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FORECASTING THE IMPACTS OF ALTERNATIVE ELECTRICITY RATE STRUCTURES: A FEASIBILITY STUDY

by

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ABSTRACT

This report is concerned with the problem of forecasting the consumption of electricity under alternative rate structures, a responsibility of the California State Energy Resources Conservation and Development Commission, as mandated by AB 4195, Section 25403.5 . A framework is suggested for obtaining forecasts which are sensitive to changes in rate structures, including changes in block rates and the introduction of seasonal and time-of-day rates. A preliminary assessment is made of the feasibility of obtaining adequate data for baseline and demand reponse calibrations. Tentative suggestions are made on structures and functional forms to simulate the market facing an electric utility. Under simplifying assumptions, a trial analysis of the long-run impacts of changes in block rate structure is carried out. This feasibility analysis makes no attempt to provide a realistic dynamic simulation of the consumption of electricity, and ignores changes in economic conditions, the possibility of time-of-day pricing, or changes in supply cost. The purpose of the analysis is to demonstrate the feasibility of the forecasting framework, and is <u>not</u> intended to be used as a basis for policy conclusions.

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FORECASTING THE IMPACTS OF ALTERNATIVE ELECTRICITY RATE STRUCTURES: A FEASIBILITY STUDY

1. Introduction

The California State Energy Resources Conservation and Development Commission is required by the Warren-Alquist Act to provide an electricity demand forecasting and planning program as an input to the Commission's facility siting decisions, and to provide a basis for more general Commission recommendations. Specifically, the Commission is "charged by the Warren-Alquist Act with responsibility for adopting a ten-year forecast as a basis for determining the need for new sites and generating facilities. In addition, the Commission is responsible for adopting a twenty-year forecast which is to be used to assess the implications of present trends and to evaluate potential conservation and benefits from alternative electricity demand and supply futures."[1]

Assembly Bill 4195 (Warren) amends the Public Utilities Code, Sect. 25403.5, to require that "the commission shall, by July 1, 1978, adopt standards by regulation for a program of electrical load management for each utility service area. In adopting the standards, the commission shall consider, but need not be limited

[1] Draft Electricity Forecasting and Planning Report, CSERCDC, Ocotber, 1976, p. I-1 .

to, the following load management techniques:

(1) Adjustments in rate structure to encourage use of electrical energy at off-peak hours or to encourage control of daily electrical load. ..."[1]

The establishment of cost-effective standards and the determination of their impact on the need for new generating facilities requires the Commission to provide forecasts of consumption and load curves which are sensitive to alternative rate structures. This need is recognized in the SERCDC Draft Electricity Forecasting and Planning Report, which states that a question to be answered by the forecasting process is "What is the expected impact of future electrical prices and rate structures?"[2]

The demand forecasts submitted to the SERCDC by major utilities in 1976, and the SERCDC Staff forecasts, fail to address the impact of rate level and structure in a satisfactory manner. The utility forecasts, employing variants of the forecasting methodology proposed by Economics Sciences Corp. and mandated by SERCDC in 1975, reflect some assumptions and analysis of the impact of the average cost of electricity on

[1] AB 4195, Chapter 1375, Sect. 25403.5 .

[2] Volume II, October 1976, p. I-3.

consumption. However, no alternative price scenarios are given. The forecasting methodology does not allow examination of the impacts of changing the rate structure, either by modifying block rates or introducing time-of-day pricing. The SERCDC staff forecasts are completely insensitive to electricity rates. It is indicative of the limits of both the current utility and SERCDC staff forecasting methodologies that neither is able to forecast the impact on demand of the "lifeline" rates mandated by the Warren bill.

Historically, almost no data were available on the response of consumption to rate structure. Consequently, there was little reason to develop forecasting methodologies which were sensitive to rate structure, since they could not be calibrated. The current utility and SERCDC staff forecasts use methodologies developed under these data limitations.

Very recently, new data sources have become available--from disaggregate customer surveys, load management studies, peakload pricing experiments, and end-use metering experiments--which allow calibration of consumption response to rate structures. An analysis and forecasting methodology which captures the important structural features of rates and of consumption is necessary to exploit effectively the information contained in these data sources, and to allow answers to a broad range of policy questions concerning rate structure.

This report proposes a methodological framework for analysis and forecasting which meets the need described above. This

general framework could accommodate alternative models of the determination of demand. Specifically, the "analytic" approach to forecasting represented by the Economic Sciences/utility methodologies can be employed, provided the methodology is augmented to provide load curves. Alternately, the "synthetic" approach to forecasting represented by the SERCDC staff methodology can be employed provided consumption by end use is made price-sensitive. This report proposes an "analytic/synthetic" approach which should combine the strong points of each method. However, it should be emphasized that the usefulness of the proposed framework for forecasting could be adapted to a variety of models of the process determining consumption.

In setting out a forecasting methodology, two obvious points must be kept in mind:

(1) If peakload pricing is to be investigated, the distribution of consumption over time--or <u>load curve</u>--is required, and the load curve must be forecast as a function of peakload rates.

(2) If alternative block rate structures are to be investigated, the distribution of customers by size--or <u>consumption frequency distribution</u>[1]--is required, and the consumption frequency distribution must be forecast as a function of block

[1] This is sometimes referred to as the <u>bill</u> frequency distribution.

rates.

A general methodology allowing both peakload pricing and changes in block rate structure must include determination of both the load curve and the consumption frequency distributions. From these schedules, revenues, average consumption, peak demand, and a load duration curve can be calculated. Further, the consumption frequency distribution allows the assessment of the costs, revenues, and effective rates of return on small and large customers.

Section 2 of this report outlines a general framework for analysis and forecasting, including the classification of customers and schedules by end use type, the determination of base year conditions, and the forecasting of impacts of alternative rate policies. A schematic overview is given of data requirements and availabilities. Section 3 sets out a framework for describing electrical rates, consumption frequency distributions, and load curves. Empirical evidence is given for the proposition that for a user type--defined by end uses of electricity, such as space heating and air conditioning--the pattern of electricity consumption over time is independent of average consumption. With this proposition, it is possible to describe completely the structure of demand from available data on consumption frequency distributions and on load curves. Section 4 describes the computation of customer bills, revenues, average demands, system load curves, and summary price measures which enter the computation of demand reponse. Section 5

describes the forecasting of average consumption and consumption frequency distributions in response to alternative rate structures. Section 6 describes the forecasting of load curves in response to peakload pricing. Section 7 describes the forecasting of user type shares--or <u>saturation curves</u>--which are rate-sensitive. A trial application of the methodology for forecasting consumption frequency distributions is carried out in Section 8 for San Diego data. This application is intended to illustrate the feasibility of the method, and is carried out under simplifying assumptions which limit its usefulness for drawing policy conclusions.

The report concludes with a series of appendices. Appendix A is an annotated bibliography of previous demand studies. Appendix B describes data available from surveys and related sources. Appendix C describes data available, or soon to be available, from peakload pricing experiments. Appendix D describes data available on consumption frequency distributions. Appendix E gives a flow chart for a computer program to carry out the forecasting methodology described in this report, and lists an APL program which implements a portion of this methodology under simplifying assumptions.

2. A Framework for Analysis and Forecasting

Electric utility customers are normally divided into three classes--residential, commercial, and industrial, distinguished by rate schedule.[1] Each customer class may be disaggregated further by user type, where a user type is characterized by a particular pattern of end use of electricity. The number and definition of user types will depend on the shape of the load curve and the sensitivity of demand to rates. Customers with similar load curves and demand response can be treated as a single user type. For residential customers, identification of eight user types--distinguished by type of water heating, space

[1] Commercial customers are normally identified as those on general service rate schedules who are not sufficiently large to qualify for Large Light and Power demand schedules. Industrial customers are identified as those on Large Light and Power schedules. While commercial customers are primarily offices and retail establishments, and while industrial customers are primarily factories, the classification between commercial and industrial customers is not normally based on functional activity. For example, large office buildings and centrally metered shopping centers will usually be on "industrial" rate schedules, while small factories with no process use of electricity will usually be on general service rate schedules. As a consequence, the numbers of commercial and industrial customers is sensitive to the rate schedules, particularly the qualifications for Large Light and Power schedules, and to the level of consumption.

heating, and air conditioning--appears to be adequate to isolate major differences in load curves.[1] These user types are listed in Table 1. Divisions of commerical and industrial customers into user types are also suggested in this table. However, these are not based on empirical investigation, and may require modification depending on the practicality and usefulness of the proposed division.

Rate schedules may differ by user type, as for example, lifeline schedules, which provide different allowances to residential customers depending on type of water and space heating. There is also the possiblity that a user type may face different rate schedules for different end uses, as for example, interruptable service for industrial process use, or separately metered timed heating service for residential customers.

Analysis of demand and revenues should be carried out for each user type, and for each end use which is billed separately. A "synthetic" approach to demand forecasting, building up total demand from demand by end use, would fit naturally into this framework, provided data were available to forecast rate-sensitive demand by end use. Alternately, an "analytic" approach to forecasting, in which total demand is forecast as a function

[1] This question has been examined carefully only for seasonal variation in consumption by residential customers in San Diego. Impressionistic inferences from peakload pricing experiment data suggest that when analyzed, these data will support the suggested eight-way classification.

TABLE 1. Proposed User Types

[Defined by customer groups with similar load curves.]

Residential

1. Electric water heating, space heating, air conditioning
2. Electric water heating, space heating, no air conditioning
3. Electric water heating, air conditioning
4. Electric water heating, no air conditioning
5. Electric space heating, air conditioning
6. Electric space heating, no air conditioning
7. Air conditioning
8. No air conditioning

Commerical

1. Electric space heating, air conditioning
2. Electric space heating, no air conditioning
3. Air conditioning
4. No air conditioning

Industrial

1. Process use, non-seasonal
2. Process use, seasonal
3. No process use, non-seasonal
4. No process use, seasonal

of rates, and then allocated to end uses, can be used, and appears to be more practical with currently available data. Since the analysis is disaggregated by user type, with the user types distinguished by the major electricity-consuming end uses, the latter approach incorporates some of the advantages of the synthetic method.

The process of forecasting can be broken into three phases, (1) calibration of the behavioral demand relationships which give the level and pattern of demand as functions of rates, income, and exogenous explanatory variables, (2) initialization of the forecast in a base year or base period, providing all system variables and application-specific parameters, and (3) simulation of the market into the future under alternative scenarios on rate policy and other variables influencing electricity consumption.

The first phase, calibration of demand response functions, can be carried out on special data sets which are not necessarily representative of the population as a whole, and not necessarily confined to the geographical region in which the analysis is to be applied. This is due to the regularities of economic behavior over space, at least within the United States, as confirmed by consumer budget studies, including studies of energy purchases.[1] The possiblity for accurate and robust calibration is increased if observations can be drawn

[1] James E. Morgan, et al., Five Thousand American Families--Patterns of Economic Progress, Survey Research Center, Institute for Social Research, Ann Arbor: University of Michigan, 1974; Sam Schurr, ed., Energy, Economic Growth, and the Environment, Baltimore, Md., Johns Hopkins Press, 1972; Dorothy Newman and Dawn Day, The American Energy Consumer, Cambridge, Ma: Ballinger, 1975.

on customer behavior from widely varying environments. Thus, it is recommended that demand functions be calibrated, insofar as is possible, using national customer surveys or combined data from disparate areas.

Appendix B describes two household surveys which are available for use in Winter, 1977: a national survey of 1455 households in 1973, reinterviewed and augmented in 1975, by the Washington Center for Metropolitan Studies; and a survey of 2000 households in 14 cities in 1976 by Midwest Research Institute. Both surveys contain detailed appliance and electricity consumption data.

Peakload pricing experiment data will become available during 1977, as described in Appendix C, for selected geographical areas, and will provide detailed load curve information under alternative rate structures for residential customers. Load experiment data will also be available in 1977 giving load curves for industrial customers, with some variation in rate structure. Satisfactory calibration data for commercial customers will not be available immediately.

Initialization of the forecasting system in a base period requires collection of data on explanatory variables such as rates and income, plus electricity consumption data as required to determine parameters which are specific to the site of application--a utility rate area, or at more aggregate levels, a total utility, a state, or a region. In practice, this requires collection of a consumption frequency distribution and load curve for each customer class and user type in the

base period, or with somewhat broader assumptions on the applicability of national behavioral patterns, collection of a consumption frequency distribution and monthly load data for each customer class. Monthly load data is available by utility from the FPC.[2] Consumption frequency distributions are collected by many utilities, and can in other cases be approximated from summary data; see Appendix D.

Simulation of the market into the future under alternative scenarios requires auxiliary forecasts of number of customers, per capita income, mix of housing types, level of business activity, and so forth. If the rate structure is to be adjusted to allow normal rates of return to utilities, then the demand system must be combined with a system describing utility and regulatory behavior, as for example, the Teknekron model.[1] Otherwise, with exogenously specified rates, the demand system can be operated independently of supply considerations.

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[1] This model is described in Economic Impact of Water Pollution Control on the Steam Electric Industry, Report to National Commission on Water Quality, by Teknekron, Inc., Berkeley, CA, 1975.

[2] Power Statistics, annual.

3. The Description of Electrical Rates and Consumption

For each user type, a consumption frequency distribution can be defined as a curve giving the proportions of customers with monthly consumption (in KWH) at various levels. Figure 1 illustrates a typical consumption frequency distribution, expressed as a cumulative distribution. Figure 2 illustrates the same distribution, expressed as a frequency. Utilities normally calculate consumption frequency distributions for rate analysis, and in a number of states, including California, summary distributions are reported to public utility commissions. The distributions may be reported by month, as a mixture of consumption levels defined over the months in a year, or by annual consumption level. Under the plausible simplifying assumption that customers of a given user type maintain their relative positions in the consumption frequency distribution over different months, it is relatively simple to derive these frequency distributions from one another. This report proposes use of the annual consumption frequency distribution, which averages out the seasonal effects for separate treatment in the analysis of the load curve. The use of this distribution also reduces data requirements relative to the collection of monthly consumption frequency distributions. Where necessary, this distribution is deduced from the distribution of all monthly consumption levels, using the assumption stated above and observations on monthly load factors. The calculations are described in further detail in Appendix D. An alternative

FIGURE 1. Base Rate Consumption Frequency Distribution, Cumulative (PG&E, Rate D-2, 1974)

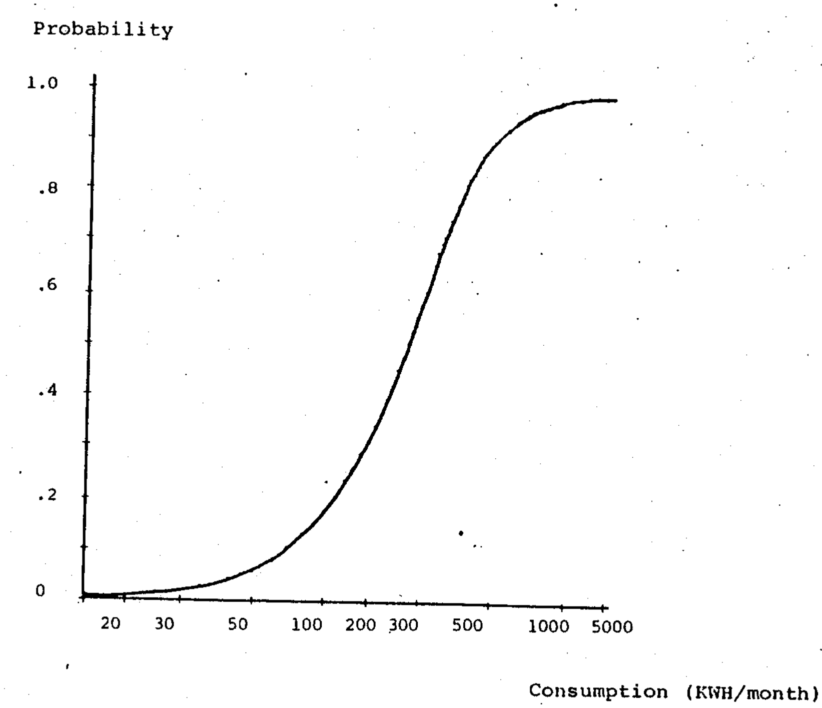
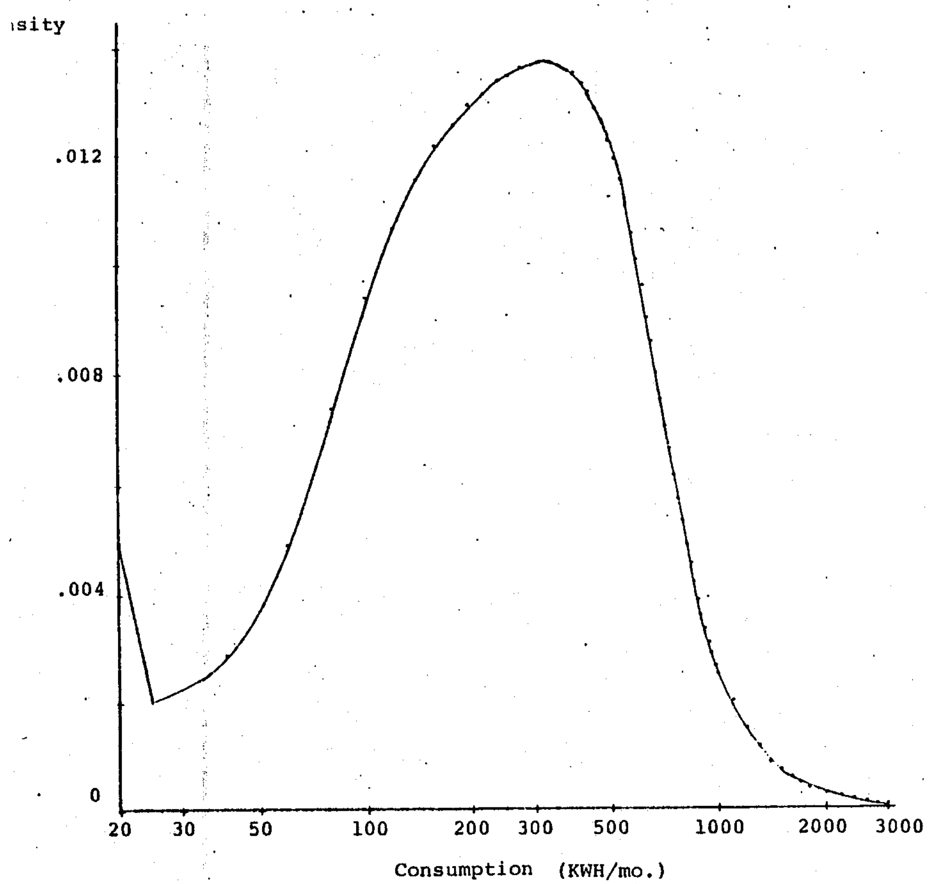


FIGURE 2. Base Rate Consumption Frequency Distribution, Density (PG&E, Rate D-2, 1974)



analysis using monthly consumption frequency distributions would avoid the necessity of assuming that customers maintain their relative position over months. This modification would be potentially more accurate, and could be implemented if monthly data were available.

The load curve for a user type specifies demand (in KW) at each moment of time. Consumption in any time interval is defined as the area under the load curve for this interval. This report assumes the load curve to be defined for each month, day of the week, and hour of the day.[1] The load curve is defined for the average, or representative, customer of each user type. Then, the area under this load curve for a year equals the average annual KWH consumption of this user type. The maximum of the load curve over the year gives peak KW demand for the user type.[2]

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[1]This definition assumes that all weeks of a month have similar demand patterns. Alternately, the load curve could be defined for each of the 8760 hours of the year, or could be simplified further. Peakload pricing experiment data and load management studies now make it possible to define load curves to almost any desired level of detail.

[2]In simulation, a forecast is made of the mean, or expected, KW demand at each point in time. There will be some variance in actual load about this expected load curve, due primarily to random variations in weather. Consequently, the expected value of peak KW demand will exceed the maximum of the expected load curve by a factor which depends primarily on variability in local weather. For forecasting purposes, this factor can be determined in a base period and assumed to remain fixed in the forecast interval. Note that this factor may differ for different user types.

In principle, the size of a customer can influence its load curve. For example, a large residential customer may have broader peaks in the load curve than does the small customer, due to the use of multiple appliances. The responsiveness of a customer to rate structure might also vary with customer size. These alternative possibilities will have little direct influence on market conditions as viewed by a utility, since it will be concerned only with the average load curve of a class of customers. However, there could be errors of aggregation in dealing with the average load curve as a stable entity whose shape is independent of the consumption frequency distribution.

Preliminary evidence suggests that there is independence between the shape of the load curve and the average consumption of a customer, <u>within a given user type</u>. Examination of San Diego data from the Miracle II survey of 12000 households shows that for user types distinguished by water and space heating, seasonal variations (in percentage terms) around annual average consumption were uncorrelated with annual average consumption; the statistical results are given in Table 2. Plots of peak demand versus average consumption for Southern California Edison are given in Figure 3, and show an almost perfect correlation. These results support the provisional conclusion that the shape of the load curve for a user type is independent of the level of average consumption. A conclusive test of this proposition should be possible, at least for residential customers, when peakload price experiment data become available.

TABLE 2. Correlation of Average Electricity Consumption and Ratio of Monthly to Average Consumption, for Selected Months (San Diego Gas and Electric, Miracle II Data)

Month	Non-Electric Water and Space Heat	Electric Space Heat	Electric Water Heat	Electric Water and Space Heat
JAN	-.019	-.019	-.018	.035
MAR	-.050	.060	-.004	.026
MAY	.011	.129	.114	.107
JUL	.107	-.017	.103	-.071
SEP	.080	-.045	.113	-.104
NOV	-.095	-.044	-.-79	-.045

All correlations are low. Customers with non-electric space heat have a positive correlation of monthly to average and average consumption in summer months, probably reflecting a positive correlation between air conditioning use and average consumption. The remaining correlations are statistically insignificant.

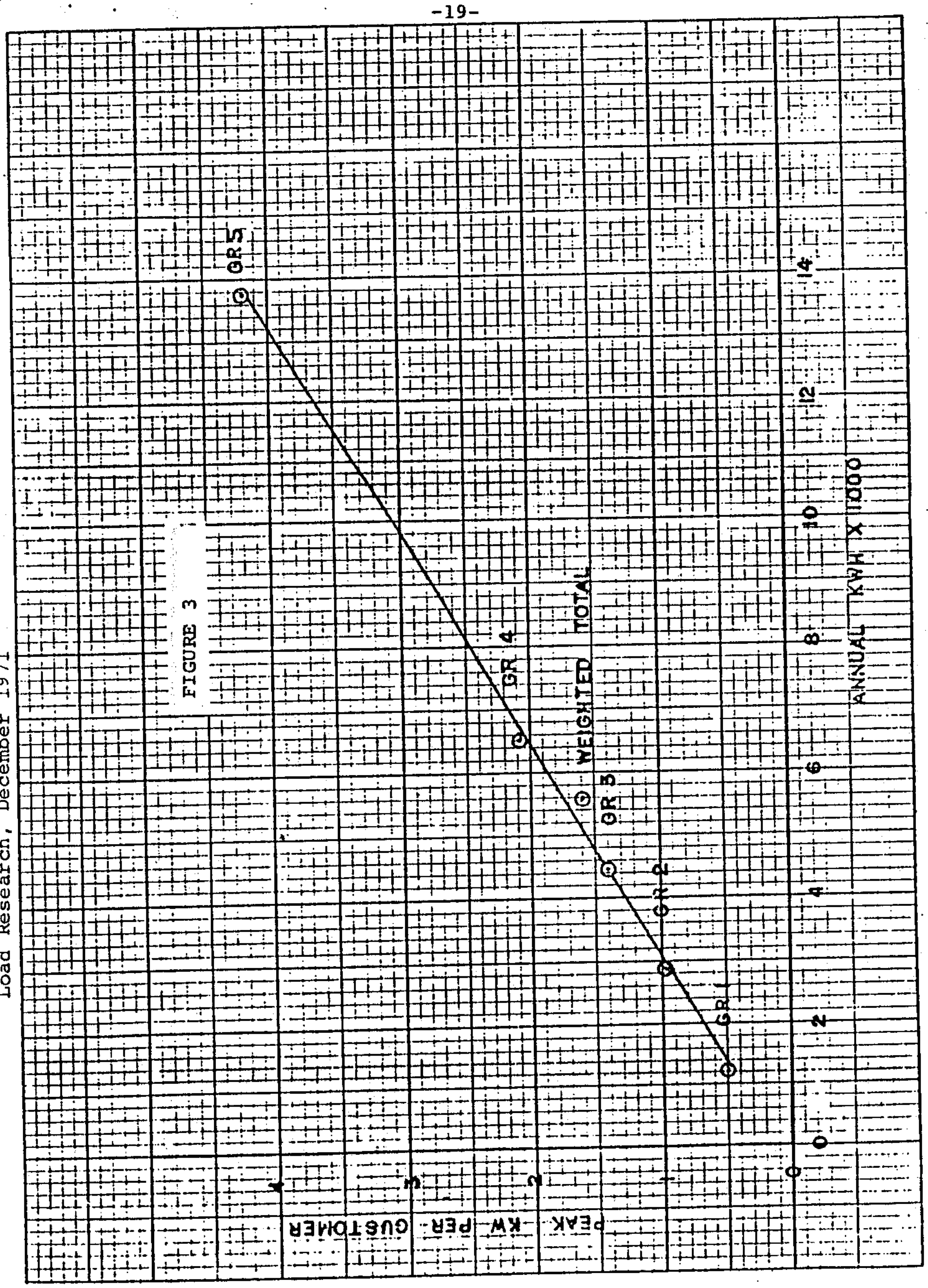

FIGURE 3

Load Research, December 1971

Electrical rates are normally comprised of three charges: (1) a <u>customer charge</u> for service, (2) an <u>energy charge</u> based on the number of KWH consumed (and, under peakload pricing, on the time profile of consumption), and (3) a <u>demand charge</u> based on the peak KW demand within some pericd. Traditionally, demand charges have applied only to large light and power rates for industrial customers. Most rate schedules specify a minimum bill, which often coincides with the customer charge. Rate schedules vary across customer classes, and may also vary by customer type, as for example, in the case of residential lifeline rates, which vary with type of water and space heat. It is also possible for a customer to face separate rate schedules for separately metered end uses, as for example, special rates for water heating or space heating service.

Energy and demand charges have normally had a declining block rate structure, with the charge per added KWH or KW, respectively, declining with increasing customer size. This structure is normally justified as reflecting the cost of service. Alternatives to declining block rate structures currently receiving consideration are increasing (inverted) block rates, flat rates, and lifeline rates, which are increasing block rates with special treatment of small customers.

Except for a few large industrial customers, there has been little U.S. experience with peakload pricing--the traditional declining block rate structure is applied to total monthly KWH. In the peakload pricing experiements, flat rates have been used, varying with time of day. Current policy discussions suggest that some combination of peakload pricing and increasing block rates is likely to receive serious attention. A likely method for the application of such rates would be computation of a basic bill from average consumption and the block rate structure, followed by an adjustment to reflect the shape of the load curve. The peak adjustment could be made by weighting points on the load curve by price factors reflecting peak costs, and then averaging over time. In the presence of flat rates such as are used in the peakload pricing experiment, this method would coincide with the methods used there to compute bills. More generally, the adjustment factor would be handled in a way comparable to the current treatment of fuel adjustment clauses.

Peakload prices will be treated in this report as entering in the manner described above. It seems reasonable to predict that when such rates are introduced in practice, they will be treated in this way.

There is great variation in the rules applying to the determination of peak KW, and to the conditions for qualification for various rate schedules. In some cases, measured peak KW (over 15 or 30 minute interval) is used for the billing month, or for some previous month or other period, or average of such periods. In other cases, a negotiated KW figure is used, with penalties if it is exceeded. Some demand charges are based on a combination of these. Furthermore, a minimum demand charge is often required to qualify for a particular rate schedule; this may particularly affect seasonal industries.

To simplify the tangle of demand charge structures for forecasting purposes, it is suggested that current year demand charges be based on the previous year's measured peak KW, and that the problem of qualification for a rate schedule be ignored except for the incorporation of a minimum bill. This is, of course, an approximation, but a reasonable one so long as KW peaks vary only slowly over time for individual customers. This simplification streamlines the computation of behavioral response, since it will make the determination of the time profile of demand recursive.

4. Computation of Bills and Revenue

For each customer class, user type, and end use schedule, a bill for a month can be computed as a function of a level of average demand for this month and the load curve of a customer with this level of average demand. Under the postulate of the independence of average demand and the shape of the load curve, a single calculation of the peakload price adjustment factor will suffice for all customers of a given user type facing a specified rate schedule. The steps below describe the computation of the peakload price factor:

A. Compute peak demand for the previous year,

$$\text{PKL} = \text{PKF} \times \max_{M,D,T} \text{PLF}(M,D,T) \quad , \tag{1}$$

where

PKL = peak demand in KW in preceding year;

PKF = an adjustment factor to relate expected peak demand to the maximum of the expected load curve;

PLF = the load curve (in KW), expressed as a function of month (M), day of the week (D), and time-of-day (T), for the preceding year.

The factor PKF may vary with user type, climate at the application site, and type of service. Note that PLF is the load curve for

the representative, or average, customer of this user type.

B. Consider a customer whose average demand is a proportion θ of that of the representative customer. Under the independence assumption, this customer will have peak demand $\theta \cdot \text{PKL}$. The demand rate schedule then gives a demand charge for this customer. This calculation is repeated for each desired level of θ. For residential and many commercial customers, this charge will not be applicable.

C. Compute the base energy charge. First, for each month M, form

$$\text{MAS} = \sum_D \sum_T \text{PLF}(M,D,T) \div 168 \ . \tag{2}$$

Since PLF is the load curve of the representative customer in KW, the sum over days of the week D and hours of the day T for month M, divided by the total number of hours under the load curve for the month (which is 168 since all Mondays in a month are assumed identical, etc.), gives the average KW over month M for the representative customer, MAS. Multiplied by 730 hours per month, this expression gives the KWH consumption in month M by the representative customer. For each customer with annual average consumption a proportion θ of that of the representative customer, the monthly consumption is $\theta \cdot 730 \cdot \text{MAS}$. From the energy consumption rate schedule, a bill for this quantity is obtained for each month.

D. Compute the peakload pricing adjustment factor. Define PEAKF(M,D,T) to be an array of price factors, reflecting the relative cost of electricity at various months, days of the week, and hours of the day. For example, PEAKF(M,D,T) may be one in off-peak periods, and greater than one in peak periods, reflecting the relatively higher cost of electricity. To obtain a peakload rate adjustment factor for month M, form

$$\text{(3)} \qquad \text{FACTOR}(M) = \frac{\sum_D \sum_T \text{PLF}(M,D,T) \cdot \text{PEAKF}(M,D,T)}{\sum_D \sum_T \text{PLF}(M,D,T)} .$$

If peakload pricing is absent, so that PEAKF(M,D,T) = 1 , then FACTOR(M) = 1 . If base rates are set for off-peak demand, with PEAKF(M,D,T) > 1 in peak periods, then FACTOR(M) will exceed one by an amount depending on the proportion of monthly consumption occuring in peak periods.

E. Compute the energy charge. For a customer with average consumption a proportion θ of that of the representative consumer, the total energy charge in month M is FACTOR(M)·BASE(M,θ), where BASE is the base energy charge for this customer in this month.

F. A total bill is obtained for each month for customer θ by summing the customer, demand, and energy charges, taking into account the minimum bill if necessary. These bills are then

accumulated over months to give annual revenue per customer of type θ .

G. Using the bill frequency distribution, average bills over customers of type θ to obtain a mean revenue per customer for the given user type and schedule.

Following completion of these steps for each schedule, revenues can be accumulated over schedules for a user type to give total revenues. From these total revenues, expressed as a function of average monthly consumption, summary average cost and marginal cost measures can be calculated. These summary cost measures, along with summary measures of the peakload pricing structure, will influence demand.

Load curves for the system, and by customer class, can be calculated from the load curves of user types. For example, the load curve of the residential class of customers is obtained from a weighted average of the load curves of the user types, weighted by their shares among all residential customers, and then multiplied by the total number of residential customers. The system load curve is obtained by summing over customer class load curves. The system load duration curve is obtained from the system load curve by determining the number of hours in the year when the system load exceeds each KW level. System peak demand is determined by the maximum of the load curve over the year; a correction factor to account for the difference between the expected value of peak demand and the maximum of the expected load curve--reflecting random variations in weather--will normally be necessary.

5. Forecasting Consumption Frequency Distributions

Electricity consumption can always be described mathematically as the product of two components--average annual consumption and a variation about this level due to the time profile of consumption. This decomposition has already been used in defining the consumption frequency distribution and the load curve of the representative consumer. In forecasting the impact of alternative rate structures on consumption, it is convenient to retain this division. In principle, this can be done without loss of generality, as all relevant explanatory factors can enter the determination of each of the two components. In practice, the decomposition makes it possible to impose plausible restrictions on behavior which are sufficiently strong to permit estimation of behavioral parameters from existing data. The resulting system is by no means the most general conceivable, or even the most general that could be calibrated using data which will become available in the coming year. However, it permits empirical estimation of the "first-order" behavioral response parameters.

The postulate of independence of the time profile of consumption and average consumption, which is supported by initial empirical tests, is assumed to continue to hold in the presence of alternative rate structures. Structurally, the time profile of consumption will be assumed to depend primarily on the time profile of prices, and not on the average level of prices. On the other hand, average consumption will be assumed to depend primarily on the average level of prices, and not on their time profile. These assumptions allow calibration of a model of average demand from nationwide and state data sources, and a separate model of the time profile of demand from peakload price experiment or load management experiment data.

The demand forecasting model implied by these assumptions exceeds in generality most of the empirical studies of electricity consumption behavior completed to date, and captures the main influences of electricity rates and other factors on demand. However, it has restrictive features which should be removed when sufficient data becomes available. For example, the time profile of demand may be influenced by average price levels if the primary effect of price increases is to reduce use of optional appliances--such as fans--which are ordinarily employed in peak periods. On the other hand, average demand may be influenced by the time profile of prices if some optional devices --such as heat pumps--become economical to operate in off-peak periods.

The annual consumption of a customer should depend on the class and user type of the customer, "average" electricity rates, the "size" of the customer, weather, and the idiosyncracies of the customer. For residential customers, family income measures customer "size," while for commercial and industrial customers, a measure of business activity--such as value-added or number of employees--seems appropriate.

Electricity rates are summarized by measures of the average

cost of electricity consumed and the marginal cost of additional KWH of consumption. The pure theory of economic consumer behavior implies that household consumption of electricity should depend on the slope and location of the budget set--"price" and "income" effects, respectively. Then, the marginal cost of electricity captures the "price" effect, while the average cost of electricity is a component of the "income" effect. The analogous economic theory of the profit-maximizing firm implies that commercial and industrial customers should be sensitive primarily to the marginal cost of electricity. For both residential and non-residential customers, consistency with the economic theory of behavior of the decomposition of demand into an average component depending on average prices and a relative load curve depending on the time pattern of prices requires that the customer's objective function have a nested structure, with the contribution of the load curve summarized in an intermediate index. For consistent aggregation, the definition of average demand should be compatible with the structure of the intermediate index.

At least for residential customers, there is considerable impressionic evidence that information on electricity costs is not available or is not processed at the level required for classical economic behavior. Consumers are unaware, except in very general terms, of the electricity consumption rates of various appliances. The absence of price indicators at time of consumption, the use of automatic appliances, and variations in the billing cycle all combine to obscure the link between consumption and cost. In these circumstances, the customer is likely to be aware only of the average monthly bill associated with his pattern of usage, combined with an impression of the cost-saving associated with particular conservation measures. This suggests that behavior will depend on average cost and marginal cost, defined as smoothed averages from monthly bills. Informal consideration of behavior under plausible information conditions then suggests a determination of average consumption similar in form to that suggested by classical economic theory when a suitable "nested" structure exists on the objective function.

Now consider the econometric specification of annual consumption for an individual customer of a given user type. This demand can be expressed as a function of the explanatory variables observed by the analyst--the rate structure, weather, "size" of the customer or of a representative customer of this user type--and of unobserved explanatory variables. Let KWH denote annual consumption, expressed on a per month basis. Without loss of generality, the log of consumption can be written as the sum of two components,

$$(4) \qquad \log \text{KWH} = f^1\begin{pmatrix}\text{observed}\\ \text{explanatory}\\ \text{variables}\end{pmatrix} + f^2\begin{pmatrix}\text{unobserved} & & \text{observed}\\ \text{explanatory} & , & \text{explanatory}\\ \text{variables} & & \text{variables}\end{pmatrix} ,$$

where f^1 gives the "representative" demand for the proportion of customers with the given observed explanatory variables, and f^2 gives a deviation from representative demand resulting from the particular values of unobserved variables occuring for

this individual customer. For economy in notation, rewrite the equation above in the form

$$(5) \qquad \log \text{KWH} = f^1(\text{RATES, SIZE, OTHER}) + \varepsilon ,$$

where RATES is a vector of variables characterizing the rate structure, SIZE is a measure of the scale of the customer unit, or of a representative customer, OTHER is a vector of other observed explanatory variables, and ε is the value of the function f^2, summarizing the impact of all unobserved variables on consumption. Note that the definition of f^2, and hence the value of ε, will depend on what variables are observed and used in a particular application. However, once the list of observed variables is specified, the function f^2 is determined, and its value ε will change when observed variables--such as the rate structure--change only if the distribution of unobserved variables in the population of customers is a function of the observed variables.

The following analysis is somewhat complicated by the fact that we will take average and marginal cost of electricity as summary characteristics of the rate schedule. The levels of these cost measures are determined by the rate structure <u>and</u> the level of KWH:[1]

$$(6) \qquad \text{AC} = f^3(\text{KWH; RATE SCHEDULE}) ,$$

$$(7) \qquad \text{MC/AC} = f^4(\text{KWH; RATE SCHEDULE}) ,$$

[1] The functions f^3 and f^4 are defined from the total revenue curve constructed in Section 4, p. 26. Average cost is defined by revenue divided by KWH, and marginal cost by incremental revenue divided by incremental KWH, where the end points are determined by minimum and maximum monthly consumption levels.

where f^3 and f^4 denote the relationships, defined in Section 4, linking the rate structure to average and marginal cost. Substituting equations (6) and (7) for the RATES variable in equation (5) yields

$$(8) \qquad \log KWH - f^1(f^3(KWH, {}^{RATE}_{SCHEDULE}), f^4(KWH, {}^{RATE}_{SCHEDULE}), SIZE, OTHER) = \varepsilon .$$

Assume, for the moment, that the left-hand-side of equation (8) is monotone increasing in KWH. (We consider later the plausibility of this assumption.) Then, the proportion of the population of customers with KWH $\leq$ k satisfies

$$(9) \qquad \text{Prob}\,[KWH \leq k] = \text{Prob}[\psi(KWH, {}^{RATE}_{SCHEDULE}, SIZE, OTHER) \leq \psi(k, {}^{RATE}_{SCHEDULE}, SIZE, OTHER)]$$

$$= \text{Prob}\,[\varepsilon \leq \psi(k, {}^{RATE}_{SCHEDULE}, SIZE, OTHER)] \quad ,$$

where ψ is the left-hand-side of equation (8); i.e.,

$$(10) \qquad \psi(k, {}^{RATE}_{SCHEDULE}, SIZE, OTHER) = \log k - f^1(f^3(k, {}^{RATE}_{SCHEDULE}), f^4(k, {}^{RATE}_{SCHEDULE}), SIZE, OTHER) .$$

Equation (9) is central to the method we propose for forecasting consumption frequency distributions. In the base year, the observed consumption frequency distribution determines the probability distribution of the unobserved component ε . If the probability distribution of ε is stable over time,

independent of the values of the observed variables, if the function f^1 is estimated econometrically, and if AC and MC can be calculated for a new rate schedule, then equation (9) determines the consumption frequency distribution correcponding to the new rate structure.

The relationship between the consumption frequency distribution and the distribution of the unobserved component ε is illustrated in Figure 4. Graphs (1) and (2) give the unobserved component (ε) frequency function and cumulative distribution function. Graph (3) gives the demand (KWH) as a function of ε , <u>given</u> the rate schedule. The demand functions for two alternative rate schedules, RATE and RATE*, are plotted. Each proportion on the unobserved component cumulative distribution function plots into the same proportion of the consumption cumulative distribution function for the specified rate schedule, as shown for the frequency 0.5 in graphs (4) and (5). In this way, the impact of the demand function response to rate differences is translated into changes in the consumption frequency distribution.

In application, the consumption frequency distribution is observed for the base rate schedule, RATE. Then, given the function $\varepsilon = \psi(KWH, RATE, \ldots)$, we can proceed from graph (4) to graph (3) to graph (2) in Figure 4, constructing the cumulative distribution function of ε which must prevail in order to yield the observed consumption frequency distribution. Once the distribution of ε is deduced for the base rate case, the consumption frequency distribution can be computed for any alternative rate schedule RATE* by going from graphs (2) to (3) to (4).

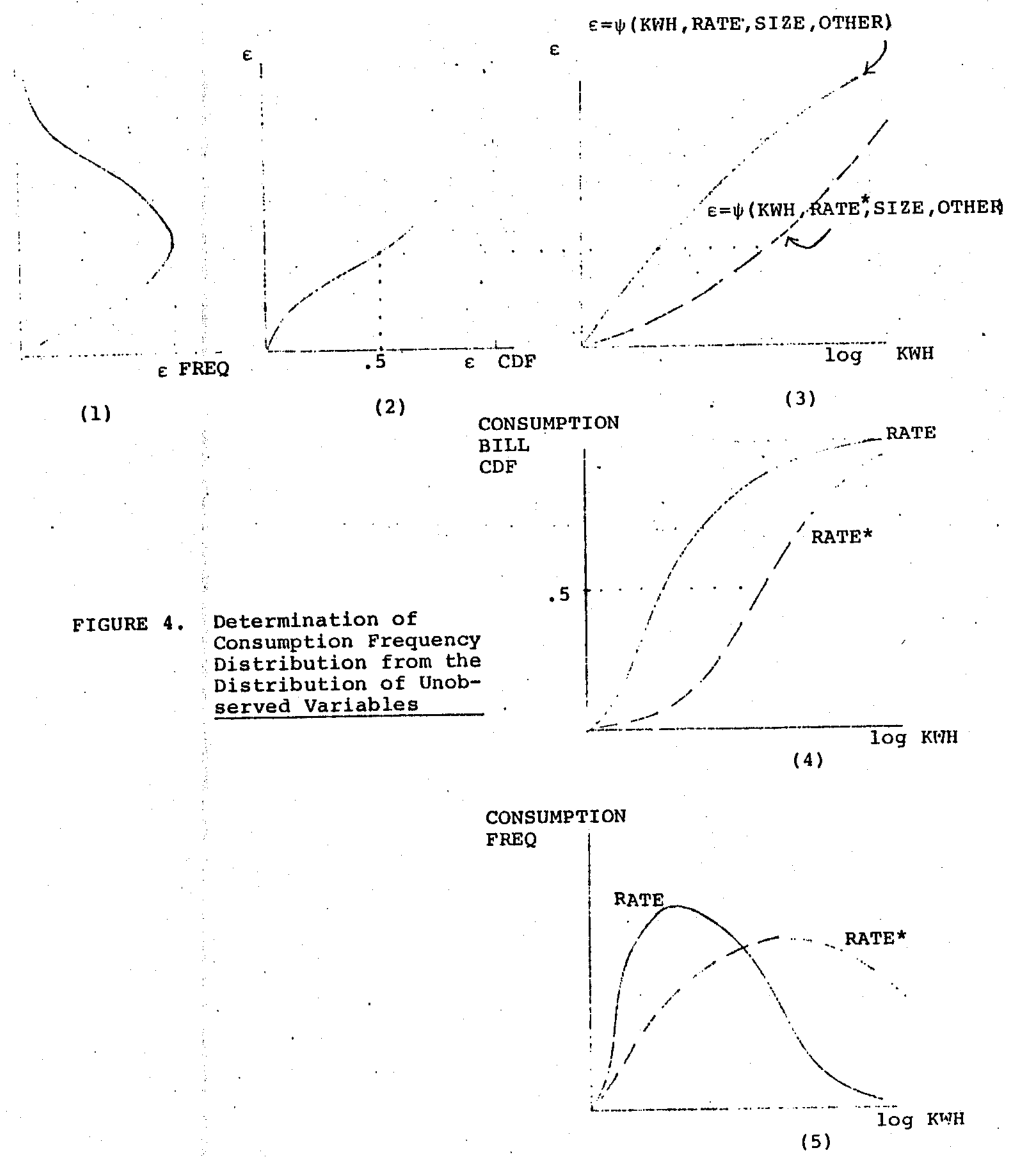

FIGURE 4. Determination of Consumption Frequency Distribution from the Distribution of Unobserved Variables

If demand is rate-elastic, then the curve in graph (3) will not change when the rate structure changes, and the consumption frequency distribution remains at the base level. Thus, this analysis includes the conventional assumption of a fixed consumption frequency distribution as a special case. More generally, the choice of demand functions allows the analyst to incorporate the degree of rate-responsiveness suggested by empirical studies.

The information needed to carry out the forecasting methodology outlined above is the base year consumption frequency distribution and the estimated function f^1 giving mean consumption as a function of observed explanatory variables. Hence, the remaining step in application of the methodology is specification and estimation of f^1.

The assumptions behind the methodology above require comment. First, the assumption that $\psi(KWH,\ldots)$ is monotone increasing in KWH is an implication of the economic theory of the consumer in the case that the rate structure is declining and electricity is a non-inferior good; i.e., if income increases and electric rates are fixed, then electricity consumption increases. For profit-maximizing commercial or industrial consumers, or for consumers in the case of non-declining rate structures, ψ must be increasing in KWH without added assumptions. This argument is developed technically in Appendix F. The monotonicity of ψ in KWH is therefore an extremely plausible assumption.

For two possible reasons, the <u>empirical</u> ψ function may fail to be monotone. The first is that observed consumption behavior may be the result of less complete information than the classical economic solution contemplates, so that a failure of monotonicity reflects an attempt to make the dependence of consumption on rates more sensitive and precise than it is in reality. A remedy is to permit smoothing of cost measures, mimicing the "mental" process of smoothing which apparently influences customer behavior. The second reason for an empirical failure of monotonicity would be inaccuracy in specification of the representative demand function f^1. Any workable econometric specification for this function will be a simplification of reality, and there may be ranges over where the approximation is not sufficiently accurate to track the monotone behavior of the underlying true ψ function. A remedy for this failure is to correct the approximation as necessary to eliminate non-monotone ranges. A proposed method for doing this is discussed later.

The second assumption underlying the forecasting methodology is that the probability distribution of the unobserved component of demand, ε, is stable over time and independent of the values of the observed variables. This assumption is analogous to the assumption in regression analysis that explanatory variables and residuals are independent, and is equally difficult to validate. In cases where failure is suspected, improved specification and measurement of explanatory variables is called for. In practice, this may require "belling the cat;" failure can be detected only by observing the variables which cannot be observed. In the absence of specific prior knowledge

of dependence of observed and unobserved variables, the independence assumption represents the "neutral" or "equal ignorance" specification. A convincing test of the validity of this assumption on the distribution of ε could be made by forecasting demand to an observed year. If the overall forecast is accurate, then except for the possibility of offsetting errors, the elements entering the forecast are plausible.

Consider now the specification of the demand function for electricity. Recent surveys of the extensive empirical literature on this subject by L. Taylor[1] summarize price and income elasticities obtained from aggregate cross-section, time series data. Table 3 reproduces selected results for residential customers from the Taylor surveys and more recent studies. Appendix A is an annotated bibliography of electricity demand studies. Note that electricity demand appears to change slowly in response to price or income changes, with low short-run elasticities, but substantial long-run elasticities. The reported studies have used either marginal cost, defined by the tail block rate, or average cost, as a price variable. Consequently, these models are unable to capture in any detail the impact of rate <u>structure</u> on demand. In particular, they do not permit a separation of the effects of the level of the rate schedule and the rate of decline of the block rates.

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[1] "The Demand for Electricity: A Survey," <u>Bell Journal of Economics</u>, Spring 1975 and "The Demand for Energy: A Survey of Price and Income Elasticities," paper prepared for the National Academy of Science Committee on Nuclear and Alternative Energy Systems.

TABLE 3.* Price and Income Elasticities of Demand in Recent Econometric Studies of Electricity Demand

Type of Demand	Type of Price	Price Elasticity Short-Run	Price Elasticity Long-Run	Income Elasticity Short-Run	Income Elasticity Long-Run	Type of Data
RESIDENTIAL						
Acton et. al. (1975)	M		(-0.70)		(0.40)	CS: small geographical areas
	M		(-0.34)		(0.41)	CS-TS: small geographical areas
Taylor, et. al. (1975)	M	-0.07	-0.78	0.10	1.18	TS: states, annual
Lacy & Street (1975)	M		(-0.45)		(1.87)	TS: area served by one utility
Wilder & Willeborg (1975)	A		(-1.00)		(0.16)	CS: individual households
Uri (1975)	A	-0.61	-1.66	0.04	0.12	TS: monthly aggregate U.S.
FEA (1976)**	A	-0.19	-1.46	0.30	1.10	CS-TS: Census regions of U.S., annual
Halvorsen (1976)	M		-1.00		.70	CS: states

Note: CS: Cross-section
TS: Time-series
M: Marginal price
A: Ex post average price
*This table is drawn in part from L. Taylor, "The Demand for Energy: A Survey of Price and Income Elasticities." 1976.
**The elasticities for the FEA model are calculated using 1975 weights for the Northeast Census region.

Halvorsen[1] fits a demand system in which the rate structure is approximated by a two-parameter family,

$$\begin{pmatrix}\text{Electricity}\\ \text{bill}\end{pmatrix} = b_0(\text{KWH})^{b_1} . \qquad (11)$$

In this formulation, the level of rates and their rate of decline could be analyzed as separate policy instruments. However, this parametric family is quite restrictive, at least in terms of proposed rate structures, and Halvorsen's econometric method does not allow the observed interstate variation in rates.

A modification of the Halvorsen model which corrects some of its difficulties has been fitted by McFadden and Puig[2]. This residential demand model consists of (1) an equation specifying the rate structure as a function of Typical Electric Bills (TEB) of 250,500, and 750 KWH per month, (2) an equation specifying demand per customer as a function of exogenous factors and the price schedule, and (3) a simple equation describing the growth of number of customers. The model was fitted using state cross-section data for 1969 collected by Halvorsen. State average TEB's at the 250,500, and 750 KWH per month levels were used to fit three-parameter functional approximations to the rate structure for each state. The parameterized rate schedule

[1]"Demand for Electricity in the United States," Southern Economic Journal, 1976.

[2]Chapter 2, "Economic Impact of Water Pollution Control in the Steam Electric Industry," Report EEED 12, Teknekron, Inc.

was then used to provide estimates of average cost and marginal cost at the average demand level for the state. The demand function was assumed to have the log linear form

$$\text{(11)} \qquad \log KWH = a_0 + a_1 \log AC + a_2 \log (MC/AC) + a_3 \log \bar{Y} ,$$

where AC and MC are the average and marginal cost of electricity, evaluated at the demand level KWH, $\bar{Y}$ is mean income, and a_0 contains in addition to the constant term, a number of variables such as gas price, temperature, and population density which can be treated as fixed for the purpose of rate analysis. The demand equation was fitted econometrically by two-stage least squares. The estimated coefficients are given in Table 4. The coefficients of price and income can be interpreted directly as long-run price elasticities; we see that the elasticity of -.71 of demand with respect to average cost (where marginal cost and average cost rise proportionately) and .989 of demand with respect to income are in the range suggested by other studies.

Studies of the dynamics of electricity demand show that adjustment to price changes is slow. An adaptive adjustment to desired demand,

$$\text{(12)} \qquad \log\begin{pmatrix}\text{KWH in}\\ \text{year t}\end{pmatrix} = (1 - \alpha) \log\begin{pmatrix}\text{Desired}\\ \text{KWH}\end{pmatrix} + \alpha \log\begin{pmatrix}\text{KWH in}\\ \text{year t-1}\end{pmatrix} ,$$

fitted by Chapman-Mount-Tyrrell[1], estimated an adjustment rate $\alpha = .886$. This implies that the "half-life" of the response to a price change is 5.7 years. The short-run elasticities implied by this response rate and the coefficients in Table 2 are -.081 for demand with respect to average price and .107 for demand with respect to income.

The demand system above has two drawbacks. The first is that the functional specification, while agreeing qualitatively with the features one would expect for electricity demand by a consumer facing a declining rate structure, is not derivable directly from such a structure. Hence, this demand function must be viewed as an approximation to a fully consistent model derived from utility foundations. We might note that it is possible to carry through exercises of the last sort analytically only for highly simplified forms which fail to incorporate expected features of demand. However, further analysis may permit the development of approximations which reflect more accurately the structure of demand.

The second difficulty is the uncertain impact aggregation over customers of different sizes will have on the coefficient estimates. The only satisfactory solution to this problem is to estimate demand functions using individual customer data.

The only disaggregate data set available to us at the time of this report was the Miracle II survey of 12,000 households

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[1] "Electricity Demand in the United States: An Econometric Analysis," Oak Ridge National Laboratory, 1973.

by San Diego Gas and Electric in 1975. Because of lack of variability of rate structures within this utility, it is not possible to obtain estimates of consumption elasticities with respect to average cost or marginal cost. However, it is possible to obtain income elasticity estimates from this data set. Table 5 gives the statistical relationship between average monthly demand levels and income by user type for customers in the D1 rate area in San Diego. The implied income elasticities are somewhat lower than those obtained from aggregate studies.

A preliminary analysis of the Washington Center for Metropolitan Studies (WCMS) survey by Temple-Barker-Sloan for FEA gave price and income elasticities well below the range of those found in aggregate studies. The model estimated by Temple-Barker-Sloan differed somewhat from that suggested here, in both functional form and variable specification, making comparison difficult. However, it appears that variables such as number of rooms, which are highly correlated with income, are capturing part of the variance explained by the income variable. This poses a choice for the demand forecaster. If the historical trend of individuals to acquire additional rooms when income rises continues in the future, then the forecast should include the trend in this variable, and the appropriate "income" elasticity for comparison with aggregate cross-section studies is a combination of the "pure" income elasticity and the elasticity of demand with respect to number of rooms. Alternately, variables such as number of rooms can be

TABLE 5. Income-Consumption Curves for Electricity, San Diego Gas and Electric, Schedule D-1, 1975[1]

Income Level	Average Monthly Demand in KWH			
	Non-electric water and space heat	Electric space heat	Electric water heat	Electric water and space heat
3000	114	161	96	144
5000	167	201	188	217
7000	219	265	254	276
10000	308	344	380	425
15000	463	469	588	777
20000	600	586	773	1093
25000	756	798	921	1267
40000	1088	1005	1580	2014
Income Elasticity[2]				
	.48	.39	.52	.62

[1]Suppose consumption (K) and income (Y) have a joint cumulative distribution function $F(K,Y)$. Let $G(K) = F(K,\infty)$ and $H(Y) = F(\infty,Y)$ be the marginal cumulative distribution functions of F , and let $K = G^{-1}(p)$ be the inverse of the function $K = G^{-1}(p)$. Define the mapping $K = G^{-1}(H(Y))$. Values of this mapping are given in the table. The income elasticity calculation is exact if there is an exact non-stochastic relationship between Y and K . If the relationship is stochastic, then these calculated income elasticities are biased toward one.

[2]Calculated between income levels of 7000 and 20000.

excluded; the resulting coefficient of income reflects jointly the direct effect of income on consumption and the indirect effect via an influence on number of rooms. If, on the other hand, the forecaster anticipates a break from the historical relationship between income and number of rooms, then the estimated model containing both variables should be used, and an auxiliary forecast should be made on number of rooms, taking into account historical patterns and the justification for supposing a change from these patterns. Note that an assumption that a variable such as number of rooms will remain unchanged when income changes should not be made without justification.

To obtain fully satisfactory demand parameter estimates, an econometric analysis should be carried out on a disaggregate data set extending over differing rate structures. The WCMS national data set or the Midwest Research Institute (MRI) data set from fourteen geographical areas would be appropriate for this purpose in the case of residential demand.

A possible functional form for the representative demand function, f^1, for residential demand is[1]

$$\log KWH = a_1 + a_2 \log AC + a_3 \log \frac{MC}{AC} + a_4 \log \overline{Y} + a_5 \log \frac{Y}{\overline{Y}} + a_6 NR + a_7 MF + a_8 RU + a_9 \log GP + a_{11} WH + a_{12} SH + a_{13} AIR + a_{14} HDD \cdot SH + a_{15} CDD \cdot AIR + a_{16} HDD + a_{17} CDD \tag{13}$$

[1] In forecasting, the term $\log Y/\overline{Y}$ will be absorbed into the unexplained residual.

where

KWH = annual KWH consumption, expressed on a per month basis;

AC = average real cost of electricity, 1975 dollars, measured at observed KWH;[1]

MC = marginal real cost of electricity, 1975 dollars, measured between seasonal high and low KWH;

$\overline{Y}$ = average income of households in the geographic region;

Y = customer income;

NR = number of rooms in household [1 if true, 0 otherwise];

MF = if multiple-family dwelling;

RU = if rural;

GP = average real price of gas, 1975 dollars;

WH = if electric water heat;

SH = if electric space heat;

AIR = if air conditioning (room or central);

HDD = heating degree days (or mean January temperature);

CDD = cooling degree days (or mean July termperature) .

An argument was made earlier that average and marginal costs are likely to be the summary measures of the rate structure to which the customer is most sensitive. The average income of households in the geographic region is included since it is this variable which can be provided in forecasting the representative demand of a region. The relative

[1] AC and MC are calculated from the rate schedule facing each observed customer. AC is given by the total bill at the observed KWH level, divided by KWH. Marginal cost is given by the ratio of the incremental bill to incremental KWH between the highest and lowest months of the year.

price variable $Y/\bar{Y}$ measures the responsiveness of demand to income relatives. If demand responds only to absolute income, then $a_4 = a_5$. Alternately, if consumers are reluctant to depart from "community norms," one may expect $a_5 < a_4$. The variables NR, MF, and RU reflect technological features of the residence affecting demand. The average real price of gas provides a coefficient estimating the cross-substitution of electricity and gas. In the absence of data on prices of other fuels, this coefficient is a proxy for all non-electric sources. The gas available variable indicates whether gas is used for some purpose in the residence. The variables WH, SH, AIR indicate the presence of specific appliances. The coefficient of one of these variables, say $a_{12} + a_{14}$ HDD, measures the increment in demand due to use of electric space heat when HDD is the number of heating degree days. The heating and cooling degree day variables also enter separately, reflecting use of auxiliary appliances (e.g., portable heaters, fans).[1]

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[1] Since space heating is likely to be roughly proportional to number of heating degree days, with a similar proportionality for air conditioning and cooling degree days, one might expect demand to have the structure

$$\begin{matrix}\text{total}\\\text{demand}\end{matrix} = \begin{matrix}\text{non-space}\\\text{conditioning}\\\text{demand}\end{matrix} + SH \cdot HDD \cdot \begin{matrix}\text{space heat}\\\text{demand per}\\\text{degree day}\end{matrix} + AIR \cdot CDD \cdot \begin{matrix}\text{cooling}\\\text{demand per}\\\text{degree day}\end{matrix} .$$

Suppose the three demand components in this equation are identical except for scale; e.g., each depends on AC in the same way. Then,

$$\begin{matrix}\text{total}\\\text{demand}\end{matrix} = \begin{matrix}\text{contribution of}\\\text{common variables}\end{matrix} \cdot (b_1 + b_2\, SH \cdot HDD + b_3\, AIR \cdot CDD) ,$$

where b_1, b_2, and b_3 reflect the scales of the demand components.

The log linear specification is selected to give coefficients interpretable as elasticities, and should be viewed as an approximation to the true demand function. The log linear specification is one special case of the general translog demand system introduced by Jorgenson and Lau[1], and shown to have desirable theoretical and empirical properties as a flexible demand system. Since we do not have a primary interest in substitution between electricity and other goods, this last generalization has not been adopted. However, it could be introduced if empirical analysis of large data sets suggests that the log linear specification is inadequate.

An econometric fit of equation (13) to cross-section data should give long-run elasticities, at least if the price and income levels facing residential customers have been stable at a steady rate over time. To obtain short-run response for dynamic forecasting, it is necessary to combine these cross-section results with an adaptive adjustment equation of the form of equation (12). Such an adjustment equation could be fitted

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(footnote 1 cont. from page 47)

Taking logs, and making a first-order expansion of the last terms,

$$\log \left[\begin{matrix}\text{total}\\\text{demand}\end{matrix}\right] = \log \left[\begin{matrix}\text{contribution of}\\\text{common variables}\end{matrix}\right] + \log b_1 + \frac{b_2}{b_1} SH \cdot HDD + \frac{b_3}{b_1} AIR \cdot CDD + \dots .$$

Thus, the suggested form can be interpreted as a first-order approximation to a formula obtained by aggregating consumption over end uses.

[1] "The Integrability of Consumer Demand Functions," Discussion Paper 425, Harvard Institute of Economic Research, 1975.

to disaggregate panel data if it were available--the only current possibility is the WCMS data set, which has observations on customers in 1973 and again in 1975. Alternatively, satisfactory adaption rates can be estimated from aggregate data at the regional, state, or national level. This method also allows the possibility of testing for the presence of an exogenous trend in electricity consumption. Let KWH* denote desired consumption, as determined by the long-run demand function estimated above, evaluated at representative values of the explanatory variables. (One might take, for example, the mean of the consumption frequency distribution, given the rate schedule and other explanatory variables.) If there is an exogenous trend in desired consumption, then $KWH_t^* \cdot e^{\lambda(t-1975)}$ will be the desired consumption in year t, where KWH_t^* is evaluated at year t explanatory variables and λ is the exogenous trend rate. A first-order adaptive expectation would then have the form

$$(14) \qquad \frac{KWH_t}{KWH_{t-1}} = \left(\frac{A \cdot KWH_t^* \cdot e^{\lambda(t-1975)}}{KWH_{t-1}} \right)^{\alpha} ,$$

where α is an adjustment rate and A is a parameter introduced to normalize KWH^* to the scale of KWH. Taking logarithms for estimation

$$(15) \qquad \log KWH_t - \log KWH_{t-1} = A' + \alpha[\log KWH_t^* - \log KWH_{t-1}] + \alpha\lambda(t - 1975) ,$$

where A' is a constant.

Commercial consumption demand specification closely resembles the residential specification outlined above. Recalling that the distinction between commercial and industrial customers is in terms of quantity of electric demand and type of service rather than functional activity, note that most commercial establishments are small office buildings and retail establishments with the same end uses as residential customers--lighting, water heating, space heating, and air conditioning. As in the residential customer case, there is likely to be considerable smoothing of average and marginal cost in the customer evaluation of cost of service. Hence, a demand specification similar to the residential specification in equation (13) seems appropriate, with the following exceptions:

--residential income variables $\overline{Y}$ and Y are replaced by establishment size measures, such as value-added, number of employees, installed horsepower, or depreciated book value of capital.

--number of rooms is replaced by square feet; multiple-family and rural indicators are dropped.

Virtually no disaggregate data is currently available for commercial customers, with the exception of a limited number of customers in the Arkansas demonstration project on four alternative rate schedules. A number of estimates of commercial demand have been made using state cross-section or time-series and cross-section data. One model estimated by McFadden and

Puig[1] on 1972 state cross-section data has substantially the form suggested above. The estimation method parallels that described earlier for the residential model in Table 4. The estimated commercial model is given in Table 6. This model gives long-run price elasticities almost identical to the residential model in Table 4, and a slightly lower elasticity with respect to per capita income.

The specification of industrial demand presents several difficulties. First, this customer class is quite heterogeneous, with wide variations in functional activity, end uses of electricity, substitutability of energy sources, and rate schedules. Industrial rates are complex, with separate demand and energy charges, various methods of calculating demand charges, and conditions for qualification for schedules. Explicit load management may be adopted by customers, including purchase of interruptable power and customer-supplied monitoring and management devices. Treatment of industrial customers as a single class or a few user types is likely to obscure a number of these variations. On the other hand, considerable aggregation seems necessary, since data sources are inadequate to estimate fully disaggregated models.

One major division of industrial customers into user types

[1] Chapter 2, "Economic Impact of Water Pollution Control in the Steam Electric Industry," Report EEED 12, Teknekron, Inc.

TABLE 6. Commercial Demand

1972 cross-section data for the 48 contiguous states, two-stage least squares.

Dependent Variable - Ln monthly demand per customer (LMUS) in KWh

Independent Variable		Units	Coefficient	Std. Error
LV16S	Ln State Population	1	-.078	.037
LAVP	Ln Average Price	S/KWh	-.728	.165
LPRT	Ln Ratio of Marginal to Average Price		-.548	.377
LV216S	Ln Per Capita Personal Income	$	.823	.387
LV196S	Ln Commercial Gas Price	¢/therm	-.253	.121
LCOL	Ln Index of Cost of Labor	$	.917	.585
LV20S	Ln July Temp.	deg.F	1.557	.548
LV21S	Ln Degree Days	0.1 deg.F-da	-.089	.075
LPOPD	Ln Population Density	1/sq.mi.	.053	.036
LRURL	Ln Per cent Rural Population	%	-.059	.114
LV218S	Ln Per cent Single Unit Housing	%	-.249	.368
TENN	Dummy		-.864	.181
	Constant		-12.099	5.855

R^2 = .800 Std. Error = .155 No. Observations = 48 is by process use of electricity. Process demand (for welders, electrolytic processes, motors, etc.) is likely to behave differently than non-process demand for lighting, space conditioning, and so forth. An alternative classification is by S.I.C. code; some rough correspondence between S.I.C. code and proportion of end use of electricity for process purposes can be made.

Selected data on industrial customers is available from load management studies. These studies generally classify customers (or groups of customers) by S.I.C. code, and provide detailed load curve information. However, information on customer "size"--as measured by value-added, number of employees, or depreciated book value of capital--is usually not reported. Information on appliance holdings, type of structures, and shares of various end uses is also usually missing. Billing demand is usually not separately reported. Consequently, it is difficult to use these data to fit a complete industrial demand model. However, these data can be used to determine price elasticities, with an unknown amount of confounding of effects due to heterogeneity of customers on different schedules.

For the purpose of forecasting the impacts of peakload pricing or load management on industrial demand, it would be extremely useful to isolate elasticities with respect to demand charges and with respect to energy charges. This does not appear to be possible from existing data sources, but could be incorporated in the analysis if data on billing demand and demand charges were collected in on-going load management studies.

Aggregate industrial demand models, fitted to state cross-section time-series data, are common in the literature. Such models raise several difficulties. The first is that manufacturing activities have some locational mobility, and will tend to locate in areas where inputs they use intensively are cheap. For example, electricty-intensive industries may locate in the Pacific Northwest or Tennessee Valley, gas-intensive industries in Texas, and labor-intensive industries in the Southeast. Then state cross-section estimates of price elasticities may suggest a responsiveness which is not present for the mix of industries in a given state and would not be observed over time except in the extremely long-run due to migration of industries. Correction requires disaggregation by homogeneous industry groups (in terms of factor intensity), or modelling of the simultaneous determination of location and factor intensities across states. McFadden and Puig[1] have estimated a combined time-series cross-section model of industrial demand and interstate location of value-added, using aggregate state data. The demand model estimates are given in Table 7. Because lagged consumption is included, the coefficients are interpretable as short-run elasticities. The corresponding long-run elasticity with respect to value-added is .729; with respect to average

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[1] Chapter 2, "Economic Impact of Water Pollution Control in the Steam Electric Industry," Report EEED 12, Teknekron, Inc.

TABLE 7. Industrial Demand

(1958-1972 combined time series,
state cross-section data, two stage least squares)

Dependent Variable - Ln monthly demand per customer (LMUS) in KWh

	Independent Variables	Units	Coefficient	Std. Error
LMUSL	Ln Lagged Monthly Demand per Customer	$/KWh	.865	.015
LVPC	Ln Value Added per Customer	$/Cust.	.099	.012
LNCU	Ln Number of Customers	Cust.	-.040	.007
LAVP	Ln Average Price	$/KWh	-.173	.043
LPRT	Ln Ratio of Marginal to Average Price		-.115	1.89
LPRB	Ln Price of Gas	¢/therm	.015	.021
LV14S	Ln Mineral Production	$	.027	.007
LV15S	Ln Hourly Earnings	$	.025	.034
C	Constant		-.0623	.206

R^2 = .961   Std. Error = .176   No Observations = 690

* This model was estimated using values of variables in current dollars. A reestimate using constant dollars had a negligible effect on coefficients, presumably because of relatively stable prices over the bulk of the period 1958-1972.

price, -1.307; with respect to the ratio of marginal to average price, -.907; and with respect to the price of gas, .114 . These elasticities are comparable to those found in other aggregate studies. As discussed in some detail in the original source, the model in Table 7 has a number of econometric deficiencies which should be corrected before the model is used, in addition to the problems of specification already discussed.

This section has suggested practical specifications for representative demand models for residential, commercial, and industrial customers. The aggregate estimates cited provide a "state-of-the-art" starting point for the specification of representative demand in the determination of consumption frequency distributions. As models are fitted to disaggregate data sets, these specifications can be upgraded.

Forecasting requires determination of the values of all explanatory variables in the base year, and auxiliary forecasts of these variables in future years. Table 8 summarizes the explanatory variables for which auxiliary forecasts are required. The auxiliary forecasting methods proposed are relatively crude. This is due in part to the fact that most of these variables are relatively highly correlated with future external projections of income and population which themselves are rather uncertain, and little is gained from high precision in the remainder of the auxiliary forecasting effort. More

(Text continues on p. 60)

TABLE 8. Explanatory Variables for which Auxiliary Forecasts are Required

	Aux. Forecasting Method
Regional per customer mean income (residential)	1
Average number of rooms	2
Proportion multiple family dwellings	2
Proportion rural	2
Gas price	3
Gas availability (hook-up present) proportion	3
Heating degree days	4
Cooling degree days	4
Value-added per customer (commercial, industrial)	5
Number of employees per customer (commercial, industrial)	6
Square footage per customer (commercial)	7
Cost of labor index	
Population density	2
Number of customers	8

Auxiliary forecastng methods (t = forecast year, 0 = base year, s = observation unit, n = national aggregate)

1. Assume the regional variable grows at the national rate, starting from base-year observed values, where the national growth rate is given by standard U.S. Bureau of Labor Statistics

long-range forecasts of GNP per household.

2. Assume proportions change at constant geometric rates, with the rates determined from 1960 and 1970 aggregate U.S. Census figures.

3. Specified by scenario. The gas availability proportion will fall under the scenario that new construction will not have gas, with

$$GAV_t = GAV_0 \cdot \text{Housing stock in } 0 \div \text{Housing stock in } t \ .$$

4. Assume constant in time.

5. Project value-added by method 1, number of customers by method 8.

6. Project employees as the product of a regional projection of population and a national projection of labor force participation rates. Project number of customers by method 8.

7. Assume proportional to value-added per customer.

8. For residential, fit the model

$$\log \text{no. customers} = a_1 + a_2 \log \text{population} + a_3 \text{ time} \ ,$$

and use it for projections with regional forecasts of population. For commercial and industrial, fit the model

$$\log \text{no. customers} = a_1 + a_2 \log \text{population} + a_3 \log \text{GNP per capita} + a_4 \text{ time} \ ,$$

and use it for projections with regional forecasts of population and national forecasts of GNP.

(End, Table 7)

importantly, for the analysis of relative impacts of alternative rate or load management policies, conclusions will be relatively insensitive to small variations in the base year and exogenous variable specifications, and even crude auxiliary forecasts will contribute errors to the final analysis of relative impacts which are small compared to errors arising from uncertainty in price coefficients. On the other hand, if the interest were primarily in absolute forecasts, say for expansion and plant siting purposes, the base year and exogenous variable forecasts would require the same care and level of effort as the demand function estimation.

6. Peakload Pricing and Forecasts of Load Curves

Let PLF(M,D,T) denote the load curve, as a function of month M, day of the week D, and time of day T, for a representative customer of a particular class and user type. The forecasting task is to determine this curve in the base period, and then to estimate a functional relationship between the shape of the curve and the profile of peakload prices which can be used to forecast changes in the load curve.

Seasonal variations in demand in the base period can be obtained for each utility and customer class from the FPC Form 1. Base data by user type, or for weekly and time-of-day variations, are available from selected load management studies of individual utilities and from FEA peakload pricing experiment control groups. A large collection of relevant data has been made by the Association of Edison Illuminating Companies (AEIC). These data are released only to member utilities, and are not available for public use. There are no standards for geographical coverage, or data structure and format, making systematic application of the results difficult. Many utilities will release the load studies which are reported in the AEIC documents. If an individual utility has engaged in a systematic program of load research, then a relatively comprehensive construction of base period PLF is possible for this utility. However, the construction of consistent comprehensive base period load curves for different utilities or geographical

regions is impossible from existing data. What is possible is to approximate base period load curves by geographical region, taking into account the general shifts introduced by variations in climate. Some errors will necessarily be introduced by this approximation; data are sufficiently sparse and varied in form to make assessment of error magnitudes almost impossible. A specific procedure for approximation is outlined below. In practice, it seems most efficient to implement this procedure in reverse, starting from widely collected system and customer class load curves, and specializing and upgrading these curves with additional data, including demonstration data.

1. Express each load curve as a <u>relative</u> to mean KW demand.
2. From the demonstration experiment control data sets and selected utility load management studies, fit <u>monthly</u> relative load curves a linear functions of absolute temperature differentials from 65°F in each month, for each user type.
3. Form a monthly relative load curve for each customer class by forming a weighted average of relative load curves over user types, the weights being proportional to <u>total annual consumption</u> by each user type.
4. Compare the results of step 3 with the monthly relative for the customer class in the geographic region from FPC Form 1, introducing a correction factor to yield equality. Apply this correction factor to the monthly relative of <u>each</u> user type for the geographic area.

5. Calculate day-of-week relatives from demonstration experiment control data sets. Test for independence of season and geographical area. If independence is accepted, apply average day-of-week relatives to all geographical regions. If independence is rejected, seek a simple parametric dependence on season, temperature, degree of urbanization, or major geographical area such that independence <u>conditional</u> on these variables holds.

6. Calculate time-of-day relatives from demonstration experiment control data sets, by day of the week and month, controlling for day/night temperature differentials if possible. Apply to each geographic region, using day/night temperature differentials.

Next consider the problem of estimating and forecasting the impact on the load curve of peakload pricing. We propose the simplest possible response function:

$$\frac{PLF^{*\prime}(M,D,T)}{PLF^{*}(M,D,T)} = \left(\frac{PEAKF'(M,D,T)}{PEAKF(M,D,T)}\right)^{\eta} , \quad (16)$$

where

PLF* = the relative load curve;

PLF*′ = a modified relative load curve with modified rates;

PEAKF = an array of relative peakload prices, normalized so that the average of PEAKF is one;

PEAKF′ = a modified array of relative peakload prices;

η = a response elasticity, which may depend on M,D,T .

Note that it will generally be necessary to re-normalize PLF*′ in equation (16) so that the area under it is one. Equation (16) is not derived from a theoretical model of time-of -day demand substitution, and is merely a convenient one-parameter approximation. For commercial and industrual customers, the very limited data available makes it difficult to refine this specification. However, we note that it is possible to add to Equation (16) variables indicating the use of various load management methods. Then, the demand system can forecast the impact on the load curve of various load management technologies.

In the case of residential demand, the demonstration experiment data is sufficiently detailed to permit exploration of somewhat more flexible models. Appendix G describes one possible theoretical foundation for a more general analysis of residential time-of-day demand patterns.

From base period load curves and Equation (16) with fitted η , the load curves for alternative peakload pricing schemes can be forecast. One deficiency in this analysis is that η represents the short-run response to a <u>temporary</u> peakload pricing structure. This elasticity may be considerably below the long-run elasticity of response to <u>permanent</u> peakload pricing, where the introduction of storage devices and other capital investments to shift demand to off-peak become attractive to the customer.

It is not possible to estimate econometrically this long-run elasticity from existing U.S. data. However, a parametric analysis, based on European experience and technological possibilities, can be carried out.

7. User Type Saturation Curves

The preceding sections describe the forecasting of a consumption frequency distribution and load curve for each user type within a customer class. To obtain overall values for the customer class, it is necessary to average these curves, using forecasts of the shares of each user type. We anticipate that much of the overall long-run price elasticity of demand for a customer class will be due to changes in user type shares rather than to demand changes within each user type.

Base period shares for residential user types can be obtained from the U.S. Census of Housing for each geographic region analyzed, for 1970. Some technical analysis of the reported data is necessary due to the fact that the three-way classification by type of water heating, space heating, and air conditioning is not reported directly. We propose use of the classical statistical method of iterative proportional fitting, starting from an initial three-way classification derived from WCMS or MRI data, to obtain a three-way classification for each geographic region which matches regional marginals.[1]

Industrial user type shares in the base period can be

[1] The method is described in Bishop, Feinberg, Holland, Discrete Multivariate Analysis, 1975, and in Duguay, Jung, McFadden, "SYNSAM: A Methodology for Synthesizing Household Transportation Survey Data," Working Paper No. 7618, Urban Travel Demand Forecasting Project, Institute of Transportation Studies, University of California, Berkeley, 1976.

obtained from the U.S. Census of Manufacturers, 1967, 1972, and 1974. No geographically uniform data base for determining commercial user type shares or estimating their responsiveness to relative energy prices has been identified.

The estimation of user share response to relative energy prices will be made using a multinomial logit model, with customer size and climate as other explanatory variables in the residential case. The form of the model is

$$P(WH,SH,AIR) = V(WH,SH,AIR) \Big/ \sum_{W,S,A=0}^{1} V(W,S,A) \quad , \tag{17}$$

where

$P(WH,SH,AIR)$ = the share of the user type with characteristics (WH,SH,AIR) .

$$\begin{aligned} \log V(W,S,A) = {} & a_1(W,S,A) + a_2(W,S,A) \log \overline{Y} \\ & + a_3(W,S,A) \log Y/\overline{Y} + a_4(W,S,A) \log RC \\ & + a_5(W,S,A)\, MF + a_6\, S \cdot NR \\ & + a_7\, S \cdot HDD + a_8\, A \cdot CDD \end{aligned} \tag{18}$$

with

$a_i(W,S,A)$ = a distinct parameter for each triple (W,S,A) , for $i = 1,\ldots,5$, with the normalization $a_i(0,0,0) = 0$.

RAC = relative cost of electricity and gas, measured in the case of electricity as the

marginal cost between 250 and 750 KWH per month, and in the case of gas as the marginal cost between 40 and 160 therms per month.

$\overline{Y}$, Y, MF, NR = mean household income, customer income, if multiple family, and number of rooms, as defined in the model of representative annual consumption.

HDD, CDD = heating and cooling degree days (or Jan., July temp)

Alternatives to a joint specification which simplify the model structure and reduce the size of the estimation problem are to assume some aspects of the user type choice to be conditional upon or independent of other aspects. For example, if the three aspects are assumed independent, one can estimate a multivariate model

$$\begin{aligned} P(WH = 1) = 1/(1 + \exp(-(a_{11} + a_{12} \log \overline{Y} + a_{13} \log Y/\overline{Y} \\ + a_{14} \log RC + a_{15}\, MF))) \; ; \end{aligned} \tag{19}$$

$$\begin{aligned} P(SH = 1) = 1/(1 + \exp(-(a_{21} + a_{22} \log \overline{Y} + a_{23} \log Y/\overline{Y} \\ + a_{24} \log RC + a_{25}\, MF + a_{26}\, NR + a_{27}\, HDD))) \; ; \end{aligned} \tag{20}$$

$$\begin{aligned} P(AIR = 1) = 1/(1 + \exp(-(a_{31} + a_{32} \log \overline{Y} \\ + a_{33} \log Y/\overline{Y} + a_{34} \log RC + a_{35}\, MF + a_{38}\, CDD))) \; . \end{aligned} \tag{21}$$

Then the joint probability is the product of three univariate probabilities. Alternately, one may assume these probabilities to give marginal distributions without assuming independence, and then assume the joint distribution contains two-factor and three-factor interaction effects estimated from a single joint calibration sample. A detailed methodology for this alternative can be adapted from standard statistical procedures.[1]

The multinomial logit model above can be fitted to WCMS or MRI data. This fitted equation should provide a long-run share equation as a function of relative prices, at least if relative prices have remained stable over time. These elasticities are probably substantially higher than short-run elasticities--since changes in user type arise primarily from changes in the housing stock, the short-run adaptation to long-run desired shares is likely to vary with the rate of gross additions to the housing stock. There may also be an unexplained trend in user type shares.

One deficiency of the model in Equation (18) is that age of housing unit is not included. It is reasonable to assume that user type choices are largely determined at the time of construction, and that relative prices at the date of construction are a determinant of the type choice. Unfortunately, the available household surveys, WCMS and MRI, do not report age of dwelling. If contemporary relative fuel costs across regions

[1] Bishop, Feinberg, Holland, Discrete Multivariate Analysis, 1975.

have maintained their historical differentials, and the age distribution of the housing stock is similar in the areas surveyed, then the specification in equation (18) should give consistent estimates. Otherwise, this specification introduces some error.

We have fitted a user type share equation to sample survey data from San Diego Gas and Electric, where age of dwelling is available, and have used as an explanatory variable the historical relative price of electricity and gas at the date of construction.[1] The results are given in Table 9. One sees that there is a very strong response of user type to relative price. The equation shows no significant exogenous historical trend.

To utilize estimated equations (17) or (19)--(21) for dynamic forecasting, we suggest introducing an adoptive adjustment to desired shares, in much the same manner as was employed for the estimates of representative annual demand. Postulate a simple adjustment equation

$$(22) \qquad \log \frac{P(W,S,A,t)}{P(0,0,0,t)} = (1-\gamma) \log \frac{P(W,S,A,t-1)}{P(0,0,0,t-1)} + \gamma \log \frac{P^*(W,S,A,t-1)}{P^*(0,0,0,t-1)}$$

where

$P(W,S,A,t)$ = observed share in period t ;

$P^*(W,S,A,t)$ = desired share in period t .

The parameter γ can be fitted from U.S. Census of Housing data on a national or regional level. The parameter γ may itself be

[1] The "relative price" is defined as the ratio of the total electric and gas bills which would result in alternative user categories, where end use shares are first determined for the sample as a whole, then scaled to the overall consumption level of each customer.

TABLE 9. Price Response of User Type Shares

(San Diego Gas and Electric, Miracle II Survey, 1975)

Dependent variable: Gas water and space heat; or otherwise
Estimation method: Binary logit analysis

Dependent Variable	Coefficient	T-Statistic
Square feet	-.000082	-.55
Income	-.0000045	-.48
If single family	1.09	4.9
If owner	.84	3.9
Age of dwelling	-.0088	-1.5
Relative cost[1]	-11.35	-7.56
Constant	12.42	8.20

Percent correctly predicted: 90.0

Likelihood ration index: .55

[1] Ratio of gas and electric bill for an all gas household to gas and electric bill for other household type, computed using sample end use shares and observed overall consumption level.

a function of the rate of gross housing starts.

8. An Illustrative Analysis of the Impact of Alternative Block Rate Structures

We have examined the feasibility of the method suggested here for forecasting consumption frequency distributions, using 1975 Miracle II Survey data from San Diego Gas and Electric. The long-run impacts of declining block, flat, inverted, two-part, and lifeline rate structures are analyzed for residential customers in the D1 rate zone (metropolitan San Diego). User types are distinguished by energy source for water and space heat. The analysis is carried out under simplifying assumptions which limite the policy conclusions which can be drawn from the results. First, the effect of peakload cost and the load curve has been ignored. Second, utility marginal cost of supplying electricity is arbitrarily assumed to equal the tail block rate prevailing in 1975. Third, the rate of return on capital is arbitrarily set to six percent in the base period, and rates are adjusted to meet the revenue requirements implied by costs and this rate of return, assuming costs and numbers of customers do not change over time. Equity and debt capital are not distinguished and taxes are ignored. In a full forecasting model, these arbitrary assumptions will be replaced by a model of utility supply, finance, and regulation.

The demand function in Table 4 is used to define the transformation ψ (see equation (10)) in this application. As noted earlier, this empirical function may fail to satisfy the theoretically plausible monotonicity condition at some

points, due to shortcomings of the approximation.

Figure 5 plots an empirical function $\psi(KWH,...)$ for the 1974 D-2 rate schedule of Pacific Gas and Electric, using the model in Table 4. One sees that this curve is generally monotonic in nature, but that small deviations from monotonicity occur near the block boundaries of 200 and 1000 KWH per month. Consequently, this fitted curve is not consistent with the implications of economic consumer theory if indeed electricity is a non-inferior good. This outcome is almost certainly due to the restricted functional form of the demand equation (f^1) and the ract that it was fitted using state stypical electric bill data using smooth parametric forms which did not display the sharp rate drops at block boundaries which characterize actual schedules and which failed to incorporate smoothing by consumers. We conclude that a portion of the demand curve like the segment AB in Figure 6 reflects deficiencies in the functional specification and transferability of the estimated demand function rather than plausible behavior. As an empirical correction for these anomolies, we replace the curve between A and B by a straight line between these points. The result is a curve which is consistent with the theoretical construction and qualitatively reasonable, and which in the absence of further information provides the simplest correction available for the inconsistency arising from the demand specification. We note from Figure 5 that this substitution has a very small effect on the overall demand curve; hence, its impact on our conclusions is small. Nevertheless, this point of the analysis could be strengthened

FIGURE 5. The Demand Function for the Base Rate Schedule

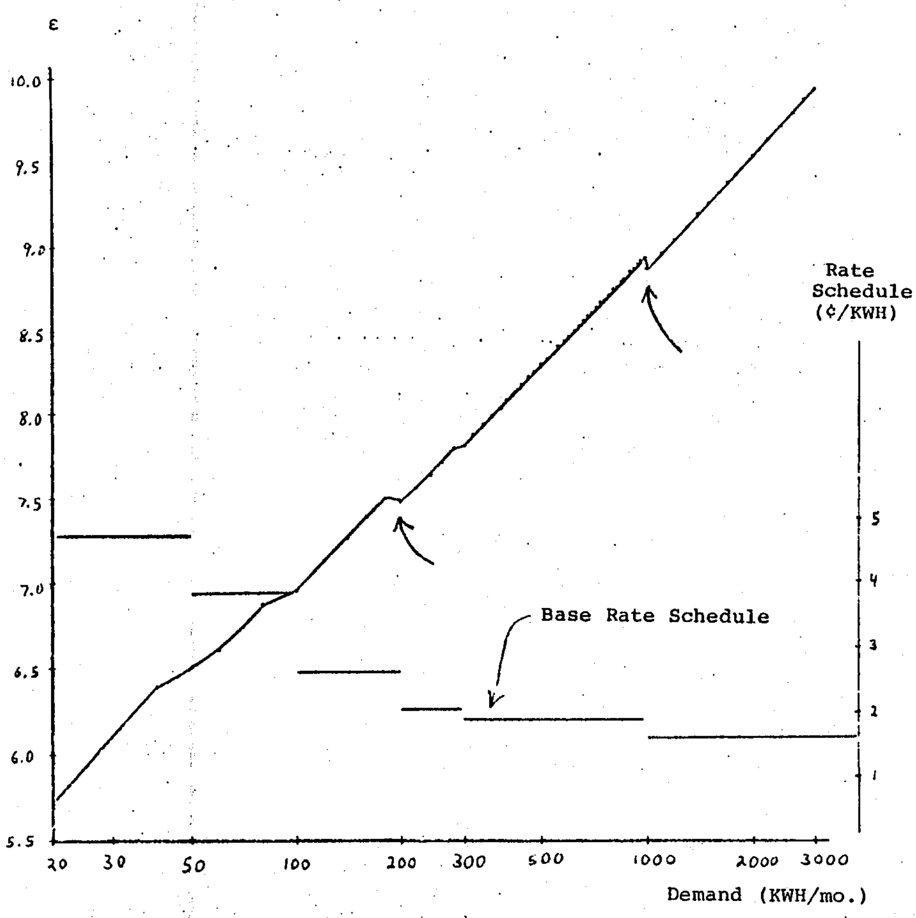
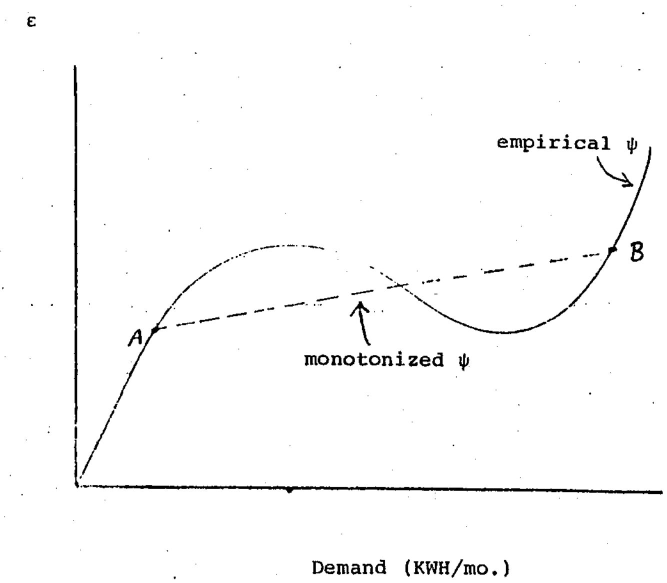


FIGURE 6. "Monotonization" of the ψ function



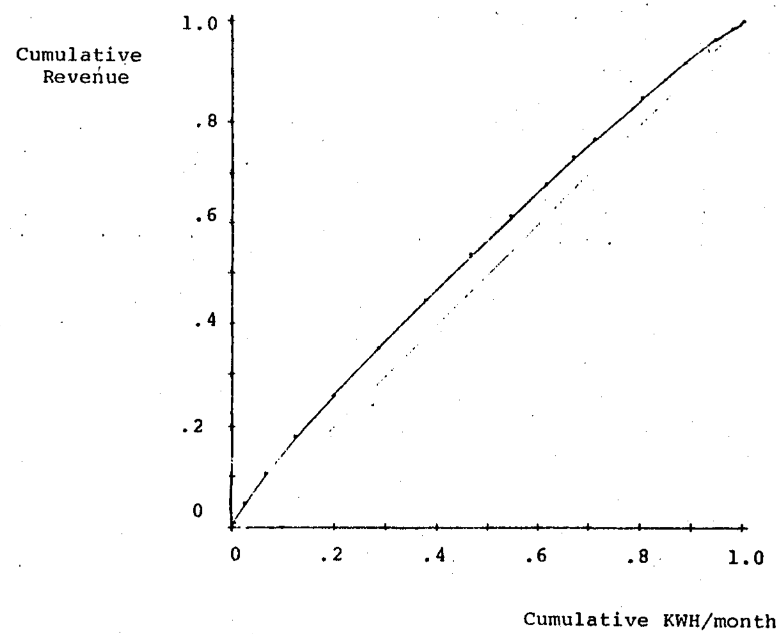
considerably with the use of disaggregate demand data to determine directly the behavior of the demand function.

The methodology described above has been implemented in an APL program listed in Appendix E. This appendix also includes a sample output from the program, and instructions for its operation. In broad outline, the program requires as inputs the base rate schedule and consumption frequency distribution. Given any alternative rate schedule, it provides the resulting consumption frequency distribution, plus the average demand and revenue per customer, and average price.

Two additional curves can be introduced which are useful in assessing the distributional effects on large and small customers of alternative rate schedules. The revenue distribution curve plots cumulative percent of total revenue collected against cumulative percent of total KWH supplied, starting with the smallest customers. When the curve lies above the diagonal, small users are paying more per KWH than large users. Lifeline rates or an inverted rate structure would tend to flatten or reverse the curvature of this curve. Figure 7 illustrates a typical revenue distribution curve for a declining block rate structure.

The net subsidy curve plots the difference in cost and revenue, expressed as a percentage of cost, for customers at various KWH demand levels. An exact definition requires a utility supply model which gives cost of providing service as a function of consumption level. We approximate the curve, for illustrative purposes, in this application, by assuming that

FIGURE 7. Revenue Distribution Curve at Base Rate
(PG & E Rate D2, 1974)



cost has a simple "two-part" structure: a connect charge plus an energy cost determined at a constant marginal cost of electricity,

(23) $\quad$ COST = CONNECT + (MARGINAL COST) × (KWH CONSUMPTION) .

The net subsidy curve then satisfies

(24) $\quad$ PERCENT NET SUBSIDY = 100 × (COST − REVENUE) ÷ COST ,

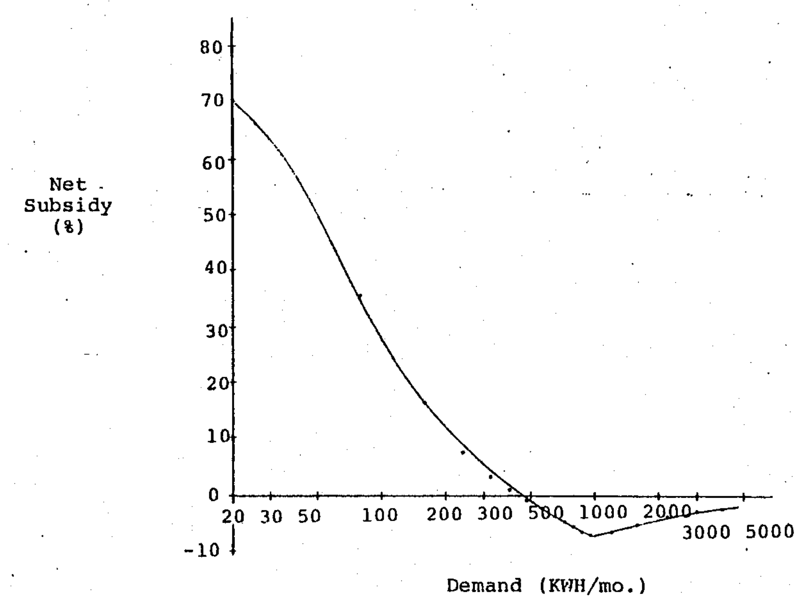
determined as a function of monthly demand. In equation (24), total revenue is normalized to equal total cost, so that averaged over all customers, the net subsidy is zero. For purposes of computation in this illustration, we assume the tail block rate to equal marginal cost, and compute CONNECT using the formula

(25) $$\text{CONNECT} = \begin{pmatrix}\text{Base average}\\ \text{revenue per}\\ \text{customer}\end{pmatrix} - \begin{pmatrix}\text{Marginal}\\ \text{cost}\end{pmatrix} \times \begin{pmatrix}\text{Average demand}\\ \text{per customer}\end{pmatrix} .$$

Figure 8 illustrates a typical net subsidy curve for a declining block rate schedule, for the simplified cost structure assumed above.

Before considering specific alternative rate schedules, we note several classical economic principles which apply to electricity pricing. The most fundamental is that a good should

FIGURE 8. Percent Net Subsidy at Base Rate
(PG & E Rate D2, 1974)



be priced at its marginal cost.[1] Any other price is inefficient and imposes a deadweight loss on the economy in the sense that the resulting allocation of goods could be adjusted to make all consumers better off. In practice, this adjustment must be carried out by lump-sum transfers between individuals or by selective taxes and subsidies on inelastically supplied goods. In an industry such as electricity generation where marginal cost is approximately constant and fixed costs are positive, marginal cost pricing will entail a loss, in the absence of a subsidy to the utility. Alternately, in the case that all individuals consume some quantity of the commodity, a "two-part tariff," in which each customer pays a connect charge plus marginal cost per unit consumed, with the connect charge equal to prorated fixed cost, is an efficient pricing scheme and allows the industry to earn normal profits. With a two-part tariff, each customer receives a zero net subsidy;[2] hence this pricing scheme cannot be utilized to effect an income redistribution policy. However, redistribution of the connect charge is possible.

In the absence of effective or politically feasible methods to carry out direct lump-sum transfers to achieve goals of income distribution, electricity rates may be used, as presently, as an indirect instrument of redistribution. However, this

[1] This statement is formally true only if all goods in the remainder of the economy are priced at marginal cost. Otherwise, specific "second-best" deviations from marginal cost pricing can be justified. It is nevertheless useful to take marginal cost pricing as the policy of choice, introducing deviations only where a persuasive case can be made for their presence.

[2] A "lump-sum" subsidy or tax can be reintroduced by varying the customer charge according to income or other social welfare criteria.

should be done with the clear understanding that such a program imposes a deadweight loss on the society, causing the inefficient use of resources. A balance is required between the social benefits of redistribution and the social cost of misallocation of resources. Most importantly, socially inefficient redistributive mechanisms should not be undertaken until all possibilities for achieving the same redistributive goals by transfers or other efficient mechanisms have been exhausted. Specifically, lifeline and inverted rate structures which seek to reduce the burden on small customers and penalize large ones should not be adopted if there are possibilities for achieving the same relief by less distortionary means such as property or income tax relief, adjustments in welfare payments, and so forth.

Aside from the question of the redistributive implications of rate structures, alternatives are often evaluated in terms of their potential for energy conservation. Economic principles suggest that there is an optimal degree of conservation, and that this degree can be achieved by setting price equal to marginal cost of production plus the marginal social loss caused by the consumption of energy resources <u>not otherwise captured in the marginal cost of production</u>. Conservation measures which deviate from this standard <u>in either direction</u> will cause deadweight loss.

Application of the principle above requires several cautions. First, care must be taken in determining the extent to which the private marginal cost subsumes factors we think of as "social cost." For example, recent studies of the economics of natural resource extraction indicate that a private market will often pursue a socially optimal pattern of extraction, and in the case of the monopolistic power on the part of the resource owner, may be inefficiently conservative, making optimal social policy anti-conservative.[1] Second, public intervention in private markets usually imposes costs of enforcement and non-compliance; these must be balanced against the benefits of intervention. For example, imposition of industrial rates in excess of marginal cost in the past decade resulted in the installation of non-regulated, inefficient private power plants in some cases.

We apply the analysis to alternative rate structures for the D-1 rate area of San Diego Gas and Electric. The base case is the 1975 declining block rate schedule given in Table 10. As alternatives to this case, we consider four alternative rate structures:

Two-part tariff
Flat rates
Inverted rate structure
Lifeline structure

The two-part tariff sets energy charges at the constant marginal cost of electricity, and achieves the required rate of return with high customer charges. The flat rate schedule sets a constant price of electricity per KWH consumed, and assumes a low customer

[1] Heal, G., "The Relationship Between Price and Extraction Cost for a Resource with a Backstop Technology," Bell Journal of Economics, Autumn 1976, 371-378. Kay, J. and Mirrlees, J.A., "The Desirability of Natural Resource Depletion," in D. Pearce and J. Rose, eds., Economic Aspects of Natural Resource Depletion, London: Macmillan, 1975. Solow, R., "Intergenerational Equity and Exhaustible Resources," Review of Economic Studies (Symposium Issue), 1974, 29-46.

charge. The inverted rate structure has rising electricity rates in successive blocks. The lifeline rate structure sets a uniform, relatively low rate for small customers, and a substantially higher rate for large customers. The lifeline rate is a modified version of a schedule studied by the State of California Energy Resources Conservation and Development Commission[1], adjusted as described below.

Table 10 gives the alternative rate schedules considered. The flat and inverted rate structures are scaled uniformly to achieve the six percent rate of return required for customers with non-electric space and water heat (eighty percent of the customers). Similarly, the customer charge for the two-part tariff and the non-lifeline block rates for the lifeline schedule were adjusted to achieve this rate of return.

For some consumption frequency distributions and lifeline schedules, we have found that the maximum collectable revenue from non-lifeline customers may be inadequate to meet the revenue requirements. (This is not the case, however, for the lifeline schedule considered here.) We conclude that it is possible for long-run demand to become elastic with respect to high tail block rates in lifeline schedules, so that beyond some point increases in tail block rates will decrease revenues and realized rate of return. This observation has substantial policy implications. It may be impossible in the long-run to maintain

[1] "Electrical Pricing Policies," Staff Report, August, 1976, p. 126.

TABLE 10. Alternative Rate Schedules

1. Base Rate

Customer charge	$.86/mo.			
Energy charge				
Blocks (KWH/mo.)	0-40	40-100	100-200	200+
Rates (¢/KWH)	5.678	4.228	4.038	2.918

2. Two-Part Tariff

Customer charge	$3.83/mo.
Energy charge	2.918¢/KWH

3. Flat Rates

Customer charge	$.86/mo.
Energy charge	3.66¢/KWH

4. Inverted Rate Structure

Customer charge	$.86/mo.		
Energy charge			
Blocks	0-200	200-300	300+
Rates (¢/KWH)	3.02	4.02	5.03

5. Lifeline Structure[1]

Customer charge	$.86/mo.	
Energy charge		
A. Non-electric water and space heat		
Blocks (KWH/mo.)	0-240	240+
Rates (¢/KWH)	2.92	5.25
B. Electric space, non-electric water heat		
Blocks (KWH/mo.)	0-400	400+
Rates (¢/KWH)	2.92	5.25
C. Electric water, non-electric space heat		
Blocks (KWH/mo.)	0-480	480+
Rates (¢/KWH)	2.92	5.25
D. Electric water and space heat		
Blocks (KWH/mo.)	0-640	640+
Rates (¢/KWH)	2.92	5.25

[1]The block boundaries for schedules B - D are determined by adding the lifeline boundary fo 240 KWH/mo. for non-electrically heated homes and the average consumption in the D1 rate area for each of the electric end uses--water heating (240 KWH/mo.) and space heating (160 KWH/mo.). These figures do not agree except in general magnitude with specific lifeline schedules proposed or adopted in California.

a lifeline rate structure which provides substantial subsidies to small users and at the same time achieves a required rate of return. In effect, even the revenues obtained by monopoly pricing of electricity to large users may be inadequate to meet the revenue requirements of the subsidy program implied by some lifeline structures. Furthermore, this problem appears to be much more than a theoretical possibility, occuring for rate structures in the range currently being proposed.

These conclusions are drawn from the long-run demand curve, which takes into account adjustment in some appliance stocks and types. Short-run elasticities are substantially lower, and the long-run inability of a lifeline rate strucutre to achieve the target rate of return will not be evident in the short-run. The dynamics one would expect are that tail rate increases would be adequate to achieve the target rate of return in the initial several years, but that declining numbers of large users would lead to successive shortfalls in revenue. After some point, further rate increases would accelerate the decline in revenues from large users. Unless the participants are aware that demand is elastic in this price range, there is a danger of successive rate increases in excess of the monopoly level which both penalize the consumers and are counter to the effort to maximize the rate of return.

Table 11 gives the long-run demand and average revenue per customer resulting from each of the rate structures in Table 10, for all residential customers in rate area D1.

TABLE 11. Forecasts of Impacts of a Once-and-for-All Implementation of Alternative Block Rate Structures

Assumptions:

1. Residential customers, San Diego Gas and Electric rate area D1
2. No change in utility costs, number of customers, income, or other exogenous variables, from 1975 level.
3. Long-run equilibrium schedules are imposed in 1977, and remain fixed thereafter.
4. The user type shares remain fixed at 1975 levels[1]:
   a. non-electric water and space heat 81.7%
   b. electric space, non-electric water heat 10.2%
   c. electric water, non-electric space heat 3.3%
   d. electric water and space heat 4.8%
5. Base demand (1975) in long-run equilibrium[2]

A. Average Demand (KWH/mo.)

Rate Schedule	Long-Run Demand User Type					
	1	2	3	4	All Residential	
Base	442	424	690	801	466	
Two-part tarriff	448	429	693	804	471	(1.1%)
Flat	399	383	609	704	419	(-10.1%)
Inverted	354	342	517	592	370	(-20.6%)
Lifeline	352	385	556	667	377	(-19.1%)

B. Average Revenue per Customer ($/mo.)

Rate Schedule	Long-Run Average Revenue/Customer User Type 1	2	3	4	All Residential	
Base	16.67	16.20	23.97	27.23	17.37	
Two-part tariff	16.83	16.28	23.99	27.23	17.51	(0.8%)
Flat	15.47	14.88	23.16	26.64	16.20	(-6.7%)
Inverted	14.11	13.37	22.07	25.79	14.86	(-14.5%)
Lifeline	14.04	13.06	20.24	23.53	14.60	(-15.9%)

[1]Full implementation of the forecasting methodology requires that user type shares be forecast from the appliance saturation model, taking into account the availability of new gas connections. Interfuel substitution, if gas is available, will lead to larger proportions of user type 1 under inverted rates, and larger proportions of user type 4 under two-part, and possibly under lifeline, rates.

[2]The assumption that 1975 consumption is in long-run equilibrium is clearly invalid, but is made in this feasibility demonstration to avoid collection of historical time-series. The qualitative, but not quantitative, implications of alternative rate structures will probably continue to hold with more realistic assumptions on base year desired demand.

Taking the base rate consumption as a standard, we see that all the alternative structures except the two-part tariff are energy-conserving, with the flat rate reducing demand ten percent, and the lifeline and inverted rates, twenty percent. Table 12 gives the dynamic path of average demand, for each rate schedule, under our simplifying assumptions.

All the alternative rates increase revenue per customer substantially in the short-run, due to the low elasticity of short-run demand. This increase is least for the two-part tariff, and most for the lifeline rate structure. In the long-run, the impact of flat, inverted, or lifeline rates is to reduce consumption sufficiently to lower revenue per customer.

In the short-run, the realized rate of return from the imposition of a long-run equilibrium rate schedule is elevated due to the low elasticity of short-run demand. This impact is greatest for the inverted and lifeline schedules. In the longer-run all the structures approach the target rate of return of six percent.

We next consider the distributional implications of the alternative rate structures in this illustrative application. The net subsidy curve displays most clearly the economic distributional effects of a rate schedule. In the absence of explicit social objectives of income redistribution or demonstrated external effects, efficient economic allocation of resources requires zero net subsidy. It is intuitive that the larger the deviation of a net subsidy curve from zero, the greater the economic loss caused

TABLE 12. Forecasts of Dynamic Impacts of Alternative Block Rate Structures

Assumptions: Same as preceding table.

A. Average demand, all residential customers (KWH/mo.)

Rate Schedule	Year				
	1975	1977	1987	1997	Long-Run
Base	466	466	466	466	466
Two-part tariff	466	467	470	471	471
Flat	466	461	433	423	419
Inverted	466	455	399	379	370
Lifeline	466	456	404	385	377

B. Average revenue per customer, all residential customers (KWH/mo.)

Rate Schedule	Year				
	1975	1977	1987	1997	Long-Run
Base	17.37	17.37	17.37	17.37	17.37
Two-part tariff	17.37	17.38	17.47	17.50	17.51
Flat	17.37	17.71	16.71	16.35	16.20
Inverted	17.37	19.17	16.31	15.29	14.86
Lifeline	17.37	20.11	16.45	15.15	14.60

by distortion of the price system.[1] Thus, inverted and lifeline rates may be judged to be desirable on grounds of income redistribution and/or energy conservations, but these benefits must be weighted against the social costs of imposing a rate structure which is substantially at variance with the cost of service. Figure 9, (a)-(c), plots the net subsidy curves for alternative rate schedules in San Diego's D1 rate area for the user type with non-electric water and space heat, defined under the simplifying assumptions made for purposes of illustration. The flat, inverted, and lifeline schedules all increase the net subsidy to users of less than 350 (lifeline) or 450 (flat, inverted) KWH/month by relatively modest amounts. The revenue requirements of these subsidies impose a substantial tax on large users. The reason for this difference in magnitude is the preponderance of small users and the elasticity of demand of large users. The two-part tariff, not graphed, gives a zero net subsidy to all users. From the standpoint of economic theory, if externalities are unimportant and fuels are correctly priced, then a two-part tariff with a connect charge varying with income class to meet objectives of income redistribution, would be preferable to the flat, inverted, or lifeline schedules.

[1] This "intuitive" condition is not always true, although it holds in many cases. The most comprehensive analyses have appeared in the economic literature on optimal tariff adjustments in international trade and on the pricing of public goods.

Figure 9. Percent Net Subsidy
Graph (a)

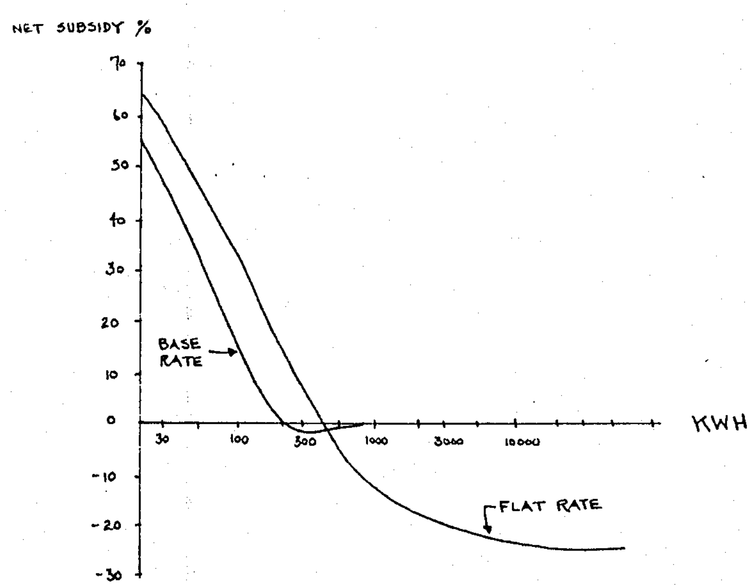


Figure 9. Percent Net Subsidy
Graph (b)

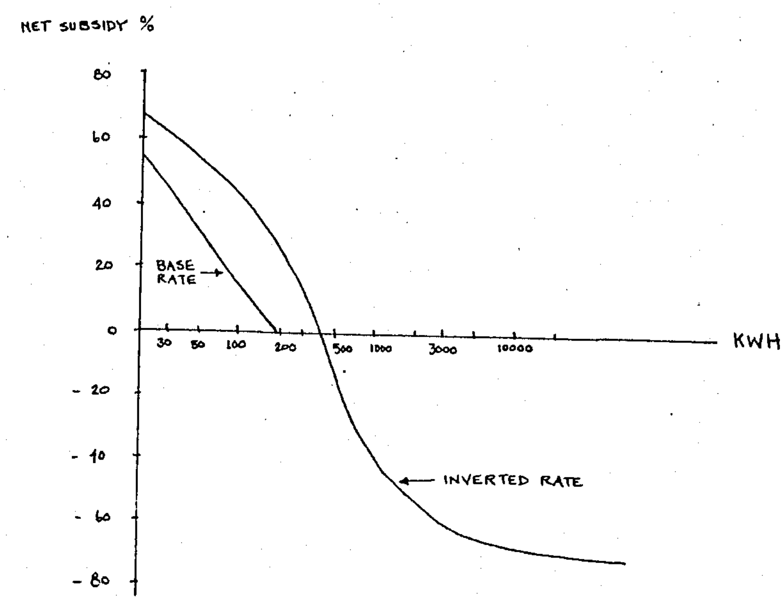
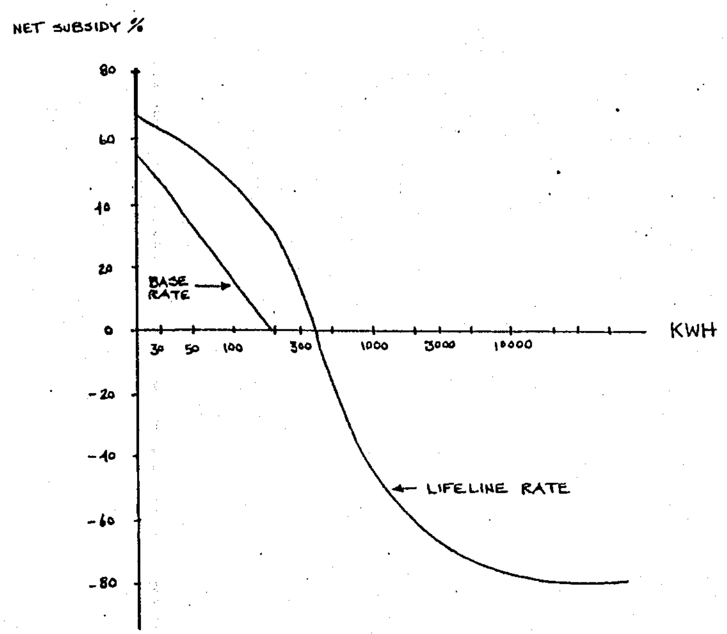


Figure 9. Percent Net Subsidy

Graph (c)



APPENDIX A

Studies in Price and Income Elasticities in
Residential Demand for Electricity: A Bibliography

Houthakker, H.S. "Some Calculations of Electricity Consumption in Great Britain," Journal of the Royal Statistical Society (A), Vol. 114, Part III, (1951) pp. 351-371.

The author seeks to estimate the determinants of demand for electricity. His model consists of linear and log-linear regression models, in which the dependent variable is the average annual amount of electricity consumed per customer, and the independent variables are the average money income per household, the marginal price of electricity, the marginal price of gas, and an estimate of the average holding of heavy domestic equipment (e.g. stock of electricity-consuming goods).

The author restricts his analysis to those customers choosing to buy electricity according to a two-part tariff, using as marginal price the second part of the tariff. His data are cross sectional observations on 42 provincial towns for 1937-38. His results are $\varepsilon_{ey} = 1.17$, $\varepsilon_{ep} = -0.89$, and $\varepsilon_{eg} = 0.21$ (not significant), and $R^2 = 0.872$.

Because a capital stock term is included, Houthakker interprets all his elasticities to be short run.

Fisher, F.M. and C. Kaysen. A Study in Econometrics: The Demand for Electricity in the United States. Amsterdam: North Holland Publishing Co., 1962.

The study is very ambitious. It is the first of its kind to differentiate between long- and short-run demand for electricity by residential end-users. The authors interprete SR demand to be the choice of a certain utilization rate of a given stock of electricity-consuming goods, whereas LR demand implies a choice of size of such stock over time.

First the authors deal with SR demand. Their SR model consists of two basic parts. First, they posit that $D_t = \sum_{i=1}^{n} K_{it} W_{it}$, where

D_t = the total metered use of electricity in kwhs by all households in the area during period t, W_{it} = the average stock during t of the ith type of "white good", measured in kwhs of electricity during an hour of normal use, and finally K_{it} = the average intensity of use of the ith white good during t (measured in units of kwh/time period • units of good). Naturally it is the K_{it}'s which are of interest here. The second part of the authors' SR model asserts that K_{it} is a log-linear function of price and per capita personal income in t, (the price term is the average price of electricity per kwh in the area in question). Putting the two parts of the model together, using averages and the approximation that the stock of white goods grows at a constant rate, the authors are able to eliminate what they feel is an ambiguous stock term from their model. Their final SR estimation equation regresses the change in $\ln D_t$ on the change in the log of average price, the change in the log of per capita income, and a constant term, the co-efficient of which is interpreted to be the constant growth rate of white goods.

Their data are observations on 47 states (North and South Carolina combined) during the years 1946-57, (e.g. pooled CS-TS). D_t is kwh sales to residential customers, reported by the Edison Electric Institute Statistical Bulletin. The price variable is (sales revenue)/kwh, that is, an ex post measure. Per capita personal income data are from the census.

Qualitatively, the authors find that the SR ε_{ep} is lower in older parts of the USA, the implication being that as the ecomony matures, the SR demand is less price sensitive. The authors further divide the "older" USA into 3 groups according to degree of urbanization. The more urban states have a higher ε_{ey}, suggesting that in poorer, rural areas, most white goods are constant use goods, necessity goods whose use varies only little with income.

Their long-run model is simpler. They reject the flow adjustment model approach, and instead regress the change in $\ln W_{it}$ on the change in the log of per capita permanent income (a' la Friedman), per capita income as before, the price of the good, the price of the relevant gas using substitute, the average prices of elasticity in kwhs and gas in

therms, and a host of demographic variables. The LR equation is estimated for five types of white goods: washers, refrigerators, ranges, water heaters, and irons. Annual data are used for the same 47 states as in the SR model, for the years 1946-7, and 1951-7. In some estimates the states are pooled into groups to reflect economic and social realities.

In general, the change in W_{it} depends on the change in permanent income, the change in population, and the change in the number of electrically wired households per capita. The price of electricity is shown to have almost no effect, the appliance prices only small ones. In general, the results support the hypothesis that in situations where the use level of appliances is almost total, only demographic variables are important, whereas when such a level is less than total, economic variables start to be important.

Houthakker, H.S. and L.D. Taylor, Consumer Demand in the United States, 2nd edition, Cambridge, Mass.: Harvard University Press, 1970.

The authors use a two part model to estimate the price and income elasticities of residential demand for electricity. The first equation in the model relates energy consumption to the stock of energy consuming goods, income, and relative price, in a simple linear fashion. The second equation relates the change in the stock of energy consuming goods to the consumption of electricity and the absolute level of appliance stock via a constant depreciation rate. The two equations are merged to yield the estimated equation, in which present consumption is related to present income, present relative price, and the lagged dependent variable.

The data used are TS on personal consumption expenditure (aggregate US) from the national income accounts.

The results are as follows: $SR\varepsilon_{ep} = -0.13$, $SR\varepsilon_{ey} = 0.13$, $LR\varepsilon_{ep} = -1.89$, $LR\varepsilon_{ey} = 1.93$.

Wilson, J.W. "Residential Demand for Electricity." Quarterly Review of Economics and Business, Vol. 11, No. 1, (Spring, 1971), pp. 7-22.

Wilson's purpose is to estimate the determinants of (1) the residential demand for electricity, and (2) the demand for six categories of electricity-using household appliances.

In his energy demand model, the average electricity consumption per household (in kwhs/year) is fitted as a simple linear function of price (a variable constructed from the FPC's Typical Electric Bills), average price of natural gas, median family income, and demographic and climatic variables. The data are CS of 77 cities for the year 1966.

His results are $\varepsilon_{ep} = -1.33$, and $\varepsilon_{ey} = -0.46$. He interprets his model as long run. His results are significant at the .01 level. His model for the demand for appliances is qualitatively identical to his energy demand model. The only difference is that the dependent variable is the percentage of households owning at least one unit of the commodity in question (ranges, water heaters, clothes dryers, home food freezers, air conditions, and electric space heating). His results here are that the elasticity of demand for the appliance with respect to electricity price is negative and significant at the .001 level for five out of six appliances. For these five it is less than -1 for all but one. Median income is less important, while gas price is important for four out of six.

Mount, T.D., L.D. Chapman, and T.J. Tyrrell. "Elasticity Demand in The United States: An Econometric Analysis." Oak Ridge National Laboratory (ORNL-NSF-49), Oak Ridge, Tenn., June 1973.

In a comprehensive study, the authors estimate LR and SR demand functions for electricity for residential, commercial, and industrial classes of end users. The data used are a pooled set of CS-TS observations on 47 states during the period 1947-1970. They employ a double logarithmic model where the energy consumed in state i during time t is the endogenous variable, and population, per capita income, electricity average price, natural gas price, and a price index for appliances are exogenous variables. Also included as an independent variable was a mean January temperature shift parameter. The presence among the explanatory variables of a lagged dependent variable value allowed for a geometric adjustment of the dependent variable to shifts in the independent variables. A summary of the results follows:

	SR	LR
ε_{epop}	0.12	0.99
ε_{ey}	0.02	0.02
ε_{ep}	-0.14	-1.20
ε_{egas}	0.02	0.19
$\varepsilon_{eappl.}$	-0.05	-0.42

Anderson, K.P. "Residential Energy Use: An Econometric Analysis." The Rand Corp. (R-1297-NSF) October, 1973.

The author estimates two models. The first is an electricity demand model; the second is a model of demand for electricity consuming capital stock. The data employed are from the Census of Housing, 1960 and 1970, and are annual observations on 50 states. The energy consumption equation is estimated using both 1960 and 1970 data while the stock equation is restricted to the 1970 data.

The electricity demand model is a simple double logarithmic equation in which the consumption of electricity per household is fitted to the price of electricity (the average price for a fixed amount of electricity), the prices of gas, oil, and coal, income per household, and several climatic and demographic variables. His energy consumption stock model is very similar, except that the dependent variable is a ratio: the fraction of total installations consuming electricity over the total number consuming energy of a different type.

The price variables in these equations are the electricity price, and the price of the corresponding alternative energy form. The other explanatory variables are very similar to those in the energy consumption equation. The stock equation is estimated for eight classes of energy consuming goods: space heating, cooking stoves, washers, dryers, air conditions, freezers, dishwashers, and television. In the stock equations, the strongest predictors are the prices of electricity and utility gas. Income is of little importance (not surprisingly, due to the ratio form of the dependent variable). The author uses his stock equation to get indirect estimates of the LR demand for electricity elasticities, which accompany a set of direct estimates obtained from the first equation. The direct result are: $LR\varepsilon_{ep} = -1.12$, $LR\varepsilon_{ey} = 0.80$.

Lyman, R.A. "Price Elasticities in the Electric Power Industry." Department of Economics, University of Arizona, October 1973.

The author seeks to estimate the determinants of the demand for electricity for each of the three major classes of end users. His model encorporates two interesting innovations: (1) a nonlinear demand function, and (2) the inclusion of income through points on the income distribution, allowing income and price elasticities to vary with the level of income. Otherwise, the variables in his model are those that have become rather standard. The dependent variable is electricity purchases per customer. The explanatory variables are average price of electricity consumed (ex post), gas prices, a price index for other goods, and other economic, demographic, and climatic variables.

His data base is a pooled CS-TS set of observations for the years 1957-68 on 67 investor-owned utilities and the regions they serve. His electricity price data are average prices obtained from Typical Electric Bills data on quantity purchased, divided into expenditures. His results can be summarized in a threefold manner; (1) a linear semilog function is suggested for residential demand, (2) the price elasticities of demand are typically elastic for each customer class, and for residential customers is positively correlated with income (in absolute values), e.g. $\varepsilon_{ep} = -0.90$, $\varepsilon_{ey} = -0.20$. and (3) income elasticity of demand is very weak; in general. The size of the income elasticity varies inversely with income level, as one might expect with necessity goods.

Houthakker, H.S., P.K. Verleger, and D.P. Sheehan. "Dynamic Demand Analyses for Gasoline and Residential Electricity" Lexington, Mass. Data Resources, Inc., 1973.

The authors attempt to obtain an estimate of price and income elasticities of demand for electricity by applying a statistical fit to a rather simple logrithmic flow adjustment model, similar to that of Houthakker and Taylor (1970). First, the ratio of actual demand in the current period to that of the last period is assumed to be linearly related to the ratio of the desired demand in this period over the actual demand in the last period. The desired demand in the current period is assumed to be related to price and income in a double logarithmic fashion. Combining the first and second parts of the model, the authors obtain the lagged adjustment equation which is estimated statistically.

The data set is a pooled CS-TS sample of annual observation on State aggregates for the years 1960-71. Electricity consumption and income are in per capita terms. The prices of electricity are the marginal rates per kwh in the 250-500 block and the 100-500 block, respectively as taken from the FPC's Typical Electric Bills. The results are as follows:

		ε_{ey}	ε_{ep}
kwh 250-500			
	SR	0.15	-0.03
	LR	2.20	-0.44
kwh 100-500			
	SR	0.14	-0.09
	LR	1.64	-1.02

Halvorsen, R. "Demand for Electric Power in the U.S." Discussion Paper #93-13, Institute for Economic Research, University of Washington, Seattle, 1973.

This is one of the first studies to tackle the simulataneity problem encountered in estimating demand for electricity when electricity is sold according to declining block rates. The author estimates a simultaneous model via 2SLS. First, $P = B(E/N)^{x-1}$, when P = average electricity rate (¢/kwh), E = energy demand in state, N = # of customers in state, x = a parameter such that $0 < x < 1$, and B = a positive parameter. The second part of the model gives E simultaneously as a function of P and other factors such as income, gas price, July temperature, and the like. Criticisms have been made of the model to the effect that (a) the price equation is not not really representative in that it does not show interstate variability, and (b) that demand depends upon marginal costs and total inframarginal costs, the latter of which are reflected in average price.

Acton, J.P., B.M. Mitchell, and R.S. Mowill. "Residential Demand for Electricity in Los Angeles: An Econometric Study of Disaggregated Data," Preliminary draft, the Rand Corporation (R-1899-NSF), December, 1975.

The model is SR in principle. It focuses on the utilization rate of a given stock of electricity-consuming appliances. An innovation is the creative use of defining effective stock as a function of heating and cooling degree days.

Pooled CS-TS data give rise to partial reflection of long-run elasticities. The marginal price of electricity, used as an argument in the utilization rate function is highly significant. The data base consists of observations on small service areas collected by the Los Angeles Department of Water and Power (LADWP) and Southern California Edison (SCE). For LADWP, the area is the service area designated by the company readbook (about 260 residential meters); for SCE, it is a special unit, encompasing about four census tracts. All of the observation points are taken within Los Angeles county, between July 1972 and June 1974. Income and demographic data come from the 1970 Census, including detailed data on seven major appliances. The results:

$\varepsilon_{ep} = -0.70$, $\varepsilon_{ey} = 0.40$.

Taylor, L.D., G.R. Blattenberger, and P.K. Verleger, Jr. "The Residential Demand for Energy." A Report to the Electric Power Research Institute (Preliminary draft), Department of Economics, University of Arizona, January, 1976.

The model is the logarithmic flow adjustment model of Houthakker and Taylor (1970). The data are TS observations on a state, annually from 1956 to 1972, from the Census. The results are as follows:

	ε_{ep}	ε_{ey}
SR	-0.07	0.10
LR	-0.78	1.18

The Marginal Price of electricity, the tailblock the actual rate schedules, was highly statistically significant.

Lacy, A.W. and D.R. Street. "A Single Firm Analysis of the Residential Demand for Electricity," Department of Economics, Auburn University, 1975.

The model is a static double logarithmic demand function with the marginal price of electricity, income, the unemployment rate, the price of natural gas, and climatic variables as predictors. The base of obervation is the area served by the Alabama Power Company. Observations were made over this area on a monthly basis during the period from January, 1967 to December, 1974. The estimated elasticities are -0.45 and 1.87 for the marginal price of electricity and income, respectively. Since the model is a static one, the elasticities are neither SR nor LR. The marginal electricity price is taken from the actual resident rate schedule in the area served. Thus the elasticities are not unambiguous; they depend upon the block designated as the marginal block. In any event, the price variable has a t-ratio of nearly 3.

Federal Energy Administration, 1976 National Energy Outlook, FEA, Washington, D.C. 1976.

The model uses average price instead of both average and marginal price. It allows explicitly the possibility of interfuel substitution. The study approaches demand for energy in each sector in the framework of a system of joint demand functions. The FEA model is two stage. In the first stage, an index of total quantity of energy consumed is specified as a function of an index of energy price, income, and a lagged value of the dependent variable. In the second stage, the ratio of demand for each specific fuel to total energy index is related to the ratio of the price of the specific fuel over the total price index, and a lagged value of the first ratio. The equations in both stages are specified in double logarithmic form.

The data are a pooled CS-TS set of observations on nine census regions annually during theperiod 1960-1972. The equations are estimated using specifications which allow for regionally specific autocorrelation in the error terms.

The total price and quantity indeces used in the first and second stage are log-linear regional expenditure weighted average of regional prices and quantities, respectively.

The methodology enables estimates of price and income elasticities and cross elasticities in different end use sectors, regions, and over different time periods, because the expenditure weights are allowed to vary. The results are as follows:

	ε_{ep}	ε_{ey}
SR	-0.19	0.30
LR	-1.46	1.10

Wilder, R.P. and J.F. Willenborg. "Residential Demand for Electricity: A Consumer Panel Approach," Southern Economic Journal, Vol. 42, No. 2, October, 1975.

The study uses household cross section data to estimate price and income elasticities of demand for electricity.

The general approach of the authors follows the notion that the demand for electricity is a derived demand, depending upon the size of the electricity consuming appliance stock, the intensity of its use, and the residence size. In the SR, a price change will lead to a change in use intensity, whereas in the LR, such a change or a change in income will lead to a changing stock level.

The general model consists of (1) a residence size-equation, (2) and appliance stock equation, (3) and electricity demand equation, and (4) an electricity price equation. Equations (1) and (2) are simple equations predicting the demand for a certain residence size and appliance stock size on the basis of income, family size, age of household head, price of electricity, price of appliance stock, and the like. They are estimated via OLS. Equation (3) contains basically the same variables as explanatory factors as (1) and (2) and includes the stock of appliances as well. Equation (4) contains only the electricity consumption level as its independent variable. The authors discuss the identification problem arising between the demand relationship (3) and the price (expenditure) relationship (4) because of the declining block rate structure, and estimate (3) and (4) simultaneously via 2SLS.

All the functions are specified in log-linear form. This enables the authors to justify their usageof average rather than marginal price of electricity; the same elasticities are yielded using either. Estimation of (3) and (4) first with stock and family size data as arguments, then without, yields SR and LR elasticities of price and income respectively.

The data used are cross section observations on household drawn from an ongoing consumer panel in a southeastern metropolitan area of over 300,000 population. The panel is representative of middle and upper income classifications. The racial compositions reflects that of households with income over $5000 per annum (e.g. downward biased). The Spring, 1973 panel of 158 households and the Summer 1973 panel of 116 households were pooled.

Income, family size, and race are all significant in determining house size and appliance stock. Age of household head is significant only in the stock equation. The $\bar{R}^2$ are uniformly low, as one would expect with household data. The elasticities are as follows:

	ε_{ep}	ε_{ey}
SR	-1.00	0.16
LR	-1.31	0.34

Halvoren, R. "Demand for Electric Energy in the United States," Southern Economic Journal, Vol. 42, No. 4, April 1976.

In many respects Halvorsen takes a similar approach to demand estimation to Wilder and Willenborg (1975). His model is a two stage, simultaneous estimation of price of electricity and demand for electricity. This is necessary because of the declining block rate structure according to which most electricity for residential use has been sold. His price equation is in the form $P = P(Q,X,u)$, where P = average price, Q = quantity, and X = variables which determine the shape and location of the price function. Two price equations are used, one incorporating in X variables influencing the costs of electricity supply, the other incorporating Typical Electric Bill (TEB) data to estimate the shape of the rate schedule directly.

Residential demand is a function of the number of customers, electricity and gas prices (marginal prices should be used but Halvorsen argues that only average prices are available and that in a logarathmic form, price elasticities will be identical. See Wilder and Willenborg (1975)), per capita income, and several demographic and climatic variables. No capital stock term is included; the static model is interpreted by the author to yield strictly long-run elasticities.

The data set consists of CS observations on the 48 continental states in 1969. When the equations are specified in a dynamic form and 1961-1969 data are used, the elasticities do not change significantly. The static model with its built-in LR assumptions is thus considered appropriate by the author.

Demand equations are also estimated for commercial and industrial end users. Price and value added (income) elasticities are estimated directly or indirectly, depending upon the availability of good data on value added by sector. All equations are specified in double log form.

The results are summarized briefly as follows: ε_{ep} is close to unity in absolute value and higher significant. ε_{ey} is highly significant but about .65 - .71 in value. Price and income variables are significant at the .001 level. The gas price cross elasticity is significant at the .05 level, but only about .15 is absolute value. A corrected and adjusted $\bar{R}^2$ value is about .91.

Halvorsen also estimates an equation predicting the number of customers, a value used as an explanatory variable in the electricity demand equation. All of the economic variables used as predictors in the customers equation, e.g. price, income, gas price, etc. are insignificant. The principal determinants here appear to be demographic, with population and number of houses per capita assuming the greatest importance.

Baughman, Martin and Paul Joskow. "The Effects of Fuel Prices on Residential Appliance Choice in the United States," Land Economics, Vol. LI, No. 1, Univ. of Wisconsin, Madison, Wisconsin, February, 1975.

The purpose of this study is to estimate the effects of fuel prices on fuel type choice for four major energy use categories: water heating, space heating, cloths drying, and cooking. The authors find the demand for a particular fuel type is a demand derived from a desire for certain house services. Most of the LR adjustment to a change in relative fuel prices is in the form of an appliance choice switch. That is, the choice of a certain fuel type is embodied in the choice of an appliance portfolio.

The model can be briefly summarized as follows:

$$P_i = \sum_{j=1}^{n} \frac{e^{I(x_i)}}{e^{I(x_j)}}$$

WHERE P_i = the probability of choice of technique i (electricity, gas, oil, etc.) and $I(x_i)$ = an index function of preference defined over a vector of characteristics of technique i, such as capital costs, fuel costs, convenience, etc. The above probability model comes out of the assumption that the joint probability distribution function related to all techniques is a Weibull Distribution. It is assumed further that $I(x_i)$ is of a linear form, e.g. $I(x_i) = b_1 + b_2C_i + b_3H_i + b_4A_i$, where C_i = effective cost of fuel i, H_i = capital cost of burning fuel via technique i, and A_i = an index of convenience associated with i.

The approach taken is the estimation of relative probabilities in a log-odds fashion. Let e = electricity, g = gas, then the relative probability of choice of electricity to that of gas is given by $P_e/P_g = e^{I(x_i)}/e^{I(x_g)}$ so that $\ln(P_e/P_g) = I(x_e) - I(x_g) = b_2(C_e - C_g) + b_3(H_e - H_g) + b_4(A_e - A_g)$.

Given frequency data across a geographic aggregate on the incidence of use of various fuel types and for different end uses, and data on area wide costs, the model can be estimated by ordinary regression techniques.

In each case the authors attempt to explain the relative proportions of residential customers choosing a particular fuel for a particular end use service, as a linear function of fuel prices and household incomes. The authors assume that to the extent that such immeasurable variables as convenience, cleanliness, etc. enter into a households' appliance preference function, that the importance of these is correlated with household income. Hence, income is essentially used as a proxy for the A_i above. Finally, the authors find that the capital cost differential variable is essentially invariant across the geographic observations. Thus the term $b(C_i - C_j)$ is reduced to a constant.

For house heating, the fuel type alternatives are oil, gas, and electricity. The proportion of households choosing a given technique is determined as a linear function of fuel price, household income, and minimum winter temperature. The latter variable serves as a proxy for the appliance utilization rate.

The house heating model is specified as follows:

$$\ln(P_g/P_e) = a_0 + b_{11}F_e + b_{12}F_g + b_{13}Y + b_{14}TEMP$$

$$\ln(P_o/P_e) = a_1 + b_{21}F_e + b_{22}F_o + b_{23}Y + b_{24}TEMP$$

The cloths drying, water heating, and cooking models are similar.

A cross section of 48 states for 1969 (Census) is used. Gas price is measured as a typical bill per 100 therms. Electricity price is measured as the difference between a TEB for 1000 kwh and a TEB for 500 kwh per month (1970). Y is effective household buying income per capita. TEMP is mean January temperature.

The authors have the following expectations for results:

(1) a higher TEMP implies lower utilization of heating unit, in turn implying a higher ratio of fixed to variable cost. On this basis, the authors predict that a higher TEMP would tend to favor electricity over gas and oil for heating fuels, as electric space heating is generally expected to include lower relative fixed costs (purchase and installation plus various types of maintenance).

(2) a higher Y should lead to a preference for gas over electricity, and electricity over oil, reflecting perceptions of reliability, cleanliness, etc.

(3) Prices (F_e, F_o, F_g) should work in the obvious direction.

The results are as expected. The Price variables are all significant at the .05 level. TEMP behaves as predicted. Y behaves generally as expected although its significance is low in the water and space heating equations. Both the own- and cross- price elasticities are large (the own elasticities are larger than unity in absolute value in all but two cases out of ten). One can conclude, then, that appliance stock choice is very sensitive to relative fuel prices.

Berg, Staford V. and James P. Herden. "Electricity Price Structures: Efficiency, Equity, and the Composition of Demand," Land Economics, Vol. LII, No. 2, Univ. of Wisconsin, Madison, Wisconsin, May 1976.

The purpose of this study is to examine the declining block structures which characterize many electric utilities, and to explore impact of alternative pricing structures, including life-line (rate inversion). It is found that the effects of changes in the block structure depend on the composition of demand, including the pattern of price elasticities and demand intensities. The investigators are interested in the effects of changing rate structures on (a) the adequacy of utility revenue, (b) the distribution of consumer expenditure, and (c) the pattern of consumer (surplus) benefits.

The study seeks to analyze how varying patterns of demand result in different aggregate consumption and expenditure patterns, as well as in differential distributions of benefits and loses. The study uses a simulation procedure to evaluate the effects of differential demand.

The authors conclude that for any given composition of demand, it is correct to conclude that rate spread reductions implies a more equal sharing of the cost burden. However, when comparing rate spreads between two utility service areas, the differential composition of aggregate demand will imply different rate spreads even if the rate schedules are the same. Thus, welfare is not easily judged from a measure of rate spread alone. The supporters of lifeline rate structures should examine more closely the distribution and allocation effects of such price structures.

APPENDIX B. Household Survey Data

Midwest Research Institute

A survey of 2000 households in 14 SMSA's in 1975 provides detailed appliance holdings, demographic, income, and behavior pattern data. As of 1/13/76, electricity consumption data was available in usable form only for April, 1975. Rate data is available in raw form. Missing data reduces the number of usable observations to approximately 1000. This data set is potentially useful for parameterizing characteristics of consumption frequency distributions in terms of weather and other explanatory variables, and provides a possible source of residential demand function estimate. This data collection was supported by FEA, and has been made available by EPRI, which is currently appending usage data.

Washington Center for Metropolitan Studies

A national survey of 1455 households in May, 1973 provides detailed appliance holdings, demographic, and income data. Monthly consumption of gas and electricity is available for one year. This data collection was supported by FEA. As of 1/13/76, FEA has not released this data for use. Rate data is reported to be available on unreleased tape versions. This data has been previously released for an extensive study of energy consumption (D. Newman and D. Day, The American Energy Consumer, 1975) and a cursory demand analysis by Temple-Barker-Sloan. This data set is potentially useful for estimating residential demand functions, and determining seasonal demand factors as functions of weather and other explanatory variables.

Miracle II

A survey of 12,000 households in 1975 by San Diego Gas & Electric in their service area provides data of monthly consumption of gas and electricity for one year. This data is available from the California State Energy Resources Conservation and Development Commission. This data set is potentially useful for the study of the independence of consumption level and seasonal effects, the effect of income on consumption, and appliance holdings. There is insufficient rate variation to permit detailed study of rate impacts.

LOAD CURVE DATA:

9 states received, 11 expected to provide data

Many studies exist on residential loads, or on category of commercial establishment loads. Only in the past four or five years has the recording equipment existed, however, to meter load continuously (i.e., at 15 minute intervals), and only in the past three years has interpreting equipment allowed the translation of multi-track, multi-meter recordings into usable data bases. Most of the studies that would yield end-use load curves, therefore, are only now delivering data, and the problem now is the sheer volume of information.

For our purposes, the load studies that break system peak days into class contributions are of limited value, but nonetheless that is primarily what we will get from most utilities. Only a few have load research in end-use detail. For the preliminary load curves, we have monthly peak day load curves for residential, coupled with "typical" or average week curves for nine (9) states. With the demonstration project data, we should be able to refine the effects of appliance holdings.

DATA FORM:

Generally graphic, Bonneville will give us punchcards detailing system load in Washington over scattered days in 1975. We will receive graphic load curves for 24 hours of system load on summer peak, winter peak, spring and winter "representative days".

LOAD CURVE DATA:

System peaks, which may be broken down as follows:

R - residential
C/I - commercial/industrial
A - by appliance grouping
L - large industrial
S - small light and power

Alabama: R 1970 12 months
Arkansas: Arkansas Power and Light; R, C/I 1975
California: Pacific Gas and Electric; San Diego Gas and Electricity; R, C/I 1975
Colorado: Just started survey, no new data
Connecticut: Northeast: don't know what's available
Georgia: Georgia Power; R C/I 1975
Maryland: C/I (L) 1970-71
Massachusetts: Boston Edison; some don't know breakdown yet
Michigan: Detroit Edison; system only - may have appliance for water heater experiment
New Hampshire: System 1975
New Jersey: R(monthly), (A) 1970-71, 8 months. R, C/I 1975
New York: Long Island: R, C/I, A 1975 by month
Con Ed: R, C/I (L,S) 1975 by month
North Carolina: Carolina Power and Light: C/I(L) 1971-72, R 1970-71, only system peak for 1975
Ohio: Cleveland Illumination; system
Columbus and Southern; system
Oregon: Partial for N. Oregon, system only
Pennsylvania: Penn. Power; C/I (S,L) 1969-70
Philadelphia; C/I (S) 1970
South Carolina: Carolina Power and Light; C/I (L) 1971-72, R 1970-71
Virginia: Virginia Electric; R(A), C/I(L,S) 1974-75
Washington: Bonneville; R,C,I Public 1975
West Virginia: Virginia Electric; R(A), C/I(L,S) 1974-75
Wisconsin: Wisconsin Electric; R

RESIDENTIAL APPLIANCE SATURATION

Data Applicable to Distributions:

1960,70 Census data - State basis

Gives breakdown on house and water fuel type; number of households with air conditioners (1 room unit, 2 or more room units, central system).

1973,74 Census data - Divisional basis (4 divisions)

Gives space heating (no water heating) types and number of households with air conditioners (room units, central system).

1970 Public Use Tape (LBL - Bureau of Census) - State level only

Can be used to calculate joint frequencies between space and water heating.

1973 WCMS - National

Calculates national joint frequencies.

Preliminary Method for Estimation of 1975 Distributions:

1) Calculate 1970,'73,'74 Divisional Marginal Distributions. Run time series regression (use logits if necessary in final stages - it probably will be necessary to do so for air conditioners data).
2) Assume:
   a) Divisional Water Heater Marginal Rate of Change = Divisional Space Heating Marginal Rate of Change
   b) Each state has its Divisional Rate of Change
3) Project to 1975.
4) Fit joints from 1973, 1970 data marginals.

Other Data Possibilities:

General

1) State Energy Data Base

   From FEA (Ken Vaght's office), it has a time series of consumption data 1960-75, by state, for the following sectors: residential, industrial (manufacturing, construction, and mining), agriculture, transportation, and commercial (everything else). This is being sent (hard copy), but we have not seen it or the documentation for it, so there's no way to know whether we can use it yet. Mostly quantity data, but there may be some price data for certain fuels/electricity.

2) AHAM (Association Home Appliance Manufacturers)

   Sales figures by state for room air conditioners in 1975. Seeking 1970-74. Total factory shipment for same.

3) Census of manufacturing data on number and total value of various types of air conditioning units. Difficult to use.

4) Some data is available on Service Life Expectancy. See Black Book.

5) Construction and other sales information. Some sought, others unknown.

6) AHAM also has internal "Consumer Survey Reports" which are available to member companies which help in financing these studies. They supposedly contain saturation data. We are currently seeking these with the aid of Teknekron.

Commercial

For 1967, 1972 -

1) Census of Retail Trade

Number of establishments, total sales, total payroll, number of employees for 50 states (each) and the United States (total).

2) Census of Wholesale Trade

Same as above.

3) Census of Selected Services

Same as above

4) Census of Construction Industries

Same as above.

5) Census of Governments

We have not yet coded anything, but can code number of employees and payroll.

6) Census of Agriculture

We have not coded anything; since this census is taken at different years than the other economic census, the data may be incompatible. We may look for non-census sources on value added, etc., but for the moment, this is the weak spot of the commercial sector.

For 1974 -

7) Faucett Report, Energy Consumption in Commercial Industries by Census Division - 1974

An FEA study which pieces together total consumption and end use splits for each commercial "industry" (retail trade, et al) for 9 census regions and the United States. Construction and agriculture are excluded. Also, the quality of estimates are questionable, particularly the end use splits.

For 1974 -

7) (continued)

At any rate, the results are not compatible with Census/EEI energy data in the sense that sectoral total are not correct (i.e. commercial + industrial + residential $>$ total electricity). However, we should be able to use data to find proportions of total energy used per "industry" in the commercial sector. No price information here. We will have to use TEB's and GAS FACTS for commercial sector.

Industrial

1) Census of Manufacturing

For 1967, 1972 we have number of firms, value added, number of production workers, and total wages, per SIC code, states and the United States. For 1967, 1974 we will get on tape (from Census Bureau) fuels and electric energy consumed, costs and quantities by fuel type, for SIC's, states and the United States. For 1971, 1972 we have quantities and costs for electricity only.

2) Census of Mineral Industries

For 1963, 1967, 1972 we can get number of firms, value added, number of production workers, total wages for mining (and 2 digit SIC) sector by state. For 1972, 1963 we can get fuel cost and quantity breakdown (as in Manufacturing), but for 1967 we can only get quantities, costs for electricity.

APPENDIX C. Peakload Pricing Data and Other Data Sources

Data on peakload pricing and demand response is available for residential customers from FEA sponsored electric demonstration experiments, and for industrial customers from utility load management studies.

Residential Peakload Pricing

FEA has indicated that demonstration data from Connecticut, Arizona, and Arkansas will be available in early January, 1977. Connecticut and Arizona data are already in the hands of some users. As of 1/13/77, this data or detailed descriptions have not been received. The data sets are described in Table C.1, taken from FEA/C&E/OUP.

Industrial Peakload Pricing

Data on peakload pricing for industrial customers has proven to be generally unavailable except for selected California load studies. Some assessment of industrial load response to peakload prices can be made by comparing utilities with different time-of-day or seasonal rate structures, using primarily load studies done for the Load Research committee of AEIC and released by individual utilities. This data has severe problems of comparability, in terms of industrial mix, date, and data collected.

ELECTRIC DEMO CHARACTERISTICS

STATE	G	CLASS	PARTI-CIPATION Vol	Man	ADJUST FACTOR	RATE	CHARGE[1/] c/Kwh Peak	Shoulder	Off-Peak	RATIO	RATE MAKING METHOD	HOURS Peak	Shoulder	Off-Peak	DATA COLL	TIME INTV COLL[1]	[2/] QUESTIONNAIRE Pre	Int	Post	CUST STRAT	CON-STANTS
ARIZONA	E	R_1	5		No	TOD	16	5	3	$5.3/_1$	Exper. Design	1400-1700	0900-1400 1700-2200	2200-0900	6/76 10/76	Month	Yes	No	Yes	KWH	N/A
	E	R_2	5		No	TOD	15	4	2	$7.5/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_3	5		No	TOD	15	7	4	$3.8/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_4	5		No	TOD	14	4	2	$7/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_5	5		NO	TOD	14	6	4	$3.5/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_6	5		No	TOD	13	3	3	$4.3/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_7	5		No	TOD	13	4	2	$6.5/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_8	5		No	TOD	13	7	3	$4.3/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_9	5		No	TOD	12	5	1	$12/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{10}	5		No	TOD	12	6	3	$4/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{11}	5		No	TOD	11	4	2	$5.5/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{12}	5		No	TOD	11	7	4	$2.8/_1$	"	"	"	"	"	"	"	"	"	"	"

1/ Customers' monthly bills are reduced by an amount which depends on how they react to these pricing schedules. However, in no case would their bill exceed what it would be under the standard pricing schedule in effect at the time.

2/ Approximately 35 consumers' consumption will be recorded every 15 minutes.

Table C.1, p. 1

ELECTRIC DEMO CHARACTERISTICS

STATE	G	CLASS	PARTICIPATION Vol	PARTICIPATION Man	ADJUST FACTOR	RATE	CHARGE ¢/KWH Peak	CHARGE Shoulder	CHARGE Off-Peak	RATIO	RATE MAKING METHOD	HOURS Peak	HOURS Shoulder	HOURS Off-Peak	DATA COLL	TIME INTV COLL	QUESTIONNAIRE Pre	QUESTIONNAIRE Int	QUESTIONNAIRE Post	CUST STRAT	CONSTANTS
ARIZONA	E	R_{13}	5		No	TOD	10	4	1	$10/_1$	Exper. Design	1400-1700	0900-1400 1700-2200	2200-0900	6/76 10/76	Month	Yes	No	Yes	KWH	N/A
	E	R_{14}	5		No	TOD	10	6	3	$3.3/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{15}	5		No	TOD	9	5	2	$4.5/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{16}	5		No	TOD	8	4	1	$8/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{17}	5		No	TOD	12	4	2	$6/_1$	"	1400-1900	0900-1400 1900-2200	2200-0900	"	"	"	"	"	"	"
	E	R_{18}	5		No	TOD	12	6	4	$3/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{19}	5		No	TOD	11	4	2	$5.5/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{20}	5		No	TOD	10	6	3	$3.3/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{21}	5		No	TOD	9	4	2	$4.5/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{22}	5		No	TOD	6	4	1	$6/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{23}	5		No	TOD	9	4	2	$4.5/_1$	"	1400-2200	0900-1400	2200-0900	"	"	"	"	"	"	"
	E	R_{24}	5		No	TOD	8	3	2	$4/_1$	"	"	"	"	"	"	"	"	"	"	"

Table C.1, p. 2

STATE	G	CLASS	PARTICIPATION Vol	PARTICIPATION Man	ADJUST FACTOR	RATE	CHARGE Peak	CHARGE Shoulder	CHARGE Off-Peak	RATIO	RATE MAKING METHOD	HOURS Peak	HOURS Shoulder	HOURS Off-Peak	DATA COLL	TIME INTV COLL	QUESTIONNAIRE Pre	QUESTIONNAIRE Int	QUESTIONNAIRE Post	CUST STRAT	CONSTANTS
ARIZONA	E	R_{25}	5		No	TOD	7	4	2	$3.5/_1$	Exper. Design	1400-2200	0900-1400	2200-0900	6/76 10/76	Month	Yes	No	Yes	KWH	N/A
	E	R_{26}	5		No	TOD	6	3	1	$6/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{27}	5		No	TOD	5	4	3	$1.6/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{28}	5		No	TOD	4	3	1	$4/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{29}	5		No	TOD	13	5	2	$6.5/_1$	"	1400-1700	0900-1400 1700-2200	2200-0900	"	"	"	"	"	"	"
	E	R_{30}	6		No	TOD	9	5	2	$4.5/_1$	"	"	"	"	"	"	"	"	"	"	"
	E	R_{31}	5		No	TOD	13	3	3	4.3/ 1	"	"	"	"	"	"	"	"	"	"	"
	E	R_{32}	8		No	TOD	7	5	2	3.5/ 1	"	"	"	"	"	"	"	"	"	"	"
	C	R_{33}	10		No	DB	85% of standard rate in effect in summer '75				"	"	"	"	"	"	"	"	"	"	"
ARKANSAS	E	R		101		TOD	$3./Mo. Cust. Chg. 5.45¢/KWH (S) P 1.30¢/KWH (S) OP					11:00- 19:00		19:00- 11:00	12/75 12/76						
							1.23¢/KWH (W) P 1.17¢/KWH (W) OP					"		"							
	E	R		121		F #1	$3.7/Mo Cust. Chg. 4.14¢/KWH (S) [illegible]					"		"	12/75 12/76						

STATE	G	CLASS	PARTICIPATION Vol	PARTICIPATION Man	ADJUST FACTOR	RATE	CHARGE	RATIO	RATE MAKING METHOD	HOURS Peak	HOURS Shoulder	HOURS Off-Peak	DATA COLL	TIME INTV COLL	QUESTIONNAIRE Pre	QUESTIONNAIRE Int	QUESTIONNAIRE Post	CUST STRAT	CONSTANTS
ARKANSAS	E	R		811		F #2	\$3./Mo Cust. Chg. 3.53¢/KWH (S) 1.79¢/KWH (W)						12/75 12/76					L KWH	
	C	R		129		TOD	3.00 cust. chg. 1.24c/kwh(w)P 8.45c/kwh (s) P 1.17c/kwh(w)OP 1.39c/kwh (s) OP	6:1(s) 1.1(w)					12/75 12/76	15 min.				"	
	C	R		115		F#1	3.00 cust. chg 4.14/Kwh (s) 1.17/Kwh (w)		(CO)				12/75 12/76					"	
						F#2	3.00 cust. chg 3.53/Kwh(s) 1.75/Kwh(w)						12/75 12/76					"	
	E	I&C		160		TOD	\$3.Cust. Chg. \$1.93KW BD (W) \$1.95KW BD (S) 1.63¢/KWH(W)P 1.65¢/KWH(S) P 1.55¢/KWH(W)OP						2/76 2/77					"	
							1.84¢/KWH(S) OP						2/76 2/77					"	
	E	I&C		225		F #1	\$3.Cust. Chg. \$2.03KW BD (W) \$6.18 KW BD (S) 1.58¢/KWH(W) 2.18¢/KWH(S)						2/76 2/77					"	
	E	I&C		357		F #2	\$3.Cust. Chg. \$2.96KW BD (W) \$4.65KW BD (S) 1.05¢/KWH(W) 2.88¢/KWH(S)						2/76 2/77					"	
	C	I&C		22		TOD							2/76 2/77					"	

STATE	G	CLASS	PARTICIPATION Vol	PARTICIPATION Man	ADJUST FACTOR	RATE	CHARGE c/KWH	RATIO	RATE MAKING METHOD	HOURS Peak	HOURS Shoulder	HOURS Off-Peak	DATA COLL	TIME INTV COLL	QUESTIONNAIRE Pre	QUESTIONNAIRE Int	QUESTIONNAIRE Post	CUST STRAT	CONSTANTS
CONNECTICUT	E	R					Peak 16¢ Shoulder 3¢ Off-Peak 1¢	16/1	Exper. design	W 09-11 17-	08-09 11-17 19-21	21-08	10/75	15 min.	8-9/75	-	10-11/75	K	K
										S 10-12 13-15	08-10 12-13 15-21	21-08	to 10/76		10-11/75			K	K
	C	K	200			DB	0-12 \$2.27 13-100 6.25¢ 101-300 5.05¢ 301-600 3.54¢ 600+ 3.24¢												

Table C.1, p. 4

APPENDIX D

Consumption Frequency Distribution Data

Data on consumption frequency distributions has been promised by selected utilities in twenty states. Collection practices vary by utility, making reduction of raw data to common form difficult. Aggregation of distributions within states will be required. Table D.1 lists the sources of distribution data.

(Text continues, p. D-7.)

TABLE D.1 Consumption Frequency Distribution (CFD) Data

ARKANSAS	Expect CFD annual 1975, with montly KWH averages, for residential general service commercial Cost: Free
CALIFORNIA	Have Pacific Gas and Electric CFD annual 1975 for a) all residential classes consolidated b) D-2 (representative of 4 of 5 R rates) PG&E CFD month 1975 for D-2 Cost: Free
COLORADO	Expect CFD annual 1975 for residential for Denver, E. Colorado. May generate CFD annual 1975 for other classes. Cost: Free
CONNECTICUT	Expect CFD annual 1975 for central CT (Hartford)
GEORGIA	Expect CFD annual 1975 for most of state. Only 4 classes.
MAINE	Expect CFD annual, CFD monthly 1975 for Boston, all classes can get as detailed CFD as we want for all classes by sending someone to their office.
MICHIGAN	Expect CFD annual 1975 for res; maybe not other classes for Detroit; need more of state.
NEW JERSEY	Only store CFD one month; may have CFD annual 1975, don't know if available. This for central New Jersey, all classes.
NEW YORK	Expect CFD annual 1975 for residential from CONED--"not representative of state," says PUC, "since so many apartments" Expect CFD annual 1975, maybe CFD month 1975 from Long Island Lighting for residential

NORTH CAROLINA	Expect CFD annual 1975, CFD month 1975 from Utilities Commission for Carolina Power, Virginia Power, Duke Power (1974) (Cost: May cost approximately $40.00)
OHIO	Expect CFD annual 1975 for Columbus, Southern, Cleveland Electric, however, may not have time to produce them for us. Will know next week.
OREGON	Expect CFD annual 1975 for all public power; checking to see if monthly data exists. Doesn't cover Portland, but most of rest of state.
PENNSYLVANIA	Don't know what to expect--letter not sent yet.
VIRGINIA	Expect CFD month from April 1975, when converted to monthly billing. Residential only.
WASHINGTON	Expect public power (20% of Seattle--omits Spokane) CFD annual. No monthly. Probably residential only.
WISCONSIN	Expect Madison CFD annual 1975--hope for breakdown of controlled WH, regular WH customers.

In general, CFD's are used for forecasting revenues, are preapred monthly (often not in a readily available format) and are not available except in annual summary form. PG&E is exceptional, a s is Boston, Long Island, North Carolina, in having printout easily available. Most say they may have to do a special run. We really won't know what they understood over the phone until January 20th.

*One vagary to be aware of--since the months are billing months, the bills cover KWH amount consumed over as many as sixty days; as if this isn't enough, often the books are closed several days early, particularly in December, so late-month billing days are carried over in the next month's accumulation.

When the CFD's arrive, we face the problem of accumulating the distributions over rate classes; we asked for separate distributions by class, so that we might see different patterns by end use. In some states the rate class is determined by geographical location, and in others by end-use or appliance holding, so the meaning of different CFD shapes is obscured.

The use of CFD shapes from one state as representative of the shapes of neighboring states is necessary, but must be justified when rate structures differ significantly in these ways.

FEA may assist us with CFD data by providing us with demonstration project tapes as soon as possible, so that we may derive CFD by user group, and apply that derived breakdown to CFD by class we receive from utilities.

Consumption Frequency Distributions: Standard Codes for Census Division

(Underlined codes indicate desirable state to survey)

AL Alabama

AK Alaska

AZ Arizona

AR Arkansas (R, C/I: Ark Power & Light)

CA California (R, C/I: PG&E)

CO Colorado (R; maybe C/I: Co. Pub. Service)

CT Connecticut (R; maybe C/I: Northeast Utility)

DE Delaware

DC Washington, DC

FL Florida

GA Georgia (R, C/I: Georgia Power)

HI Hawaii

ID Idaho

IL Illinois

IN Indiana

IA Iowa

KS Kansas

KY Kentucky

LA Louisiana

ME Maine

MD Maryland

MA Massachusetts (R, C/I: Boston Edison)

MI Michigan (R: Detroit Edison)

MN Minnesota

MS Mississippi

MO Missouri

MT Montana

NE Nebraska

NV Nevada

NH New Hampshire (Not available)

NJ New Jersey (R, C/I: Public Service)

NM New Mexico

NY New York (R, C/I: ConEd, Long Island)

NC North Carolina (R, C/I: Util. Commission, Virgina Power, Carolina Power and Light)

ND North Dakota

OH Ohio (R, C/I: Columbus and Southern, Cleveland Electric Illuminating)

OK Oklahoma

OR Oregon (R, annual: Bonneville)

PA Pennsylvania (?: Pennsylvania Power and Light)

RI Rhode Island

SC South Carolina

TE Tennessee

TX Texas

UT Utah

VT Vermont

VA Virginia (R, C/I: Virginia Electric)

WA Washington (R (annual): Bonneville, Snohomish)

WV West Virginia

WI Wisconsin (R, C/I: Wisconsin Electric)

WY Wyoming

Base State Consumption Frequency Distributions

$M \equiv$ number of rate zones

$N_i \equiv$ number of customers in rate zone i

$N_i(c) \equiv$ total bill for consumption at level c in rate zone i

$f_i(c) \equiv$ density of consumption (for fixed time period) in rate zone i

$\overline{x} \equiv$ state average demand (i.e., consumption)

$\overline{r} \equiv$ state average revenue = total revenue/number of customers in state

Assume all customers have consumption > 0 .

$$\text{State consumption density} \equiv f(c) = \frac{\sum_{i=1}^{M} N_i f_i(c)}{\sum_{i=1}^{M} N_i}$$

We want $r(c)$ ($\equiv$ state rate schedule) such that

$\overline{r} = \int_0^\infty r(c) f(c) dc$ for all possible $f(c)$. By definition

$$\overline{r} = \frac{\sum_{i=1}^{M} N_i \int_0^\infty r_i(c) f_i(c) dc}{\sum_1^M N_i}$$

$$= \int_0^\infty \frac{\sum_1^M N_i r_i(c) f_i(c)}{\sum_1^M N_i} \times \frac{\sum_1^M N_i f_i(c)}{\sum_1^M N_i f_i(c)} dc$$

$$= \int_0^\infty \frac{\sum_1^M N_i r_i(c) f_i(c)}{\sum_1^M N_i f_i(c)} f(c) dc$$

Therefore, $r(x) = \dfrac{\sum_1^M N_i r_{i(c)} f_i(c)}{\sum_1^M N_i f_i(c)}$.

Approximate $f(c) = \dfrac{c^{\alpha-1} e^{-c/\beta}}{\beta^{\alpha}\Gamma(\alpha)} = f_{\alpha}(c)$

$$r(x) \approx a + bx^c$$

then

$$\bar{r} = a + b\int_0^{\infty} \frac{x^{\alpha+c-1} e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)}\, dx$$

$$= a + \frac{b\Gamma(\alpha + c)\beta^c}{\Gamma(\alpha)} \int_0^{\infty} \frac{x^{\alpha+c-1} e^{-x/\beta}}{\beta^{\alpha+c}\Gamma(\alpha + c)}\, dx$$

$$\bar{r} = a + \frac{b\beta^c\Gamma(\alpha + c)}{\Gamma(\alpha)}$$

$$\bar{x} = \int_0^{\infty} \frac{x^{\alpha} e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)}\, dx = \alpha\beta$$

$$\Longrightarrow \cancel{\beta = \bar{x}/\alpha}$$

Therefore, $\dfrac{\bar{r} - a}{b(\bar{x})^c} = \dfrac{\Gamma(\alpha + c)}{\alpha^c\Gamma(\alpha)}$

if $c = 1$ then since $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$

LHS = 1

if $r(x) = a_i + b_i x$ on $[d_i, d_{i+1})$ $\quad i = 1,\ldots,I$; $d_1 = 0$

then

$$\bar{r} = \sum_{i=1}^{I}\left(a_i\int_{d_i}^{d_{i+1}} f_{\alpha}(x)\,dx + \frac{b_i\alpha^2}{\bar{x}}\int_{d_i}^{d_{i+1}} f_{\alpha+1}(x)\,dx\right)$$

Relation of Various Consumption Frequency Distributions

The monthly consumption levels for a randomly selected customer are given by random vector $K = (K_1,\ldots, K_{12})$, where K_m is the KWH consumption in month m. are interested in (1) the distribution of consumption in a given month, (2) the stribution of annual consumption, and (3) the distribution of the mixtures of l consumption levels across months.

A simple relationship is obtained if the following assumption is valid: $= (K_1,\ldots, K_{12})$ is distributed with $K_m = \mu_m X$, where μ_m is a monthly scale actor and X is a random variable which is common to all months.

This assumption implies that if a customer is at the .8 probability level n January, then he is at the .8 probability level in every month. This rules ut, for example, the presence of "winter peakers" and "summer peakers" within he same user type.

Let F(x) be the cumulative distribution function of X. Then,

$$\text{Prob } [K_m \leq k] = \text{Prob } [X \leq k/\mu_m] = F(k/\mu_m).$$

Define $\bar{K} = \frac{1}{12}\sum_{m=1}^{12} K_m = \bar{\mu} \cdot X$, where $\bar{\mu} = \frac{1}{12}\sum_{m=1}^{\mu}\mu_m$.

hen,

$$\text{Prob } [\bar{K} \leq k] = \text{Prob } [X \leq k/\bar{\mu}] = F(k/\bar{\mu}).$$

Next consider the distribution of the mixture of consumption levels across nths. Let K* denote a consumption level obtained by drawing a customer and a nth at random. Then,

$$\text{Prob } [K^* \leq k] = \sum_{m=1}^{12} \text{Prob } [K_m = k/m \text{ chosen}] \cdot \text{Prob}[m]$$

$$= \sum_{m=1}^{12} \text{Prob } [K_m \leq k] \cdot \text{Prob}[m]$$

$$= \sum_{m=1}^{12} F(k/\mu_m)/12.$$

Given a parametric form for F, empirical information from any of the three pes of consumption frequency distributions can be utilized to determine the para- ters, provided the scale factors μ_m are known.

Case 1. Assume X to have the gamma density $f(x) = x^{\alpha} e^{-\alpha x}/\alpha^{-\alpha}\Gamma(\alpha)$, which has mean one and variance $1/\alpha$. Then μ_m is monthly mean consumption, and one empirical dispersion measure is sufficient to calculate α. For example,

$$\text{Prob } [\bar{K} \leq k] = \int_0^{k/\bar{\mu}} x^{\alpha-1} e^{-\alpha x} dx/\alpha^{-\alpha}\Gamma(\alpha)$$

$$= \int_0^{\alpha k/\bar{\mu}} y^{\alpha-1} e^{-\alpha} dy/\Gamma(\alpha)$$

$$= \Gamma(\alpha k/\bar{\mu};\alpha),$$

where $\Gamma(p;\alpha)$ is the incomplete standard gamma with parameter α. Given a fractile of the empirical distribution of $\bar{K}$, and a sub-routine for the incomplete gamma, this formula can be iterated to determine α.

A similar calculation can be carried out from data on the K* distribution,

$$\text{Prob } [K^* \leq k] = \frac{1}{12}\sum_{m=1}^{\mu} \Gamma(\alpha k/\mu_m;\alpha).$$

The following cases can be worked out if they are needed:

Case 2. log X has a gamma density

Case 3. log X has a normal density

Case 4. log log X has a normal density

Case 5. log X has a logistic density

PG&E appears to have the case 2 shape; however, Case 1 may also give a reasonable fit.

An Alternative to the Gamma Approximation to Consumption Frequency Distributions

Consider the cumulative distribution function

$$F(x;\alpha,\beta) = \exp(-\alpha/x^{\beta}) \qquad x,\alpha,\beta > 0 \quad ,$$

with frequency function

$$f(x;\alpha,\beta) = \alpha\beta x^{-\beta-1}e^{-\alpha/x^{\beta}} \quad .$$

This frequency function is bell-shaped and skewed to the right, with a mode at

$$x_{mode} = (\alpha\beta/(\beta + 1))^{\beta} \quad .$$

It has finite moments of order $k < \beta$, with

$$\begin{aligned} Ex^{k} &= \int_{0}^{\infty} \alpha\beta x^{k-\beta-1}e^{-\alpha/x^{\beta}}dx \\ &= \alpha^{k/\beta}\int_{0}^{\infty} y^{-k/\beta}e^{-\alpha y}dy\alpha^{1-k/\beta} \\ &= \alpha^{k/\beta}\Gamma(1 - (k/\beta)) \quad . \end{aligned}$$

The advantage of this distribution over the gamma is that it permits fitting of the distribution to empirical fractiles without extensive iteration.

1. Suppose the distribution is to be fit to two fractiles, x_{θ} and x_{γ} ; i.e., $F(x_{\theta};\alpha,\beta) = \theta$. (One might take $\theta = .75$ and $\gamma = .25$, for examples.) Then, $\theta = \exp(-\alpha/x_{\theta}^{\beta})$ implies $\alpha = x_{\theta}^{\beta} \log(1/\theta) = x_{\gamma}^{\beta} \log(1/\gamma)$. Hence, $\beta = \log(\log\gamma/\log\theta)/\log x_{\theta}/x_{\gamma}$. Given β , α is obtained from the preceding equation. A check on the accuracy of the approximation is obtained by computing $Ex = \alpha^{1/\beta}\Gamma(1 - (1/\beta)) = x_{\theta}(\log(1/\theta))^{1/\beta}\Gamma(1 - (1/\beta))$, and comparing it with the empirical mean $\overline{x}$.

2. Alternately, suppose the distribution is to be fit to the empirical mean $\overline{x}$ and variance $\overline{\sigma}^2$. Then,

$$\overline{x} = \alpha^{1/\beta}\Gamma(1 - (1/\beta))$$

$$\overline{\sigma}^2 + \overline{x}^2 = \alpha^{2/\beta}\Gamma(1 - (2/\beta))$$

or

$$\begin{aligned} 1 + \overline{\sigma}^2/\overline{x}^2 &= \Gamma(1 - (2/\beta))/\Gamma(1 - (1/\beta))^2 \\ &= 1/(1 - (2/\beta))B(1 - (1/\beta), 1 - (1/\beta)) \quad , \end{aligned}$$

where B is the beta function. The right-hand-side of this expression approaches one as $\beta \to +\infty$, and approaches $+\infty$ as $\beta \to 2$. Hence, a solution always exists. I have not determined if the right-hand-side is monotone--if not, the largest solution β , which minimizes the upper tail, should give the best overall approximation to the empirical distribution. A library Γ function will be needed in iterating to a solution.

3. Finally, suppose the fit is to the empirical mean $\overline{x}$ and a fractile x_θ. Then $\alpha = x_\theta^{\beta} \log (1/\theta)$ and

$$\overline{x}/x_\theta = (\log (1/\theta))^{1/\beta}\Gamma(1 - (1/\beta)) \quad .$$

The existence of a solution to this equation is not guaranteed. For $\theta > .3679$, the right-hand-side is increasing as $\beta \to 1$ from above, and approaches $+\infty$. It is also increasing as $\beta \to +\infty$, and approaches 1. Hence, there is a minimum value of the right-hand-side, less than one, for some β between one and $+\infty$. For consistency, $\overline{x}/x_\theta$ must exceed this minimum value. There will be multiple solutions if any exist; the largest β solution should be chosen. Computationally, locate the minimum of the right-hand-side, check for consistency, then increase β until a solution is obtained.

For $\theta \leq .3679$, the right-hand-side approaches $+\infty$ as $\beta \to 1$, and is decreasing, with a limit of 1, as $\beta \to +\infty$. Hence, a solution exists if $x_\theta < \overline{x}$.

4. In fitting empirical consumption frequency distributions, I propose using Method 1 to obtain an initial $\hat{\beta}_1$ estimate, employing the .25 and .75 fractiles. If $\hat{\beta}_1 > 1$, the computed mean can be compared with the empirical mean to determine direction of adjustment required (i.e., $\overline{x} > Ex$ implies β must be lowered). Then use Method 3 with the .25 fractile and iterate to determine a revised β estimate, starting from $\hat{\beta}_1$.

APPENDIX E, Part 1: Computer Software Design for the Demand Module

Interface Between the Demand Module and the Rest of the FEA Simulation Model

Top Level Interface to Demand Module

The following is a preliminary description of the interface between the demand module and the regulatory module for the FEA simulations. Your comments and suggestions are invited.

The demand module will have four main entry points (subroutines) as follows:

I. Intialization

CALL DMINIT (STATE, YEAR, CUSCLS)

Inputs: STATE - Integer type state code variable.
YEAR - Integer type year code variable.
CUSCLS - Integer type customer class code variable.

Outputs: none

Function: Requests that the module be intialized for a particular state, year, and customer class. Routine should be called once at the beginning of each set of independent revenue/load curve calculations.

II. Definition of Rate Structures

CALL DMRATE (NOSCHD, SCHDSC, SCHEDS)

Inputs: NOSCHD - Integer type number of rate schedule variables.
SCHDSC - Integer type array of rate schedule characteristics and dimensions.
SCHEDS - Real type array containing customer, charge, minimum bill, demand blocks and rates, and time-of-day/seasonal surcharges for each rate schedule.

Outputs: none

Function: Passes basic rate structure information to the demand module. Rates are pre-processed into a form more useful for internal calculations, but revenues and load curves are not computed.

. Calculation of Revenues and Peak from Scaled Rate Schedules

CALL DMRVPK (SCALEF, DEMAND, REVNUE, PEAK)

Input: SCALEF - Real type arry of scale factors, one for each rate schedule last entered by DMRATE.

Outputs: DEMAND - Real type variable for total demand for the current class of customer in KWH,

REVNUE - Real type variable for total revenues for the current class of customer in 1975 dollars.

PEAK - Real type variable for annual peak KW load for the current class of customer.

Function: After applying the scale factors to the rate schedules previously entered, this subroutine uses the demand models to estimate total revenue and annual peak. The subroutine is the "heart" of the demand module.

/. Updating of Internal Demand Records

CALL DMFINL (LOADCR)

Input: none

Output: LOADCR - Multi-dimensional real type array containing load in KW at specified times during the year.

Function: A call to this subroutime fixes the rate schedules, revenues, consumption distributions, load curves, and other,internal variables in the demand modules at the values determined by the last call to DMRVPK. These values form the basis for the simulation of the following year's demand. The final load curve is also returned as a parameter.

FLOW CHART FOR OVERALL DEMAND MODULE OPERATION

The operation of the demand module within the FEA simulation model is depicted in Figure 1. In general, the procedure blocks (rectangles) are part of the demand module itself, but all of the decision blocks (diamonds) are external to the demand module. Decision blocks 7, 8, 9, and 10 would probably be incorporated into the regulatory module, while blocks 12 and 13 would probably form part of the main model driving routine.

Procedure blocks 1,2, 3, 4, 5, 6, and 11 will be implemented by means of the four subroutines described in the attached documentation on the top level inter--

face to the demand module. Intialization (1, 2, 3, and 4) will be carried out in DMINIT. A trial rate schedule structure will be passed to the demand module via DMRATE (block 5) and the corresponding revenues and peak will be retrieved through DMRVPK (block 6). If revenue requirements are not met, the rate schedules can be scaled uniformly; then revenue and peak can be computed again with DMRVPK without altering the basic structure of the rates (block 6). After the final price levels have been set, the final load curve can be retrieved with DMFINL; internal module variable records are also updated in this step (block 11).

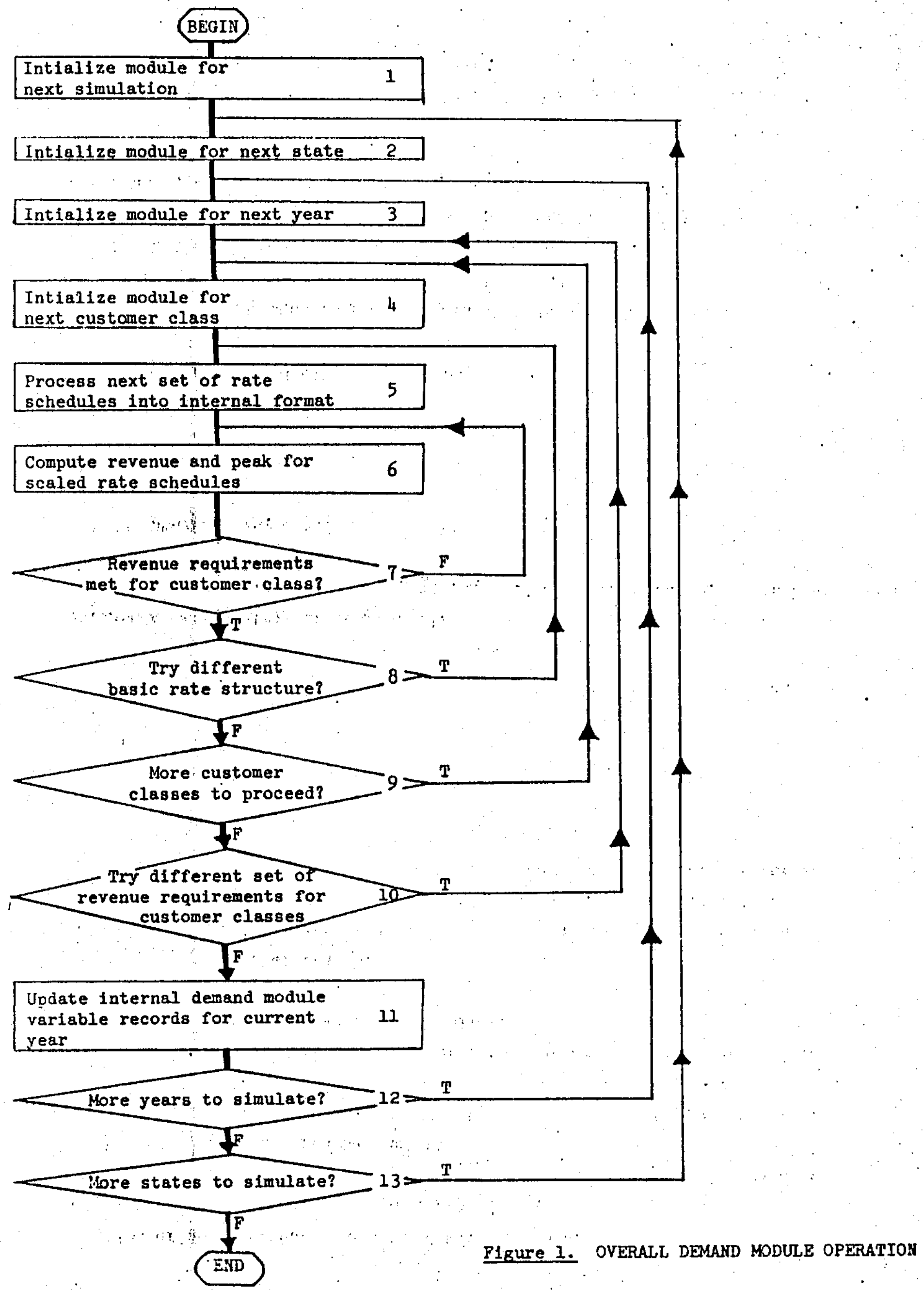


Figure 1. OVERALL DEMAND MODULE OPERATION

Structure of Rate Schedules

Rate schedules are passed to the demand module via subroutine DMRATE. This subroutine has three parameters, as previously described. The first is a simple integer variable NOSCHD indicating the number of different rates for which information is provided. The second is an integer array SCHDSC that describes the customer class and end uses to which each schedule applies, as well as the type of charges included in the schedule. Finally, the third is a long array SCHEDS that contains the actual rate information. If more than one rate schedule is supplied, then the last two parameters contain the arrays for all the schedules stacked end-to-end. Thus, it suffices to describe the structure of the second and third parameters for a single schedule.

The second parameter SCHDSC comprises the following six simple integer variables:

(1) customer class code (see next section)
(2) end use description (see next section)
(3) number of demand blocks (may be equal to 0)
(4) number of energy blocks (must be GT 0)
(5) type of energy charge flag: 1, if energy charges are per KW demand;
0, otherwise
(6) peak-load pricing flag: 1, if peak-load surcharges are to be included;
0, otherwise

If there are NOSCHD rate schedules, then there will be a two-dimensional, 6 by NOSCHD array.

The third parameter SCHEDS has the following structure (all type real variables):

(1) minimum bill per month in dollars
(2) customer charge in dollars
(3) vector giving the lower boundary in KW of each demand charge block starting with 0 KW. (Present only if SCHDSC (3) is greater than 0)
(4) vector giving the corresponding demand charge in dollars per KW above each lower block boundary (Present only if SCHDSC (3) is greater than 0)
(5) vector giving the lower boundary in KWH of each energy charge block
(6) vector giving the corresponding energy charge in dollars per KWH above each lower block boundary
(7) array giving the peak-load surcharge factors for seasonal, weekly and time-of-day pricing. For a description of the usage of this array, see D. McFadden's memo of 12/20/76 (array PEAKF). Its structure should correspond to that of LOADCR (see later section). (Present only if SCHDSC(6) is equal to 1.)

This composite vector will have length

L = 2 + 2*SCHDSC(3) + 2*SCHDSC(4) + LCRLEN*SCHDSC(6)

where LCRLEN is the length of a load curve. If there are multiple schedules, this third parameter will be a two-dimensional L by NOSCHD array.

Characterization of Rate Schedules by Customer Class and End Use

The first two elements of the SCHDSC vector characterize completely the appliciability of the rate schedule to particular end uses. The first element is a customer class code as follows:

1 for residential
2 for commercial
3 for industrial

The definitions of these three customer classes will be consistent with the energy totals in the 1975 FPC Form 1 and 1-M tapes.

The second element, an end use descriptor, has a more complex structure. As indicated by D. McFadden on p. 4 of his memo #2 of 12/20/76, several end uses will be considered for each customer class. These are denoted by the following codes:

Residential - G W S A
Commercial - G S A
Industrial - P N

In general, a rate schedule may apply to any combination of these end uses. In the case of residential schedules, one may find special schedules for electric water heating (W) and for all-electric homes (GWSA), as well as a general purpose "catch-all" schedule for all other end use combinations. Using the order given in the list above, each schedule can be described by a binary string, such as 1 1 1 1 for GWSA and 0 1 0 0 for W, with ones in the slots for the special end uses covered and 0's elsewhere. The catch-all schedule would have the code 0 0 0 0 . The end use descriptor included in SCHDSC is simply the decimal equivalent of these binary codes. E.g.,

$15 = 1\,1\,1\,1_2$ for GWSA
$8 = 0\,1\,0\,0_2$ for W
$0 = 0\,0\,0\,0_2$ for "catch-all".

The method for applying such multiple rate schedules to specific user types will be described in a later memo. At that time, the same procedure will be used to accurately describe each user type.

In many cases, it will not be entirely clear which combination of schedules should apply to a particular useer type. Different combinations of rate schedules may cover all of the end uses. Therefore, it is also necessary to rank the rate schedules in order of their applicability. For example, suppose that there exist three schedules for a residential class: general (G), water heating (W), and all-electric (AE). An all-electric home could be covered either by a combination of G and W or by AE alone. To eliminate this ambiguity, the schedules must be ranked in some order from first applicable to last applicable. If, the order were AE, W, G, then the choice for an all-electric home would be AE; if, on the other hand, the schedules were ranked W, AE, G, the choice would be W and G. Insofar as such ambiguities arise, the order of the rate schedules in the arrays SCHDSC and SCHEDS is important. Priorities will be assigned to the different schedules from highest to lowest in the order in which they appear in the arrays.

Although the examples used above relate to the residential customer class, it should be easy to see how the same method can be extended to the other customer classes or to different lists of possible end uses.

Structure of the Load Curves

The structure of the load curves LOADCR returned by subroutine DMFINL will remain fixed throughout a given simulation. The structure of the curves may vary from simulation to simulation as the module develops. It is likely that early versions of the module will provide considerably less detail than the final version.

The structure of the array LOADCR will be completely defined in a common block /LOADDF/. The array will be three dimensional

LOADCR (NDINT, NWINT, NSINT)

and the common block will contain seven variables

```
COMMON /LOADDF/  NDINT, NWINT, NSINT, LCRLEN, LNDINT(NDINT),
                 LNWINT(NWINT), LNSINT(NSINT)
```

follows:

LOADCR - average load in KW within the specified period

NDINT - number of intervals into which a typical day is divided

NWINT - number of intervals into which a typical week is divided

NSINT - number of seasons into which a typical year is divided

LCRLEN - the product NDINT*NWINT*NSINT

LNDINT - the number of hours within each of NDINT intervals in a typical day The first interval starts with 0000 hours.

LNWINT - the number of days within each of NWINT intervals in a typival week The first interval starts with Monday.

LNSINT - the number of weeks within each of NSINT intervals in a typical year The first interval starts with the first Monday in January.

mory Management

In order to achieve maximum flexibilty with minimum memory utilization, the mand module will alaocate memory dynamically for large arrays. Workimg storage ll be provided in blank common, while permanent storage will be allocated in LCM. coordinate memory usage between the demand module and the rest of the program special common block will be used:

COMMON / MEMDMD/ MINBCM, MAXBCM, MINLCM, MAXLCM

ere

MINBCM - minimum
MAXBCM - maximum
index of blank common array to be used by module

MINLCM - minimum
MAXLCM - maximum
index of LCM array to be used by module

e common block /MEMDMD/ must be set by the main simulation module before the rst call to the demand module. In case the space allocated to the demand dule is insufficient, the module will abort the run with an appropriate error ssage.

mpiler Standards

The demand module will be developed with the MNF4 compiler at LBL. Subroutine nkage will follow the FTN convention.

APPENDIX E, Part 2:

A Provisional APL Program for
the Consumption Frequency Distribution
Forecasting Methodology

APPENDIX E, Part 2: A Provisional APL Program for the Consumption Frequency Distribution Forecasting Methodology

```
      ∇ADJCR[⎕]∇
    ∇ R←Z ADJCR AR;N;M;T
[1]   N←2×⍳0.5×⍴AR
[2]   M←1↓N-1
[3]   N←1,N
[4]   →(0=LIFE)/L
[5]   T←+/LIFE≥,AR[M],0
[6]   N←((~ABS)/1),T↑N
[7]   →L1
[8]  L:N←((~ABS)/1),N
[9]  L1:R←AR
[10]  R[N]←Z×R[N]
    ∇

      ∇AKI[⎕]∇                          ⍝SYSTEM FUNCTION
DEFN ERROR
      ∇AKI
         ^

      ∇AKN[⎕]∇
    ∇ R←AKN P
[1]   R←'YN×'=1↑(AKI P),'  '
[2]   →R/ 5 5 4
[3]   →1
[4]   →
[5]   R←R[2]
    ∇

      ∇ARBOUT[⎕]∇                       ⍝SYSTEM FUNCTION
DEFN ERROR
      ∇ARBOUT
           ^
```



```
      ...  K←H;N;K;NN;SN;SNN;I;W
[1]   N←⍴K←,KWH
[2]   NN←⍴R1←,RATE[1;]
[3]   SN←N⍴1
[4]   SNN←NN⍴1
[5]   T←K∘.≥R1
[6]   W←(SN∘.×⍳NN)=(,+/[2] T)∘.×SNN
[7]   MC←+/[2](SN∘.×,RATE[2;])×W
[8]   TC←+/[2](SN∘.×R1)×W
[9]   TC←MC×K-TC
[10]  TC←TC++/[2] T×(SN∘.×,RATE[3;])
[11]  AC←TC÷K⌈1E¯7
    ∇

      ∇CUM[⎕]∇
    ∇ C←CUM F
[1]  ⍝CUMULATES FREQ FUNCTION
[2]   N←⍴,F
[3]   I←1
[4]   C←N⍴0
[5]  L:C[I]←+/I↑F
[6]   I←I+1
[7]   →(I≤N)/L
    ∇

      ∇DINV[⎕]∇
    ∇ Y←RATE DINV A
[1]  ⍝COMPUTES LOG INCOME PTS FOR DEMAND LEVELS A, GIVEN RATES
[2]   RATE COSTCR((⍴A),1)⍴A
[3]   Y←BCOEFF[1]+(BCOEFF[2]×⍟AC)+(BCOEFF[3]×⍟MC÷AC)+BCOEFF[4]×⍟A⌈1E¯6
    ∇

      ∇FRCR[⎕]∇
    ∇ FR←RATE FRCR A
[1]   Y←RATE DINV A
[2]   Y←MONO Y
[3]   FR←Y INTER(2,⍴YB)⍴YB,YF
    ∇

      ∇FRE[⎕]∇
    ∇ F←FRE CC
[1]  ⍝DIFFERENCE CDF
[2]   F←CC-0,¯1↓CC
    ∇
```

```
    ∇FREQUENCY[⎕]∇                                    MAIN DRIVING FUNCTION
  ∇ BILLING FREQUENCY ANALYZER
[1]   L10:'1. ENTER DEMAND LEVELS AT WHICH BILLING FREQUENCY IS RECORDED'
[2]    A←,⎕
[3]   L2:'2. ENTER CUMULATIVE DISTRIBUTION OF BILLS AT BASE RATES'
[4]    BFR←,⎕
[5]    →(((ρBFR)≠ρA)∨(0≥⌊/A)∨(1<⌈/BFR)∨0>⌊/BFR)/ERR
[6]   L3:'3. ENTER BASE RATE SCHEDULE'
[7]    BRATE←,⎕
[8]    RATE←RCR BRATE
[9]    FR←BFR
[10]  L4:TRUEMC←ΔFI AKI 'ENTER BASE MARGINAL COST OF ELECTRICITY '
[11]  L5:RETURN←0.01×ΔFI AKI '5. ENTER BASE RATE OF RETURN (PERCENT) '
[12]   STACK
[13]   KAP←0.5×AREV÷RETURN
[14]   DENSITY←(¯1↓FF)÷/-0,¯1↓A
[15]   BKWH←KWH
[16]   BAREV←AREV
[17]   BRPK←RPK
[18]   4 1 ρ' '
[19]   'STATISTICS FOR BASE RATES'
[20]   MES[1;]:KWH
[21]   MES[2;]:AREV
[22]   MES[3;];RPK
[23]   2 1 ρ' '
[24]   YB←MONO RATE DIV 1,A,3000+¯1↑A
[25]   YB←YB,10+¯1↑YB
[26]   YF←0,BFR,1,1
[27]   XPLOT← 2 3 4 4
[28]   →(AKN 'WANT A PLOT OF BILLING DENSITY?')/K1
[29]   ABS←(ρA) SELECT A
[30]   ORD←(ρA) SELECT DENSITY
[31]   0 1 PPR 1
[32]   ' '
[33]   'ABSCISSA IS KWH, WITH + AT VALUES ';AMARK
[34]   'ORDINATE IS DENSITY, WITH + AT VALUES ';OMARK
[35]   PLOT XPLOT
[36]  K1:BCUMKWH←CUM(FF×AT)÷KWH
[37]   BCUMREV←CUM(FF×REV)÷AREV
[38]   →(AKN 'WANT A PLOT OF THE REVENUE DISTRIBUTION CURVE?')/K2
[39]   ABS←(ρBCUMKWH) SELECT BCUMKWH
[40]   ORD←(ρBCUMREV) SELECT BCUMREV
[41]   1 1 PPR 1
[42]   ' '
[43]   'ABSCISSA IS CUMULATIVE KWH, ORDINATE IS CUMULATIVE REVENUE'
```



```
[46]   'BASE CONNECT COST IS ';CONNECT
[47]   COST←CONNECT+AT×TRUEMC
[48]   BNET←1-REV÷COST
[49]   →(AKN 'WANT A PLOT OF THE NET SUBSIDY CURVE?')/K3
[50]   ' '
[51]   ABS←(ρAT) SELECT AT
[52]   ORD←(ρAT) SELECT BNET
[53]   0 1 PPR 1
[54]   'ABSCISSA IS KWH, WITH + AT VALUES ';AMARK
[55]   'ORDINATE IS PERCENT NET SUBSIDY, WITH + AT VALUES ';100×OMARK
[56]   PLOT XPLOT
[57]  K3: 3 1 ρ' '
[58]  K4:→(~AKN 'WANT TO ANALYZE AN ALTERNATIVE RATE SCHEDULE?')/K5
[59]   →(AKN 'WANT TO SUSPEND BEFORE EXITING?')/0
[60]   'TO REENTER, TYPE→K4';÷0
[61]  K5:CADJ←0.01×ΔFI AKI 'OPERATING COST AS A PERCENTAGE OF THE BASE CASE'
[62]   KAP←CADJ×KAP
[63]   TRUEMC←CADJ×TRUEMC
[64]   CONNECT←CADJ×CONNECT
[65]   IT←AKN 'WANT THIS RATE SCHEDULE SCALED UP OR DOWN TO ACHIEVE A SPECIFIC RATE OF RETURN?'
[66]   →IT/L7
[67]   ABS←AKN 'IS CONNECT CHARGE TO BE ADJUSTED?'
[68]   LIFE←0
[69]   LIFE←ΔFI AKI 'ENTER THE LIFELINE LEVEL BELOW WHICH RATES ARE NOT TO BE ADJUSTED'
[70]   AROR←0.01×ΔFI AKI 'ENTER ALLOWED RATE OF RETURN (PERCENT)'
[71]  L7:'7. ENTER ALTERNATIVE RATE SCHEDULE'
[72]   ARATE←,⎕
[73]   ARATE←ARATE,100000,¯1↑ARATE
[74]   →(2|ρARATE)/L71
[75]   →(LIFE=0)/L72
[76]   T←1+¯1+2×⍳0.5×ρARATE
[77]   ADJ←2×+/LIFE≥,ARATE[T],0
[78]   →(LIFE∊,ARATE[T])/L72
[79]   ARATE←(ADJ↑ARATE),LIFE,ARATE[ADJ],ADJ↓ARATE
[80]   →L72
[81]  L71:'LENGTH ERROR'
[82]   →L7
[83]  L72:RATE←RCR ARATE
```

```
[83] L72:RATE←RCR ARATE
[84]  →(IE=1)/L7
[85]  ' ' ◇ FR←BFR
[86]  STACR
[87]  'STATISTICS FOR THIS ALTERNATIVE IF DEMAND WERE RATE INELASTIC'
[88]  OUTCR 2
[89] K6: 3 1 ρ' '
[90]  FR←RATE FRCR A
[91]  STACR
[92]  'STATISTICS FOR THE SPECIFIED ALTERNATIVE RATE SCHEDULE'
[93]  OUTCR 1
[94]  →IT/K7
[95] ⍝ITERATE TO ALLOWED ROR
[96]  LOOK
[97]  ADJ←1
[98] K20: 3 1 ρ' '
[99]  'STATISTICS FOR THE ADJUSTED RATE SCHEDULE'
[100] '  THE ADJUSTED RATE SCHEDULE: ';¯2↓CRATE
[101] '  PERCENTAGE ADJUSTMENT REQUIRED ';ADJ PCTCR 1
[102] ' '
[103] OUTCR 1
[104]K7:→(AKN 'DO YOU WANT A PLOT OF BILLING DENSITY?')/K21
[105] ABS←ABS,ABS←(ρA) SELECT A
[106] ORD←((ρDENSITY) SELECT DENSITY),(ρDENSITY) SELECT(¯1↓FF)÷A-0,¯1↓A
[107] 0 1 PPR 2
[108] ' '
[109] 'ABSCISSA IS KWH, WITH + AT VALUES ';AMARK
[110] 'ORDINATE IS DENSITY, WITH + AT VALUES ';OMARK
[111] PLOT XPLOT
[112] ABS VALUES ORD
[113]K21:CUMKWH←CUM(FF×AT)÷KWH
[114] CUMREV←CUM(FF×REV)÷AREV
[115] →(AKN 'DO YOU WANT A PLOT OF THE REVENUE DISTRIBUTION CURVE?')/K22
[116] ABS←((ρCUMKWH) SELECT BCUMKWH),(ρCUMKWH) SELECT CUMKWH
[117] ORD←((ρDENSITY) SELECT BCUMREV),(ρDENSITY) SELECT CUMREV
[118] 1 1 PPR 2
[119] ' '
[120] 'ABSCISSA IS CUMULATIVE KWH, ORDINATE IS CUMULATIVE REVENUE'
[121] PLOT XPLOT
[122] ABS VALUES ORD
[123]K22:COST←CONNECT+AT×TRUEMC
[124] NET←1-(REV÷COST)×(+/COST×FF)÷+/REV×FF
[125] →(AKN 'DO YOU WANT A PLOT OF THE NET SUBSIDY CURVE?')/K23
[126] ABS←ABS,ABS←(ρAT) SELECT AT
[127] ORD←((ρAT) SELECT BNET),(ρAT) SELECT NET
```

```
[129] 'ABSCISSA IS KWH, WITH + AT VALUES ';AMARK
[130] 'ORDINATE IS PERCENT NET SUBSIDY, WITH + AT VALUES ';OMARK
```



```
[131] PLOT XPLOT
[132] ABS VALUES ORD
[133]K23:→K4
[134]ERR:'ERROR IN BILLING INPUT'
    ∇

      ∇INTER[⎕]∇
    ∇ FR←Y INTER Z
[1]    YB←Z[1;]
[2]    YF←Z[2;]
[3]    N←ρ,Y
[4]    I←1
[5]    FR←Nρ0
[6]   L:T←+/Y[I]≥YB
[7]    S←(Y[I]-YB[T])÷YB[T+1]-YB[T]
[8]    FR[I]←(S×YF[T+1])+YF[T]×1-S
[9]    I←I+1
[10]   →(I≤N)/L
    ∇

      ∇MAKERATE[⎕]∇
    ∇ RATE←RATES MAKERATE BLOCKS
[1]    →((ρ,RATES)≠2+ρ,BLOCKS)/ERROR
[2]    RATE←(3,ρ,RATES)ρ0
[3]    RATE[1;]←0,0,BLOCKS
[4]    RATE[2;]←RATES
[5]    RATE[3;]←(1↑RATES),0,1↓¯1↓RATES
[6]    RATE[3;]←RATE[3;]×RATE[1;]-¯1,¯1↓,RATE[1;]
[7]    →0
[8]   ERROR:'LENGTH ERROR'
[9]    →
    ∇

      ∇MCR[⎕]∇                                ⍝AUTOMATIC AXIS MARKING FUNCTION FOR PLOTS
    ∇ QMARK←TY MCR Q
[1]    J←1
[2]    TOP←⌈/Q
[3]    BOT←⌊/Q
[4]    →TY/LIN
[5]    [illegible]
```

7

```
[12]  ADJ←(TOP-BOT)÷ADJ
[13]  RS←⌊¯2+BOT÷J
[14]  QMARK←RS+(⍳65)×0.5+(0.5×ADJ>5)-0.3×ADJ<2
[15]  QMARK←QMARK×1E¯6<|QMARK
[16]  TOP←TOP÷J
[17]  BOT←BOT÷J
[18]  →LOG
    ∇

      ∇MONO[⎕]∇
    ∇ YY←MONO Y;I;N;YU;YL;YR;K;RR
[1]   ⍝MONOTONIZES THE VECTOR Y
[2]    I←1
[3]    RR←0
[4]    N←⍴,Y
[5]    YR←⍳0
[6]    YU←YL←N⍴0
[7]   L:YU[I]←⌈/I↑Y
[8]    YL[I]←⌊/(I-1)↓Y
[9]   ⍝TEST IF RUN IN PROGRESS
[10]   →(RR>0)/RUN
[11]  ⍝TEST IF RUN STARTS
[12]   RR←YL[I]=YU[I]
[13]   →RR/START
[14]   YR←YR,0
[15]   →ADV
[16]  START:K←,1⍴1
[17]   →ADV
[18]  ⍝RUN IN PROGRESS, TEST IF STOPS
[19]  RUN:RR←RR+1
[20]   K←K,RR
[21]   →(I=N)/ST
[22]   →(YL[I]=YU[I])/ADV
[23]  ⍝RUN STOPS
[24]  ST:K←K÷⌈/,K
[25]   YR←YR,K
[26]   RR←0
[27]  ADV:I←I+1
[28]   →(I≤N)/L
[29]   YY←(YL×1-YR)+YU×YR
    ∇
```

8

```
    ∇ OUTCR J
[1]    →J+1
[2]    MSS[1;];KWH;' (';KWH PCTCR BKWH;')'
[3]    MSS[2;];AREV;' (';AREV PCTCR BAREV;')'
[4]    MSS[3;];RPK;' (';RPK PCTCR BRPK;')'
[5]    RROR←RRORCR
[6]    MSS[4;];100×RROR;' (';RROR PCTCR RETURN;')'
    ∇

      ∇PCTCR[⎕]∇
    ∇ R←U PCTCR V
[1]    R←100×(U-V)÷V
    ∇

      ∇RCR[⎕]∇
    ∇ R←RCR RATE;Z;W
[1]    IE←0
[2]    Z←⍴,RATE
[3]    →(2|Z)/ERROR
[4]    W←1,2×⍳0.5×Z
[5]    Z←1+1↓¯1↓W
[6]    R←RATE[W] MAKERATE RATE[Z]
[7]    →0
[8]   ERROR:'LENGTH ERROR'
[9]    IE←1
    ∇

      ∇STACR[⎕]∇
    ∇ STACR;B
[1]    B←-(⍟÷/1E¯12⌈1-¯2↑FR)÷⍟÷/¯2↑A
[2]    B←1.1⌈B
[3]    AT←A,(¯1↑A)×B÷B-1
[4]    FF←(FRE FR),1-¯1↑FR
[5]    RATE COSTCR AT
[6]    REV←AT×AC
[7]    KWH←+/FF×AT
[8]    AREV←+/FF×REV
[9]    RPK←AREV÷KWH
    ∇

      ∇YCR[⎕]∇
    ∇ BFR YCR A
[1]   ⍝CONSTRUCTS CUMULATIVE Y DISTRIBUTION FROM BFR AND A IN BASE CASE
[2]    YB←(BRATES MAKERATE BBLOCKS) DINV 0,A,3000+¯1↑A
[3]    YB←MONO YB
```

```
      ∇FI                                   ⍝SYSTEM FUNCTION
[1]   ⎕∇
    ∇ FI
    ∇

      ∇MI[⎕]∇                               ⍝SYSTEM FUNCTION
DEFN ERROR
      ∇MI
       ^

      ∇VS[⎕]∇                               ⍝SYSTEM FUNCTION
DEFN ERROR
      ∇VS
       ^

      ∇PLOT[⎕]∇                             ⍝PLOT ROUTINE FOR DTC300 TERMINAL, USASCII CODED (REQUIRES
    ∇ PLOT Y;N;I;AVECT;W                     PLOT MODE AND PLOT SWITCH ON, DOES NOT REQUIRE SUPERPLOT)
[1]   N←(⍴X)[2]
[2]   X←X,[1](2,N)⍴0
[3]   W← 60 48 60 ¯48 ×4↑Y
[4]   W[2]←W[2]-W[4]
[5]   W[3 4]←W[3 4]÷,1E¯6⌈((⌈/X[1 2 ;])-⌊/X[1 2 ;]
[6]   X[1 2 ;]←X[1 2 ;]-(,⌊/X[1 2 ;])∘.×N⍴1
[7]   X[1 2 ;]←X[1 2 ;]×W[3 4]∘.×N⍴1
[8]   X[1 2 ;]←X[1 2 ;]+W[1 2]∘.×N⍴1
[9]   X[1 4 ;]← 0 6 ⊤⌊X[1;]+0.5
[10]  X[2 5 ;]← 0 8 ⊤⌊X[2;]+0.5-2×X[3;]=46
[11]  W←(7⍴0),W[2]
[12]  I←1
[13]  ARBOUT 12
[14] L1:W[6 7]←(∧/W[1 4]=,X[1 4 ;I]),∧/W[2 5]=,X[2 5 ;I]
[15]  W[⍳5]←,X[;I]
[16]  →W[6 7]/(L2,L3)
[17]  AVECT← 27 6 27 84 ,W[1], 27 116 ,W[2],6,(W[4]⍴32),(W[5]⍴10),W[3]
[18]  →L4
[19] L2:AVECT← 27 6 27 116 ,W[2],6,(W[5]⍴10),W[3]
[20]  →L4
[21] L3:AVECT← 27 6 27 84 ,W[1],6,(W[4]⍴32),W[3]
[22] L4:I←I+1

      →(I≤N)/L1
[25]  X←0
[26]  AVECT← 27 6 27 84 1 27 116 ,(W[8]+8), 10 10 10
[27]  ARBOUT AVECT
    ∇
```

10

```
      ∇PPR[⎕]∇
    ∇ TR PPR K                              ⍝PREPARES DATA ARRAY FOR PLOT
[1]   →((⍴ORD)≠⍴ABS)/ERROR
[2]   X←(3,⍴ORD)⍴ABS,ORD,(⍴ORD)⍴46
[3]   AMARK←TR[1] MCR ABS
[4]   OMARK←TR[2] MCR ORD
[5]   ALOC←TR[2]∧(0≤⌈/OMARK)∧0≥⌊/OMARK
[6]   OLOC←TR[1]∧(0≤⌈/AMARK)∧0≥⌊/AMARK
[7]   ALOC←(ALOC,~ALOC)/0,⌊/OMARK
[8]   OLOC←(OLOC,~OLOC)/0,⌊/AMARK
[9]   BOT←⍴AMARK
[10]  TOP←⍴OMARK
[11]  ALOC←(3,BOT)⍴AMARK,(BOT⍴ALOC),BOT⍴45
[12]  OLOC←(3,TOP)⍴(TOP⍴OLOC),OMARK,TOP⍴45
[13]  X←OLOC[;⌽⍳TOP],X,ALOC
[14]  →TR[1]/L1
[15]  X[1;]←10⍟X[1;]
[16] L1:→TR[2]/0
[17]  X[2;]←10⍟X[2;]
[18]  →0
[19] ERROR:'INCONSISTENT LENGTHS'
[20]  →
    ∇

      ∇RRORCR[⎕]∇
    ∇ RROR←RRORCR
[1]   RROR←RETURN+(AREV-CONNECT+KWH×TRUEMC)÷KAP
[2]   RROR←1E¯9⌈RROR
    ∇

      ∇VALUES[⎕]∇
    ∇ ABS VALUES ORD;N
[1]   ' '
[2]   FS←'4F10.4'
[3]   FS CENTER 'SELECTED VALUES'
[4]   FS COLNAMES ' BASE BASE NEW NEW'
[5]   FS COLNAMES ' ABSCISSA ORDINATE ABSCISSA ORDINATE'
[6]   ' '
[7]   N←(⍴ORD)÷2
[8]   FS ΔFMT⍉(4,N)⍴ABS[⍳N],ORD[⍳N],ABS[N+⍳N],ORD[N+⍳N]
[9]   ' '
    ∇
```

```
      ∇SEARCH[⎕]∇
    ∇ TOL SEARCH N                                ⍝SECANT TYPE SEARCH ALGORITHM FOR RATES TO
[1]   X←(-F←EVAL 0),0                             ACHIEVE ALLOWED RATE OF RETURN...WORKS ONLY IF
[2]   F←(EVAL X[1]),F                             REVENUE IS MONOTONE IN RATES...CURRENTLY REPLACED
[3]   I←1                                         BY  LOOK  IN FREQUENCY
[4]  LOOK:→(0≥×/×F)/STRAD
[5]   →((×F[1])=×F[1]-F[2])/NON
[6]   X←(X[1]-2×F[1]×(X[1]-X[2])÷F[1]-F[2]),X[1]
[7]   F←(EVAL X[1]),F[1]
[8]   I←I+1
[9]   →(I≤N)/LOOK
[10]  'FAILURE TO STRADDLE'
[11]  →CONV
[12] STRAD:X←(0.5×+/X),X
[13]  F←(EVAL X[1]),F
[14]  →(1E¯12>|F[1])/CONV
[15]  →(∨/1=(⍋F)[1 3])/NON
[16]  B1←(((F[3]-F[1])÷X[3]-X[1])×((X[2]-X[1])÷X[2]-X[3]))+((F[2]-F[1])÷X[2]-X[1])×(X[1]-X[3])÷X[2]-X[3]
[17]  B2←((F[3]-F[1])÷X[3]-X[1])-(F[2]-F[1])÷X[2]-X[1]
[18]  X←((T←X[1]-F[1]÷B1)+F[1]×F[1]×B2÷B1×B1×B1),X
[19]  →((∧/1≠(⍋X)[1 4])∧(×X[1]-X[2])=×T-X[1])/OK
[20]  X[1]←T
[21] OK:F←(EVAL X[1]),F
[22]  T←F≥0
[23]  T← 2 3 -×¯2++/T
[24]  T←(⍋F)[T]
[25]  T←T[⍋T]
[26]  X←X[T]
[27]  F←F[T]
[28] ITER:I←I+1
[29]  →(TOL≥|-/X)/CONV
[30]  →(I≤N)/STRAD
[31]  'FAILURE TO CONVERGE'
[32] CONV:X←0.5×+/X
[33]  F←F[1]
[34]  →0
[35] NON:'NON-MONOTONIC FUNCTION'
[36]  →
    ∇
```



```
      ∇ΔFMT[⎕]∇                                   ⍝SYSTEM FUNCTION
DEFN ERROR
      ∇ΔFMT
          ∧

      ∇CENTER[⎕]∇
    ∇ R←A CENTER B
[1]   A←+/×/ 0 ¯2 ↑RWTD A
[2]   R←(1,A)⍴A↑((⌈0.5×0⌈A-⍴B)↑' '),B←,B
    ∇

      ∇COLNAMES[⎕]∇
    ∇ R←A COLNAMES B;C;D;E;S
[1]   C←R←0⍴A←((E←~Z1)⍴0),,RWTD A
[2]   →(1≥⍴A)⍴12
[3]   →(A[3]≥4)⍴7
[4]   C←C,A[1]⍴-A[2]
[5]   A←4↓A
[6]   →2
[7]   C←C,(0.5×4≠A[3])+×/A[1 2]
[8]   →(1≥⍴A←4↓A)⍴12
[9]   →(A[3]< 4 5)/ 2 7
[10]  C←(¯1↓C),(¯1↑C)+×/A[1 2]
[11]  →8
[12]  A←+/|⌊C
[13]  S←0,E+(B=¯1↑B)/⍳⍴B←1⌽,B
[14]  →(1 0.5 =(⍴S),1|1↑C)/ 22 19
[15]  R←R,(|1↑C)↑(((⍴D)>|1↑C)/(|1↑C)⍴'*'),(1↑C)↑D←¯1↓S[1-E]↓S[2-E]↑B
[16]  S←1↓S
[17]  C←1↓C
[18]  →14
[19]  R←R,(⌊1↑C)↑' '
[20]  C←1↓C
[21]  →14
[22]  R←(1,A)⍴A↑R
    ∇

      ∇RWTD[⎕]∇                                   ⍝SYSTEM FUNCTION
```

```
      ∇LOOK[⎕]∇                                                   AMANUAL SEARCH ALGORITHM FOR RATES TO ACHIEVE ALLOWED
    ∇ LOOK;I;M;FS;B1                                              RATE OF RETURN
[1]   'ENTER ADJUSTMENT PERCENT'
[2]   L:FS←0.01×100+,⎕
[3]   M←⍴FS
[4]   JJ←1
[5]   B1←⍳0
[6]   L2:CRATE←FS[JJ] ADJCR ARATE
[7]   RATE←RCR CRATE
[8]   FR←RATE FRCR A
[9]   STACR
[10]  RROR←RRORCR
[11]  B1←B1,RROR
[12]  JJ←JJ+1
[13]  →(JJ≤M)/L2
[14]  B1×100
[15]  →(AKN 'STOP?')/L
[16]  L1:CRATE
    ∇

      BILLING

      ANALYZER

      APLSYM
¨)<≤=>]∨∧≠÷,+./0123456789([;×:\¯⍺⊥∩⌊∊_∇∆⍳∘'⎕|⊤○⋆?⍴⌈~↓∪⍵⊃↑⊂←⊢→≥-⋄ABCDEFGHIJKLMNOPQRSTUVWXYZ{⊣}⍞

      MES
 AVERAGE DEMAND (KWH/MONTH).......
 REVENUE PER CUSTOMER (¢/MONTH)...
 AVERAGE PRICE (¢/KWH)............

      MSS
 AVERAGE DEMAND (KWH/MONTH) [PERCENT CHARGE FROM BASE]..........
 REVENUE PER CUSTOMER (¢/MONTH) [PERCENT CHANGE FROM BASE].......
 AVERAGE PRICE (¢/KWH) [PERCENT CHANGE FROM BASE]...............
 REALIZED RATE OF RETURN (PERCENT) [PERCENT CHANGE FROM BASE]...

      ∇EVAL[⎕]∇
    ∇ F←EVAL X
[1]   RATE←RCR CRATE←(*X) ADJCR ARATE
[2]   FR←RATE FRCR A
[3]   STACR
[4]   RROR←RRORCR
[5]   F←⍟RROR÷AROR
    ∇

      ∇SELECT[⎕]∇
    ∇ AA←N SELECT A
[1]   AA←A[1,(4×⍳⌊N÷4),N]
    ∇

      )GRP ECDC
ADJCR   AKI     AKN     ARBOUT  COSTCR  CUM     DINV    FRCR    FRE     FREQUENCY       INTER   MAKERATE
MONO    OUTCR   PCTCR   RCR     STACR   YCR     ∆FI     ∆MI     ∆VS     PLOT    PPR     BILLING ANALYZER
MES     MSS     BCOEFF  RRORCR  VALUES  SSEARCH EVAL    SELECT  ∆FMT    CENTER  COLNAMES        RNTO    LOOK
                                                                                                        MCR
                                                                                                        APLSYM

      BCOEFF
1.4701  0.7179  0.48635  1.0111

      BRATE
60  4.777  50  3.484  100  2.65  200  2.056  300  1.9057  1000  1.6233
```

```
⍕3 64⍴(⍳64),A,1000×PGEBFRCUS
```

	KWH/MO.	CUMUL. FREQ. OF CUSTOMERS×10^3
1	20	9.94
2	40	15.64
3	60	25.49
4	80	40.26
5	100	59.09
6	120	80.47
7	140	103.63
8	160	128.06
9	180	153.78
10	200	179.2
11	220	205.56
12	240	232.4
13	260	259.45
14	280	286.73
15	300	314.16
16	320	341.65
17	340	369.17
18	360	396.55
19	380	423.82
20	400	450.95
21	420	477.73
22	440	504.15
23	460	530.04
24	480	555.38
25	500	579.99
26	520	603.88
27	540	626.58
28	560	649.11
29	580	670.31
30	600	690.5
31	620	709.71
32	640	727.8
33	660	744.97
34	680	761.04
35	700	776.14

36	720	790.25
37	740	803.43
38	760	815.54
39	780	827.05
40	800	837.62
41	820	847.36
42	840	856.37
43	860	864.76
44	880	872.44
45	900	879.56
46	920	886.14
47	940	892.25
48	960	897.95
49	980	903.18
50	1000	908.09
51	1100	928.01
52	1200	942.59
53	1300	958.53
54	1400	962.03
55	1500	968.67
56	1600	974.03
57	1700	978.33
58	1800	981.83
59	2000	987.07
60	2200	990.66
61	2400	993.17
62	2600	994.93
63	2800	996.2
64	3000	997.11

```
      )OFF
027  14.17.19 10/21/76 ECD
CONNECTED     0.39.32  TO DATE   54.30.42
CPU TIME      0.00.01  TO DATE    0.07.05
```

APPENDIX F

Monotonicity of ψ(KWH,...)

Let $u + U(z,k,\theta)$ be the utility function of a consumer, where k is electricity consumption, z is consumption of all other goods, and θ is a variable defining tastes. The budget constraint is $z + E(k) = Y$, where $E(k)$ is the expenditure required for electricty demand k. Substituting in the budget constraint and differentiating, the first and second order conditions are

$$\frac{du}{dk} = -U_z E' + U_k = 0 \quad , \tag{1}$$

$$\frac{d^2u}{dk^2} = \underbrace{-U_z E''}_{(1)} + \underbrace{U_{kk} - 2U_{zk}E' + U_{zz}(E')^2}_{(2)} < 0 \quad . \tag{2}$$

The change in the demand for electricity with respect to Y is given by

$$\frac{d^2U}{dk^2}\frac{\partial k}{\partial Y} = -\frac{\partial}{\partial Y}\left(\frac{dU}{dk}\right) = U_{zz}E' - U_{kz} = -U_z \frac{\partial}{\partial z}\left(\frac{U_k}{U_z}\right) \quad . \tag{3}$$

The condition for electricity to be non-inferior is

$$\frac{\partial}{\partial z}\left(\frac{U_k}{U_z}\right) = \frac{\partial}{\partial z}\left(-\left.\frac{dz}{dk}\right|_{U\,=\,\text{constant}}\right) > 0 \quad .$$

Then, from the second order condition, $\partial k/\partial Y > 0$. Note however in the second order condition that while the term (2) in equation (3) is always negative, since its sign is determined by the sign of

$$\det \begin{vmatrix} U_{kk} & U_{kz} & U_k \\ U_{zk} & U_{zz} & U_z \\ U_k & U_z & 0 \end{vmatrix} > 0 \quad , \tag{4}$$

the term (1) in equation (3) is positive when the rate structure is declining (i.e., $E'' < 0$). Hence, there may be ranges of k where the second order condition for a maximum fails. Then, individual demand will have the form shown in Figure 5. The curve OABCDEFG solves the first-order condition. In the range CD, the second order condition for a maximum fails. For Y between A and C, there are two local maxima, on the legs AC and DF of the curve. The global maximum occurs on the left leg for Y between A and B, and on the right leg for Y above B (= E). Then, observed demand, indicated by the heavy line, jumps from B to E. We have thus shown that individual demand must be non-decreasing in Y, although it may contain jumps. Therefore, aggregate demand which is the average of individual demands must be non-decreasing in Y. Further, averaging over tastes will usually smooth aggregate demand so that it does not have jumps. For example, suppose the utility function has the form $U(z - \theta, k)$, with the taste parameter θ distributed smoothly in the population with a frequency function $g(\theta)$. Then the demand function has the form $k = f(Y - \theta)$, with at most a countable number of arguments where jumps occur. Aggregate demand satisfies $KWH = \int f(Y - \theta)\phi(\theta)d\theta$. Since the jumps in f occur on a set of measure zero, KWH will be a smoothly increasing function of Y without jumps.

We consider, finally, the consumption frequency distributions which result under alternative rate structures. One caution is necessary in interpreting these curves. The method used to construct these schedules assumes that all customers have identical tastes, and that each customer sets precisely his monthly demand, evaluating average and marginal cost at the exact point of consumption. As Figure F shows, this will lead to a jump in demand in a region around a block boundary where there is a discrete drop in rates, and conversely, a constant demand at the block boundary where there is a discrete increase in rates. If the distribution of income has a smooth density function, then the bill frequency distribution obtained from these rate structures will in the case of declining rates have zero denisty in a region around the block boundaries, and in the case of increasing rates will have a "spike" of high probability at the block boundary.

Now consider the impact of this relationship on the methodology used here. We observe a base consumption frequency distribution that has a smooth density without gaps or spikes. Since the base rate structure is declining block, this can occur in our construction only if the E distribution has a

FIGURE F. Demand at a Block Boundary Where Rates Decline

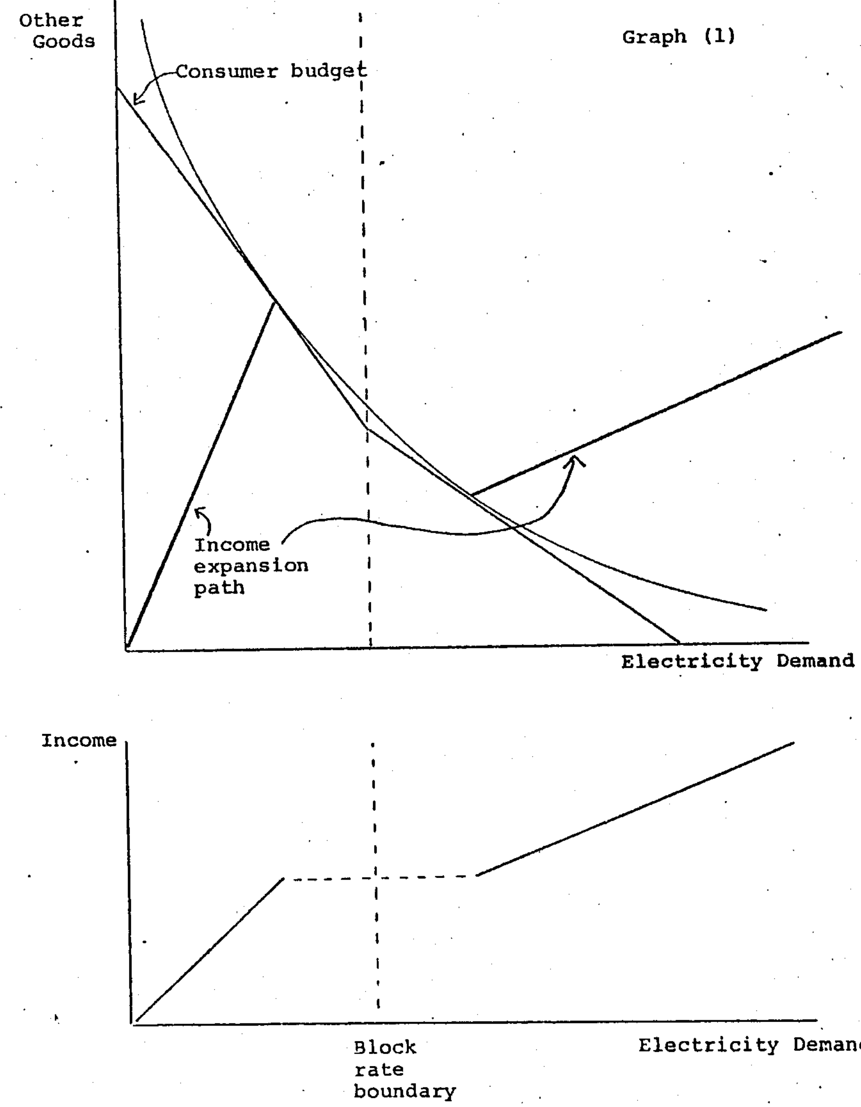
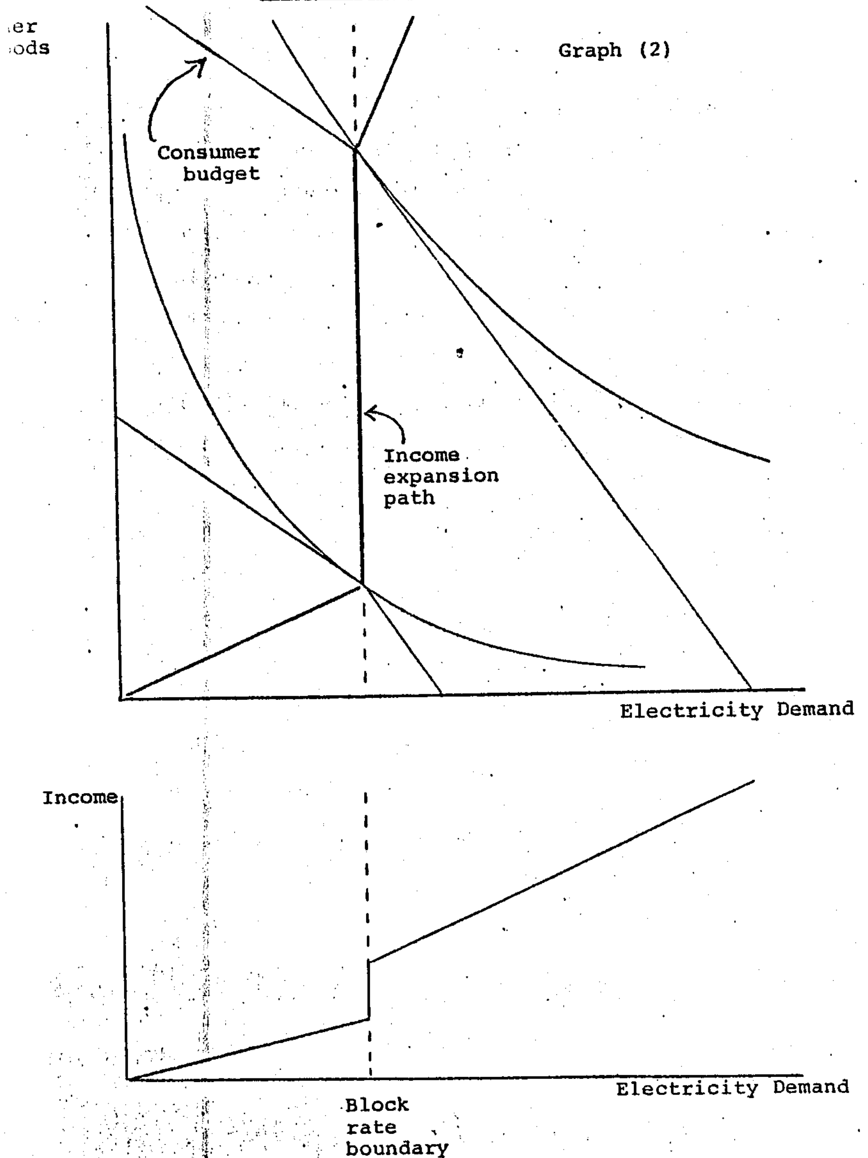


FIGURE F . Demand at a Block Boundary Where Rates Increase



density with spikes at the income levels which yield demands "at," or bracketing, the block boundary. Our method then constructs from the base bill frequency destribution an income distribution with spikes at 50, 100, 200, 300, and 1000 KWH per month. When this income density is applied to derive the consumption frequency distribution for alternative rate structures, we will now obtain a consumption density which will tend to have spikes at block boundaries where rates increase, and at the base block boundaries and will tend to drop at block boundaries where rates decrease. If old and new block boundaries coincide, the net effect will be a combination of these effects. However, it must be emphasized that this fine structure of the densities is almost certainly spurious, a product of the restrictive assumptions of common customer tastes and precise setting of demand levels. Fortunately, the major policy indicators with which we are concerned for alternative structures--average demand, revenue per customer, average price, the revenue distribution curve, and the net subsidy curve, are quite insensitive to this fine structure, and will be essentially unchanged under relatively drastic changes in the fine structure. Thus, the question of constructing consumption densities without spurious spikes or zeros, while important from the standpoint of theoretical consistency, is not important for policy analysis.

The evidence from the base consumption / frequency distribution suggests strongly that factors which smooth demand must be at work. The true ϵ-distribution is believed to have a smooth density. Since this is not translated under the base declining block rate structure into a consumption / densities with zeros at the block boundaries, it must be the case that our assumption that customers choose an exact demand level, evaluating average and marginal cost at this level, is incorrect. A more reasonable alternative is that customers choose a demand "strategy" or "target" demand level rather than a specific demand level. The strategy will establish appliance holdings and general patterns of electricity use. Actual demand will be a random variable whose distribution is determined by the consumption strategy. Average and marginal costs are then assessed for alternative strategies. If the distribution of demand resulting from a particular strategy is itself smooth, then the expected values of average and marginal costs will be smooth functions of the target demand level, even at block boundaries. If customers are somewhat vague and generalized in their assessment of costs, or tastes vary between customers, this will have a further smoothing effect on the bill densities derived from alternative rate structures. We have not attempted to build this theoretically sounder structure into the current method, since it would not be expected to change policy conclusions.

APPENDIX G

A Model of Time-of-Day Demand

Suppose residential customers have a utility function $u = z + a\cdot x - \frac{1}{2} x\cdot B\cdot x$, where x is a vector of electricity demands at various time, a is a commensurate vector which may depend on weather and other exogenous variables, B is a positive definite matrix, and z is an index of all other commodities consumed. With a budget constraint $z + p\cdot x = y$, where p is a vector of electricity prices at various times of day, and y is income, the customer maximizes $u = y + (a - p)\cdot x - \frac{1}{2} x\cdot B\cdot x$. Ignoring the possibility of corner solutions, the demand for x satisfies $B\cdot x = (a - p)\cdot x$, or $x = c - CP$, where $c = B^{-1}a$ and $C = B^{-1}$. Thus, we have a linear (in relative prices) demand system for consumption at various times of day.

Suppose now that the consumer has a nested homogeneous utility function, with x interpreted as the ratio of demand at various times to average demand, and the utility function $a\cdot x - \frac{1}{2} x\cdot B\cdot x$ appearing as an argument in the top-level utility function. Then, x will be determined as before, $x = c - Cp$, with p defined relative to expenditure on electricity per average unit of consumption. We propose this system, or more complex variants as necessary, for the empirical analysis of time-of-day demand.