would use information about the value of demand parameter \( b \) to decide whether the two products were in the same economic market.

Consider what happens as \( b \) goes to 0: each firm’s demand gets closer to \( P = 120 - q \). In this case, we would consider each to be in a separate market, since the output/price in one market has no affect on the other market. As \( b \) increases, the cross-elasticity between the two markets also increase (in absolute terms). A change in the output/price in one market has a greater affect the greater \( b \), making them closer substitutes.

e) Write down the profit for firm 1 and then verify that firm 1’s best response curve to firm 2’s quantity is:

\[r_1(q_2) = 60 - \frac{1}{2}bq_2. \]

Be certain to show each step of your derivation.

Firm 1’s profit is given by:

\[
\pi_1 = (P - c)q_1 - F \\
= (120 - q_1 - bq_2 - 0)q_1 - F \\
= 120q_1 - q_1^2 - bq_2q_1 - F \\
\frac{d\pi_1}{dq_1} = 120 - 2q_1 - bq_2 = 0 \\
r_1(q_2) = 60 - \frac{1}{2}bq_2
\]

f) Solve for the Cournot-Nash equilibrium quantities for an arbitrary value of \( b \).

Plug firm 1’s best response into firm 2’s best response (which will be symmetric):

\[
q_2 = r_2(q_1) = 60 - \frac{1}{2}bq_1 \\
= 60 - \frac{1}{2}b(60 - \frac{1}{2}bq_2) \\
q_2 = 60 - 30b + \frac{1}{4}b^2q_2 \\
(1-1/4)b^2)q_2 = (60 - 30b) \\
q_2 = (60 - 30b) / (1-\frac{1}{4}b^2)
\]

Note that \( b = 1 \) is the usual form of the Cournot game, where \( q_2 = 30/(3/4) = 40 \).
g) Suppose $b = \frac{1}{2}$, draw the new best response curves on your graph from part b

See part h).

h) What has happened to each firm’s equilibrium quantities? Give your economic intuition that explains this result.

- Intuitively, as $b$ becomes smaller, the two markets become more independent, thus we would expect each firm to look more like a monopolist, increasing output and prices.
- From part c) for $b = \frac{1}{2}$, $q_A^* = q_B^* = 48$.
- Best response curves. As $b$ goes from 1 to $\frac{1}{2}$, the best response curve goes from $q_2 = 60 - \frac{1}{2}q_1$ to $q_2 = 60 - \frac{1}{4}q_1$. That is, each firm becomes less responsive to the other firm’s output. Thus Nash equilibrium has each firm producing more.
- Graphically, the best-response curves rotate out and we can see that quantities produced increases.

5. You are given the table below which contains sales figures for three industries in the most recent calendar year.

<table>
<thead>
<tr>
<th>Firm</th>
<th>Industry 1</th>
<th>Industry 2</th>
<th>Industry 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$70</td>
<td>$80</td>
<td>$120</td>
</tr>
<tr>
<td>B</td>
<td>40</td>
<td>80</td>
<td>120</td>
</tr>
<tr>
<td>C</td>
<td>40</td>
<td>80</td>
<td>120</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>80</td>
<td>120</td>
</tr>
<tr>
<td>E</td>
<td>10</td>
<td>20</td>
<td>120</td>
</tr>
<tr>
<td>All Others</td>
<td>30</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>200</strong></td>
<td><strong>400</strong></td>
<td><strong>600</strong></td>
</tr>
</tbody>
</table>

Figures in millions of current dollars
a) Compute the C4 for each industry. Which industry is most concentrated according to the C4?

<table>
<thead>
<tr>
<th>Firm</th>
<th>Industry 1</th>
<th>Industry 2</th>
<th>Industry 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>s_i</td>
<td>s_i^2</td>
<td>s_i</td>
</tr>
<tr>
<td>A</td>
<td>.35</td>
<td>0.1225</td>
<td>.20</td>
</tr>
<tr>
<td>B</td>
<td>.20</td>
<td>0.04</td>
<td>.20</td>
</tr>
<tr>
<td>C</td>
<td>.20</td>
<td>0.04</td>
<td>.20</td>
</tr>
<tr>
<td>D</td>
<td>.05</td>
<td>0.0025</td>
<td>.20</td>
</tr>
<tr>
<td>E</td>
<td>.05</td>
<td>0.0025</td>
<td>.05</td>
</tr>
<tr>
<td>All Others</td>
<td>.15</td>
<td>.15</td>
<td>0</td>
</tr>
</tbody>
</table>

The C4 for each industry is 80%, thus all are equally concentrated according to the C4 measure.

b) Consider Industry 1. If each of the firms in the “All Others” category have sales smaller than Firm E, what is the largest that the HHI could be? What is the smallest that it could be? Calculate the HHI range for industry 2. Calculate the HHI for industry 3.

The HHI is calculated as the sum of the squares of market shares for all firms in an industry. Through the first 5 firms (A-E), that sum is 0.2075. The smallest number of “all other” firms there could be in Industry 1 would be three firms, each with 5% of the market, and would increase the HHI by \( 3 \times 0.0025 = 0.0075 \). The highest the HHI could be is 0.2150.

To find the lowest level of HHI, imagine 100 firms sharing the remaining 15% (each then having a market share of 0.15%). Each firm would then contribute \((0.0015)^2 = 0.00000225\) to the HHI. Even when we multiply this by 100, the HHI only raises by 0.000225, or: not very much. As we increase the hypothetical number of “all others,” this contribution will go to zero, thus the min is just 0.2075.

Similarly, the min for Industry 2 is 0.1625 and a max of 0.1700. The HHI for industry 3 is 0.2000.

c) Considering that the DOJ’s critical value for HHI is 1800, why is Industry 1 considered more of a threat to consumer welfare than Industry 2 (i.e., why is the HHI better at describing the distribution of concentration within the top 4 firms)? What are economists assuming about Firm A’s ability to raise prices in Industry 1 versus Industry 2? Do you find this persuasive? Explain.

Comparing Industry 1 and 2, we can see that what raises Industry 1 over the “critical value” is the market share of Firm A. Thus even though the top four firms have the same total market share, the cutoff suggests that firm A can more easily raise prices, to the detriment of consumers, in market A than B.

Looking forward, we can compare the symmetric Cournot model (e.g. Industry 2) with the Dominant Firm or Stackelberg Leader models (possibly Industry 1) and examine whether larger firms have a greater ability to raise prices.