A TWO-LEVEL ELECTRICITY DEMAND MODEL

Evaluation of the Connecticut Time-of-Day Pricing Test

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A two-stage budgeting model is developed for electricity demand where consumption in each period is treated as a different commodity. A relative household demand model is first estimated, a consistent price index for electricity is constructed, and then a total electricity consumption model is estimated. Econometric procedures are derived which permit application of the model to both time-of-day price situations and also declining block price situations which result in non-linear budget sets. The model is applied to both types of situations – the data from the Connecticut time-of-day pricing test as well as data from the declining block rate situation of the previous year. The model is also tested in a forecasting application to time-of-day customers.

1. Introduction

Time-of-day (TOD) electricity pricing has received increased attention in the post-1974 era of sharply increased energy prices. Generating capacity for electricity can be divided into three main types: base load capacity, intermediate capacity, and peak capacity. The marginal costs of generation rise greatly as system demand requires utilization of intermediate and peak load capacity at certain times of day or during certain seasons of the year. Thus, time-of-day prices for electricity offer a possible method of decreasing average generation cost per kWh of demand by causing peak demand to shift into other hours of the day when only base load capacity is required. Also, by setting time-of-day prices in a pattern which has a shape similar to that of marginal costs of generation, the price patterns prescribed by economic theory to achieve a welfare optimum might be closely approximated.[1] Another possible outcome is to decrease energy consumption of oil

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[1]It is incorrect, however, to conclude on this basis that the introduction of TOD pricing would lead to a welfare improvement. Since the cost of metering equipment is significant, this

and natural gas by electric utilities since these fuels tend to be used primarily for intermediate and peak capacity while coal and nuclear generation are more economic for longer load durations. The increased attention by both regulatory commissions, electric utilities, and the federal government has led to the initiation of at least six time-of-day demonstrations or experiments which attempt to estimate the effect on household electricity demand of time-of-day prices.

Here we analyze the results of the Connecticut Peak Load Pricing Test, which was conducted in the Connecticut Light and Power Service Area during the period October 1975 to October 1976. Meters were installed in 199 households and readings of household electricity consumption were recorded every fifteen minutes for the one-year period. Households faced a time-of-day pricing structure with price set on an hourly basis into one of three categories corresponding to peak price (16¢/kWh), intermediate price (3¢/kWh), and off-peak price (1¢/kWh). The TOD pattern changed between weekdays and weekends and between summer and winter. Previous analyses in the Final Report (1977) and papers in the EPRI conference volume (1977), which have concentrated on the time pattern of response have demonstrated that significant shifting of household electricity demand from peak to intermediate and off-peak periods occurred. The peak demand in January for the experimental group of households occurred in an off-peak price period while peak demand for the control group occurred during a peak-price period. Similar load shifting occurred during other periods of the year resulting in the conclusion that TOD prices had the expected effect on the pattern of household electricity demand.

Other important results are not conveniently analyzed within a time series framework, however. Total electricity consumption of the experimental group was 5% less than the control group during the period of the experiment. Also, given the short time duration of the experiment, the appliance stock is taken as fixed, so that the medium- and long-run response cannot be inferred from the analysis. Our approach, which falls more within the tradition of econometric consumer demand estimation, has the potential to explain these questions and others which arise regarding TOD prices. We treat electricity demand within a two stage budgetary context. Letting electricity demand in each period be a different commodity, we estimate relative household demands across periods conditional on relative prices, the appliance stock, socioeconomic characteristics of the household, and the weather. From these relative demand estimates, we then estimate a price index for electricity that corresponds to the unit cost function of a subutility function in a weakly

extra cost must be included in any cost–benefit calculation, see Wenders and Taylor (1976). Also, the significant fixed costs of electricity generation raise questions in the absence of a two-part tariff which need to be considered in any welfare calculation.

separable utility specification. Given the price index for electricity and the prices for other goods, the household determines how much total electricity to consume. The household's absolute demand in each time period is then derived from the product of its relative demand in the period and its total demand for electricity. Since initial analysis demonstrated that relative demands could be estimated more precisely than absolute demands by households across time periods, this approach permits quite precise estimates of the relative load schedule by households. Also, since the estimated price index for the experimental household is higher than for control households, the model explains why total electricity consumption is lower for households which face time-of-day prices. As the appliance mix of households changes over time, forecasts of hourly demand can be generated by the model, so long as the technology of the appliances does not shift markedly. Lastly, interactions of prices, weather, and the appliance mix are accounted for so that the model can be used in a general load forecasting framework.

In section 2 of the paper, we describe the theoretical model for time-of-day electricity consumption. Beginning with a specification of a weakly separable utility function, we derive an equation for relative electricity demand. The equation is derived by applying a Taylor expansion to the demand function derived from Roy's identity for each hour of the day during the winter and summer price periods. We then construct the price index and in a similar manner derive an equation for total electricity demand. In section 3 we estimate both relative hourly demand and total demand using the equations developed from the demand theory in the previous section. For the time period in which households faced declining block prices rather than time-of-day prices, we use appropriate procedures to deal with the non-linear budget set. These procedures have been previously developed in work on labor supply where an identical problem arises due to progressive taxation and government tax and transfer programs and in work on aggregate demand for electricity under declining block rates. In the last section we compare forecasts of relative demand, total demand, and absolute hourly demands to actual demands for a sample of twenty households which were not used in the estimation of section 3. The model appears to perform relatively well in the forecast environment, especially in predicting peak period demands. We conclude that it is worthwhile to consider using such a model in the other time-of-day experiments where cross elasticities could also be estimated since the data design in the Connecticut test did not permit these elasticities to be estimated.

2. Theoretical derivation of time-of-day model

Given the preliminary empirical observation from the time-of-day experimental data that the distribution of the relative load across households is more stable than are absolute consumption levels, it was decided to fit a two-level budgeting model to the Connecticut data. An advantage to this approach is that it reduces the complication, inherent in the control customers, that price depends on quantity consumed, i.e., the declining block rate structure. The main disadvantage to the approach is that a linear homogeneity assumption is required on the relative load which is unlikely to be strictly correct. Considered as an approximation, the usefulness of our approach is an empirical matter which we test in the process of estimating the model. We also take what is essentially a single equation approach to estimation (mainly due to computer software limitations). Thus, given the insufficient variation in experimental prices in the Connecticut experiment, only the own price effect will be estimated. By utilizing a multi-variate approach and imposing strict assumptions about the form of cross-price effects within the framework of consumer theory, it is possible to estimate cross-price effects. We will indicate the direction of this approach after discussing the two-level scheme used in the actual estimation.

2.1. *Two-level budgeting model*[2]

The model considers electricity consumption in a representative day. For simplicity, the household appliance holdings are assumed to be predetermined, and the dynamics of electricity consumption behavior across days or seasons is ignored. Thus, response of the appliance mix to time-of-day pricing is ignored. In time-of-day experiments of longer duration than the Connecticut experiment, it will be necessary to consider this important source of consumer response. Interday dynamics are less important; an example of such a response is the shifting of clothes washing to take advantage of weekend rates. However, in future detailed analyses of interactions among time-of-day rates, serial weather patterns, and 'storage' appliances, interday dynamics will require modeling.

Suppose the day is divided into a series of periods $t_1, t_2, \ldots, t_N$. These may correspond to fifteen-minute measurement periods, or may be longer intervals which aggregate a number of measurement periods. Let x_n denote the consumption of electricity (in kWh) during period t_n. Then, $x=(x_1, \ldots, x_n)$ is the vector of electricity consumption levels over the day. Under time-of-day pricing, there is a corresponding vector of electricity rates, $p=(p_1, \ldots, p_N)$ with p_n the cost (¢/kWh) in period t_n. For simplicity, assume time-of-day rates are constant with respect to kWh consumption levels, as is the structure of prices in the Connecticut experiment. Much of the model structure below would continue to hold, however, if the vector p specified only relative time-of-day

[2]The model structure described here was suggested by McFadden (1976); see also McFadden, Puig, and Kirschner (1977). Two level budgeting models were pioneered by Gorman and Strotz.

prices, with the overall level determined by a block rate structure depending on overall consumption.

The household is assumed to have a utility function depending on the vector of electricity consumption levels x and on the quantities consumed of all other commodities, denoted by x_0. (We shall treat x_0 as a single variable; the formulae below have an obvious reinterpretation when it is a vector.) Let p_0 denote the price of all other commodities, and let I denote the daily expenditure of the household. On an average, I should also equal daily income. The household then has the budget constraint

$$p_0x_0+p_1x_1+\ldots+p_NX_n=I. \tag{1}$$

Let $u=U(x_0,x)$ denote the household utility function, and let

$$\begin{aligned} v&=V\left(\frac{p_0}{I},\frac{p_1}{I},\ldots,\frac{p_N}{I}\right)\\ &=\max\left\{U(x_0,x)\left|\frac{p_0}{I}x_0+\frac{p_1}{I}x_1+\ldots\frac{p_N}{I}x_N=1\right.\right\} \end{aligned} \tag{2}$$

be the indirect utility function giving the maximum utility obtainable at prices $p_0,p_1,\ldots,p_N$ and expenditure I.

The two-level budgeting procedure imposes separability between electricity consumption and the consumption of other commodities. The basic idea behind the approach is that the household decides the total to be spent on electricity along with how much to spend on other commodities. This decision uses a price index for electricity consumption at different times of day along with the prices of all the other commodities. Given the total expenditure on electricity, the share of expenditure allocated to electricity consumption in each time period depends on relative electricity prices only. The utility function then has the special form $U(x_0,x_1,\ldots,x_N)\equiv W(x_0, f(x_1,\ldots,x_N))$. For the price index scheme to be feasible, it is sufficient to have f homogeneous of degree one.[3] Since f is homogeneous, the indirect utility function corresponding to f can be written in separable form as $r\phi(p_1,\ldots,p_N)$ where $r=\sum_{n=1}^N p_nx_n$, the total expenditure on electricity and ϕ might be thought of as the inverse of a price index. Thus the linear homogeneity assumption implies that if all electricity prices were to double, the relative allocation among periods of the day would not change, although total consumption would change. This is the restrictive assumption to which we previously referred.

Given these assumptions, the two-stage budgeting process corresponding

[3]Gorman (1974) has shown that this condition can be weakened slightly, but only to the extent of shifting the intercept of the Engel curve.

to the utility function $W(x_0, f(x_1, \dots x_N))$ leading to the corresponding indirect utility function is

$$
\begin{aligned}
&\max_x \left\{ W(x_0, f(x_1, \dots, x_n)) \middle| \sum_{n=0}^{N} p_n x_n = I \right\} \\
&= \max_r \{ W(x_0, r\phi(p_1, \dots, p_n)) | x_0 = (I - r)/p_0 \} \qquad (3) \\
&= V(p_0/I, 1/\phi(p_1/I, \dots, p_n/I)),
\end{aligned}
$$

where the homogeneity of degree one of ϕ has been used and the fact that $r = f(x, \dots, x_n)/\phi(p_1, \dots, p_n)$. Roy's identity gives demands from the indirect utility function as $x_n = -(\partial v/\partial p_n)/(\partial v/\partial I) \equiv -v_n/v_I$. Then, again using the linear homogeneity assumption and Euler's theorem so that $\partial\phi(p_1/I, \dots, p_n/I)/\partial I = -\phi(p_1/I, \dots, p_n/I)/I$, we have the demand function

$$x_n = -(V_2 \phi_n/\phi^2)/(p_0 V_1/I + V_2/\phi), \qquad n = 1, \dots, N. \tag{4}$$

Defining total electricity consumption as $\bar{X} \equiv \sum_{n=1}^{N} x_n$, the proportion of electricity consumption in period t_n is

$$s_n = \phi_n(p_1, \dots, p_N) \Big/ \sum_{m=1}^{N} \phi_m(p_1, \dots, p_n), \qquad n = 1, \dots, N, \tag{5}$$

or consumption in period n, relative to consumption in a 'base period' N, is

$$
\begin{aligned}
h_n &= s_n/s_N = x_n/x_N \\
&= \phi_n(p_1, \dots, p_N)/\phi_N(p_1, \dots, p_N) \\
&= \phi_n(p_1/p_N, \dots, p_{N-1}/p_N)/\phi_N(p_1/p_N, \dots, p_{N-1}/p_N), \qquad (6) \\
&\qquad n = 1, \dots, N-1,
\end{aligned}
$$

where the last expression follows using the homogeneity of ϕ. Taking a first order Taylor expansion to this function gives

$$h_n \simeq \theta_n + \sum_{m=1}^{N-1} \alpha_{mn} \frac{p_m}{p_N}, \qquad n = 1, \dots, N-1. \tag{7}$$

The effects of appliance holdings, socioeconomic factors, and weather are incorporated both through the intercept parameters θ_n and through the slope parameter by specifying α_{mn} as a (linear-in-parameter) function of these variables. Thus, adding a stochastic term ε_n would lead to an equation of the form

$$\begin{aligned} h_n = &\sum_{j=1}^{J} \beta_{jn}\text{app}_j + \sum_{k=1}^{K} \gamma_{kn}\text{soc}_k + \delta_n \text{weather} \\ &+ \sum_{m=1}^{N-1} \psi_{mn}\frac{p_m}{p_N} + \sum_{j=1}^{J}\sum_{m=1}^{N} b_{jmn}\frac{p_m}{p_N}\text{app}_j \\ &+ \sum_{k=1}^{K}\sum_{m=1}^{N-1} c_{kmn}\frac{p_m}{p_N}\text{soc}_k + \sum_{m=1}^{N-1} d_{mn}\frac{p_m}{p_N}\text{weather} + \varepsilon_n, \end{aligned} \tag{8}$$

where app_j are appliance variables, soc_k are socioeconomic variables, and ε_n is the disturbance. In principle both own price effects and cross price effects could be estimated from this relative consumption equation. Note that normalization has eliminated the statistical problem (at this stage) of having prices dependent on quantity for the control observations when electricity price depends on total consumption $\bar{X}$. Unfortunately, due to lack of sufficient variation of the p_m/p_N variable in the Connecticut sample design, only own-price effects can be recovered using this approach. The column rank of the price variable submatrix is one so that all but the own price effect is absorbed into the constant. We will later show that by imposing additional assumptions using consumer theory, these cross-price effects could be estimated in restricted form. But since the restrictions could not be tested in the present dataset, we leave this model to other experiments where the restrictions could be tested.

This specification permits estimation of both the relative consumption and the proportion of electricity consumption in each period. To estimate total expenditure on electricity we form the price index to use in the first stage of the budgeting process. The linear homogeneity of $f(x_1,..,x_N)$ implies that the correct price index to use is the estimated demand weighted average price of electricity since it corresponds to the unit cost function for utility

$$\begin{aligned} \bar{p} &= r/\bar{X} \\ &= \sum_{n=1}^{N} p_n x_n \Big/ \sum_{n=1}^{N} x_n \\ &= \sum_{n=1}^{N} p_n\left(\theta_n + \sum_{m=1}^{N-1} \alpha_{mn}\frac{p_m}{p_N}\right) \Big/ \sum_{n=1}^{N}\left(\theta_n + \sum_{m=1}^{N-1} \alpha_{mn}\frac{p_m}{p_N}\right), \end{aligned} \tag{9}$$

where the estimates from the previous stage are used for the θ_n and α_{mn} parameters. The price index corresponds to the unit cost of electricity consumption and is now used with eq. (3) to derive a demand function for total daily electricity consumption. Again, using a Taylor expansion on the demand $\bar{X}$ derived from using Roy's identity to eq. (3) gives total consumption $\bar{X}$ for day l,

$$\begin{aligned}\bar{X}_l &= \sum_{n=1}^{N} x_{nl} \\ &= \sum_{j=1}^{J} \beta_j \text{app}_j + \sum_{k=1}^{k} \gamma_k \text{soc}_{kl} + \delta \text{weather}_l + \psi \bar{p} \\ &\quad + \sum_{j=1}^{J} b_j \bar{p}_l \text{app}_j + \sum_{k=1}^{k} c_k \bar{p}_l \text{soc}_{kl} + \delta \bar{p}_l \text{weather}_l + \varepsilon_l. \end{aligned} \tag{10}$$

For the pre-experimental period when a declining block rate structure was in use rather than time of day pricing, p_n is constant across all periods so that $\bar{p} = p_n$. However, an additional complication arises because $\bar{p}$ is a function of total monthly consumption, $p = g(\sum_{l=1}^{M} \bar{X}_l) = g(\bar{X})$ due to the block structure which causes marginal price to depend on monthly demand. A question arises of which is the appropriate price from the rate structure to use and how to avoid the simultaneity problem of having price, a right-hand side variable, depend on quantity consumed, the left-hand side variable. These problems are resolved in the next section when the demand system is estimated.

2.2. *Multivariate approach*

The single equation approach used in our empirical analysis ignores two potential sources of information: cross-equation restrictions which arise because the ϕ function is common to all the h_n equations, and restrictions which arise from the maximization hypothesis of consumer theory. We now outline a fairly general approach which imposes this information in the form of parameter restrictions. Here we do not follow the two-stage budgeting approach, with the result that the indirect utility function has the unrestricted form $V(p_0/I, p_1/I, \ldots, p_N/I)$. Then by Roy's identity

$$x_n = v_n \Big/ \sum_{m=0}^{N} \frac{p_m}{I} v_m. \tag{11}$$

Therefore, total expenditure on electricity

$$r = \sum_{n=0}^{N} p_n x_n = \sum_{n=1}^{N} p_n v_n \Big/ \sum_{m=0}^{N} \frac{p_m}{I} v_m. \tag{12}$$

The proportion of electricity consumption in period n is

$$s_n = x_n/\bar{X} = v_n \Big/ \sum_{m=1}^{N} v_m, \tag{13}$$

and relative to the base period N,

$$h_n = s_n/s_N = v_n/v_N. \tag{14}$$

Once an assumption is made in the form of the indirect utility function v. any of the above equations could be estimated. A relatively general function form which serves as a linear-in-parameter approximation to v is the translog form of Jorgenson and Lau,

$$v = \sum_{n=0}^{N} \theta_n \log \frac{p_n}{I} + \tfrac{1}{2} \sum_{n=0}^{N} \sum_{m=0}^{N} \alpha_{nm} \left(\log \frac{p_n}{I}\right)\left(\log \frac{p_m}{I}\right). \tag{15}$$

This functional form leaves $N^2 + N$ parameters to be estimated. However, the maximization hypothesis of consumer theory implies symmetry ($\alpha_{nm} = \alpha_{mn}$), which gives $N(N-1)/2$ restrictions. Still, any of the three equations determining x_n, s_n, or h_n is non-linear in parameters and therefore complicated to estimate. A sufficient assumption to attain linearity in parameters is to assume that all income elasticities are unity, or equivalently that V is linear homogeneous in prices. This additional restriction implies $\sum_{n=0}^{N} \theta_n = 1$, and $\sum_{m=0}^{N} \alpha_{mn} = 0$ which gives $N+1$ further restrictions. Then

$$x_n = \frac{I}{p_n}\left[\theta_n + \sum_{m=0}^{N} \alpha_{mn} \log \frac{p_m}{I}\right], \tag{16}$$

and expenditure shares are

$$s_n = \frac{p_n x_n}{I} = \theta_n + \sum_{m=0}^{N} \alpha_{mn} \log \frac{p_m}{I}. \tag{17}$$

Now the cross-price effects arise from the α_{mn}, which has $N(N+1)/2$ independent parameters, while the Connecticut data has a price submatrix of rank $2N-1$. Thus, to identify all price effects $(N-2)(N-1)/2$, restrictions must be imposed *a priori* and *cannot be tested*. If the own-price effects are unrestricted then $N-1$ cross-price effects are permitted. Two possible sets of restrictions come to mind: $\alpha_{mn} = \gamma_m \gamma_n$ for $m \neq n$ with $\sum_{n=1}^{N} \gamma_n = 0$ so the cross-price effect has a multiplicative form from 'spillover' propensities for each period, or $\alpha_{mn}=\beta_{(m-n)}$ so only temporal distance matters. This approach then would include socioeconomic factors, appliance holdings, and weather in the θ_n and α_{mn} specification, as did our earlier approach. While the current approach relies heavily on consumer theory and the linear homogeneity assumption, it does permit estimation of all desired price effects while the earlier single equation approach requires $N(N-2)$ additional restrictions to identify any price effect beyond an own-price effect and a single cross-price effect between a given period and all other periods.

3. Estimation of the two level demand system

Given the development of the theoretical two level demand system we now proceed to estimate it on a sample of 150 households using data from both the pre-experimental period and the time-of-day experimental period. An additional 20 households in the sample are not used to estimate the unknown parameters; instead they are used for testing and validation of the model. Two levels of household demand are estimated. The first level which corresponds to relative demands for electricity is estimated on daily data on an hourly basis using the specification of eq. (8). Then the price index $\bar{p}$ is estimated by eq. (9) for the experimental period or instrumental variable estimates for the declining block rate period (pre-experiment) are used to estimate a daily consumption function for electricity using the specification of eq. (10). Using those two equations allows us to forecast both the relative system load and the absolute system load for different hours, days, and seasons of the year. Thus, the estimation procedure allows analysis and forecasts of household demand using a consistent aggregation scheme on the household level for the important price variable during the experiment and a consistent estimation scheme for the demand determined marginal price during the pre-experimental period.

Before presenting the actual estimates some potentially important econometric issues need to be discussed. First, the sample of households is not a random sample in two respects. The sample was chosen by stratification on an endogenous variable, annual consumption for the year 1973. Since the residuals from almost any specification of demand over time will be correlated due to unobserved permanent household effects, this stratification will affect the distribution of residuals in all later periods. For instance, large users in the highest stratum are ten times as likely to occur in the sample as customers in the lowest stratum (*Final Report*, p. 2). Thus, even two years later, it is likely that the conditional expectation of the error term is positive rather than zero. Thus least squares type techniques will lead to inconsistent estimates. To counteract this problem; inverse sample weights are used in a weighted least squares procedure to yield consistent estimates. As Hausman and Wise (forthcoming) demonstrate, these estimates are not efficient: more important is the fact that the reported standard errors are likely to be downward biased.[4] This downward bias should be kept in mind when interpreting the results. The second respect in which the sample is not random is potentially more important and cannot be so easily fixed. Individual households voluntarily agreed to participate in the experiment. Over 12% of the households interviewed declined to participate, raising the possibility that the experimental participants anticipated more advantage from the experimental TOD rates than those people who declined. While an analysis (*Final Report*, p. 26) was conducted to check on the representative nature of the sample, it is likely that unobserved attributes are important in consumer response and these attributes cannot be checked. In a demand study of the Arizona TOD experiment, Aigner and Hausman (1978) found that the peak period elasticity declined significantly when the voluntary choice was modelled. Thus, the possibility exists that the elasticities of response are overstated relative to the population as a whole. The last potential econometric problem is that the appliance stock is taken as predetermined and unchanging, while only the pre-experimental marginal price is treated as endogenous. While the short-term nature of the experiment precludes any other treatment, inspection of the daily consumption equations indicates the likely event that some high electricity using appliances, such as air conditioners, were purchased during the course of the experiment. Since the choice of model characteristics of the appliances are affected by the price regime during the experiment, these appliances (which enter the error term) are likely to create correlation with the right-hand side variables. This problem is unlikely to be too important, since the occurrence of such events seems to be relatively small as an inspection of the estimated residuals in the daily consumption equation indicates.

3.1. First level demand estimation

The first level of the two level demand system was estimated on the basis of four periods: weekdays and weekends during two months in the winter (January–February) and two months in the summer (July–August) which correspond to the system peak demand periods. The total number of periods used are $N=17$ from eq. (5). Each hour during the day was treated as a separate demand period with the night-time hours of 11 p.m. to 7 a.m. being aggregated to form the base period. Thus, in the pre-experimental period the price relative to the base period is a constant set to one since price varies only with total consumption, not with relative consumption across periods.

[4]Unfortunately, the correction procedure is quite complicated so we did not attempt to use it here. The appropriate technique is to use maximum likelihood as Hausman–Wise do for their particular case of the Gary NIT experiment.

During the experiment, the hourly relative rates differ due to the TOD structure of hourly prices. Fig. 1 indicates the relative prices for the 17 periods of winter weekdays. The specification for the relative demand eq. (8) has prices, appliance holdings, socioeconomic and weather variables entering both in level form and as interactions. With the addition of day of week variables, the total number of coefficients for winter weekdays is 36. Since each of the 16 hours has its own set of estimated coefficients, presentation of the complete model is left to the final Charles River Associates (CRA) report. Instead, we present the results for three hours during winter weekdays, corresponding to one peak hour, one intermediate hour, and one off-peak hour.

The estimated coefficients for one peak period, one intermediate period, and one off-peak period are presented in table 1. In interpreting the coefficients, it is important to remember that these demand equations are relative to the base period which is the middle of the night when presumably little substitution occurs.[5] In the long run, considerably increased substitution would occur in this period due to the use of storage heaters and timed appliance use. That is, P_N in eq. (7) is the off-peak rate, and we have aggregated the eight night hours into a base period. The coefficient of the variables corresponding to appliances generally have the correct sign and are estimated precisely. An appliance like an electric range or clothes dryer should have a positive coefficient since they are operated during the day, and thus they raise consumption relative to the base period if they are present in a household. For instance, 60% of the households have electric dryers and relative consumption rises by 22% during the 9–10 a.m. period if a dryer is present. Alternatively, an appliance like a freezer should have a negative coefficient since it is operated at all times of day. But a freezer increases consumption in the numerator period less proportionately than it increases consumption in the denominator period and since log of the left-hand side variable is used to reduce heteroscedasticity, the coefficient is negative. Most of the appliance coefficients fall into this expected pattern; while dishwasher may initially look incorrect, its interaction with the electric water heater has the expected sign pattern. The socioeconomic variables of home-during-the-day and number-of-persons have the expected signs. The interpretation of the income coefficient is not totally in one direction, but its sign should be positive as is found.

The effect of price comes through mainly in the interaction terms. Higher prices cause people to reduce their heating consumption in periods when these higher prices are in effect, and the effect spills over to the off-peak

[5]It is possible that people could buy timing devices to operate appliances at night or use heat storage devices to take advantage of the low rate. Virtually no appliances of this sort were purchased during the experiment, however, although a repurchase agreement by the electric utility has been made.

	A.M.					P.M.											P.M.–A.M.
	7–8 (1)	8–9 (2)	9–10 (3)	10–11 (4)	11–12 (5)	12–1 (6)	1–2 (7)	2–3 (8)	3–4 (9)	4–5 (10)	5–6 (11)	6–7 (12)	7–8 (13)	8–9 (14)	9–10 (15)	10–11 (16)	11–7 (17)
Pre-experiment relative price	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Experimental relative price	1	3	16	16	3	3	3	3	3	3	16	16	3	3	1	1	1

Fig. 1

Table 1
Winter weekdays parameter estimates.[a]

Variable	Period 3 (peak) 9–10 a.m.	Period 8 (intermediate) 2–3 p.m.	Period 16 (off-peak) 10–11 p.m.
(1) Constant	−2.23	−2.37	−1.38
	(0.049)	(0.082)	(0.073)
(2) Heat[b]	0.039	−0.163	−0.356
	(0.063)	(0.098)	(0.088)
(3) Range	0.166	0.374	−0.126
	(0.035)	(0.058)	(0.053)
(4) Water heater	0.0100	0.545	0.058
	(0.045)	(0.076)	(0.068)
(5) Dishwasher	−0.064	−0.215	0.152
	(0.032)	(0.055)	(0.050)
(6) Clothes washer	−0.079	−0.899	−0.411
× water heater	(0.050)	(0.084)	(0.075)
(7) Clothes dryer	0.222	0.450	0.705
	(0.027)	(0.046)	(0.041)
(8) Dishwasher	0.307	0.266	0.192
× water heater	(0.064)	(0.108)	(0.096)
(9) Freezer	−0.283	−0.407	−0.462
	(0.029)	(0.048)	(0.043)
(10) Home during day	0.257	0.079	−0.228
	(0.033)	(0.057)	(0.051)
(11) Home Sq. Ft.	0.084	0.114	−0.076
	(0.031)	(0.052)	(0.046)
(12) Home type[c]	−0.395	−0.206	−0.316
	(0.026)	(0.045)	(0.040)
(13) Persons	0.070	0.038	0.065
	(0.013)	(0.021)	(0.019)
(14) Income (1000's)	0.0058	0.0074	0.0020
	(0.0021)	(0.0036)	(0.0032)
(15) Relative price (p)	0.011	0.137	0.019
	(0.004)	(0.037)	(0.033)
(16) $p \times$ heat	−0.012	−0.283	−0.021
	(0.005)	(0.042)	(0.038)
(17) $p \times$ range	−0.020	−0.031	0.073
	(0.003)	(0.023)	(0.021)
(18) $p \times$ water heater	−0.018	−0.208	−0.200
	(0.004)	(0.031)	(0.028)
(19) $p \times$ dishwasher	0.014	0.062	0.066
	(0.003)	(0.025)	(0.022)
(20) $p \times$ clothes washer	0.043	0.379	0.266
× water heater	(0.004)	(0.035)	(0.031)
(21) $p \times$ clothes dryer	−0.022	−0.181	−0.141
	(0.002)	(0.021)	(0.018)
(22) $p \times$ dishwasher	−0.027	−0.017	−0.067
× water heater	(0.005)	(0.048)	(0.042)
(23) $p \times$ freezer	0.018	0.123	−0.036
	(0.002)	(0.021)	(0.018)
(24) $p \times$ home days	−0.005	0.061	0.118
	(0.003)	(0.024)	(0.022)

Table 1 (continued)

Variable	Period 3 (peak) 9–10 a.m.	Period 8 (intermediate) 2–3 p.m.	Period 16 (off-peak) 10–11 p.m.
(25) $p \times$ home sq. ft.	−0.010	−0.084	0.019
	(0.002)	(0.022)	(0.019)
(26) $p \times$ home type	0.021	−0.024	0.030
	(0.002)	(0.019)	(0.017)
(27) $p \times$ persons	−0.002	0.0098	−0.025
	(0.001)	(0.0094)	(0.008)
(28) $p \times$ income	−0.0006	−0.0065	−0.0028
	(0.0002)	(0.0015)	(0.0014)
(29) Temperature	−0.0006	0.0024	−0.012
	(0.021)	(0.0045)	(0.003)
(30) $p \times$ temperature	0.0002	−0.0002	0.004
	(0.0002)	(0.0017)	(0.001)
(31) Temperature $\times$ heat	0.037	−0.047	0.034
	(0.011)	(0.022)	(0.017)
(32) $p \times$ temperature $\times$ heat	0.0006	0.024	−0.020
	(0.0008)	(0.008)	(0.007)
(33) Tuesday	−0.011	−0.107	0.002
	(0.021)	(0.025)	(0.023)
(34) Wednesday	−0.043	−0.079	0.051
	(0.022)	(0.026)	(0.023)
(35) Thursday	−0.011	−0.109	0.032
	(0.022)	(0.026)	(0.022)
(36) Friday	−0.029	−0.153	0.050
	(0.021)	(0.024)	(0.021)
NOBS	8198	7986	7973
S.E.E.	0.786	0.906	0.808
R^2	0.861	0.835	0.772

[a]The reported standard errors (given in parentheses) here are from the weighted least squares estimates. Since we use a time series of cross sections with 60 observations per household, however, the standard errors are understated since no account is taken of the non-independence of the observations. In a variance components framework including a household effect and a random effect, the household effect comprises between 14% and 35% of the total variance for the different regressions in the paper. Thus, an upper bound on the understatement of standard errors is between 3 and 4 times. Generalized least squares is the appropriate estimator, but due to a constrained budget, it was not undertaken. The parameter estimates would hardly change due to the large number of observations. On the other hand, GLS would lead to more efficient estimates so the final result could well be *lower* standard errors than we reported. Thus, the only firm conclusion we can draw is that the parameter estimates would not change; the final direction of change in standard errors cannot be calculated on *a priori* grounds.

[b]Corresponds to a dummy variable indicating that an electric appliance of the type indicated is owned by the household.

[c]Corresponds to a dummy variable indicating a multiple family dwelling.

period in a weaker form through non-adjustment of the thermostat. Likewise, households lessen their use of electric ranges during peak periods and shift their use to off-peak periods which goes along with the sign reversal in table 1. On the other hand, the sign pattern of the price-freezer interaction has exactly the opposite pattern since during the high-priced periods the numerator of relative consumption falls more than the denominator. We find the coefficient of the price-income interaction difficult to interpret. Overall price elasticity is negative in each hour, as expected. The morning peak elasticities for 9–11 a.m. average −0.22 while the evening peak elasticity for 5–7 p.m. averages −0.13. Thus, daily household tasks seem more substitutable than supper-time activities. The intermediate period elasticities are lowest near the peak periods and attain a maximum value of −0.21 from 2–3 p.m. Temperature seems to have little effect on relative consumption, although the coefficients here can be contaminated due to slowly changing temperature conditions. Lastly, the day-of-the-week variables, all measured relative to Monday, reinforce the presumption that people still wash and dry their clothes more on Monday than any other day of the week. However, on this interpretation they did do more of their wash in intermediate price periods than in peak price periods as we would expect. As we noted in section 2, relative consumption seems much more stable than absolute consumption which appears in our R^2's of around 0.8 in table 1, which is quite high for cross section data. Thus, the first stage equation seems to be quite reasonable. We will further investigate its properties in the next section when we discuss model validation.

3.2. *Construction of appropriate prices*

Given the estimates of relative consumption at the first level in our two level system, we proceed to estimate the aggregate daily eq. (10). This daily equation when combined with the hourly relative equations estimates the absolute system load during each hour of the day. Before presenting estimates from the daily consumption equations, we must specify how the price of electricity was entered into the demand equations. During the pre-experimental period, residential electricity rates were not time differentiated; rather a declining block rate structure was in effect. In this structure, after an initial connection fee the price per kWh declines as quantity consumed increases. The pre-experimental rate structure in Connecticut during the pre-experimental period had the form (October 1975) shown in fig. 2. Thus, it is not immediately evident which is the correct price to enter into the household demand function. Given the household demand, one might use either average price or some marginal price, but this procedure introduces a simultaneous equation problem as Taylor (1976) and others have pointed out. Exactly the same problem occurs with a progressive tax system in the

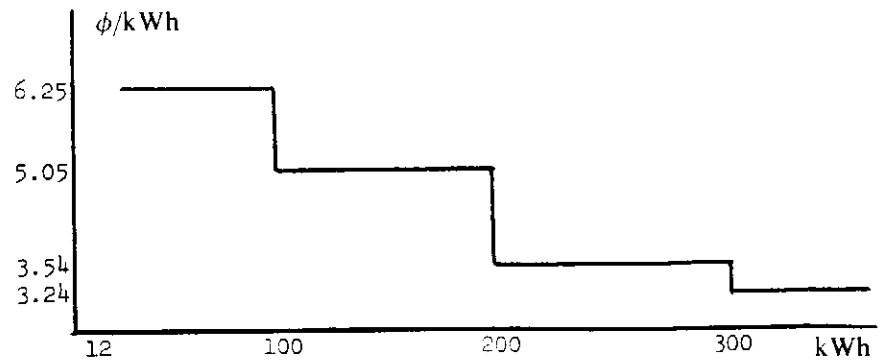

Fig. 2

estimation of labor supply. Thus, we use a technique analogous to the one used by Hausman and Wise (1976), and Rosen (1976) in the labor supply case and by McFadden, Puig, and Kirschner (1977) in the electricity demand case.[6] The basic idea is to linearize a budget set at a reduced form prediction of monthly consumption and then use instrumental variables to estimate daily consumption using predicted marginal price and the lump sum payment to characterize the budget set.[7] The familiar two good diagram of fig. 3 where consumption of electricity is on the horizontal axis and consumption of the composite good illustrates the situation. Here a three-part declining block rate shows each segment linearized with marginal prices $p_1 > p_2 > p_3$. The appropriate income for each of the linearized budget sets is $y_1, \tilde{y}_2, \tilde{y}_3$ where the latter two correspond to 'virtual' income since they are not actually observed. It is straightforward to show by integration that $y_1 - \tilde{y}_2$ equals the corresponding lump sum payment from fig. 2 so that its coefficient in the demand equation should be the same magnitude as the income coefficient with an opposite sign. For instance, if predicted monthly consumption is between 200 and 300 kWh then in fig. 2 the marginal price is 3.54¢/kWh with associated lump sum payment of the fixed fee ($2.27) plus the amount paid in excess of the marginal price for the amount consumed up to the block where predicted demand falls. For fig. 2 this lump sum amounts to $6.16. This linearized budget set then is equivalent to the declining block rate structure the household faces, and the instrumental variable procedure eliminates the potential simultaneity problem.

During the experimental period, the household faced three different prices depending on the time of day. The question of the correct price to enter into

[6]One aspect of the problem which this approach does not capture adequately is the non-convexity of the budget set which leads to intervals around the kink points being non-utility maximizing points. Due to the non-convexities individual (utility maximizing) demand functions are discontinuous in the price space at certain points. A treatment of this problem within the labor supply context is given by Burtless and Hausman (1978).

[7]It is important to use a true instrumental variable estimator in this situation and not a two-stage least squares procedure since the latter estimator is inconsistent because of the nonlinear price schedule.

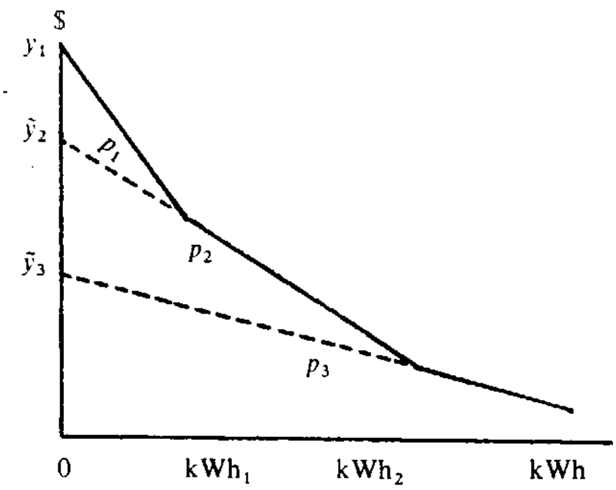

Fig. 3

a daily demand equation is basically a question of index number theory. An hourly weighted average on demand weighted averages of the Laspyres or Paasche type are possible choices. However, within our two-level demand approach, the correct choice arises from eq. (9) which gives a weighted average using the estimated coefficients from the first level of demand. This estimated demand weighted average price indicates the amount of substitution that each household practiced to miss the higher price periods. For instance, during winter weekdays for the price schedule shown in fig. 1, the mean and standard deviation of the price index are 3.89¢/kWh and 0.320 respectively with a high estimated price index of 4.66 and a low value of 3.25. These price indices can be compared to the winter of the pre-experimental period when the average marginal price was 3.34¢. During this period 137 of the 150 households were forecast to be in the same block with a marginal price of 3.31¢/kWh and a lump sum payment of \$4.07. Thus, comparing the price indices, it appears that households faced a *higher* marginal price during the experiment of about 16%. Since electricity is certainly a normal good with a positive income elasticity, unless removal of the \$4 lump sum payment induces sufficient additional consumption to overcome the price effect, we would expect to see a decrease in daily consumption during the experiment. A decrease in consumption did occur with approximately a 1% decline from the previous year and a 5% decline compared to the controls during the same year which allows us to control for weather differences.[8] Thus, the higher marginal prices faced during the experiment

[8]During the month of January 1976, experimental demand is significantly greater than both the previous year and than the controls. Thus, even accounting for weather, we would have an elasticity with the wrong sign. For February and the average of the two winter months, the elasticity has the correct sign. Both summer months have the correct sign since marginal price again increased during the experiment while consumption decreased.

seem to outweigh the removal of the lump sum payment and result in a decline in overall consumption.

3.3. Second level demand estimation

Four different daily demand equations are estimated using the specification of eq. (10). These equations correspond to weekday and weekend demand for the winter and summer. Table 2 presents the weekday consumption for summer and winter, leaving discussion of the weekend result for the final CRA report. The first point to note is that the daily demand equations are much less precise than the first level hourly relative demand equations. The R^2's fall dramatically, and this problem in the absolute demand equation is one reason that we found the two level approach to be attractive. Next to be noted is that the appliance coefficients generally have the correct total effect. For instance, at the mean of the sample presence of electric heat adds 50.7 kWh/day during the winter while a 1% rise in the marginal price leads to a reduction in weekday electricity consumption due to electric heating of 2.45 kWh/day or about 4.3%. It is important to note in calculating these demand derivatives that a change in price not only affects the marginal price terms, but also affects the lump sum payment. Both effects need to be accounted for in the calculations. Thus, households did seem to lower their thermostats during the TOD experiments relative to the previous year. On the other hand, the presence of a freezer adds 1.24 kWh/day to electricity consumption, but the price elasticity is only 0.68% which is not significantly different from zero, which reflects the non-discretionary behavior of freezer electricity consumption. The one significant disappointment in the estimates arises from the demand effect of the lump sum income effect. As demonstrated above, its total effect should be equal in magnitude but opposite in sign to the income effects. During the winter weekdays, the income elasticity at the mean of the sample is 0.230 while the demand elasticity with respect to the lump sum payment is 0.207, which has the wrong sign. For summer weekdays the income elasticity is 0.118 while the lump sum elasticity is estimated to be 0.029. The likely reason for the failure of the elasticities to have the correct sign probably lies with the lack of variation in the lump sum payments during the pre-experimental period. Since 137 out of 150 households are forecast to be in the same block, while, of course, the lump sum payment is an identical $2.00 during the experiment, the variable is very close to becoming a dummy variable for the two periods and thus merely representing a time effect. The lack of sufficient variation in prices does not permit accurate estimates to be made of this lump sum payment effect. Furthermore, since average income in the sample is around $14,000 the income effect of the lump sum payment is miniscule compared to yearly income. Unless the income elasticity of electricity is extremely high, change in

Table 2

Weekday consumption estimates (standard error given in parentheses).

Variable	Winter	Summer
(1) Constant	−136.8	−29.6
	(6.67)	(12.0)
(2) Heat	−24.9	—
	(10.0)	
(3) Central air conditioner	—	−375.5
		(84.9)
(4) Window air conditioner	—	−76.7
		(26.8)
(5) Range	46.2	17.4
	(3.43)	(1.92)
(6) Water heater	−389.9	4.85
	(11.3)	(2.78)
(7) Dishwasher	−3.36	19.2
	(3.04)	(3.25)
(8) Clothes washer × water heater	392.2	−1.64
	(11.2)	(3.98)
(9) Clothes dryer	7.92	6.96
	(2.61)	(2.09)
(10) Dishwasher × water heater	13.1	−17.32
	(6.46)	(7.90)
(11) Freezer	−31.4	−0.421
	(2.74)	(2.05)
(12) Home during day	29.6	26.9
	(3.60)	(3.48)
(13) Home sq. ft.	−1.52	0.182
	(3.09)	(1.88)
(14) Home type	14.4	−9.39
	(2.65)	(1.76)
(15) Persons	−0.067	−0.468
	(1.25)	(0.828)
(16) Income (1000's)	3.19	0.577
	(0.370)	(0.184)
(17) Marginal price (p)	30.9	4.48
	(1.71)	(3.31)
(18) Lump sum payment	4.85	0.828
	(0.234)	(0.109)
(19) $p \times$ heat	20.98	—
	(2.86)	—
(20) $p \times$ central air conditioner	—	103.0
		(24.3)
(21) $p \times$ window air conditioner	—	17.4
		(7.95)
(22) $p \times$ range	−11.14	−3.58
	(0.902)	(0.559)
(23) $p \times$ water heater	108.72	−0.717
	(3.13)	(0.804)
(24) $p \times$ dishwasher	−1.21	−5.89
	(0.815)	(0.922)
(25) $p \times$ clotheswasher × water heater	−106.8	2.75
	(3.11)	(1.15)

Table 2 (continued)

Variable	Winter	Summer
(26) $p \times$ clothes dryer	−0.298	0.512
	(0.687)	(0.583)
(27) $p \times$ dishwasher	−0.007	5.06
$\times$ water heater	(1.76)	(2.36)
(28) $p \times$ freezer	8.69	0.239
	(0.739)	(0.582)
(29) $p \times$ home days	−6.40	−6.71
	(0.964)	(1.00)
(30) $p \times$ home sq. ft.	2.00	1.88
	(0.828)	(0.533)
(31) $p \times$ home type	−4.41	1.13
	(0.699)	(0.467)
(32) $p \times$ persons	1.04	0.626
	(0.338)	(0.238)
(33) $p \times$ income	−0.760	−0.123
	(0.094)	(0.050)
(34) Lump sum $\times$ income	−0.114	−0.034
	(0.015)	(0.0071)
(35) Temperature	−0.313	—
	(0.214)	
(36) $p \times$ temperature	0.037	—
	(0.055)	
(37) Temperature $\times$ heat	14.78	—
	(1.61)	
(38) $p \times$ temperature	−4.60	—
$\times$ heat	(0.463)	
(39) Temperature humidity	—	0.056
index (THI)		(0.168)
(40) $p \times$ THI	—	0.016
		(0.046)
(41) THI $\times$ central air conditioner	—	6.04
		(1.26)
(42) $p \times$ THI	—	−1.66
$\times$ central air conditioner		(0.361)
(43) THI $\times$ window air conditioner	—	1.71
		(0.399)
(44) $p \times$ THI	—	−0.427
$\times$ window air conditioner		(0.118)
(45) Tuesday	0.142	0.102
	(0.331)	(0.232)
(46) Wednesday	−0.127	0.175
	(0.347)	(0.235)
(47) Thursday	−0.462	0.090
	(0.330)	(0.232)
(48) Friday	0.016	−0.372
	(0.314)	(0.233)
NOBS	8116	9338
S.E.E.	12.16	9.41
R^2	0.187	0.382

the lump sum payment should have almost no effect on demand, which is in accord with our empirical finding. This effect could be potentially important. since a lump sum payment scheme might be an important part of a time-of-day pricing structure where the lump sum payment covered part or all of the fixed costs of electricity generation. Then the time of day change would cover the marginal costs of generation, and the pricing system might more closely resemble the pattern which arises from economic theory at a first best optimum.

In this section we have presented the estimates of our two level demand system. The first level which estimates the relative load curve is quite successful in capturing the pattern of hourly demands for electricity. From these first level demands we construct the price index which enters the second level daily consumption demand equations. Our specification is considerably less successful at this level in capturing demand behavior. In the next section we report some limited attempts at validation of the forecasting properties of the two level system. Again, the performance in forecasting the relative load curve is superior to forecasting the absolute levels.

4. Model validation

Twenty households from the sample, including four from each of the five strata, were not used in the estimation procedure. Instead, they are used to attempt to validate the results of the demand system estimation in the previous section.[9] Forecasts are made using the estimated coefficients for the last five days in February and the last five days in August although here we present only the winter results. First, we consider the relative load forecasts using eq. (8) and the estimates from table 1. Then the price indices are estimated using eq. (9) and used in eq. (10) along with the coefficient estimates from eq. (10) to forecast daily consumption. Lastly, these two sets of forecasts are combined to forecast absolute system load.

In fig. 4 we first present the actual average relative consumption and the forecast average relative consumption for the validation sample of twenty households. The four worst forecasts (% error greater than 20%) are the periods from 7–8 a.m., 11–12 a.m., 1–2 p.m., and 7–8 p.m. The forecasts do well in periods of peak TOD prices with the average absolute percentage error being about 12%. A $\chi^2(4)$ test of prediction confidence intervals, taking the estimated coefficients as known with certainty, yields a value of 9.08 which is not significant at the 5% level. Thus, our peak demand forecasts do fairly well in forecasting the relative load curve. The intermediate period

[9]The mean consumption levels of the validation sample differ significantly from the corresponding estimation sample mean so that the validation exercise is not bound to succeed through the law of large numbers given unbiased estimates. The estimates are made using *both* pre-experimental and experimental data, while the forecasts use only the latter data.

	A.M.					P.M.											P.M.–A.M.
	7–8 (1)	8–9 (2)	9–10 (3)	10–11 (4)	11–12 (5)	12–1 (6)	1–2 (7)	2–3 (8)	3–4 (9)	4–5 (10)	5–6 (11)	6–7 (12)	7–8 (13)	8–9 (14)	9–10 (15)	10–11 (16)	11–7 (17)
Relative price	1	3	16	16	3	3	3	3	3	3	16	16	3	3	1	1	1
Actual demand	0.046	0.036	0.036	0.032	0.045	0.037	0.043	0.033	0.034	0.041	0.047	0.048	0.046	0.055	0.068	0.067	0.285
Forecast demand	0.063	0.039	0.032	0.027	0.034	0.033	0.030	0.027	0.031	0.048	0.056	0.048	0.062	0.066	0.067	0.068	0.267

Fig. 4

forecasts are less successful with three out of nine periods having relatively large forecast errors. The off-peak forecasts do well at night with the average absolute forecast error equal to 2.9% during the three night periods. However, the morning period off-peak forecast during 7–8 a.m. has the largest forecast error of the entire day of nearly 38%. It is of interest to note that the best forecasts are made during the peak periods since these periods are of the most importance in designing system capacity and pricing schemes since system peaks involve the use of the highest unit cost capacity for electricity generation.

Another way to consider the share forecasts is to aggregate them into peak, intermediate, and off-peak periods. These results presented in fig. 5 show remarkable accuracy – except for rounding error they are exactly correct! Since actual peak usage in the validation group occurs in an off-peak price period, this forecasting accuracy seems extremely good. Thus, we may conclude that while our forecasts do not capture all the hourly dynamics, they certainly explain the economics of demand shifts among the three periods when prices differ in a time of day pattern.

Period	Forecast	Actual
(1) Peak	0.163	0.163
(2) Intermediate	0.370	0.370
(3) Off-peak	0.465	0.466

Fig. 5

The next step in the forecast validation procedure involves computing the price index $\bar{p}$ from eq. (9). The mean $\bar{p}$ over the twenty validation households is 4.15¢ for winter weekdays which is 0.8 standard deviations greater than the mean 3.89¢ of the estimation sample. These prices are then used in eq. (10) to forecast average daily electricity demand. In fig. 5 we present the mean forecasts for the four households in each of the five sample strata and then give the demand weighted average which represents the appropriate population forecast.

Fig. 6 gives the expected result of overprediction for stratum 1 and underprediction for stratum 5. This result occurs because of the sample stratification used to design the experiment. Since part of the reason that a household falls in stratum 1 is due to unobserved individual effects which persist over time, the expectations of the forecast error is negative in stratum 1 and positive in stratum 5. Thus performing a χ^2 test on each stratum or the arithmetic average across strata is incorrect for the same

Strata	1	2	3	4	5	Weighted average
% of population	0.46	0.22	0.16	0.10	0.05	–
Actual demand (kWh)	14.5	17.5	35.0	52.2	107.6	26.9
Forecast demand	18.1	15.5	25.6	56.1	78.3	25.5

Fig. 6

reason that ordinary least squares on the original data yields biased estimates. Thus, the analogous procedure to that discussed at the beginning of section 3, which forms a *weighted* average, using the strata weights, will lead to an unbiased forecast. The weighted average forecast is 25.5 kWh which is quite close to the actual valve of 26.9 kWh. When we take a weighted average across strata we find a forecast error of only 5.3% with the $\chi^2(1)$ forecast test equal to 0.52 which is not significant at any reasonable level test. Thus, while the second level demand equation is not as accurate as the first level equation in the estimation, the daily demand forecasts do considerably better in the forecast validation test. We probably do not want to be overconfident that similar excellent performance would hold in future tests.

That last step of our validation procedure is to combine the relative load forecasts with the daily consumption forecasts to predict absolute system load. These forecasts are presented in fig. 7. The average absolute forecast error is about 16% with somewhat better performance during the peak periods where the absolute forecast error is 14%. In terms of the increment to system peak design, our model forecasts an increment of 1.43 kWh while the actual increment in period 12 is 1.32 kWh which is an overestimate of 8%. Thus, the model seems to forecast reasonably well in a situation where time of day prices are in effect.

Now to present the final economic forecasts of the experiment, the hourly demands are aggregated into peak, intermediate, and off-peak periods. The results are presented in fig. 8. The model underpredicts actual usage in each case by about 5%. Given that actual peak usage shifted from 6–7 p.m. in the pre-experimental year, which is a peak price period in the test year, to 9–10 p.m. during the test year, which is the first off-peak price period at night, the model forecasts the demand shift quite well. One reason for the underprediction may be the exclusion of higher order terms for temperature in the specification since the test year winter was extremely cold by historic standards. Nevertheless, the forecasts capture the shift from peak periods to off-peak periods; and the model seems valuable for revenue forecasts under a time-of-day price system.

The approach presented here is thus a flexible procedure to model time-of-day electricity demand within a two-level budgeting framework. It represents

	O (1)	I (2)	P (3)	P (4)	I (5)	I (6)	I (7)	I (8)	I (9)	I (10)	P (11)	P (12)	I (13)	I (14)	O (15)	O (16)	O (17)
Actual demand (kWh)	1.24	0.97	0.97	0.86	1.21	1.00	1.16	0.89	0.91	1.10	1.26	1.32	1.23	1.48	1.83	1.80	7.69
Forecast demand	1.61	0.99	0.82	0.69	0.87	0.84	0.77	0.69	0.79	1.22	1.43	1.22	1.58	1.68	1.71	1.73	6.80
% difference	−26%	−3%	17%	22%	33%	16%	41%	25%	14%	−10%	−12%	7%	−24%	−12%	7%	4%	12%

Fig. 7. P = Peak period, I = Intermediate period, and O = Off-peak period.

Period	Forecast (kWh)	Actual	% Difference
(1) Peak	4.16	4.41	5.8%
(2) Intermediate	9.43	9.95	5.4%
(3) Off-peak	11.85	12.56	5.8%

an alternative approach to both the time series approach and the single level budgeting approach which have been used previously. The main advantage of the method would occur in future applications if the model can continue to forecast the relative load curve better than alternative approaches which attempt to forecast the absolute load curve. In situations where the right-hand side variables change as both appliance holdings and prices would change if time-of-day pricing is adopted, it may be superior to the other methods so long as the second level daily consumption equation continues to forecast well in the aggregate. Application to other time-of-day experimental data where more price variation exists would help resolve the question of whether this alternative approach is a valuable tool for analyzing and forecasting time-of-day electricity demand.

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