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by

Daniel McFadden

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AN EXAMPLE OF THE NON-EXISTENCE OF MALINVAUD PRICES IN A TIGHT ECONOMY

Daniel McFADDEN*

University of California, Berkeley, Calif., U.S.A.

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Malinvaud (1953, 1963) has shown that an efficient production program in an infinite-horizon economy can be supported by a sequence of prices, provided this program is *non-tight*; i.e., at each time, the output of producible resources can be increased with lower levels of each non-producible input by use of increased inputs of producible goods. Recent proofs and generalizations of the Malinvaud price theorem by McFadden (1965), Peleg and Yaari (1970), and Kurz and Majumdar (1972) all employ the non-tightness assumption. This note gives an example of a tight economy in which Malinvaud prices fail to exist, establishing that some condition like non-tightness is needed in these theorems.[1]

Consider an intertemporal economy with one producible good ('machines') and one non-producible good ('labor'). Suppose one unit of labor is supplied each period, and is used solely as an input to production. Let a_t, b_t, and $y_t = b_t - a_t$ denote, respectively, the input, gross output, and net output of machines per unit of labor in period t. The production function for machines, illustrated in the figure, is

$$b_{t+1} = f(a_t) = \begin{cases} 2-2(1-a_t)^2 & \text{for} \quad 0 \leqq a_t \leqq 1, \\ 2 & \text{for} \quad a_t \geqq 1. \end{cases}$$

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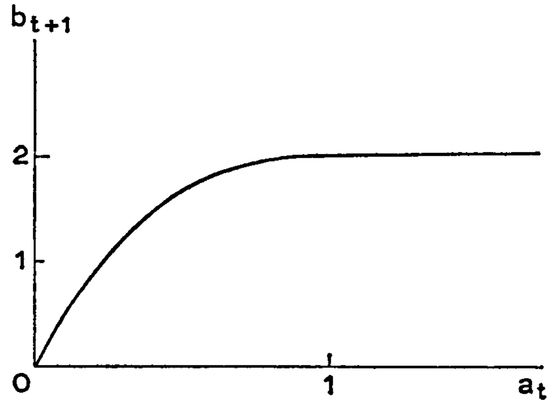
[1]The set of all feasible consumption programs in this example is a convex subset of l_∞, and while the constructed efficient program cannot be supported by a competitive price *sequence*, it is present-value maximizing relative to a price system in the form of a non-negative (non-zero) linear functional on l_∞. In a sense, examples of this type set the point of departure for exploring other concepts of price systems that can define criteria for evaluating alternative feasible programs, and in this case, the price system only looks at the 'long-run' or 'asymptotic' behavior of the program.



Assume $b_0 = \frac{3}{2}$. Let T denote the set of time periods $t_n = 3^n$ for $n = 1, 2, \ldots$; i.e., $T = \{3, 9, 27, 81, \ldots\}$. Define a program (a_t, b_t, y_t) as follows:

$$\bar{a}_t = \begin{cases} 1 & \text{if} \quad t \in T, \\ \frac{1}{2} & \text{if} \quad t \notin T, \end{cases} \qquad t = 0, 1, 2, \ldots;$$

$$\bar{b}_t = \begin{cases} 2 & \text{if} \quad t-1 \in T, \\ \frac{3}{2} & \text{if} \quad t-1 \notin T, \end{cases} \qquad t = 0, 1, 2, \ldots;$$

$$\bar{y}_t = \begin{cases} \frac{1}{2} & \text{if} \quad t \in T, \\ \frac{3}{2} & \text{if} \quad t-1 \in T, \\ 1 & \text{otherwise}, \end{cases} \qquad t = 0, 1, 2, \ldots .$$

This program is feasible, since $\bar{y}_t = \bar{b}_t - \bar{a}_t$ and $\bar{b}_{t+1} = f(\bar{a}_t)$. We will now show that it is efficient. Suppose, to the contrary, that there exists a second feasible program $(\tilde{a}_t, \tilde{b}_t, \tilde{y}_t)$ with $\tilde{y}_t \geqq \bar{y}_t$ for all t, and with the inequality strict in some period τ. If τ denotes the first period with a strict inequality, then

$$\bar{y}_t = \tilde{b}_t - \tilde{a}_t = \bar{b}_t - \bar{a}_t \quad \text{for} \quad t = 0, \ldots, \tau - 1$$

and $\tilde{b}_0 = \bar{b}_0$ imply

$$\delta_\tau \equiv \bar{a}_\tau - \tilde{a}_\tau = (\bar{b}_\tau - \bar{y}_\tau) - (\tilde{b}_\tau - \tilde{y}_\tau)$$
$$= \tilde{y}_\tau - \bar{y}_\tau > 0.$$

Define $\delta_t = \bar{a}_t - \tilde{a}_t$. Then

$$0 \leqq \tilde{y}_{t+1} - \bar{y}_{t+1} = f(\tilde{a}_t) - f(\bar{a}_t) - \tilde{a}_{t+1} + \bar{a}_{t+1},$$

or

$$\delta_{t+1} \geqq f(\bar{a}_t) - f(\tilde{a}_t)$$
$$= \begin{cases} 2\delta_t^2 & \text{for} \quad t \in T, \\ 2\delta_t(1+\delta_t) & \text{for} \quad t \notin T, \end{cases} \qquad t = \tau, \tau+1, \ldots .$$

Let t_n be the first element of T satisfying $t_n \geqq \tau$. The above inequality implies

$$\delta_{t_n} \geqq \delta_\tau, \qquad \delta_{t_n+1} \geqq \delta_{t_n}^2,$$

and

$$\delta_{t_{n+1}} \geqq (2^{(t_{n+1}-t_n)})\delta_t^{2_n}.$$

Hence,

$$\delta_{t_{n+j}} \geqq (2^{(t_{n+j}-t_{n+j-1}+2(t_{n+j-1}-t_{n+j-2})+\ldots+2^{j-1}(t_{n+1}-t_n))})\delta_{t_n}^{2^j}$$
$$\geqq (2^{(t_{n+j}-t_{n+j-1})})\delta_\tau^{2^j}.$$

But, for j sufficiently large,

$$t_{n+j}-t_{n+j-1} = 3^{n+j}-3^{n+j-1}$$
$$= \tfrac{2}{3}\, 3^{n+j} > 2^j[(-\log \delta_\tau)/(\log 2)],$$

implying $\delta_{t_{n+j}} > 1$, or

$$\tilde{a}_{t_{n+j}} = \bar{a}_{t_{n+j}} - \delta_{t_{n+j}} < 0.$$

This contradicts the supposition that $(\tilde{a}_t, \tilde{b}_t, \tilde{y}_t)$ is feasible, establishing that $(\bar{a}_t, \bar{b}_t, \bar{y}_t)$ is efficient.

Suppose now that $(\bar{p}_t)$ were a sequence of Malinvaud prices for the program $(\bar{a}_t, \bar{b}_t, \bar{y}_t)$. These prices would satisfy

$$\bar{p}_t = \bar{p}_{t+1} f'(\bar{a}_t),$$

or

$$\bar{p}_t = \begin{cases} 0 & \text{for} \quad t \in T, \\ 2\bar{p}_{t+1} & \text{for} \quad t \notin T. \end{cases}$$

Hence, since T is infinite, $p_t = 0$ for all t, and no Malinvaud prices exist.

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