ESSAYS ON
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On some facets of betting

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C4.2.1. Introduction
A standard proposition of the theory of choice under uncertainty is that two individuals whose personal probabilities of a future event differ can make a mutually advantageous wager. On the other hand, empirical observation suggests that widespread betting is absent on events where individuals' personal probabilities apparently differ widely. Professor Hildreth's interesting paper suggests several possible explanations of this inconsistency. Extending this analysis, we look for answers in the nature of beliefs, the structure of markets for wagers and the impact of market form on beliefs. In each case, we must ask whether the postulated phenomenon is likely to be prevalent in reality, and whether it is sufficient to imply the observed paucity of wagers.

C4.2.2. The nature of beliefs
Professor Hildreth has suggested that when individuals consider wagers against the background of the 'grand lottery of life', they may not view as independent the events determining the outcomes of the 'grand' lottery and the wager. We first ask whether it is likely that personal probabilities would tend to display this non-independence; in particular, more likely than 'objective' probabilities determined by relative frequencies. An examination of human psychology suggests an affirmative answer. Chance jolts the harmony of conscious belief; relief from this dissonance is gained by imposing an order over chaos, weaving a fabric of cause and effect, out of the jumbled coincidences of random events.

It is so much easier to assume than to prove; it is so much less painful to believe than to doubt; there is such a charm in the repose of prejudice, when no discordant voice jars upon the harmony of beliefs. . . .

W. E. H. Lecky, *A History of Rationalism* (1900).

Nothing is so easy as to deceive one's self; for what we wish, we readily believe.

Demosthenes, *Third Olynthiac* (348 B.C.).

The mind accepts and emphasizes those coincidences which reaffirm the perceived order of the universe, ignores and forgets inconsistent data.

L. Festinger (*A Theory of Cognitive Dissonance*, pp. 162–176) has carried out a study of subjects given an opportunity to accept a series of wagers involving a complex random event, and has examined the willingness of the subjects in the course of play to accept information dissonant with their beliefs about the random event. He concludes that 'the interaction between the amount of dissonance which exists and the expectation concerning some particular source of new information in determining whether or not a person will expose himself to, or avoid, this source of information is [clearly consistent with the theory of dissonance reduction]'. In an experimental test of the von Neumann–Morgenstern axioms, D. Davidson and P. Suppes (*Decision Making*, p. 53) report that 'Winning or losing several times in a row made subjects sanguine or pessimistic and tended to produce altered responses to the same offers'.

Thus, the evidence is persuasive that personal probabilities will tend to distort the independence properties of 'objective' probabilities, implying correlations between events which are in fact independent. A simple model of personal probability determination with selective memory gives a final illustration of this point. Suppose two events E_1 and E_2 yield favorable outcomes to the individual in repetitive play, and that these events are in fact independent, each occurring with probability one-half. Suppose the individual computes personal probabilities from observed relative frequencies, remembering coincidences of favorable or unfavorable outcomes perfectly but forgetting a proportion θ of the observations when a coincidence does not occur. Then the probability limit of the individual's personal probability of the joint event (E_1, E_2) as the number of repetitions goes to infinity equals $1/(4 - 2\theta)$, greater than the objective probability $\frac{1}{4}$. Note in this example that we not only have non-independence, but also that the personal correlation between events corresponds to the individual view that luck occurs in runs, so that favorable results tend to go together. This last observation has some further implications, which we shall return to later.

We next ask whether pervasiveness of non-independence in personal probabilities of events is itself sufficient to explain the paucity of wagers. We employ Professor Hildreth's notation, and for concreteness assume further that the underlying Bernoulli utility indicator exhibits constant risk aversion and that the personal probability for the current prospect

X and new venture Y is multivariate normal with means (μ_X, μ_Y) and covariance matrix

$$\begin{bmatrix} \sigma_X^2 & \rho_{XY}\sigma_X\sigma_Y \\ \rho_{XY}\sigma_X\sigma_Y & \sigma_Y^2 \end{bmatrix}.$$

Then, the expected utility of the current prospect plus a share s in the new venture is a monotone increasing function of

$$\lambda(s) = K + (\mu_Y - q + \alpha\rho_{XY}\sigma_X\sigma_Y)s - \alpha\sigma_Y^2 s^2/2, \qquad (C4.1)$$

where α is the degree of absolute risk aversion, $K = \mu_X - \alpha\sigma_X^2/2$, and q is the price of a unit share in the new venture (assumed zero by Professor Hildreth).

Even if the new venture is actuarially favorable $(\mu_Y - q > 0)$, the individual believing $\rho_{XY} > 0$ may find it undesirable to acquire a positive share; only if

$$\mu_Y - \alpha\rho_{XY}\sigma_X\sigma_Y - q > 0 \qquad (C4.2)$$

will a positive share be chosen. This argument would seem to support Professor Hildreth's conclusions. Note, however, that two individuals A and B with the preference structure above and differing personal probabilities satisfying

$$\mu_Y^A - \alpha^A\rho_{XY}^A\sigma_X^A\sigma_Y^A > \mu_Y^B - \alpha^B\rho_{XY}^B\sigma_X^B\sigma_Y^B \qquad (C4.3)$$

can find a price q between these quantities at which it is mutually advantageous for B to sell a share of the new venture to A. Thus, we see that non-independence alone is not sufficient to rule out widespread betting.

The psychological argument we made earlier implied more than non-independence of personal probabilities, however; it implied an 'irrational' belief that the probabilities of events depend on the desirability of outcomes, with 'luck' running in 'streaks'. One might incorporate this phenomenon into the example above by postulating that the parameters of the personal probability distribution $(\mu_Y, \sigma_Y, \rho_{XY})$ depend on the individual's decision variable, the net share purchase s. If, in particular, $\rho_{XY} > 0$ when $s > 0$ and $\rho_{XY} < 0$ when $s < 0$, implying the outcome of the wager is likely to be good when the outcome of the current prospect is good, and *vice versa*, no matter which way the wager is laid, then individuals with differing personal probabilities may find no grounds for a mutually advantageous wager. This explanation is of

course inconsistent with the Savage axioms, and seems to have the same behavioral implications as a pure 'distaste for gambling', the mirror image of the phenomenon claimed by V. Smith to be necessary to explain compulsive gambling. It is worth noting that the effect we have postulated is 'rational' in the sense that it can result from rational preference maximization over acts, and in the sense that one cannot engage the individual in a series of wagers that would result in his taking a sure loss.

C4.2.3. The structure of markets for wagers

The paucity of wager markets could result from the presence of high organizational costs, transactions costs or redundancy. It is plausible that the costs of searching for potential traders and enforcing contracts, particularly time costs, are a significant deterrent to the placing of small wagers. Since risk aversion lowers the desirability of large wagers where transactions costs are relatively unimportant, the combination of effects may be sufficient to explain the lack of markets. A second possible explanation is that most wager markets are in fact redundant; the individual can achieve any desired risk position through the operation of a few well-organized markets such as securities markets. This phenomenon has been noted in papers by K. Arrow and by P. Diamond on the allocation of risk-bearing showing that generally a system with N commodities and S states of nature needs only $(N-1)(S-1)$ markets, of which $S-1$ are wager markets, instead of the maximum possible number of barter markets, $NS(NS-1)/2$, including $N^2S(S-1)/2$ wager markets. In a study of the existence of equilibrium under uncertainty, R. Radner points out that wagers can be made only on information that will be common to the participants, reducing further the number of wager markets that can form. We conclude that the absence of widespread wager markets may be the result of redundancy or transactions costs rather than individual aversion to betting.

C4.2.4. Market effects on beliefs

Thus far, we have considered only the possibility that beliefs are affected by actions via a psychological mechanism of selective memory. There is the additional possibility that the events on which an individual might wager could be affected by the actions of his opponent; the problem of *moral risk*. The presence of such an effect will introduce a

dependence of the expected payoff of a new venture on the position held by the oponent. This can have the effect of eliminating the possibility of a mutually advantageous wager; the argument is the same as in the paragraphs following.

We next examine the role of the market itself in providing information and influencing beliefs. G. Akerlof has pointed out the *lemon principle*, which states that in the presence of uncertainty about the quality of a commodity unit, the fact that it is offered in a market may be taken as information on its quality. The usual example of operation of this principle is in the used car market, where the fact that a vehicle is in the market suggests that it may be below average quality for vehicles of the same identifiable type; i.e. the seller of the vehicle may have information, withheld from the potential buyer, that the car is a 'lemon'. Applied to a market for wagers, this principle suggests that a potential buyer of a lottery ticket may suspect that the seller holds inside information unavailable to himself which indicates the yield of the ticket will be low, and takes the fact that the ticket is being offered in the market as evidence supporting this suspicion. Symmetrically, a potential seller may suspect that an individual soliciting a wager has inside information. It is clear that the presence of such suspicions will inhibit the trading of wagers. In terms of eq. (C4.1) expressing the desirability of a net share s in the new venture, the expected return μ_Y will be considered a function of s, with $\mu_Y(1) < \mu_Y(-1)$ for the reason above. If, by contrast to eq. (C4.3),

$$\mu_Y^A(1) - \alpha^A \rho_{XY}^A \sigma_X^A \sigma_Y^A < \mu_Y^B(-1) - \alpha^B \rho_{XY}^B \sigma_X^B \sigma_Y^B,$$
$$\mu_Y^B(1) - \alpha^B \rho_{XY}^B \sigma_X^B \sigma_Y^B < \mu_Y^A(-1) - \alpha^A \rho_{XY}^A \sigma_X^A \sigma_Y^A,$$

then no mutually advantageous wager is possible.

The same psychological phenomenon as discussed earlier may tend to reinforce belief in the lemon principle. Actions leading to losses suggest bad judgment, generating dissonance, which can be reduced psychologically by attributing 'inside information' or 'unfair advantage' to the opponent. Further, there is a 'Gresham's law' aspect to the lemon principle; one can show that it will tend to drive out of the market a disproportionate number of 'honest' lottery tickets. Thus, the lemon principle becomes a 'self-fulfilling prophecy'. A strong argument can hence be made that the lemon principle will operate to inhibit many wager markets.

C4.2.5. Summary

This discussion has pointed out the following possible explanations for the paucity of wager markets in the presence of differing personal probabilities:

(1) The psychology of cognitive processes suggests that individuals will tend to believe in 'runs of luck', with an effect similar to that caused by a pure 'distaste for gambling'.

(2) Transactions costs, of consequence for small transactions, combined with the inhibiting effect of risk aversion on large transactions, may prevent wager markets from forming.

(3) Many wager markets may fail to form because they are redundant.

(4) Moral risks may inhibit the offering of wagers on some events.

(5) The lemon principle may operate to indicate the presence of inside information to be used to the disadvantage of potential traders, inhibiting formation of a market.