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A Simple Remark on the Second Best Pareto Optimality of Market Equilibria

DANIEL MCFADDEN [1]

Department of Economics, University of California, Berkeley, Berkeley, California 94720

1. INTRODUCTION

Suppose that a private ownership market economy has classical characteristics: no externalities, infinitely divisible commodities, and convex production possibilities and consumer preferences. Suppose, further, that all economic agents are price takers following competitive behavior rules: in response to a quoted price vector, each firm offers a net supply bundle which maximizes its profits, and each consumer demands a commodity bundle which maximizes his preferences subject to the budget constraint he faces at these prices. Ruling out pathologies, a competitive allocation will then exist and will be Pareto optimal.

Suppose now that a group of economic agents become deviants, following noncompetitive behavior rules, so that a market equilibrium is no longer Pareto optimal. When direct intervention to modify the behavior rules of the deviants is impossible, the economy is constrained to the class of allocations which are consistent with the offers of the deviants at some prices. One may ask whether an allocation in this class is second best Pareto optimal, in the sense that any alternative allocation *in the class* which leaves at least one consumer better off leaves at least one other consumer worse off. Davis and Whinston [2] argue that if a market equilibrium allocation exists in which all nondeviants continue to follow their competitive behavior rules, then this allocation will be second best Pareto optimal, independently of the rationale underlying the construction of the deviant's offer curves.

The relevance of this conclusion to economic policy has been challenged by MacManus [6], who argues ". . . the typical situation in a 'free' but 'manipulated' economy is that most or even all the behavior functions are given and must be added to the list of constraints. At the same time, the authorities have at their command a set of policy instruments which

[1] I am indebted to Steve Goldman, Sidney Winter, and the members of Gruppe Montag for extensive discussions which stimulated this research, and to a referee for useful comments on the objectives of second-best theory.

enter these functions as arguments. The optimum is then found by maximizing welfare subject to all the constraints over the domain of these instruments. Indeed the whole problem has little practical interest without such explicit policy formulation." Extending this view, the prototype welfare economics theorem states that for a given set of policy instruments with a specified domain, there exist policies with certain characteristics which are optimal relative to some complete or partial (e.g. Pareto) welfare ordering over the subset of allocations satisfying specified behavioral constraints.

In the language of this prototype, the Davis–Whinston theorem assumes that the economic agents in the society can be divided into two distinct groups. The first group, including those defined as deviants, is essentially independent of the control of the authorities: the net supply of commodities from the sector defined by this group is determined solely by quoted market prices, and there are no discretionary specific taxes or transfers available as policy instruments to manipulate this net supply. The second group of agents is essentially completely controlled by the authorities, the policy instrument being the selection of a behavior rule for each controlled agent from a specified domain which includes the competitive behavior rule. The definition of the controlled group could come from either the basic limitations of government policy or from a restriction of attention *by the authorities* to the limited range of instruments deemed relevant to a particular piecemeal welfare economics analysis. The second interpretation is favored by Davis–Whinston [3] on the pragmatic grounds that policy decisions are in fact made on a piecemeal basis, presumably because the informational and computational costs of a general equilibrium analysis are prohibitive. In the language above, the Davis–Whinston theorem concludes that the Lange–Lerner policy in which all controlled agents behave competitively will be optimal relative to the Pareto partial welfare ordering, subject to the behavior constraints on noncontrolled agents, provided only that a market equilibrium exist and that there be no externalities between the controlled and noncontrolled sectors.

Aside from a question of taste on the importance of establishing norms for piecemeal analysis *vis-à-vis* striving to establish the more fundamentally correct general equilibrium analysis as a practical tool, the Davis–Whinston result raises three substantive issues. *First*, while this result is directly applicable to problems where the instruments under piecemeal control affect a small number of agents (e.g. the establishment of operating rules for a public firm), it is not relevant to a second class of common cases where an *instrument* under piecemeal control affects all agents (e.g. the modification of an excise tax or an income tax rate schedule). Consequently, the result can provide only one part of a general prescription for "optimal" piecewise policy. *Second*, economists are generally interested in Pareto optima only as a convenient half-way house in a two-stage optimization of social welfare. This should be as true in piecemeal analysis as it is in the general equilibrium analysis. The Davis–Whinston result characterizes only one Pareto optimum (albeit one which may have some advantages in minimizing information and computation cost or bureaucratic interference in consumer decisions), and consequently will normally fall short of providing the information necessary for the second stage of optimization. *Third*, the validity of the Davis–Whinston conclusion has been challenged on technical grounds by an example constructed by Gruppe Montag [7]. In this note, we shall outline additional conditions on the behavior of deviants which make the Davis–Whinston result valid, and present an alternative proof of the corrected theorem. The additional conditions required, along with the reservations listed above, would seem to limit severely the applicability of the corrected result to questions of practical piecemeal policy.

2. A COUNTER-EXAMPLE

The difficulty with the Davis–Whinston result can best be seen in a modification of the example of Gruppe Montag. Consider a two-consumer, two-commodity exchange economy, illustrated by the box diagram in Figure 1. From an initial allocation E, the consumers Mr. A and Mr. B have

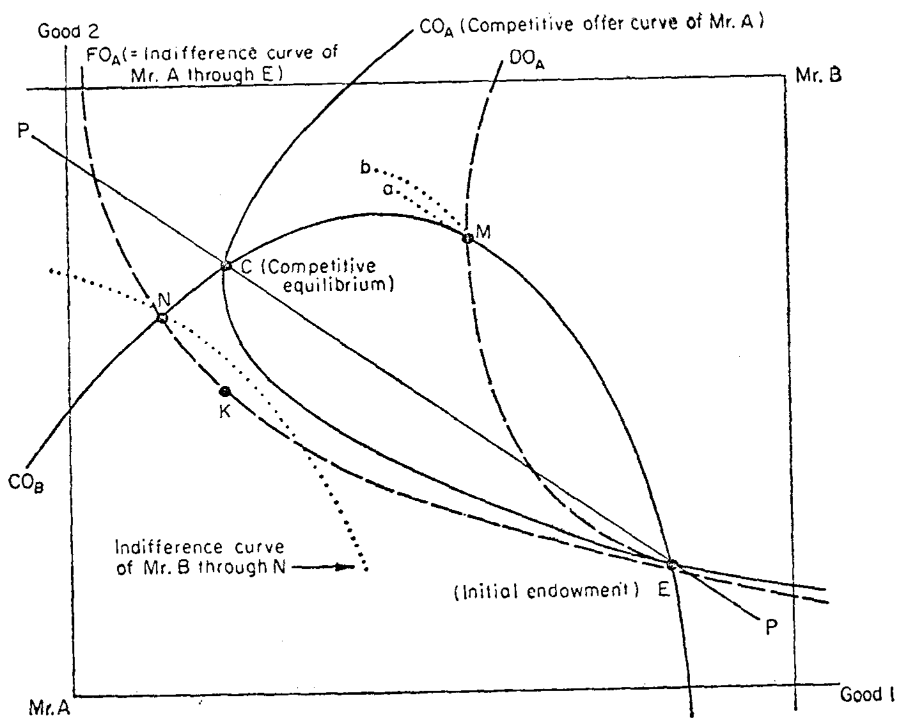

FIG. 1.

competitive offer curves, CO_A and CO_B, respectively, characterizing their competitive behavior rules: for any budget line through E, such as PP, the intersection of the budget line and the offer curve corresponds to a tangency of the highest possible indifference curve of a consumer with the budget line. The intersection of CO_A and CO_B at the point C then corresponds to a competitive allocation with prices given by the budget line PP. The allocation C is Pareto optimal. Suppose now that Mr. A becomes a deviant, with his behavior described by a noncompetitive offer curve DO_A. The intersection of CO_B and DO_A at the point M then corresponds to a market equilibrium with prices given by a budget line through E and M. Although M is not Pareto optimal, it is a second best Pareto optimum: there is no point in DO_A lying inside the lens aMb (bounded by the consumers' indifference curves through M) of points Pareto preferable to M. However, now consider the case in which the deviant Mr. A has a noncompetitive offer curve FO_A, coinciding with his indifference curve through the initial allocation E. The intersection of CO_B and FO_A at N again defines a market equilibrium with a budget line through E and N. But the point K in FO_A leaves Mr. B better off than at N, and Mr. A as well off. Hence, N is not a second best Pareto optimum.

How do the cases DO_A and FO_A differ? From the definition of competitive behavior, a bundle which Mr. B prefers to a market equilibrium allocation must have a higher value at the equilibrium prices than his equilibrium income. At N, the income of Mr. A could be reduced without making him worse off, and a point K Pareto preferable to N was attainable. However, at M any reduction in Mr. A's income leaves him worse off, and there is no way to make Mr. B better off except at the expense of Mr. A. Hence, a sufficient condition for the validity of the Davis–Whinston conclusion at a market equilibrium in an exchange economy is that all bundles in the deviant offer curve which the deviant considers at least as good as the market equilibrium bundle cost at least as much. Now, the conventional theorem in the theory of value establishing the Pareto optimality of competitive equilibrium assumes these conditions hold for the equilibrium budget set (preference maximization implies that a bundle better than the equilibrium bundle costs more, and local nonsatiation implies that a bundle as good as the equilibrium bundle costs at least as much). The sufficient condition above can then be interpreted as requiring that deviant behavior be "orthodox pseudo-competitive", in the sense that the deviant offer curve coincides with the competitive offer curve of a hypothetical consumer with the following preferences: among bundles in the deviant offer curve, his preferences coincide with the deviant's preferences, he prefers any bundle in the deviant offer curve to any bundle outside it, and he exhibits local nonsatiation in the equilibrium budget set.

(Note: provided the deviant's preferences satisfy local nonsatiation within the deviant offer curve, we can always assign him preferences outside the deviant offer curve to satisfy this condition.)

What types of deviant behavior will be orthodox pseudo-competitive, thus guaranteeing that a market equilibrium is second best Pareto optimal? In Figure 1, the offer curve DO_A, which restricted the quantities Mr. A would supply at various prices, yielding him some monopoly return, is orthodox pseudo-competitive, since Mr. A's preferences rise steadily along DO_A. On the other hand, the offer curve FO_A, in which Mr. A, for altruistic, masochistic, or trade-maximizing reasons makes the largest possible offers which leave him as well off as at E, is not orthodox pseudo-competitive. From this example and the example of Gruppe Montag, we see that a necessary condition for a deviant to be orthodox pseudo-competitive is that he be at least as well off at the market equilibrium as he is at his initial endowment, and in the case of positive trade that he be better off. This condition rules out extreme masochistic/altruistic behavior, and suggests that deviant behavior must be based on some degree of self-interest. However, selfishness is not sufficient for pseudo-competitive behavior. Figure 2 illustrates a market equilibrium R in an exchange economy where the deviant Mr. A is better off than at the competitive equilibrium C, but R is Pareto inferior to an allocation S in the deviant

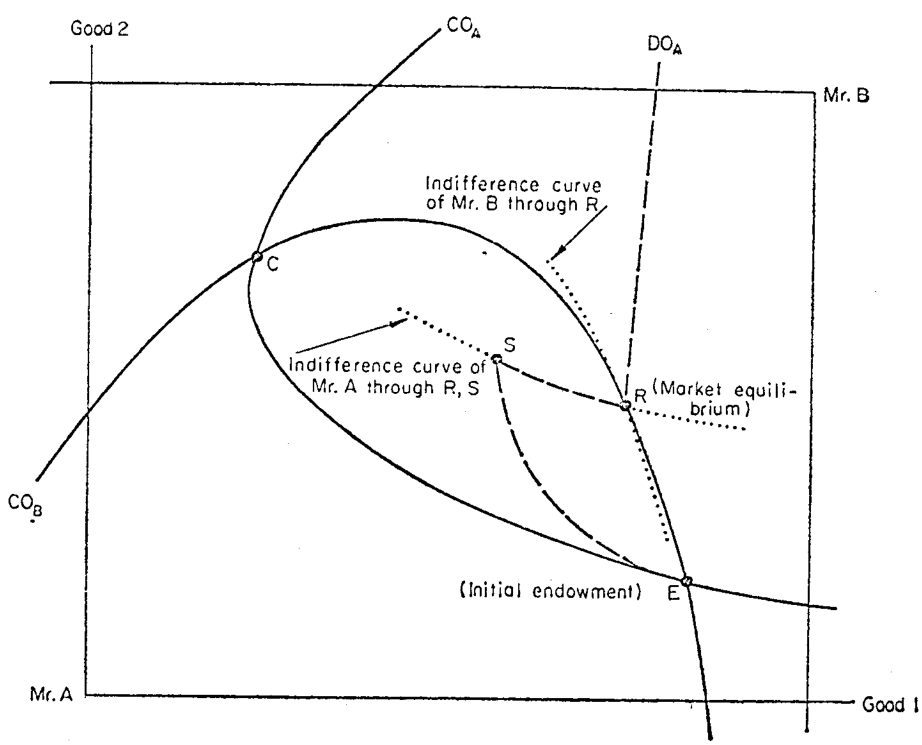

FIG. 2.

offer curve. We conclude that deviant behavior based on self-interest will usually, but not always, be orthodox pseudo-competitive (implying that a market equilibrium is second best Pareto optimal), and that deviant behavior which is sufficiently altruistic or masochistic to violate self-interest will usually lead to market equilibria which are not second best Pareto optima.

3. THE CASE OF PRODUCTION

Next consider extending the results stated above for an exchange economy to the case of production. A condition on the behavior of deviant firms analogous to the one for consumers is found to be sufficient for the validity of the Davis–Whinston result. This condition is that deviant firms which do not follow the competitive profit maximizing behavior rule are "pseudo-competitive" in the following sense: there is no net supply bundle which the firm has offered at any prices which yields higher profits at the market equilibrium prices than its equilibrium net supply bundle. A pseudo-competitive firm has the characteristic that there is no evidence from its market net supply schedule which directly refutes the hypothesis that it is a competitive profit maximizer at the market

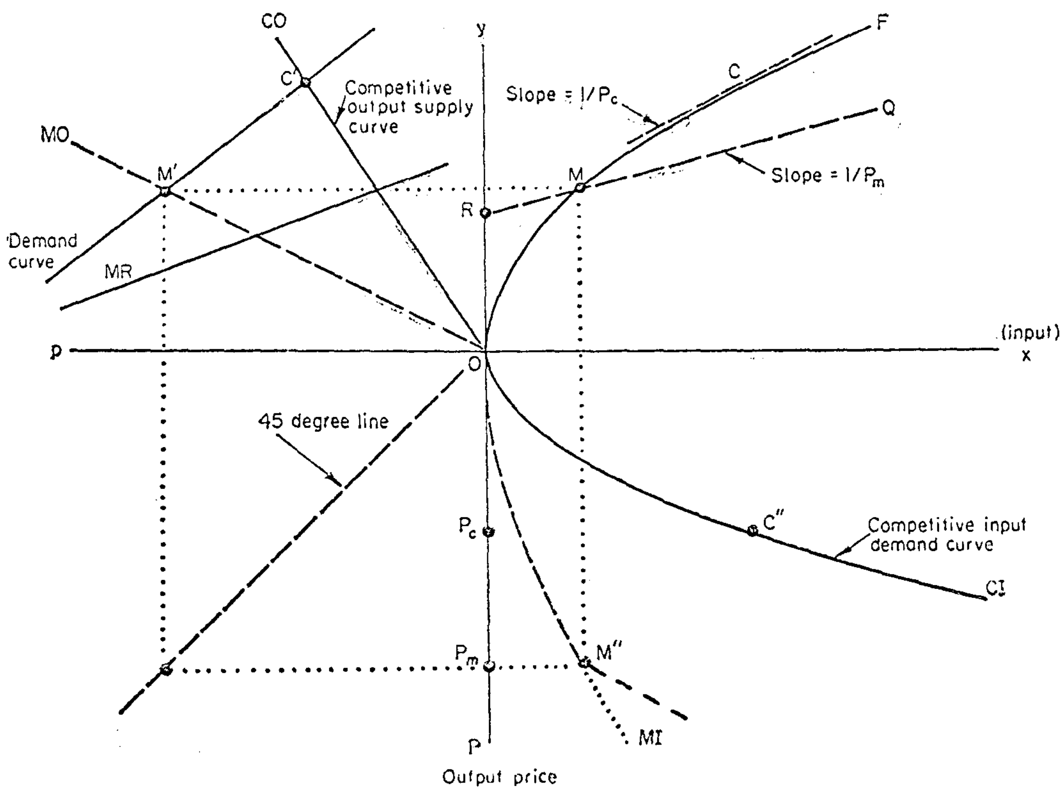

FIG. 3.

equilibrium, such evidence would have to come from disclosure of the technological capabilities of the firm.

What types of deviant firm behavior are pseudo-competitive? Some insight into this question can be gained by examining the one-input, one-output firm illustrated in Figure 3. Let x and y denote input and output quantities, respectively, and the curve OMCF (satisfying $y = 2\sqrt{x}$ in this example) denote the production frontier. Let p denote output price, with the input as numériare. The firm following the competitive profit maximization rule will offer an output supply curve CO (orthant II) and an input demand curve CI (orthant IV), with the reciprocal of the slope of the production frontier at a profit maximizing point C equal to the price p_c at which the corresponding output C′ and input C″ are offered. With an output demand curve D, C will be the competitive equilibrium operating point of the firm.

4. A DIGRESSION ON THE BEHAVIOR OF MONOPOLISTS

Now suppose the firm conjectures correctly that it is faced by output demand curve D, and attempts to attain equilibrium at the point M which maximizes its "monopoly" profit. In this case, what is the nature and form of the firm's "behavior rule"? In the conventional theory of monopoly, the firm operates directly as a "market manager", quoting prices and observing demands until a market equilibrium is reached. The firm's "behavior rule" is then a strategy for quoting a sequence of trial prices which in general incorporates all the information it has gathered on previous trials. Questions of second best Pareto optimality become very difficult in this circumstance, since allowing competitive agents to consider strategies other than the one imposed by the competitive rules makes operation of the economy a complex bargaining game. However, the observed equilibrium behavior of a monopolistic firm can be explained in an alternative theory employing much simpler behavior rules: each firm submits to a neutral "market manager" an offer curve specifying the net supply bundle it will produce at each price, and the market manager then calculates an equilibrium. On the basis of its conjecture on the demand curve it faces, a monopolistic firm will submit an offer curve such that marginal revenue and true marginal cost are equated in the conjectured equilibrium. The firm is then committed to this offer curve even if its conjecture proves false. Unless direct intervention to modify the behavior of this firm is possible, the economy is then constrained to seek second best allocations consistent with the firm's offer curve.

The market mechanism required in this alternative theory of monopoly is not unreasonable in an economy with administered prices, but is contrived and unrealistic in a free market economy. Nevertheless, it is with the type of behavior rule postulated in this alternative theory that the second best analysis of Davis and Whinston deals. In exploring the validity of their results, we shall adopt this approach, leaving open the question of whether similar conclusions can be obtained in more realistic economic games.

5. THE CASE OF PRODUCTION CONTINUED

In Figure 3, the monopolist might submit the output supply curve MO and input demand curve MI. Is he pseudo-competitive at the resulting equilibrium M? Profit at this operating point, expressed in output units, equals the distance RO, and the line RQ is the locus of input–output combinations yielding this profit level. A pseudo-competitive firm will make no offers which would yield a higher profit than RO at the equilibrium price p_m; i.e. it will never offer to produce a bundle in the region QMCF. Thus, the firm behaves as if its production frontiers were OMQ rather than OMCF. The offers MO and MI are pseudo-competitive, since MI is determined from MO by the requirement that the offered point lie on OMQ. However, had the monopolist been less careful in understating the productivity of inputs beyond M, offering, say, the same MO curve and a MI curve which extended smoothly beyond M″, he would have offered, at some output price higher than p_m, a bundle near C violating the pseudo-competitive condition. It is clear from this example that the pseudo-competitiveness of a monopolist at an equilibrium will depend critically on the offers to which it is committed in other price ranges. Thus, a monopolist may be pseudo-competitive, but there is no simple pattern of behavior guaranteeing that it will be.

A second example of deviant firm behavior is sales maximization subject to a nonnegative profit constraint, illustrated in Figure 4. The output supply curve of the sales maximizing firm is SO, its input demand curve is SI. At the equilibrium offer S for the demand curve D, it is *not* pseudo-competitive: the point C, which the firm offers at a price p such that the slope of the line OC equals $1/p$, yields a higher profit at the price p_s than does S. A further argument shows that the Davis–Whinston result fails for this case; S is not second best Pareto optimal. Suppose the economy has a single consumer with a positive endowment of commodity x, and that in the absence of profit income his competitive offer curve is OST, with preferences increasing to the northwest as he gives up less x and receives more y. The point C is preferred to S by the consumer, and is in the deviant's offer curve, as noted above. Hence, the consumer can achieve a Pareto preferable allocation by submitting a deviant offer curve passing through C, rather than his competitive offer curve. The point C in this case is actually the first best Pareto optimum.

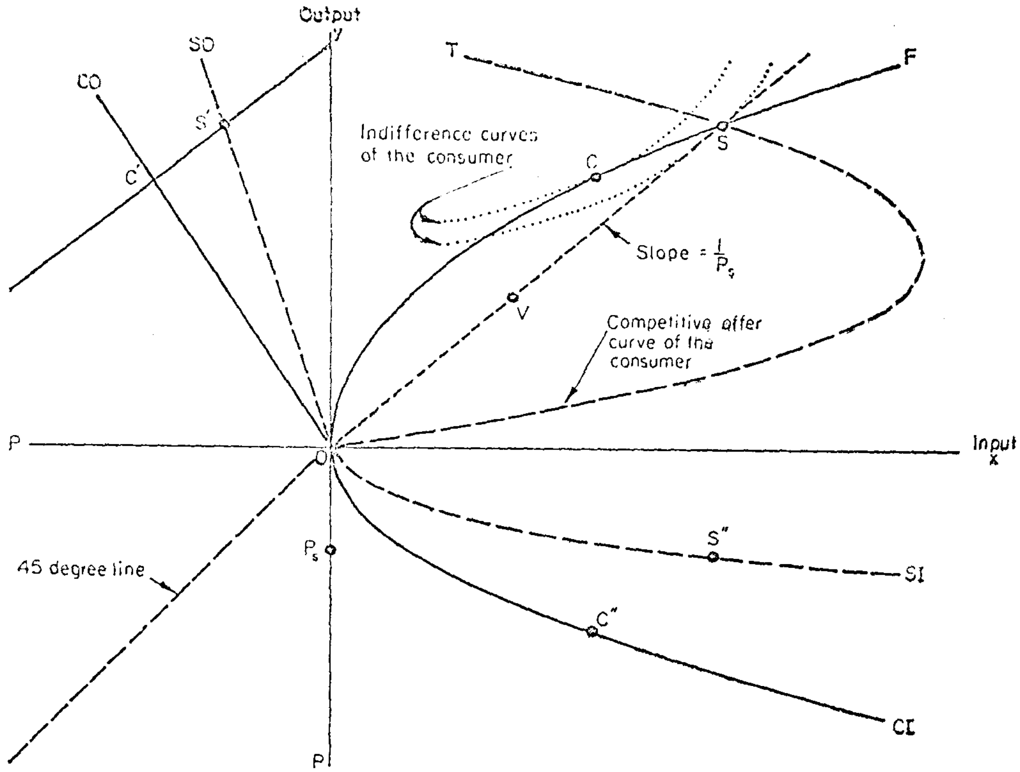

FIG. 4.

Although the sales maximizing firm is not pseudo-competitive at S, there are variants of its offer curves which are. An overstatement of input requirements (given SO) which confines the firm's offers to OVSF makes it pseudo-competitive. So does the variant in which the firm offers only S at all prices.

In a third example of deviant firm behavior, the use of "mark-up" pricing rules, the reader can verify again that the firm typically will not be pseudo-competitive, and an allocation Pareto preferable to a market equilibrium can be obtained if consumers abandon competitive behavior.

From these examples, it is clear that unless there is some institutional factor in the economy which gives deviant firms an incentive to overstate input requirements in the offers they submit, their behavior will usually not be pseudo-competitive, and the Davis–Whinston conclusion will often fail to hold. Thus, the corrected theorem is likely to have limited applicability.

6. A THEOREM ON THE SECOND BEST PARETO OPTIMALITY OF MARKET EQUILIBRIUM

In the remainder of this paper, we shall formulate a correct version of the Davis–Whinston theorem for an abstract economy and outline a simple formal proof. All the informal conclusions reached earlier are implied by this theorem. We employ the model of an abstract decentralized private ownership economy developed by Arrow [1] and Debreu [4], using the form in which it is discussed by Koopmans [5].

Consider an economy with an N-dimensional commodity space $\mathbf{X}$, J firms, indexed $j = 1, \ldots, J$, and K consumers, indexed $k = J+1, \ldots, J+K$. With firm j we shall associate a *net output vector* y_j (inputs negative, outputs positive) in $\mathbf{X}$, and with consumer k we shall associate a *consumption vector* x_k and an initial endowment vector w_k, both in $\mathbf{X}$. An *allocation* is a $(J+K)$-dimensional vector of commodity vectors in $\mathbf{X}$,

$$\mathbf{a} = (y_1, \ldots, y_J, x_{J+1}, \ldots, x_{J+K}),$$

prescribing a net output vector y_j for each firm j and a consumption vector x_k for each consumer k. By taking appropriate subvectors of allocations, one may also speak of allocations to subsets of economic agents. Let p denote an N-dimensional price vector, and $\mathbf{P}$ the non-negative unit simplex of prices.

An allocation is *possible* for firm j if it is technologically capable of producing its prescribed net output vector. The firm is *externality-free* if the possibility of an allocation is determined solely by its own net output vector, and not by the allocation to the remaining economic agents. When this is the case, we let Y_j denote the set of possible net output vectors of the firm. A *behavior rule* for firm j is a correspondence which specifies, for each price vector in a domain $\mathbf{P}_j$ contained in $\mathbf{P}$ and each allocation to the remaining economic agents, a nonempty subset of the possible net output vectors of the firm, interpreted as the set of net supply vectors it is willing to offer in this environment.[2] The behavior rule is *price-determined* if the correspondence does not depend on the allocation to the remaining economic agents.[3] In this case, we let $S_j(p)$ denote the behavior rule correspondence. The firm's profit associated with a net output vector y_j is $p \cdot y_j$. The behavior rule is *isoprofit* at a price vector $p \in \mathbf{P}_j$ and allocation to the remaining agents if all net output vectors offered in this environment yield the same profit.[4] Hereafter, we shall make

ASSUMPTION 1. Each firm $j (j = 1, \ldots, J)$ is externality-free and has a price-determined behavior rule. Each firm's behavior rule is isoprofit on $\mathbf{P}_j$.

[2] The price domain P_j consists of the price vectors for which the firm's offer contains one or more finite bundles.

[3] In general, an economic agent which is externality-free might fail to have a price-determined behavior rule, and vice versa.

[4] This assumption is tacit in most general equilibrium analyses, and is satisfied trivially if the behavior rule specifies a unique offer in the given environment. However, there are reasonable noncompetitive behavior rules of the firm where it may fail; e.g. a firm may maximize a utility function of profits and one of its inputs, and the function may not be strictly quasi-concave.

Define $Y_j^* = \{y_j | y_j \in S_j(p) \text{ for some } p \in P_j\}$, the set of all net output bundles which firm j offers at some prices. The behavior rule of firm j is *competitive* at a price vector p if its profits are maximized over Y_j at $S_j(p)$; i.e. $y_j \in S_j(p)$ and $y'_j \in Y_j$ imply $p \cdot y_j \geqq p \cdot y'_j$. Any firm whose behavior rule is not competitive at all price vectors in P_j is called a *deviant*. If a deviant firm's behavior rule has the property at a price vector p that its profits are maximized over Y_j^* at $S_j(p)$, it is called *pseudo-competitive* at p.

An allocation is *possible* for consumer k if it is physically possible for him to consume the prescribed commodity vector. The consumer has a total, reflexive, transitive preference relation over his set of possible allocations. The consumer is *externality-free* if his set of possible allocations and preference relation depend only on his own consumption vector, and not on the allocation to other economic agents. When this is the case, we let X_k denote the set of his possible own consumption vectors and $\gtrsim_k$ denote the preference relation induced on this set by his preferences over allocations.

In a private ownership economy, consumer k owns an initial endowment w_k and a claim to a proportion

$$\Theta_{kj} \left(\sum_{k=J+1}^{J+K} \Theta_{kj} = 1 \right)$$

of the net output of each firm j. For each price vector $p \in P$ and each allocation to the remaining economic agents, he has an income

$$M_k = p \cdot w_k + \sum_{j=1}^{J} \Theta_{kj} p \cdot y_j,$$

and is constrained to a budget set consisting of all his possible consumption vectors x_k satisfying $p \cdot x_k \leqq M_k$. When the behavior rules of firms satisfy Assumption 1, the consumer's income is determined completely once a price vector p is specified in the set P' of prices common to the domains of the behavior rules of the firms

$$\left(P' = \bigcap_{j=1}^{J} P_j \right),$$

and will be denoted by $M_k = M_k(p)$. A behavior rule for consumer k is a correspondence which specifies, for each price vector in a domain P_k contained in P' and each allocation to the remaining economic agents, a nonempty subset of his budget set, interpreted as the set of consumption vectors he is willing to purchase in this environment. If this behavior rule is price-determined so that the correspondence depends only on p, it will be denoted by $D_k(p)$.[5] Hereafter, we shall make

[5] The price domain P_k consists of the price vectors in P' for which the budget set is non-empty and the consumer's offer contains one or more finite bundles. Under Assumption 1, the consumer's budget set is completely determined by prices, and he can have a price-determined behavior rule even if he is subject to externalities.

ASSUMPTION 2. Each consumer $k(k = J+1, \ldots, J+K)$ is externality-free and has a price-determined behavior rule.

Define $\mathrm{X}_k^* = \{x_k | x_k \in \mathrm{D}_k(p) \text{ for some } p \in \mathrm{P}_k\}$, the set of all consumption bundles which individual k offers at some prices. The behavior rule of consumer k is *competitive* at price vector p if his preferences are maximized over his budget set at $\mathrm{D}_k(p)$; i.e. $x_k \in \mathrm{D}_k(p)$, $x_k' \in \mathrm{X}_k$, and $p \cdot x_k' \leqq M_k(p)$ imply $x_k \gtrsim_k x_k'$. If he is a deviant, exhibiting noncompetitive behavior at some prices in P_k, but at a price vector p his preferences are maximized over $x_k \in \mathrm{X}_k^*$ satisfying $p \cdot x_k \leqq M_k(p)$ at $\mathrm{D}_k(p)$ (i.e. $x_k \in \mathrm{D}_k(p)$, $x_k' \in \mathrm{X}_k^*$, $p \cdot x_k' \leqq M_k(p)$ imply $x_k \gtrsim_k x_k'$), then he is called *pseudo-competitive* at p. We term a competitive [resp., pseudo-competitive] consumer *orthodox* at p if he spends all his income at any $x_k \in \mathrm{D}_k(p)$, and no less expensive vector in his budget set $\mathrm{B}_k(p) = \{x_k \in \mathrm{X}_k | p \cdot x_k \leqq M_k(p)\}$ [resp., in his restricted budget set $\mathrm{B}_k^*(p) = \{x_k \in \mathrm{X}_k^* | p \cdot x_k \leqq M_k(p)\}$] is as desirable. A sufficient condition for a competitive consumer to be orthodox is that he have continuous preferences and exhibit local nonsatiation in his budget set.

Let P'' denote the set of price vectors common to the domains of the behavior rule of all consumers

$$\left(\mathrm{P}'' = \bigcap_{k=J+1}^{J+K} \mathrm{P}_k \subseteq \mathrm{P}'\right).$$

An allocation a is in *material balance* if

$$\sum_{k=J+1}^{J+K} (x_k - w_k) = \sum_{j=1}^{J} y_j.$$

A *market equilibrium* (p, a) is a price vector $p \in \mathrm{P}''$ and an allocation a such that a is in material balance; $y_j \in \mathrm{S}_j(p)$, $j = 1, \ldots, J$; and $x_k \in \mathrm{D}_k(p)$, $k = J+1, \ldots, J+K$.

An allocation a is *feasible* if it is possible for each economic agent and is in material balance. Let A denote the set of feasible allocations. An allocation a is *Pareto optimal* relative to a subset A_1 of A if every allocation a' in A_1 which makes at least one consumer better off also makes at least one consumer worse off. If $\mathrm{A}_1 = \mathrm{A}$, we say a is (first best) Pareto optimal. When A_1 is a proper subset of A, then a is a second best Pareto optimum.

Partition the set of firms $\{1, \ldots, J\}$ into a subset F exhibiting competitive behavior and a subset F' with deviant behavior. Similarly, partition consumers into a subset $\mathbf{C}$ exhibiting competitive behavior and a subset $\mathbf{C}'$ with deviant behavior. We are now prepared to state the Davis–Whinston theorem.

THEOREM. *Suppose Assumptions* 1 *and* 2 *hold, and a market equilibrium* (p^*, a^*) *exists. Suppose that all deviants are pseudo-competitive and all consumers are orthodox at* p^*. *Suppose that the behavior rules of deviants* *cannot be modified, so that the economy is restricted to a subset of the feasible allocations,* $\mathbf{A}_1 = \{\mathbf{a} \in \mathbf{A} | [y_j \in \mathbf{S}_j(p),\ j \in \mathbf{F}'$ *and* $x_k \in \mathbf{D}_k(p),\ k \in \mathbf{C}']$ *for some* $p \in \mathbf{P}''\}$. *Then,* $\mathbf{a}^*$ *is a second best Pareto optimum relative to* $\mathbf{A}_1$.

Proof. The standard proof of the Pareto optimality of competitive equilibrium can be applied to this theorem essentially without modification. Suppose $\mathbf{a} \in \mathbf{A}_1$. Then, $y_j \in \mathbf{Y}_j$, $j \in \mathbf{F}$ implies $p^* \cdot y_j \leqq p^* \cdot y_j^*$ and the condition that the firms in $\mathbf{F}'$ be pseudo-competitive implies $p^* \cdot y_j \leqq p^* \cdot y_j^*$, $j \in \mathbf{F}'$. Hence, by material balance,

$$\sum_{k=J+1}^{J+K} p^* \cdot x_k = \sum_{k=J+1}^{J+K} p^* w \cdot_k + \sum_{j=1}^{J} p^* \cdot y_j$$
$$\leqq \sum_{k=J+1}^{J+K} p^* \cdot w_k + \sum_{j=1}^{J} p^* \cdot y_j^* = \sum_{k=J+1}^{J+K} p^* \cdot x_k^*.$$

By the condition that all consumers be orthodox, if consumer k is to be at least as well off in $\mathbf{a}$ as in $\mathbf{a}^*$, he must spend at least as much; i.e. $p^* \cdot x_k \geqq p^* \cdot x_k^*$. By the condition that he be competitive ($k \in \mathbf{C}$) or pseudo-competitive ($k \in \mathbf{C}'$), if he is to be better off at $\mathbf{a}$, he must spend more; i.e. $p^* \cdot x_k > p^* \cdot x_k^*$. For $\mathbf{a}$ to be Pareto preferable to $\mathbf{a}^*$, it would then have to satisfy

$$\sum_{k=J+1}^{J+K} p^* \cdot x_k > \sum_{k=J+1}^{J+K} p^* \cdot x_k^*,$$

contradicting the inequality obtained previously. This proves that $\mathbf{a}^*$ is second best Pareto optimal.

Q.E.D.

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