Economics 204–Final Exam–August 21, 2006, 2-5pm
Each question is worth 25% of the total
Please use *separate* bluebooks for each of the two Parts

Part I

1. Prove that if two vector spaces X and Y over the same field F are isomorphic, then $\dim X = \dim Y$.

2. Consider the differential equation

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}' = \begin{pmatrix} -1 & 4 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

   (a) Compute the eigenvalues of the matrix. Briefly discuss what this tells you about the qualitative nature of the solutions of the differential equation.

   (b) Explain how we know there must be an orthonormal basis of $\mathbf{R}^2$ composed of eigenvectors of the matrix. Compute this orthonormal basis.

   (c) Sketch the qualitative behavior of the solutions of the equation in a phase plane diagram.

   (d) Find a solution of the Initial Value Problem formed by combining the differential equation with the initial conditions $y_1(0) = A, y_2(0) = B$. Is the solution of the Initial Value Problem unique?

Part II

3. Let Ψ be a correspondence from X to Y which is compact-valued and upper hemicontinuous, C a compact subset of X. Let $\Psi(C) = \cup_{x \in C} \Psi(x)$. Prove that $\Psi(C)$ is compact. For full credit, you must use the open set definitions of compactness and upper hemicontinuity; a correct proof using the sequential formulations of compactness and upper hemicontinuity will receive three-fourths credit.

4. Prove that for all $n \in \mathbf{N}$,

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$