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A TECHNICAL NOTE ON CLASSICAL GAINS FROM TRADE

J.M. GRANDMONT and D. McFADDEN *

CEPREMAP, Paris, and University of California, Berkeley

1. Introduction

A venerable proposition of international trade theory is the conclusion dating from Ricardo that autarkic nations can engage in competitive (free) trade to mutual advantage. [1] The result is generally held to be valid in the following three world environments: (1) one-consumer or centrally planned nations, (2) multiple-consumer nations with decentralized competitive domestic markets which are 'small' traders in the family of nations, [1a] and (3) the previous case with the added restriction that nations are 'infinitesimal' traders in international markets.

A classical paper by Samuelson (1939), extended by Kemp (1962), Kenen (1959), and Samuelson (1956, 1962), provides a logically rigorous argument for the result in the case of centrally planned nations. The reasoning is that the refuge of autarky remains available when

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[1] The distributive question contained in the gains-from-trade theorem was first formulated by Pareto in 1896; Ricardo dealt only with the expansion of aggregate consumption possibilities. Among contemporary economists, Viner was the first to perceive the nature of the problem, in 1937. Samuelson's 1939 article first stated the question precisely. The first sketch of a rigorous proof did not come until the Kemp and Samuelson articles of 1962.

[1a] A nation is 'infinitesimal' in international markets if it cannot influence world prices or disturb world trade equilibrium, 'small' if it treats world prices as parameters but influences the determination of world market equilibrium, and 'large' if it treats world prices as variable. The traditional partial equilibrium analysis of gains from trade tacitly assumes the infinitesimal case. The analysis of this paper will include the small case, but not the large case where the competitive equilibrium is a less useful solution concept.

trade is possible, implying that (1) given an allocation achieved under autarky the nation can select, from a set of allocations feasible with trade, either the original allocation or one that is at least as satisfactory for every consumer, and (2) given an allocation attained with trade which is optimal in the Pareto sense, any alternative allocation feasible under the added restriction of autarky which improves the lot of some consumers in a nation must worsen the lot of others. It is worth noting that this argument does *not* require that nations be 'infinitesimal' or 'small' in international markets, that non-increasing returns to scale prevail, that all commodities be tradeable, or that factors be immobile.[2]

For decentralized multiple-consumer nations with competitive domestic markets, the gains-from-trade proposition can be stated in the following form:

A. Given a world competitive trade equilibrium allocation, any alternative allocation feasible under autarky[2a] which makes some consumers in a nation better off must make some other consumers in that nation worse off.

B. Given an allocation achieved under autarky, a system of world trade prices and *domestic* lump-sum transfers can be found for which a competitive equilibrium allocation will exist and will be at least as satisfactory as autarky for every consumer.

As Samuelson and Kemp have noted, proposition *A* follows quite generally from properties of competitive equilibria. It is known (Debreu (1959), Koopmans (1957)) that a competitive equilibrium allocation at which all consumers are locally non-satiated and externalities are absent is in the core of an economy, i.e., no feasible alternative for any coalition can be preferable in the Pareto sense for all coalition members. Applied to the coalition of consumers in each nation for a world competitive trade equilibrium, this result implies proposition *A*. This conclusion requires only the mild (from a neoclassical viewpoint) conditions of local non-satiation and no externalities. There are no explicit requirements that traders be 'small', that non-increasing returns prevail, or that factors be immobile. In particular, this argument applies inde-

[2] Of course, in the absence of conditions such as non-increasing returns to scale, it will generally be impossible to quote world prices which will sustain the trade allocation; the argument does *not* imply the existence of a decentralized equilibrium allocation.

[2a] The autarkic allocation may be obtained by a domestic competitive equilibrium with lump-sum transfers, or by other means.

pendently of whether nations are 'infinitesimal' traders in world markets, and covers the second and third cases of world environments in which the gains-from-trade proposition is held to be valid.

A useful result related to proposition B follows from a second elementary theorem on competitive equilibria. With some regularity conditions, including convexity, any optimal allocation in the Pareto sense can be sustained by a competitive price system (Koopmans (1957)). Choosing a world allocation which is Pareto optimal and at least as satisfactory as autarky for each consumer in each nation, one can then establish the existence of a world competitive price system satisfying B, but requiring in general *international* as well as domestic transfers. [3] Because of the last possibility, this argument cannot be used to establish proposition B, in which trade balance (no international transfers) is required.

A direct argument for proposition B has been given by Kenen (1959) and Samuelson (1939, 1956). Define a 'social welfare function' for a nation by selecting ordinal utility functions representing consumers' preferences, and then forming the minimum over individuals of (weighted) net increases in utility from the levels achieved at the autarkic allocation. [4] Given world trade prices and domestic resources, maximize this social welfare function subject to the trade balance con-

[3] This optimality theorem applies more generally to establish the existence of a competitive world trade equilibrium (with domestic and international transfers) which is Pareto non-inferior to a specified allocation resulting from *any* pattern of restricted trade.

[4] In the notation of sect. 2, let $u_{ik}(\boldsymbol{x}_{ik})$ be a continuous ordinal utility function representing the preferences of consumer i in nation k over commodity vectors $\boldsymbol{x}_{ik}$, and let $\boldsymbol{a}_k$ denote an allocation in nation k specifying the commodity vectors for each consumer. Define a 'social welfare function

$$w_k(\boldsymbol{a}_k) = \min_{i \in \boldsymbol{I}_k} \theta_{ik}[u_{ik}(\boldsymbol{x}_{ik}) - u_{ik}(\boldsymbol{x}^*_{ik})] ,$$

where $\boldsymbol{I}_k$ is the set of consumers, $\boldsymbol{x}^*_{ik}$ is consumer i's commodity vector in the autarkic allocation, and the θ_{ik} are positive weights. When the preferences of all individuals are convex and locally non-satiated, the function w_k is continuous, quasi-concave, and locally non-satiated. If we carry out any constrained maximization of w_k in which the autarkic allocation $\boldsymbol{a}^*_k$ is feasible, then an optimal allocation $\boldsymbol{a}_k$ will satisfy $w_k(\boldsymbol{a}_k) \geqslant w_k(\boldsymbol{a}^*_k) = 0$, implying $\boldsymbol{a}_k$ Pareto non-inferior to $\boldsymbol{a}^*_k$. (Samuelson uses an alternative construction of the social welfare function, a sum of individual utility functions weighted so that the maximand is always Pareto non-inferior to the autarkic allocation. This procedure yields a similar result.)

straint. [5] An allocation achieving this optimum will have the following properties: (1) By the construction of the social welfare function and the possibility of no trade, it will be at least as satisfactory as autarky for every consumer. (2) Provided some regularity conditions on individual preferences and production possibilities are met, it will be a domestic competitive equilibrium allocation at world prices for some implicitly defined system of domestic lump-sum transfers. (3) It will determine a net import bundle for the nation (as a function of the world price vector). If this net import bundle coincides with an equilibrium net export bundle from the rest of the world at the quoted world trade price vector, then the argument needed to establish proposition B is complete. In particular, B holds in the case of the third world environment in which each nation is an 'infinitesimal' trader who cannot affect world prices by his own actions, and thus can offer any trade-balanced net import bundle without upsetting world trade equilibrium. Finally as an alternative to this 'partial equilibrium' assumption, consider the case in which nations are 'small' but not 'infinitesimal', facing net export supply functions from the rest of the world and treating world trade prices as parameters. Given the 'social welfare' maximization above, each nation behaves *as if* it had a single consumer with a net import demand function of world prices. The only remaining question is then whether world prices exist which equate net export supply and net import demand. A number of standard theorems on the existence of market equilibrium (Nikaido (1956), Debreu (1959), Mc-

[5] We continue the notation of footnote 4. The trade balance constraint is

$$\Sigma_{i\in I_k} \boldsymbol{p}\cdot\boldsymbol{x}_{ik} = T_k(\boldsymbol{p}),$$

where $\boldsymbol{p}$ is the given world price vector and $T_k(\boldsymbol{p})$ is the maximum value of national income at world prices. Since the supply to the consumption sector attained under autarky remains feasible when trade is possible, it follows that $\Sigma_{i\in I_k} \boldsymbol{p}\cdot\boldsymbol{x}_{ik}^* \leqslant T_k(\boldsymbol{p})$. Hence, the autarkic allocation $\boldsymbol{a}_k^*$ is feasible. Provided all consumers are locally non-satiated, maximization of $w_k(\boldsymbol{a}_k)$ subject to this constraint is equivalent to maximizing $u_{1k}(\boldsymbol{x}_{1k})$ subject to

$$u_{ik}(\boldsymbol{x}_{ik}) - u_{ik}(\boldsymbol{x}_{ik}^*) \geqslant \frac{\theta_{1k}}{\theta_{ik}} (u_{1k}(\boldsymbol{x}_{1k}) - u_{1k}(\boldsymbol{x}_{1k}^*))$$

for $i = 1, ..., I_k$ and the trade balance constraint, and the maximand will define a Pareto optimal allocation which is Pareto non-inferior to autarky. With some regularity conditions, including convexity, on consumer's preferences, this optimal allocation will define a competitive equilibrium at world prices. One can see this by applying an arbitrage argument or by writing out the first-order conditions in the maximization problem above. (The standard conditions for a Pareto optimum to be sustained by competitive prices (Koopmans (1957)) apply. Since the trade balance constraint defines a constant-expenditure hyperplane at world prices, this equilibrium can be defined at the world price vector.)

Kenzie (1954), Arrow and Hahn (1971), or Theorem 4 below) provide sufficient conditions for an affirmative answer. Surprisingly, there appears to be no technically rigorous proof of proposition B in the literature which combines the Samuelson–Kenen construction of 'socially optimal' allocations with the existence theorems required in the case of 'non-infinitesimal' nations. The purpose of this note is to fill the gap, concentrating on the general equilibrium case not treated by Samuelson and Kenen.

One description of the allocation mechanism implicit in the Samuelson–Kenen construction is that the government collects information on tastes, makes a centralized calculation of a decentralizable Pareto optimal allocation, and then assigns incomes (lump sum transfers) to carry out the decentralization. The result is a national net import function with the properties of a one-consumer excess demand function. There is an alternative mechanism for generating allocations Pareto non-inferior to autarky, which requires less centralized information and more closely resembles the operation of the modern 'competitive welfare state'. Define a distribution policy to be a rule for distributing national income by lump-sum domestic transfers at each price vector. Consider a distribution policy which always gives each consumer income sufficient to purchase a bundle at least as good as the bundle he obtained under autarky. (E.g., assign each consumer an income at least equal to the value of his bundle under autarky.) This policy is feasible since national income with trade at given world prices will be at least as large as the value of the aggregate autarkic allocation at these prices (Kemp (1962), p. 805). Given this policy, competitive behavior of the nation's producers and consumers leads to a national net import function of world prices. This function will not generally have the properties of a one-consumer excess demand function, but will have the properties of aggregate excess demand usually required in analyzing the existence of market equilibrium. From this point, the argument for proposition B is the same as is required in the Samuelson–Kenen construction. The remainder of this note is devoted to a formal restatement of the arguments outlined in the preceding section, and elaboration of their technical points. The results require mostly minor variations on textbook theorems, contain few surprises, and confirm the intuition of previous writers on the gains from trade proposition.

The argument for proposition B will utilize Theorems 1 and 2 below on the existence of competitive equilibria for given distribution policies.

These results are of some independent interest for the analysis of the 'competitive welfare state', as contrasted with the private ownership laissez faire economy stipulated in most standard existence theorems. Theorem 3 below which establishes proposition B can be interpreted more generally as a result in welfare economics specifying conditions under which a potential Pareto improvement can be attained in a competitive economy by a decentralized distribution policy. Our notation, examples, and analysis draw heavily on the work of Debreu (1956, 1959, 1962).

2. A formal model

The model in which these results will be proved will now be defined. Consider a world economy with N commodities, indexed $n = 1, ..., N$; K nations, indexed $k = 1, ..., K$; and I_k consumers in nation k, indexed $i = 1, ..., I_k$. The symbols $\boldsymbol{N}$, $\boldsymbol{K}$, and $\boldsymbol{I}_k$ will also be used to denote the sets containing these indices. Bundles of commodities are represented by vectors $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ in the N-dimensional Euclidean space, $\boldsymbol{R}^N$.

Nation k has a net supply possibility set $\boldsymbol{Y}_k$ containing the commodity vectors which can be supplied, net, to consumers and traders from the resource endowment and production possibilities of the economy.[6] We make the conventional assumption that individual net consumption vectors are non-negative, taking the net supply vectors in $\boldsymbol{Y}_k$ to include all resources in the economy, not excepting endowments such as leisure-hours 'owned' by consumers. By definition, we take the set $\boldsymbol{N}$ of world commodities to include only those which the world economy can supply without external inputs; i.e., for each $n \in \boldsymbol{N}$, there exist $\boldsymbol{y}_k \in \boldsymbol{Y}_k$

[6] The commodity classification will, in general, index goods by their location and date of delivery as well as by their generic characteristics. Most commodities can be consumed in a nation only if they are located in the nation. (An exception might be stock commodity whose services can be consumed at a distance, such as gold bullion in Zurich.) Thus, consumers in a nation will desire only a subset of the set of world commodities. Transport costs between locations are included in the description $\boldsymbol{Y}_k$ of the possibilities for transforming and supplying commodities. Note that when commodities are dated, competitive world markets allow borrowing between nations (international capital flows). The model allows international labor mobility, but for purposes of calculation of national welfare and distribution policy assumes all individuals are identified with nation of origin. $\boldsymbol{Y}_k$ may be disaggregated into a sum of individual resource holder supply sets and firm production possibility sets; we leave this generalization to the reader.

such that $\Sigma_{k\in K}\, \boldsymbol{y}_k \geqslant \mathbf{0}$ and $\Sigma_{k\in K}\, y_{kn} > 0$. We shall employ the following conditions on the net supply possibility sets:

(Y.1) $\boldsymbol{Y}_k$ is a closed subset of $\boldsymbol{R}^N$.

(Y.2) Inaction is possible: i.e., $\mathbf{0} \in \boldsymbol{Y}_k$.

(Y.3) $\boldsymbol{Y}_k$ is convex.

(Y.4) There is free disposal: i.e., $\boldsymbol{y} \in \boldsymbol{Y}_k, \boldsymbol{y}' \leqslant \boldsymbol{y}$ implies $\boldsymbol{y}' \in \boldsymbol{Y}_k$.

(Y.5) World resources and production possibilities are limited: i.e., $\{(\boldsymbol{y}_1, ..., \boldsymbol{y}_K) \in \boldsymbol{Y}_1 \times ... \times \boldsymbol{Y}_K \,|\, \Sigma_{k=1}^{K} \boldsymbol{y}_k \geqslant \mathbf{0}\}$ is a bounded set.

The least plausible of these assumptions are (Y.3) and (Y.4). Condition (Y.3) is critical to our results. Condition (Y.4) can be largely dispensed with (see Debreu (1962)), at the cost of considerably complicating our analysis; this generalization is left to the reader. Further discussions of these conditions can be found in Debreu (1959) and Arrow and Hahn (1971).

A vector $\boldsymbol{y} \in \boldsymbol{Y}_k$ is efficient if there is no distinct vector $\boldsymbol{y}' \in \boldsymbol{Y}_k$ with $\boldsymbol{y}' \geqslant \boldsymbol{y}$. We show later that there is no loss of generality in adding to (Y.5) the facilitating assumption

(Y.6) The set of efficient points in $\boldsymbol{Y}_k$ is bounded.

Let $\boldsymbol{N}_k$ denote the subset of world commodities which could be supplied in nation k under autarky: $\boldsymbol{N}_k = \{n \in \boldsymbol{N} | y_n > 0$ for some $\boldsymbol{y} \in \boldsymbol{Y}_k, \boldsymbol{y} \geqslant \mathbf{0}\}$. A commodity price is a non-negative extended real number, with the value of infinity possible in an autarkic economy in which the commodity is unobtainable. Define the sets of price vectors

$$\boldsymbol{P} = \{(p_1, ..., p_N) | 0 \leqslant p_n < +\infty \text{ for } n \in \boldsymbol{N}\} \text{ and}$$

$$\boldsymbol{P}_k = \{(p_1, ..., p_N) | 0 \leqslant p_n < +\infty \text{ for } n \in \boldsymbol{N}_k, p_n = +\infty \text{ for } n \notin \boldsymbol{N}_k\}.$$

The set $\boldsymbol{P}$ will be the domain of possible world price vectors, while $\boldsymbol{P}_k$ will be the domain of possible price vectors in nation k under autarky. Let $\overline{\boldsymbol{P}}_k = \boldsymbol{P} \cup \boldsymbol{P}_k$.

The national income [6a] of country k under autarky at $\boldsymbol{p} \in \boldsymbol{P}_k$ is $T_k(\boldsymbol{p}) = \max\{\boldsymbol{p}\cdot\boldsymbol{y} | \boldsymbol{y} \in \boldsymbol{Y}_k, y_n = 0 \text{ for } n \notin \boldsymbol{N}_k\}$, achieved on the set of net supply vectors $\boldsymbol{S}_k(\boldsymbol{p}) = \{\boldsymbol{y} \in \boldsymbol{Y}_k | \boldsymbol{y}$ efficient, $y_n = 0$ for $n \notin \boldsymbol{N}_k$, $T_k(\boldsymbol{p}) = \boldsymbol{p}\cdot\boldsymbol{y}\}$. With trade at $\boldsymbol{p} \in \boldsymbol{P}$, $T_k(\boldsymbol{p}) = \max_{\boldsymbol{y}\in\boldsymbol{Y}_k} \boldsymbol{p}\cdot\boldsymbol{y}$ and $\boldsymbol{S}_k(\boldsymbol{p}) = \{\boldsymbol{y} \in \boldsymbol{Y}_k | \boldsymbol{y}$ efficient, $T_k(\boldsymbol{p}) = \boldsymbol{p}\cdot\boldsymbol{y}\}$. Assumptions (Y.1)–(Y.4), (Y.6) imply T_k is a continuous convex positively linear homogeneous func-

[6a] This definition of national income differs from the conventional accounting concept in that it includes the value of endowments held but not traded (e.g., leisure).

tion and S_k is a positively homogeneous of degree zero, upper hemicontinuous correspondence, with $S_k(p)$ a convex set, on P_k and on P.[7]

Consumer i in nation k has a consumption set C_{ik} containing all the commodity vectors on which he can subsist, and a preference order ρ_{ik} of the vectors in C_{ik}. When confronted with an income t and price vector p defining a budget set $B_{ik}(t, p) = \{x \in C_{ik} \mid p \cdot x \leqslant t\}$, he offers to consume the subset of commodity vectors maximizing his preference order. This defines a set-valued demand function $D_{ik}(t, p) = \{x \in B_{ik}(t, p) \mid x \rho_{it} x' \text{ for } x' \in B_{ik}(t, p)\}$. For a price vector $p \in \bar{P}_k$, there is a subsistence income $\tau_{ik}(p) = \inf_{x \in C_{ik}} p \cdot x$ below which the budget set is empty. Recalling that commodity bundles in C_{ik} are non-negative, we make the assumption:

(D.1) Consumer i of nation k can subsist under autarky: i.e., there is a vector $x \in C_{ik}$ with $x_n = 0$ for $n \notin N_k$.

If (D.1) holds, then τ_{ik} is a non-negative, finite, concave, positively linear homogeneous function on $\bar{P}_k$ which is continuous on P and continuous on P_k.[8]

A special case of (D.1) worth noting for its simplicity is the assumption that the consumption set of each individual is the non-negative orthant of the commodity space; then subsistence income is identically zero. The reader may find it useful to make this simplification in interpreting the results below.

Define an income-price domain on which income exceeds the subsistence level and values of the demand function are non-empty sets: $Q_{ik} = \{(t, p) \in R \times \bar{P}_k \mid t > \tau_{ik}(p) \text{ and } D_{ik}(t, p) \neq \phi\}$. Then D_{ik} is a demand correspondence from Q_{ik} into subsets of C_{ik}. We shall assume the consumer satisfies, in addition to (D.1), the following conditions:

[7] A real-valued function G defined on a set $A \subseteq R^M$ is convex if A is convex and $z, z' \in A$, $0 < \theta < 1$ implies $G(\theta z + (1-\theta)z') \leqslant \theta G(z) + (1-\theta)G(z')$; and is positively linear homogeneous if $z \in A$, $\theta > 0$ implies $\theta z \in A$ and $G(\theta z) = \theta G(z)$. A correspondence (non-empty set-valued function) H from a set $A \subseteq R^M$ into subsets of R^N is positively homogeneous of degree zero if $z \in A$, $\theta > 0$ implies $\theta z \in A$ and $H(\theta z) = H(z)$; is upper hemicontinuous if its graph $Gr(H) = \{(z, x) \in A \times R^N \mid x \in H(z)\}$ is closed relative to $A \times R^N$; and is closed upper hemicontinuous if its graph $Gr(H)$ is closed relative to $R^M \times R^N$. Note that the set A for a closed upper hemicontinuous correspondence need not be closed. The properties stated for T_k and S_k are well-known results in the theory of convex functions; one survey listing these properties is McFadden (forthcoming).

[8] The elementary theory of convex functions yields the concavity, homogeneity, and continuity on the interior of P (or relative interior of P_k) of τ_{ik}. A result of Gale et al. (1968) implies that τ_{ik} is continuous on P or on P_k. In general, τ_{ik} need not be continuous on $\bar{P}_k$; however, this will be the case if C_{ik} is closed and bounded (apply the proof of McFadden (forthcoming, Lemma 12)).

(D.2) Values of the demand function are non-empty for positive prices and incomes above subsistence; i.e., $p \in \bar{P}_k$, $p \gg 0$, $t > \tau_{ik}(p)$ imply $(t,p) \in Q_{ik}$.

(D.3) D_{ik} is positively homogeneous of degree zero on Q_{ik}.

(D.4) $D_{ik}(t,p)$ is a convex subset of $B_{ik}(t,p)$ for each $(t,p) \in Q_{ik}$, and all income is spent; i.e., $p \cdot x = t$ for all $x \in D_{ik}(t,p)$.

(D.5) D_{ik} is a closed upper hemicontinuous correspondence for the set of $p \in P$ (resp., $p \in P_k$) and incomes above subsistence: Suppose a sequence $(t^m, p^m) \in Q_{ik}$ with $p^m \in P$ (resp., $p^m \in P_k$) converges to (t^o, p^o) with $p^o \in P$ (resp., $p^o \in P_k$) and $t^o > \tau_{ik}(p^o)$, and suppose $x^m \in D_{ik}(t^m, p^m)$ converges to x^o. Then, $(t^o, p^o) \in Q_{ik}$ and $x^o \in D_{ik}(t^o, p^o)$.

An elementary lemma whose proof we leave to the reader is that if C_{ik} is a closed convex set, ρ_{ik} is a continuous convex preference order exhibiting local non-satiation, and (D.1) holds, then (D.2)–(D.5) must hold.

A distribution policy for nation k is a rule for the distribution of national income among its members; i.e., functions T_{ik} on $\bar{P}_k$ for $i \in I_k$ satisfying $\Sigma_{i \in I_k} T_{ik}(p) = T_k(p)$ for $p \in \bar{P}_k$. We shall require a distribution policy to satisfy

(E.1) T_{ik} is a positively linear homogeneous function on $\bar{P}_k$, and is continuous on P_k, for $i \in I_k$.

Note that (E.1) is consistent with the homogeneity and continuity of T_k.

Nation k is termed autarkically viable if for each $p \in \bar{P}_k$ with $\Sigma_{n \in N_k} p_n > 0$, national income exceeds the minimum needed for subsistence; i.e., $T_k(p) > \Sigma_{i \in I_k} \tau_{ik}(p)$.[9] A distribution policy in nation k is termed sagacious if all consumers are assigned incomes above the minimum needed for subsistence whenever national income exceeds the aggregate subsistence minimum; i.e., $T_{ik}(p) > \tau_{ik}(p)$ for $i \in I_k$ whenever $T_k(p) > \Sigma_{i \in I_k} \tau_{ik}(p)$. (Note that a private ownership, laissez faire economy will not be sagacious unless each consumer alone is autarkically viable.)

In principle, it is possible to have a 'dual' world economy in which the only commodities obtainable in a 'have-not' nation have zero value in world markets. In such an economy, a secondary domestic price system may develop in the 'have-not' nation; the numéraire for this

[9] A sufficient condition for nation k to be autarkically viable is that for each $n \in N_k$ there exists $y_k^* \in Y_k$ and $x_{ik}^* \in C_{ik}$ for $i \in I_k$ satisfying $\Sigma_{i \in I_k} x_{ik}^* \leqslant y_k^*$ and $\Sigma_{i \in I_k} x_{nik}^* < y_{nk}^*$.

secondary system will be a nondesired free good in the world market. While it is of some theoretical interest to investigate the existence of market equilibria with lexicographic price systems for such dual economies, this possibility is a technical nuisance in analyzing the gains from trade proposition. A number of reasonable assumptions rule out the 'dual-economy' case; we shall impose the following:

(E.2) There is at least one commodity n' which can be supplied under autarky in every nation, and at least one consumer in each nation who, when given an income above his subsistence level and a price vector p with $p_{n'} = 0$ will demand an unbounded amount of some commodity (i.e., there exists $n' \in N$ such that $n' \in N_k$ for $k \in K$, and there exists $i \in I_k$ such that $p \in \bar{P}_k, p_{n'} = 0, t > \tau_{ik}(p)$ implies $(t, p) \notin Q_{ik}$).

An allocation in nation k is a vector $a_k = \langle x_{1k}, ..., x_{I_k k}, y_k \rangle$ satisfying $x_{ik} \in C_{ik}$ for $i \in I_k$ and $y_k \in Y_k$. A world allocation is a vector of allocations for each nation, $a = \langle a_1, ..., a_K \rangle$. Allocation a_k is a Pareto non-inferior to allocation a'_k in nation k if it makes each consumer in k no worse off (i.e., $x_{ik} \rho_{ik} x'_{ik}$ for all $i \in I_k$), and is Pareto preferable if, in addition, at least one consumer is better off (i.e., not $x'_{ik} \rho_{ik} x_{ik}$ for at least one $i \in I_k$).

An autarkic competitive equilibrium in nation k is a vector $\langle p_k, a_k \rangle$, where $p_k \in P_k$ satisfies $\Sigma_{n \in N_k} p_{nk} > 0$ and $a_k = \langle x_{1k}, ..., x_{I_k k}, y_k \rangle$ is an allocation for nation k satisfying the autarkic material balance condition $\Sigma_{i \in I_k} x_{ik} \leqslant y_k$, the income balance condition $T_k(p_k) = \Sigma_{i \in I_k} t_{ik}$ with $t_{ik} = p_k \cdot x_{ik}$, and the behavioral equilibrium conditions $x_{ik} \in D_{ik}(t_{ik}, p_k)$ for $i \in I_k$ and $y_k \in S_k(p_k)$. A world competitive equilibrium is a vector $\langle p, a \rangle$, where $p \in P$ satisfies $p \neq 0$ and $a = (a_1, ..., a_k)$ is a world allocation satisfying the world material balance condition $\Sigma_{k \in K} (\Sigma_{i \in I_k} x_{ik} - y_k) \leqslant 0$, and the same nation-by-nation income balance and behavioral equilibrium conditions as in the definition of autarkic competitive equilibrium. (The income balance conditions for each nation imply balance of international payments. Note that when the model is given an intertemporal interpretation, short and long term capital flows are consistent with payments balance.)

An allocation a^*_k in nation k which is in autarkic material balance is termed a non-fringe allocation if the value of each consumer's commodity vector x^*_{ik} exceeds his subsistence income at each non-zero price vector in P_k for which national income is maximized at y^*_{ik}; i.e., if $p \in P_k$ with $\Sigma_{n \in N_k} p_k > 0$ satisfies $p \cdot y^*_k = T_k(p)$, then $p \cdot x^*_{ik} > \tau_{ik}(p)$ for $i \in I_k$. The property of a non-fringe allocation a^*_k which explains its role in our analysis is that a sagacious distribution policy can be defined by assigning to each individual the income just sufficient to allow him to purchase the commodity vector he received in the autarkic allocation (i.e., $p \cdot x_{ik}^*$), and then distributing to each consumer a positive share of the 'social dividend' (i.e., $T_k(p) - \Sigma_{i \in I_k} p \cdot x_{ik}^*$) left after this initial allocation.

Mild sufficient conditions can be given for an autarkic allocation a_k^* to be non-fringe; any one of the following conditions implies this result:

(1) Each consumer has a vector $x_{ik}^{**} \in C_{ik}$ which is strictly smaller than x_{ik}^* in each component $n \in N_k$.

(2) y_k^* is not a profit-maximizing vector in Y_k for any price vector.

(3) a_k^* is an autarkic competitive equilibrium allocation obtained with a sagacious distribution policy, and Y_k is smooth at y_k^* (i.e., Y_k has a unique tangent plane at y_k^*, or equivalently, $T_k(p)$ is strictly quasi-convex at the competitive price vector).

In the special case where the consumption set of each consumer is the non-negative orthant of the commodity space, a sufficient condition for an autarkic competitive equilibrium allocation to be non-fringe is that each consumer hold either a positive quantity of the commodity n' defined in assumption (E.2) or a positive quantity of some commodity (say, leisure) whose marginal product in the production of n' is positive.

Fig. 1 illustrates a case of a fringe autarkic competitive equilibrium allocation in nation 1 in which the gains from trade proposition B fails to hold. Note that the world Pareto optimum in this figure defines a 'quasi-equilibrium' in the sense that a price line exists such that profits are maximized and the cost of attaining at least as satisfactory commodity vectors are minimized for each nation-consumer at this allocation (the terminology is that of Debreu (1962)). However, this quasi-equilibrium fails to be an equilibrium because the consumer in nation 1 can purchase more desirable vectors at the same cost as the vector he is allocated. This phenomenon can occur for a locally non-satiated consumer only if he has no less-expensive bundle in his consumption set; i.e., only if his income falls to the subsistence level incompatible with a sagacious distribution policy.

3. Gains from competitive trade

For the model above of a family of nations with competitive domes-

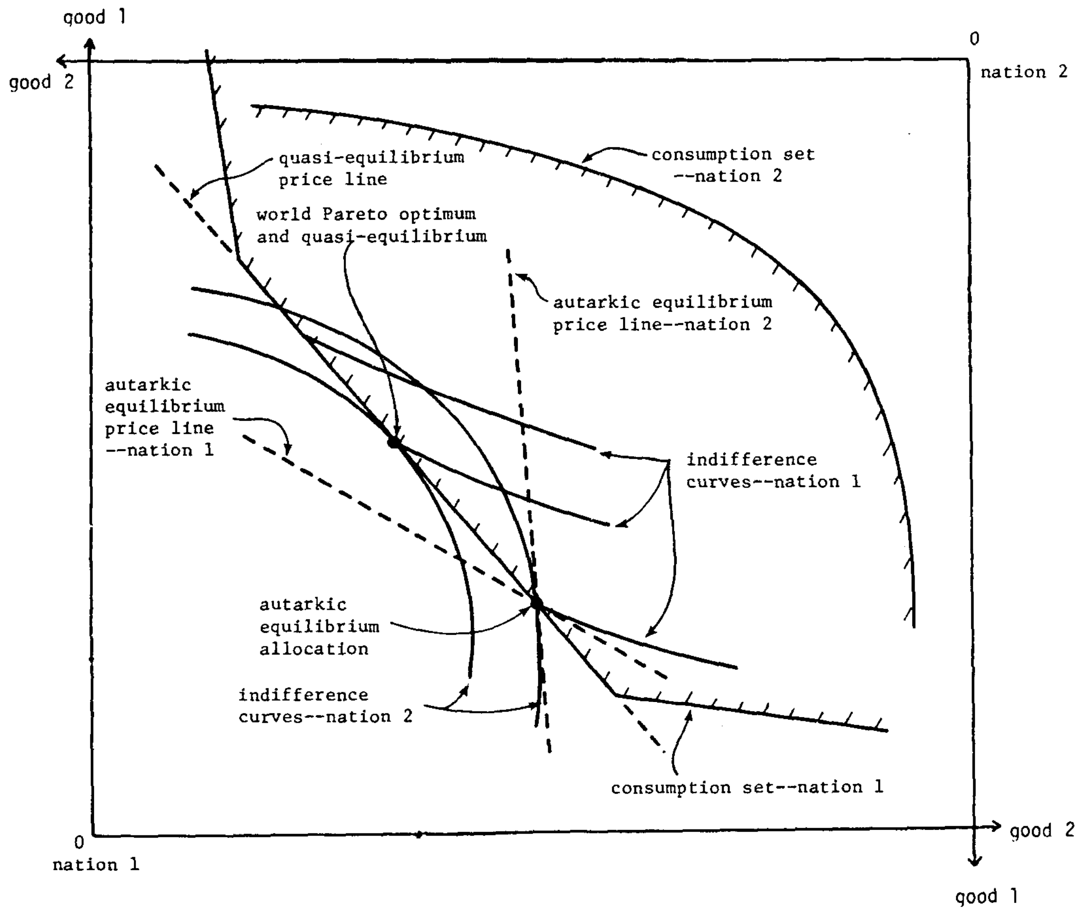

Fig. 1. Autarkic competitive equilibria exist but no world competitive equilibrium exists. Note that the autarkic equilibrium in nation 1 is a fringe equilibrium ... the optimum allocation and quasi-equilibrium price line define a quasi-equilibrium at which nation 1 receives subsistence income.

tic economies, two preliminary results establish the existence of equilibrium under autarky or free trade when nations follow sagacious distribution policies. These theorems also provide general equilibrium existence conditions for 'mixed' economies with social welfare distribution policies. Note that these theorems use only the properties (D.1)–(D.5) of the demand correspondences and corresponding properties of the supply correspondences, and not competitive behavioral assumptions underlying these properties. Hence, the theorems can be interpreted as giving market equilibrium existence conditions for any pattern of behavior of economic agents yielding 'offer correspondences' with these properties.

Theorem 1. Suppose that nation k satisfies conditions (Y.1)–(Y.4), (Y.6) on the supply sector and conditions (D.1)–(D.5) on each consumer $i \in I_k$, is autarkically viable, and follows a sagacious distribution policy satisfying (E.1). Then an autarkic competitive equilibrium exists.

Corollary 1.1. The conclusion of Theorem 1 continues to hold if (Y.6) is replaced by the condition that $\{y \in Y_k \mid y \geqslant 0\}$ be bounded.

Theorem 2. Suppose that nations $1, \ldots, K$ each satisfy conditions (Y.1)–(Y.4), (Y.6) on the supply sector and conditions (D.1)–(D.5) on each consumer $i \in I_k$, are autarkically viable, and follow sagacious distribution policies satisfying (E.1). Suppose (E.2) holds. Then a world competitive equilibrium exists. Further, provided every consumer is locally non-satiated, the equilibrium allocation is Pareto optimal for the world's consumers, and an autarkic allocation in nation k which makes some consumers in k better off must also make some consumers in k worse off.

The gains from trade proposition is established for multiple-consumer economies with competitive domestic markets by the following result.

Theorem 3. Suppose that nations $1, \ldots, K$ each satisfy conditions (Y.1)–(Y.4), (Y.6) on the supply sector, conditions (D.1)–(D.5) on each consumer $i \in I_k$, and condition (E.2), and are autarkically viable. Suppose a non-fringe autarkic allocation a_k^* prevails in each nation k. Define a distribution policy $T_{ik}(p) = p \cdot x_{ik}^* + \theta_{ik}\,[T_k(p) - \Sigma_{i \in I_k} p \cdot x_{ik}^*]$ for $i \in I_k$ with $\theta_{ik} > 0$, $\Sigma_{i \in I_k} \theta_{ik} = 1$, for each $k \in K$. Then this distribution policy satisfies (E.1) and is sagacious, and implies the existence of a world competitive equilibrium $\langle \bar{p}, \bar{a}_1, \ldots, \bar{a}_K \rangle$. This equilibrium is Pareto non-inferior for the consumers of each nation to the allocation achieved under autarky. Further, if it is not the case that $\langle \bar{p}, a_1^*, \ldots, a_K^* \rangle$ is a world competitive equilibrium and if each consumer is locally non-satiated at the autarkic allocation, then the allocation achieved under trade is Pareto preferable for the world's consumers to the autarkic allocation.

A slightly stronger version of this theorem can be obtained by refining the distribution policy. Define an autarkic allocation a_k^* to be non-extreme if the value of each consumer's commodity vector x_{ik}^* exceeds his subsistence income at each non-zero price vector in P_k for which national income is maximized at y_k^* and the minimum cost of purchasing a bundle as good as x_{ik}^* costs at least as much as x_{ik}^* for each consumer. Clearly every non-fringe allocation is also non-extreme. For consumer i in nation k, $x \in C_{ik}$, and $p \in \bar{P}_k$, define an expenditure function $M_{ik}(\boldsymbol{x},\boldsymbol{p}) = \inf\{\boldsymbol{p}\cdot\boldsymbol{x}' \mid \boldsymbol{x}' \in C_{ik}, \boldsymbol{x}'\rho_{ik}\boldsymbol{x}\}$ giving the least cost of purchasing a commodity vector as good as $\boldsymbol{x}$ at price vector $\boldsymbol{p}$. This function is concave and positively linear homogeneous on $\bar{\boldsymbol{P}}_k$, and is continuous on $\boldsymbol{P}$ and on $\boldsymbol{P}_k$.

Corollary 3.1. The conclusions of Theorem 3 continue to hold if in each nation the allocation $\boldsymbol{a}_k^*$ is non-extreme, and in any nation in which $\boldsymbol{a}_k^*$ is a fringe allocation, the distribution policy

$$T_{ik}(\boldsymbol{p}) = M_{ik}(\boldsymbol{x}_{ik}^*,\boldsymbol{p}) + \theta_{ik}\,[T_k(\boldsymbol{p}) - \sum_{i\in \boldsymbol{I}_k} M_{ik}(\boldsymbol{x}_{ik}^*,\boldsymbol{p})]\ ,$$

$$\theta_{ik} > 0, \quad \sum_{i\in \boldsymbol{I}_k} \theta_{ik} = 1\ ,$$

is used.

Corollary 3.2. The conclusions of Theorems 2 and 3 and Corollary 3.1 continue to hold when condition (Y.6) is replaced by condition (Y.5).

4. Appendix: Proofs of theorems

Theorems 1–3 can be proved as corollaries of the following basic law of supply and demand.

Theorem 4.[10] Define $\bar{U} = \{\boldsymbol{p} \in \boldsymbol{R}^M \mid \boldsymbol{p} \geqslant 0, \Sigma_{n\in M} p_n = 1\}$ and $U^{\text{o}} = \{\boldsymbol{p} \in \bar{U} \mid \boldsymbol{p} \gg 0\}$. Suppose there exists a subset U of $\bar{U}$ containing U^{o} and a function ζ from U into non-empty subsets of $\boldsymbol{R}^M$ which satisfies

(a) ζ is bounded below; i.e., there exists $\boldsymbol{b} \in \boldsymbol{R}^M$ such that $\boldsymbol{b} \leqslant \boldsymbol{x}$ for all $\boldsymbol{x} \in \zeta(\boldsymbol{p})$, $\boldsymbol{p} \in U$.

(b) For each $\boldsymbol{p} \in U$, $\zeta(\boldsymbol{p})$ is a convex set, and $\boldsymbol{p}\cdot\boldsymbol{x} \leqslant 0$ for all $\boldsymbol{x} \in \zeta(\boldsymbol{p})$.

(c) ζ is closed upper hemicontinuous on U; i.e., if a sequence $\boldsymbol{p}^m \in U$ converges to $\boldsymbol{p}^{\text{o}}$ and $\boldsymbol{x}^m \in \zeta(\boldsymbol{p}^m)$ converges to $\boldsymbol{x}^{\text{o}}$, then $\boldsymbol{p}^{\text{o}} \in U$ and $\boldsymbol{x}^{\text{o}} \in \zeta(\boldsymbol{p}^{\text{o}})$.

Then, there exists a $\boldsymbol{p}^* \in U$ and a $\boldsymbol{x}^* \in \zeta(\boldsymbol{p}^*)$ such that $\boldsymbol{x}^* \leqslant \boldsymbol{0}$.

Proof: Let U^m be a non-decreasing sequence of compact convex subsets of U^{o} such that $U^{\text{o}} \subseteq \cup_{m=1}^{\infty} U^m$. Hypotheses (a) and (b) imply the range of ζ on U^m is bounded. A standard argument (Debreu (1957)) establishes the following lemma; a proof is outlined for completeness.

Lemma. Under the hypotheses of Theorem 4, there exists $\boldsymbol{p}^m \in U^m$ and $\boldsymbol{x}^m \in \zeta(\boldsymbol{p}^m)$ such that $\boldsymbol{p}\cdot\boldsymbol{x}^m \leqslant 0$ for all $\boldsymbol{p} \in U^m$.

[10] This result specializes a theorem of Grandmont (1970).

Proof outline: Let X^m denote the convex hull of the set $\{x \in \zeta(p) | p \in U^m\}$; X^m is compact. For $(x, p) \in X^m \times U^m$, define the function υ into non-empty subsets of $X^m \times U^m$ by $\upsilon(x, p) = \{(x', p') | x' \in \zeta(p)$ and p' maximizes $p'' \cdot x$ on $U^m\}$. It is immediate that υ is upper hemicontinuous and that $\upsilon(x, p)$ is a convex set. Hence, the fixed point theorem of Kakutani (1941) establishes the existence of $(x^m, p^m) \in \upsilon(x^m, p^m)$. Then $x^m \in \zeta(p^m)$ and $0 \geqslant p^m \cdot x^m \geqslant p \cdot x^m$ for $p \in U^m$.

Returning to the proof of Theorem 4, note that the x^m given by the lemma are bounded by $x^m \geqslant b$ and $p^1 \cdot x^m \leqslant 0$. Hence, there exists a subsequence of (p^m, x^m) (retain notation) converging to $(\bar{p}, \bar{x})$. By (c), $\bar{p} \in U$ and $\bar{x} \in \zeta(\bar{p})$. For $p \in \bar{U}$, there is a sequence $r^m \in U^m$ with $p = \lim r^m$. Since $r^m \cdot x^m \leqslant 0$, in the limit $p \cdot \bar{x} \leqslant 0$. This holds for all $p \in \bar{U}$; hence, $\bar{x} \leqslant 0$. This proves Theorem 4.

Proof of Theorem 1: Define $\bar{U} = \{p \in P_k | \Sigma_{n \in N_k} p_n = 1\}$, $U^o = \{p \in U | p \gg 0\}$, $U = \{p \in \bar{U} | (T_{ik}(p), p) \in Q_{ik}, i \in I_k\}$. Since the distribution policy is sagacious, $U^o \subseteq U$. Define $\zeta(p) = \Sigma_{i \in I_k} D_{ik}(T_{ik}(p), p) - S_k(p)$. From (Y.6), ζ is bounded below. (Y.3) and (D.4) imply $\zeta(p)$ is convex and $p \cdot x = 0$ for $x \in \zeta(p)$. The continuity of T_{ik} from (E.1) and the closed upper hemicontinuity of the D_{ik} and S_k correspondence imply ζ closed upper hemicontinuous. Hence, Theorem 4 implies the existence of $p_k^* \in U$, $x^* \in \zeta(p_k^*)$ with $x^* \leqslant 0$ and $p_k^* \cdot x^* = 0$. Let a_k^* be an allocation with $x_{ik}^* \in D_{ik}(T_{ik}(p_k^*), p_k^*)$ and $y_k^* \in S_k(p_k^*)$ yielding x^*. Then $\langle p_k^*, a_k^* \rangle$ is an autarkic competitive equilibrium.

Proof of the corollary to Theorem 1: Suppose there exists a scalar β such that $y \in Y_k$, $y \geqslant 0$ implies $y_n \leqslant \beta/2$. Define $Y_k' = \{y \in Y_k | |y_n| \leqslant \beta\}$ and $Y_k'' = \{y \in R^N | y \leqslant y' \in Y_k'\}$. Y_k'' satisfies (Y.1)–(Y.4) and (Y.6). Apply Theorem 2 to obtain an autarkic competitive equilibrium for Y_k''. One can then verify that the supply vector in the competitive allocation also maximizes national income over the original possible supply set Y_k.

Proof of Theorem 2: Define $\bar{U} = \{p \in P | \Sigma_{n \in N} p_n = 1\}$, $\zeta(p) = \Sigma_{k \in K} [\Sigma_{i \in I_k} D_{ik}(T_{ik}(p), p) - S_k(p)]$, and $U = \{p \in \bar{U} | \Sigma_{n \in N_k} p_n > 0$ for $k \in K$ and $\zeta(p) \neq \phi\}$. By the same argument as in Theorem 1, $U^o = \{p \in U | p \gg 0\} \subseteq U$, ζ is bounded below, and $\zeta(p)$ is convex and $p \cdot x = 0$ for $x \in \zeta(p)$, $p \in U$. The continuity of T_{ik} from (E.1) and the closed upper hemicontinuity of the D_{ik} and S_k correspondences establish the following upper hemicontinuity property for ζ: Suppose a sequence (p^m, x^m) converges to (p^o, x^o), where $p^m \in U$ and $x^m \in \zeta(p^m)$. If $\Sigma_{n \in N_k} p_n^o > 0$ for all $k \in K$, then $x^o \in \zeta(p^o)$. Note for this sequence that there exist allocations a_k^m with $x^m = \Sigma_{k\in K}[\Sigma_{i\in I_k} x_{ik}^m - y_k^m]$. Then there is a subsequence (retain notation) with a_k^m converging to a_k°. From the definition of the set of world commodities, $\Sigma_{k\in K} T_k(p) > 0$ for every $p \in U$. Hence, for some k, $T_k(p^{\circ}) > 0$. We next argue that $T_{ik}(p^{\circ}) > \tau_{ik}(p^{\circ})$ for all $i \in I_k$ in this nation. If $\Sigma_{n\in N_k} p_n^{\circ} = 0$, then $T_k(p^{\circ}) > \Sigma_{i\in I_k} \tau_{ik}(p^{\circ}) = 0$ and the result follows because the distribution policy is sagacious. If $\Sigma_{n\in N_k} p_n^{\circ} > 0$, then the result follows from autarkic viability and a sagacious distribution policy. Condition (D.5) then implies $(T_{ik}(p^{\circ}), p^{\circ}) \in Q_{ik}$ for each $i \in I_k$, since x_{ik}^m converges to x_{ik}°. Now, consider the commodity n' postulated in (E.2). The supposition $p_{n'}^{\circ} = 0$ implies $(T_{ik}(p^{\circ}), p^{\circ}) \notin Q_{ik}$ for some consumer i in nation k, contradicting the results obtained above. Hence, $p_{n'}^{\circ} > 0$, implying $\Sigma_{n\in N_k} p_n^{\circ} > 0$ for each $k \in K$ and thus, $T_{ik}(p^{\circ}) > \tau_{ik}(p^{\circ})$ for all $i \in I_k$, $k \in K$. Then (D.5) implies $(T_{ik}(p^{\circ}), p^{\circ}) \in Q_{ik}$ for all $i \in I_k$, $k \in K$, and hence, $p^{\circ} \in U$, $x^{\circ} \in \zeta(p^{\circ})$. Hence, the hypotheses of Theorem 4 hold, and this result establishes the existence of $\bar{p} \in U$, $\bar{x} \in \zeta(\bar{p})$ with $\bar{x} \leqslant 0$. Let $(\bar{a}_1, \ldots, \bar{a}_K)$ be a world allocation with $\bar{x}_{ik} \in D_{ik}(T_{ik}(\bar{p}), \bar{p})$ and $\bar{y}_k \in S_k(\bar{p})$ yielding $\bar{x}$. Then, $\langle \bar{p}, \bar{a}_1, \ldots, \bar{a}_K \rangle$ is a world competitive equilibrium.

The Pareto optimality of the world competitive equilibrium is a standard result (e.g., Koopmans (1957)).

Proof of Theorem 3: Note that $T_k(p) - \Sigma_{i\in I_k} p\cdot x_{ik}^* \geqslant p\cdot y^* - \Sigma_{i\in I_k} p\cdot x_{ik}^* \geqslant 0$ by income maximization and the autarkic material balance condition $y^* \geqslant \Sigma_{i\in I_k} x_{ik}^*$. If $T_k(p) = p\cdot y^*$, then $p\cdot x_{ik}^* > \tau_{ik}(p)$ for $i \in I_k$ by the hypothesis that a^* is non-fringe. If $T_k(p) > p\cdot y^*$, the 'social dividend' is positive, and each individual receives an income $T_{ik}(p) > p\cdot x_{ik}^* \geqslant \tau_{ik}(p)$. Hence, the distribution policy is sagacious, and can easily be seen to satisfy (E.1). Therefore, Theorem 2 implies the existence of a world competitive equilibrium $\langle \bar{p}, \bar{a}_1, \ldots, \bar{a}_K \rangle$. Since each consumer has the option of purchasing the vector he attained under autarky, the world equilibrium allocation is Pareto non-inferior to the autarkic allocation for the consumers of each nation.

Suppose the two equilibrium allocations are Pareto indifferent. Then, $x_{ik}^* \rho_{ik} \bar{x}_{ik}$ for all consumers, implying by local non-satiation that $\bar{p}\cdot x_{ik}^* \geqslant \bar{p}\cdot \bar{x}_{ik}$ for all consumers. The inequalities $\bar{p}\cdot y_k^* \leqslant \bar{p}\cdot \bar{y}_k = \Sigma_{i\in I_k} \bar{p}\cdot \bar{x}_{ik} \leqslant \Sigma_{i\in I_k} \bar{p}\cdot x_{ik}^* = \bar{p}\cdot y_k^*$ then imply $\langle \bar{p}, a_1^*, \ldots, a_K^* \rangle$ is a world competitive equilibrium.

Proof of Corollary 3.1: Note that $T_k(p) - \Sigma_{i\in I_k} M_{ik}(x_{ik}^*, p) \geqslant p\cdot y^* - \Sigma_{i\in I_k} p\cdot x_{ik}^* \geqslant 0$. If the first inequality is strict, then the 'social dividend' is positive and each individual receives an income greater than $M_{ik}(x^*_{ik}, p) \geqslant \tau_{ik}(p)$. If equality holds, then by the hypothesis that the allocation a^*_k is non-extreme, $M_{ik}(x^*_{ik}, p) = p \cdot x^*_{ik} > \tau_{ik}(p)$. Hence, the distribution policy is sagacious. The proof then is the same as in Theorem 3.

Proof of the Corollary 3.2 to Theorems 2 and 3: By (Y.5), there exists a scalar β such that $|y_{kn}| \leqslant \beta/2$ for any $y_k \in Y_k$ with $\Sigma_{k \in K} y_k \geqslant 0$. Define $Y'_k = \{y \in Y_k \,|\, |y_{kn}| \leqslant \beta\}$ and $Y''_k = \{y \in R^N \,|\, y \leqslant y' \in Y'_k\}$. Then, Y''_k satisfies (Y.6). The equilibria established by Theorems 2 and 3 can then be shown to also be national income maximizing for the original Y_k.

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