ONLINE APPENDIX TO EXTERNAL ECONOMIES AND INTERNATIONAL TRADE REDUX: COMMENT

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Abstract

This appendix provides the proofs and derivations of theoretical results used in the sections 2.1, 2.2 and 2.3. The figures referred to in these sections are also included.

1 Complete specialization equilibrium in Section 2.1

Proof of Lemma 1. If Home serves both markets, the equilibrium entails prices p_0 and $p_0^* = tp_0$, with p_0 equal to the unit cost in Home given total consumption, with the unit cost given by

$$p_0 \equiv \frac{wa}{(x(p_0) + x^*(tp_0))^\phi}. \tag{1}$$

To establish that this is an equilibrium, we need to consider the possible deviations by Home and Foreign firms.

A preliminary result is that, under the assumptions on demand and technology, Assumption $\phi < \frac{1}{2}$ is sufficient to imply profits are increasing in prices. This, in turn, implies that the best possible deviation entails "shaving" prices p_0 and p_0^*, i.e., it is never optimal to charge strictly lower prices than p_0 and p_0^*. To see this, consider the case of a firm that sells in both markets. A Home firm that sells at prices p and p^* in Home and Foreign makes profits of

$$\pi(p, p^*) \equiv \left[p - \frac{wa}{(x(p) + x^*(p^*))^\phi}\right] x(p) + \left[p^* - \frac{wat}{(x(p) + x^*(p^*))^\phi}\right] x^*(p^*).$$

Using Assumptions on demand along with simple differentiation reveals that, $\pi_2(p, p^*) > 0$ if $\phi < 1$, and $\pi_1(p, p^*) > 0$ if $\phi < \frac{x(p)+x^*(p^*)}{x(p)+tx^*(p^*)} = \frac{1+t^{-1}}{2}$. Both conditions are satisfied by Assumption $\phi < \frac{1}{2}$.

Consider now a deviation by a Home firm. Since prices p_0 and $p_0^* = tp_0$ imply that Home firms make zero profits in both markets, then our result that profits are increasing in prices guarantees that a Home firm cannot make positive profits with any alternative set of prices. So this establishes that there is no profitable deviation for a Home firm. Turning to Foreign firms, we need two conditions: first, no firm from Foreign should find it optimal to take over the world market by undercutting Home firms in both markets, and second, no firm from Foreign should find it optimal to displace Home firms from the Foreign market. Writing x

and x^* as shorthand for $x(p_0)$ and $x^*(tp_0)$, a sufficient condition for it to be unprofitable for Foreign firms to take over both markets is given by

$$\left[\frac{wa}{(x+x^*)^\phi} - \frac{w^*a^*t}{(x+x^*)^\phi}\right] x + \left[\frac{wat}{(x+x^*)^\phi} - \frac{w^*a^*}{(x+x^*)^\phi}\right] x^* \leq 0. \tag{2}$$

In turn, a sufficient condition for it to be unprofitable for Foreign firms to displace Home firms from the Foreign market (only) is

$$\left[\frac{wat}{(x+x^*)^\phi} - \frac{w^*a^*}{(x^*)^\phi}\right] x^* \leq 0. \tag{3}$$

The previous arguments establish that if conditions (2) and (3) are satisfied for p_0, then there is an equilibrium with complete specialization in which Home firms serve both markets. ■

Proof of Proposition 1. First note that $g^*(t) = \frac{1}{g(t)}$ and $h^*(t) = \frac{1}{h(t)}$. For future reference we refer to this as "symmetry". To prove the existence of $\widetilde{t}$ and t_{CS} we use two results: first, that $g(1) = 1$ and $g(t)$ is increasing for $t > 1$, and second, that $h(1) > 1$ and $h(t)$ is decreasing for $t > 1$ with $\lim_{t\to\infty} h(t) = 0$. To prove the first result, note that $2g'(t) = 1 - \frac{1}{t^2} > 0$ implies $g'(t) > 0$. To prove the second result, note that $\frac{\partial \ln h(t)}{\partial \ln t} = \phi\frac{t}{1+t} - 1$. Since $\phi < \frac{1}{2}$ by Assumption, then this is negative. Also, note that $\lim_{t\to\infty} h(t) = \lim_{t\to\infty} \frac{1}{(1+t)^{1-\phi}} = 0$, where the first equality follows from L'Hospital's Rule. These two results along with the continuity of both $g(.)$ and $h(.)$ imply that there exists a unique $\widetilde{t}$ which is higher than 1, with $g(\widetilde{t}) > 1$. Moreover, for any $\beta < 1$ there exists a unique t_{CS}. Symmetry implies that $\widetilde{t}$ uniquely satisfies $g^*(t) = h^*(t)$ and $g^*\left(\widetilde{t}\right) < 1$. Symmetry also implies that $g^*(1) = 1$ and $g^*(t)$ is decreasing for $t > 1$, and second that $h^*(1) < 1$ and $h^*(t)$ is increasing for $t > 1$ with $\lim_{t\to\infty} h^*(t) > 1$. One can also verify that there exists a unique t such that $h(t) = h^*(t) = 1$ and $t > \widetilde{t}$. The result then follows. ■

2 Equilibrium with no trade in Section 2.2

Proof of Lemma 2. Let's now consider the conditions for there to be an equilibrium with no trade. This equilibrium would have the Home price $p_A = \frac{wa}{(x(p_A))^\phi}$, and Foreign price $p_A^* = \frac{w^*a^*}{(x^*(p_A^*))^\phi}$, so that both Home and Foreign firms make zero profits. Since profits are increasing in prices, clearly no Home or Foreign firm can make positive profits by charging any alternative set of prices when targeting their domestic market only. The conditions necessary for this to be an equilibrium is that neither Home nor Foreign firms find it profitable to sell in both markets. As explained above in the proof of Lemma 1, the best deviation would be to charge the highest possible price while serving both markets. Thus, writing x_A and x_A^* as shorthand for $x(p_A)$ and $x^*(p_A^*)$, respectively, the condition that Home firms do not make profits from this deviation is

$$\left[\frac{wa}{(x_A)^\phi} - \frac{wa}{(x_A + x_A^*)^\phi}\right] x_A + \left[\frac{w^*a^*}{(x_A^*)^\phi} - \frac{wat}{(x_A + x_A^*)^\phi}\right] x_A^* \leq 0. \tag{4}$$

Similarly, the condition that Foreign firms do not make profits from a deviation to sell in both markets is given by

$$\left[\frac{wa}{(x_A)^\phi} - \frac{w^*a^*t}{(x_A + x_A^*)^\phi}\right] x_A + \left[\frac{w^*a^*}{(x_A^*)^\phi} - \frac{w^*a^*}{(x_A + x_A^*)^\phi}\right] x_A^* \leq 0. \tag{5}$$

The result then follows. ■

3 Equilibrium with mixed strategies in Section 2.3

High transport costs (existence of no pure strategy region)

We now establish that for an industry with $\beta \leq 1$ there exists a range of trade costs for which no pure strategy in which production is either concentrated in a single country or one in which there is domestic only production can be sustained as an equilibrium. Recall first that by Proposition 2 we know that an equilibrium with no trade exists if and only if $t \geq \max\{t_{NT}(\beta), t^*_{NT}(\beta)\}$. Since we considering the case $\beta \leq 1$, this is equivalent to $t \geq t_{NT}(\beta)$, which, in turn, is equivalent to $\beta \geq l(t)$.[1] The curve $l(t)$ is decreasing and intersects the horizontal line with $\beta = 1$ at point $t_{NT}(1) = 2^{1+\phi} - 1$. It is readily verified that $t_{CS}(1) < t_{NT}(1)$. To see this recall that $t_{CS}(1)$ is defined implicitly by $1 = h(t) \equiv \frac{(1+t)^\phi}{t}$. Since $h(.)$ is strictly decreasing, to show that $t_{NT}(1) > t_{CS}(1)$, it is sufficient to show that $1 > \frac{(1+t_{NT}(1))^\phi}{t_{NT}(1)}$, which is equivalent to $t_{NT}(1) > (1 + t_{NT}(1))^\phi$, or $2^{1+\phi} - 1 > \left(2^{1+\phi}\right)^\phi$. But this is satisfied for all $\phi < \frac{1}{2}$, a restriction satisfied by Assumption.

We now establish that the curve $h(t)$ is always below the curve $l(t)$, so that $t_{CS}(\beta) < t_{NT}(\beta)$. This further implies that for $\beta \leq 1$ there is no pure strategy equilibrium for $t \in (t_{CS}(\beta), t_{NT}(\beta))$. First, recall that $l(t)$ implicitly solves $t = \frac{2\left(1+l(t)^{\frac{1}{1-\phi}}\right)^\phi - 1}{l(t)^{\frac{1}{1-\phi}}}$. Since the RHS of the expression is monotonically decreasing in l, to show that $l(t) \geq h(t)$ is equivalent to showing that $\frac{2\left(1+h(t)^{\frac{1}{1-\phi}}\right)^\phi - 1}{h(t)^{\frac{1}{1-\phi}}} - t \geq 0$, or

$$2\left(1 + \tau\left(1+\tau\right)^\gamma\right)^\phi \geq \left(1 + \left(1+\tau\right)^\gamma\right)$$

where $\gamma \equiv \frac{\phi}{(1-\phi)}$ and $\tau \equiv \frac{1}{t}$. Note that $\phi \in \left[0, \frac{1}{2}\right]$ implies $\gamma \in [0,1]$, and $t \geq 1$ implies $\tau \in [0,1]$.

If $\phi = 0$ then $\gamma = 0$ and the inequality is obviously true. Otherwise, let $\eta \equiv \frac{1}{\phi} \geq 2$ and rewrite the inequality as

$$2^\eta \left(1 + \tau\left(1+\tau\right)^\gamma\right) \geq \left(1 + \left(1+\tau\right)^\gamma\right)^\eta.$$

Let $s \equiv (1+t)^\gamma$. Then $s \geq 1$, $\tau = s^{\frac{1}{\gamma}} - 1$ and the inequality above becomes

$$2^\eta \left(1 + s^\eta - s\right) \geq \left(1 + s\right)^\eta$$

[1] Since $t_{NT}(.)$ and $t^*_{NT}(.)$ are monotone, their inverses $l(t) \equiv \left(t_{NT}\right)^{-1}(t)$ and $l^*(t) \equiv \left(t^*_{NT}\right)^{-1}(t)$ are well defined.

since $1+\frac{1}{\gamma}=\eta$.

We will show that the inequality immediately above holds for all $s \geq 1$ by showing that $f(s) \geq 0$ for all $s \geq 1$ where

$$f(s) \equiv 2^{\eta}\left(1+s^{\eta}-s\right)-(1+s)^{\eta}.$$

This we will do by showing that $f(1)=0$, $f'(1) \geq 0$ and $f''(s) \geq 0$ for $s \geq 1$.

Obviously $f(1)=0$. Differentiating we have,

$$f'(s)=2^{\eta}\left(\eta s^{\eta-1}-1\right)-\eta(1+s)^{\eta-1}$$

so

$$f'(1)=2^{\eta}(\eta-1)-2^{\eta-1}\eta=2^{\eta-1}(\eta-2) \geq 0$$

since $\eta \geq 2$. Finally,

$$\begin{aligned}
f''(s) &= 2^{\eta}\eta(\eta-1)s^{\eta-2}-\eta(\eta-1)(1+s)^{\eta-2} \\
&= \eta(\eta-1)s^{\eta-2}\left(2^{\eta}-\left(\frac{1+s}{s}\right)^{\eta-2}\right) \\
&= 2^{\eta}\eta(\eta-1)s^{\eta-2}\left(1-\frac{1}{4}\left(\frac{1+s}{2s}\right)^{\eta-2}\right) \\
&\geq 0 \text{ since } \frac{1+s}{2s} \leq 1. \textbf{ QED}
\end{aligned}$$

Proof of Proposition 3. We begin by deriving $F(p)$. Home firms earn zero profits when they pursue their local strategy in all states of nature. Thus, a Home firm pursuing the global strategy should also expect zero profits. Moreover, for a Home firm to be willing to randomize over $p \in [s, p_A]$, the expected profits for any such p should also be zero. To derive the expected profits given p in the global strategy, suppose first that the other Home firm pursues the local strategy. The profits are then $\Phi(p)+\Phi^*(p)$. If the other firm pursues the global strategy, expected profits associated with a Home price of p are

$$\left[\Phi(p) + \frac{\Phi^*(p)}{2}\right](1 - F(p)) + \int_s^p \frac{\Phi^*(y)}{2} dF(y).$$

where the second term emerges because the unit cost (and therefore profits earned abroad) will depend on the random price charged by the other firm. Thus, expected profits for a Home firm setting prices p and p_A^* when the other Home firm pursues the proposed mixed strategy are

$$\begin{aligned}\Pi(p) &\equiv q\left(\Phi(p) + \Phi^*(p)\right) \\ &+ (1-q)\left\{\left[\Phi(p) + \frac{\Phi^*(p)}{2}\right](1 - F(p)) + \int_s^p \frac{\Phi^*(y)}{2} dF(y)\right\}\end{aligned}$$

Our mixed strategy requires $\Pi(p) = 0$ for all $p \in [s, p_A]$. Differentiating $\Pi(p)$ with respect to p, setting $\Pi'(p) = 0$ and solving for $F'(p)$ yields

$$(1-q)F'(p) = q\frac{\Phi'(p) + \Phi^{*\prime}(p)}{\Phi(p)} + (1-q)\left[\frac{\Phi'(p) + \frac{\Phi^{*\prime}(p)}{2}}{\Phi(p)}\right](1 - F(p)).$$

The solution to this differential equation is

$$F(p) = \frac{q}{1-q}\int_s^p \zeta(y)\frac{M(y)}{M(p)}dy + 1 - \frac{M(s)}{M(p)}.$$

Noting that $M(s) = 1$, setting $F(p_A) = 1$ and solving for q yields (8). Plugging this back into the expression above yields (7). Finally, we also need that $\Pi(s) = 0$. This implies that

$$\Phi(s) + \left(q + \frac{1-q}{2}\right)\Phi^*(s) = 0.$$

This equation together with (8) can then be solved to yield the equilibrium value of s.

We need to study all possible deviations by Home and Foreign firms. A Foreign firm could deviate by going global, shaving prices p_A and p_A^*. If both Home firms pursue their local strategy, which happens with probability q^2 , the Foreign firm would capture both markets and make profits of

$$\pi^*(p_A, p_A^*) \equiv \left[p_A - \frac{w^* a^* t}{(x(p_A) + x^*(p_A^*))^\phi}\right] x(p_A) + \left[p_A^* - \frac{w^* a^*}{(x(p_A) + x^*(p_A^*))^\phi}\right] x^*(p_A^*).$$

Otherwise, the Foreign firm would simply sell in the local market and make zero profits. So we need to establish that $\pi^*(p_A, p_A^*) < 0$. One can readily verify that for $\beta < l_F(\widehat{t})$ (Home has a superior comparative advantage) we have $t_{CS}(\beta) > t_{NT}^*(\beta)$ (Figure III illustrates this). Since our mixed strategy applies for $t \in (t_{CS}(\beta), t_{NT}(\beta))$, then we have $t > t_{NT}^*(\beta)$ which is equivalent to $\pi^*(p_A, p_A^*) < 0$.

To describe the possible deviations by Home firms, we use notation $p^* \lesssim p_A^*$ to mean a firm shaves p_A^* (recall that firms always deviate by shaving since profits are increasing in prices). There are four possible types of pricing strategies by Home firms: (i) $p > p_A$ and $p^* > p_A^*$ (no entry- yields zero profits); (ii) $p \leq p_A$ and $p^* \lesssim p_A^*$ (competing for the global market); (iii) $p > p_A$ and $p^* \lesssim p_A^*$ (competing for foreign market only); and (iv) $p \leq p_A$ and $p^* > p_A^*$ (competing for domestic market only). But pricing strategy (i) strictly dominates pricing strategy (iii) since Home firms make losses on export sales. This implies that we can rule out strategy (iii). The conjectured equilibrium above essentially involves mixing across the part of (iv) which entails a home price $p = p_A$, and strategy (ii) for different domestic prices $p \leq p_A$. Importantly, note also that Home firms are indifferent between strategy $p = p_A$ and $p^* > p_A^*$, and (i) since both strategies yield zero expected profits. Hence, our final step is to explicitly rule out the portion of (iv) with $p < p_A$ and $p^* > p_A^*$ as a possible deviation.

First, note that if $p < s$, the expected profits are

$$q\left[p - \frac{wa}{(x(p))^{\phi}}\right] x(p) + (1-q)\,\Phi(p).$$

We need this expression to be non-positive. But since this is increasing in p (recall that $px(p) = 1$ and that $\frac{x(p)}{(x(p))^{\phi}}$ is decreasing by the Assumption that $\phi < \frac{1}{2}$), it is enough to check that the expected profits of this type of deviation are non-positive for $p \geq s$. For this case, the expected profits are

$$\widetilde{\Pi}(p) \equiv q\Gamma(p) + (1-q)(1-F(p))\,\Phi(p),$$

where $\Gamma(p) \equiv \left[p - \frac{wa}{(x(p))^{\phi}}\right] x(p)$. Since $\Gamma(p_A) = 0$ and $F(p_A) = 1$ then $\widetilde{\Pi}(p_A) = 0$. We now show that $\widetilde{\Pi}'(p) \geq 0$, implying that $\widetilde{\Pi}(p) \leq 0$ for all p. First, $\Pi'(p) = 0$ implies

$$(1-q)\,\Phi(p)F'(p) = q\left[\Phi'(p) + \Phi^{*\prime}(p)\right] + (1-q)\left[\Phi'(p) + \frac{\Phi^{*\prime}(p)}{2}\right](1-F(p)).$$

Second,

$$\widetilde{\Pi}'(p) = q\Gamma'(p) + (1-q)\,\Phi'(p)\,(1-F(p)) - (1-q)\,F'(p)\Phi(p).$$

Combining these two expressions yields

$$\widetilde{\Pi}'(p) = q\left[\Gamma'(p) - \Phi'(p) - \Phi^{*\prime}(p)\right] - (1-q)\left(\frac{\Phi^{*\prime}(p)}{2}\right)(1-F(p)).$$

One can easily verify that $\Phi^{*\prime}(p) < 0$. Hence, a sufficient condition for $\widetilde{\Pi}'(p) \geq 0$ is that

$$\Gamma'(p) - \Phi'(p) - \Phi^{*\prime}(p) \geq 0.$$

Simple differentiation reveals that $\Gamma'(p) - \Phi'(p) - \Phi^{*\prime}(p) \geq 0$ if and only if

$$(1-\phi)\left[\left(1+\frac{p}{p_A^*}\right)^{1+\phi} - 1\right] \geq (1-\phi t)\,\frac{p}{p_A^*}.$$

But $t > 1$, so $0 < \phi < \phi t$, hence the previous inequality is satisfied. We conclude that $\widetilde{\Pi}\,(p) \leq 0$ for any p. ■

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