Economics 201b  
Spring 2010  
Problem Set 3  
Due Thursday April 8

Unless otherwise specified, all equilibria are understood to mean without transfers.

1. Consider a Robinson Crusoe economy with a linear technology \( f(x_1) = \alpha x_1 \) and linear preferences: \( U(x_1, x_2) = x_1 + \beta x_2 \) where \( 0 < \alpha, \beta < \infty \). Let the endowment be \( \omega = (0, L) \). Give a complete case-by-case analytic characterization of all equilibria. For each case, draw a picture.

2. In an Arrow-Debreu economy with strongly monotone preferences, consider a 4-tuple \((p, x, y, T)\) where \( T \) is an income transfer and \( x_i \in D_i(p, y, T) \) for all \( I \) consumers. Suppose all but one market clear - without loss of generality, assume the first \( L - 1 \) markets clear, then show explicitly that, in fact, all markets clear. When writing your solution use the standard notation (e.g. \( l, i, x_i, y_j, \theta_{ij} \)) for Arrow-Debreu economies found in the notes for lecture 4. Do not assume it is a pure exchange economy.

3. Consider a two-consumer, one-firm Arrow-Debreu economy. The technology of the firm is \( Y = \{ (y_1, y_2) | y_1 \leq 0, y_2 = e \log(1 - y_1) \} \). The endowments are \( \omega_1 = (e, 0) \) and \( \omega_2 = (e^2, 0) \), and the utilities are \( U_1(x_{11}, x_{21}) = \frac{\log(x_{11})}{e} + x_{21} - \frac{1}{e} \) and \( U_2(x_{12}, x_{22}) = \log(x_{12}) + x_{22} - 2 \). Let \( \theta_1 \) and \( \theta_2 \) be the two agent’s shares of the firm’s profit.

   (a) Give an analytic characterization of all equilibria. Show your work in detail; in particular find a simple, clean expression for equilibrium labor.

   (b) Suppose that the agents bargain for their shares \( \theta_i \) of the firm’s profits. What is the Nash bargaining solution for the shares? Explain. (Recall, the Nash Bargaining solution is the split of shares - \((\theta_1^*, \theta_2^*)\) - that solves the following maximization problem

\[
\text{argmax}_{\theta_1, \theta_2} \quad (U_1^{\theta_1} - \bar{U}_1)(U_2^{\theta_2} - \bar{U}_2)
\]

where \( U_i^{\theta_i} \) is agent \( i \)'s equilibrium utility when the shares are \((\theta_1, \theta_2)\), and \( \bar{U}_i \) is agent \( i \)'s utility when there is no access to the firm’s technology.)

4. Consider a two-person, two-good exchange economy with the following nonconvex preferences: \( U_i(x_{1i}, x_{2i}) = \max\{x_{1i}, x_{2i}\} \) for \( i = 1, 2 \). Suppose the social endowment is \( \bar{\omega} = (1, \gamma) \) with \( \gamma > 0 \).

   (a) Give a careful analytic characterization of all exact Pareto Optimal allocations. Answer will depend on \( \gamma \). Draw pictures demonstrating the different possibilities.

   (b) Are there any values of \( \gamma \) for which the Second Welfare Theorem fails? Prove your answer.