Problem Set #2
(due 4/3/01)

1. In class, we considered the impact of announced changes in tax rules on the incentive to invest, and noted that the impact of an anticipated change in the corporate income tax depends on the schedule of depreciation allowances. This problem revisits the issue.

Suppose that firms invest in a single capital good and face no adjustment costs or uncertainty. Capital depreciates exponentially at rate $\delta$. All prices and depreciation schedules are constant over time. Firms receive no tax benefit from investment other than depreciation allowances, and the corporate tax rate is scheduled to change from its current value of $\tau_0$ to $\tau_1$ at a future date $T$. Firms are equity-financed and equity holders require a constant rate of return $r$.

A. Write down the expression for the firm’s user cost of capital at date $t < T$.

B. Based on your expression for the user cost, derive a condition based on the term $\Gamma$ (the present value of tax savings from depreciation) and its first and second time derivatives that must be satisfied for the user cost to be constant as $t$ changes.

C. Express $\Gamma$ in terms of the tax rates $\tau_0$ and $\tau_1$ and the depreciation schedule per initial dollar of capital of age $a$, $\{D(a) a \in [0, \infty]\}$. Using this expression, show that the condition you derived in part B holds for all $t < T$ only if the depreciation schedule is of the form $D(a) = xe^{-\delta a}$, where $x$ is a constant (i.e., a multiple of economic depreciation).

D. Suppose that actual depreciation allowances are accelerated, following a pattern $D(a) = xe^{-\gamma a}$, where $\gamma > \delta$. What will happen to the user cost over time, prior to date $T$?

2. Consider a linear labor supply model,

$$L = \alpha + \beta w(1-t) + \gamma Y$$

where $Y$ is “other” income, including “virtual” income. Suppose that a flat tax is imposed, with the marginal tax rate $t$ applying for all income levels above $Y^*$. 

A. Draw the household’s budget constraint, labeling the different segments and the value of labor supply at which a kink point occurs, say $L^*$.

B. Derive a restriction on $\beta$ and $\gamma$ based on the Slutsky equation at $L^*$.

C. Show that this restriction implies that the labor supply equation will provide at most one solution on a flat section of the household’s budget constraint. (Hint: relate this restriction to the relationship between the level of optimal labor supply assuming $L < L^*$ and the level of optimal labor supply assuming $L > L^*$).
3. Until 1991, Sweden provided businesses with a scheme known as an investment fund. Under this scheme, a business could, through a pure accounting transaction, earmark a fraction \( f \) of its earnings for the fund and deduct this contribution from tax. In exchange, it had to place a fraction \( h \) of this investment fund contribution in a non-interest-bearing account at the central bank, from which withdrawals could be made only to finance future investment. Upon investing in the future, the firm could draw on the investment fund, releasing the holdings in its central bank account in the same proportion as they were initially contributed, i.e., funds from the central bank account paid for a fraction \( h \) of new investment. However, investment funded in this manner received no other investment incentives, such as depreciation allowances. Generally, \( h \) was less than the corporate tax rate, but greater than the present value of tax savings available to investment not financed by fund releases, i.e., \( \tau > h > \Gamma \).

A. Would firms wish to contribute to the investment fund?

B. Assume that a firm always contributed the maximum amount allowed to its investment fund, and that firm’s future investment could be financed by withdrawals from its investment fund – that, in each period, the balance in the investment fund exceeded the firm’s investment. Assuming that tax parameters were constant over time, derive an expression for the firm’s user cost of capital, ignoring interest deductibility and personal taxes. (Hint: what is the marginal value to the firm of deposits at the central bank?)

C. Based on the expression you derived in part B, derive the sign of the impact of an increase in the corporate tax rate, \( \tau \), on the user cost. Do the same for an increase in the contribution rate, \( h \).

4. The Tax Reform Act of 1986 aimed to reduce marginal tax rates while maintaining the tax burden distribution by income class. Yet there is a question whether this is actually possible.

Let \( T(\cdot) \) be the tax function before the reform, and let \( T^*(\cdot) \) be the tax function after the reform. Suppose that there are \( H+1 \) distinct income groups in the population. Group 0 has before-tax income equal to 0, and each group \( i>0 \) has income \( Y_i \), with \( 0<Y_1<Y_2<\ldots<Y_H \). Ignore behavioral responses to taxation; i.e., assume that each group’s before-tax income is fixed. Define the marginal tax rate for each group \( i>0 \) as \( \frac{T(Y_i) - T(Y_{i-1})}{Y_i - Y_{i-1}} \).

A. Show that if each income class’s share of the overall tax burden remains constant, then all marginal tax rates must change by the same proportion.

B. Show that if, in addition to constant relative shares, the overall tax burden remains constant, then all marginal tax rates must remain constant.

C. Now, suppose that another element of the tax reform was “base broadening” – that prior to the tax reform a fraction \( \alpha \) of group \( i \)'s income was taxed, but that after the reform all of the income was taxed. How would this change your answers to parts A and B?