NBER WORKING PAPER SERIES

TECHNOLOGY ENTRY IN THE PRESENCE OF PATENT THICKETS

Bronwyn H. Hall
Christian Helmers
Georg von Graevenitz

Working Paper 21455
http://www.nber.org/papers/w21455

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 2015

We are grateful to the Intellectual Property Office of the United Kingdom for research support and to the National Institute of Economic and Social Research for hospitality while this paper was being written. The views expressed here are those of the authors. They are not necessarily those of the UK Intellectual Property Office or NIESR. The revision of this paper has benefitted from comments by participants in seminars at the UKIPO, the USPTO, the NBER Summer Institute, the University of California at Berkeley, TILEC, Tilburg University, the University of Wuppertal and the TIGER conference 2014 in Toulouse. We would like to thank Jonathan Haskel, Scott Stern, Dietmar Harhoff, Stefan Wagner and Megan MacGarvie for comments, and Iain Cockburn, Dietmar Harhoff, and Stefan Wagner for very generous support with additional data. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

At least one co-author has disclosed a financial relationship of potential relevance for this research. Further information is available online at http://www.nber.org/papers/w21455.ack

Technology Entry in the Presence of Patent Thickets
Bronwyn H. Hall, Christian Helmers, and Georg von Graevenitz
NBER Working Paper No. 21455
August 2015
JEL No. K11,L20,O31,O34

ABSTRACT

We analyze the effect of patent thickets on entry into technology areas by firms in the UK. We present a model that describes incentives to enter technology areas characterized by varying technological opportunity, complexity of technology, and the potential for hold-up in patent thickets. We show empirically that our measure of patent thickets is associated with a reduction of first time patenting in a given technology area controlling for the level of technological complexity and opportunity. Technological areas characterized by more technological complexity and opportunity, in contrast, see more entry. Our evidence indicates that patent thickets raise entry costs, which leads to less entry into technologies regardless of a firm's size.

Bronwyn H. Hall
123 Tamalpais Road
Berkeley, CA 94708
and NBER
bhhall@nber.org

Christian Helmers
Santa Clara University
christian.r.helmers@gmail.com

Georg von Graevenitz
Queen Mary University of London
g.v.graevenitz@qmul.ac.uk

1 Introduction

The past two decades have seen an enormous increase in patent filings worldwide (Fink *et al.*, 2013). There are signs that the level of patenting in certain sectors has become so high as to discourage innovation (Federal Trade Commission, 2011; Bessen and Meurer, 2008; Jaffe and Lerner, 2004; Federal Trade Commission, 2003). The main reason is that companies inadvertently block each other's innovations because of multiple overlapping patent rights in so-called "patent thickets" (Shapiro, 2001). Patent thickets arise where individual products draw on innovations protected by hundreds or even thousands of patents, often with fuzzy boundaries. These patents belong to many independent and usually competing firms. Patent thickets can lead to hold-up of innovations, increases in the complexity of negotiations over licenses, increases in litigation, and they create incentives to add more and weaker patents to the patent system (Allison *et al.*, 2015). This increases transaction costs, reduces profits that derive from the commercialization of innovation, and ultimately reduces incentives to innovate.

There is a growing theoretical (Bessen and Maskin, 2009; Clark and Konrad, 2008; Farrell and Shapiro, 2008; Fershtman and Kamien, 1992) and legal literature on patent thickets (Chien and Lemley, 2012; Bessen et al., 2011). Related work analyzes firms' attempts to form patent pools to reduce hold-up (Joshi and Nerkar, 2011; Lerner et al., 2007; Lerner and Tirole, 2004) and the particular challenges posed in this context by standard essential patents (Lerner and Tirole, 2013).

The existing empirical evidence on patent thickets is largely concerned with showing that they exist and measuring their density (Graevenitz et al., 2011; Ziedonis, 2004). There is less evidence on the effects patent thickets have for firms. Cockburn and MacGarvie (2011) demonstrate that patenting levels affect product market entry in the software industry. They show that a 1 per cent increase in the number of existing patents is associated with a 0.8 per cent drop in the number of product market entrants. This result echoes earlier findings by Lerner (1995) who showed for a small sample of U.S. biotech companies that first-time patenting in a given technology is affected by the presence of other companies' patents. Meurer and Bessen (2005) suggest that patent thickets also lead to increased litigation related to hold-up. Patent thickets have remained a concern of antitrust agencies and regulators in the United States for over a decade (Federal Trade Commission, 2011, 2003; USDoJ and FTC, 2007). Reforms that address some of the factors contributing to the growth of patent thickets have recently been introduced in the U.S. (America Invents Act (AIA) of 2011) [1] and by the European Patent Office.

---

[1] For further information see http://www.gpo.gov/fdsys/pkg/BILLS-112hr1249enr/pdf/BILLS-112hr1249enr.pdf

In spite of the available theoretical and empirical evidence, it is frequently argued that patent thickets are a feature of rapidly developing technologies in which technological opportunities abound (Teece *et al.*, 2014). Thickets are thus seen as a reflection of fast technological progress that is paired with increased technological complexity (Lewis and Mott, 2013). This suggests that a trade-off between technological opportunity and growth on the one hand and increased transaction costs due to the emergence of patent thickets on the other may exist. The challenge in assessing this trade-off is to develop a framework that captures the main incentives that lead to patent thickets as well as the most important effects of thickets.

This paper contributes to the literature by analyzing the effect of patent thickets on entry into new technology areas. Our focus on entry into patenting captures the positive effects of greater technological opportunity and negative effects of greater transaction costs imposed by a complex patent landscape characterized by thickets. We are able to quantify both effects empirically. The paper makes two key contributions: first, we extend the theoretical model of patenting in complex technologies introduced by Graevenitz *et al.* (2013) to free entry and the interaction between incumbents and entrants. Our model shows that technological complexity and technological opportunity increase entry, but that the potential for hold-up in patent thickets reduces entry in complex technologies. While complexity and opportunity are shown to have countervailing effects on patenting incentives in Graevenitz *et al.* (2013), we find that both factors increase the incentive to enter. However, hold-up potential clearly reduces entry incentives. This reflects the fact that patent thickets arise due to rising technological opportunity and complexity but create the potential for hold-up. The second contribution of the paper consists of an empirical test of these predictions using data on UK firms. Our analysis confirms that entry increases in technology areas characterized by greater technological opportunity and complexity. However, we also show that the hold-up potential of patent thickets has negative and economically significant effects on entry into patenting. While we cannot quantify the overall net welfare effect, our results do suggest that thickets raise entry costs for large and small firms alike. We argue that this is likely to have negative long-run consequences on innovation and product market competition.

The remainder of this paper is organized as follows. Section 2 presents a model of entry into patenting in a technology area and derives several testable predictions. Section 3 describes the data, and the empirical measurement of the key concepts in the model. Section 4 discusses our results and Section 5 provides concluding remarks.

2 Theoretical Model

This section presents a two-stage model of entry into patenting and of subsequent patenting decisions. In the first stage of the model, firms choose to enter if they expect non-negative profits from entry. In the second stage firms simultaneously choose how many technological opportunities to research and how many patents per opportunity to apply for. The complexity of the technology determines how many patents (facets) per opportunity are available. The degree of technological complexity also determines how many opportunities exist and therefore how intense competition is within each opportunity. The value of a firm's patent portfolio within a given technological opportunity depends on the number of facets in that opportunity that have been patented overall and the share of those patents held by the firm.[2] In deciding how many patent applications to submit each firm takes into account costs of researching an opportunity, costs of upholding the patent and legal costs of exploiting the patent portfolio including expected costs of hold-up.

In order to analyze the effect of technological complexity, opportunity, and transaction costs in form of thickets on entry, we extend the model by Graevenitz *et al.* (2013) in three ways:

1. We distinguish between the effect of technological complexity and the effect of hold-up by patent-holders.
2. We allow the value of patent portfolios in bargaining to exhibit decreasing returns to scale.
3. We account for the effects of increasing fixed costs of R&D as more firms undertake R&D in the same technological opportunities.

The key variables of the model are the complexity of a technology, measured by $(F \in \mathbb{R}_0^+)$, the degree of technological opportunity, measured by $(O \in \mathbb{R}_0^+)$, and hold-up potential h_k. Complexity increases when the number of patentable facets F_k per technological opportunity O_k increases. The model spans discrete technologies, for which F_k is 1 or a very low positive number and complex technologies for which F_k can be an arbitrarily large positive number. Hold-up potential in form of thickets enters the model through the legal costs of patenting. This is discussed in detail further below.

The value of all F_k patents in an opportunity is V_k. In the simplest discrete setting this is the value of the one patent (facet) that covers each technological opportunity. In more complex technologies this is the value of controlling all patents (facets) on a technological opportunity.

We analyze this model assuming that firms (indexed by i) choose the number of opportunities o_i to invest in and the number of facets f_i per opportunity to patent, subject to costs which we discuss next. Which facets a firm can patent depends on how many facets rival firms are attempting to patent. We assume that firms choose the opportunities to invest in and facets per opportunity to patent randomly. The patent office then allocates facets that have been chosen by multiple firms randomly. This modeling structure implies that of the $\mathrm{F_k}$ patents available per technological

[2] Note we assume that all opportunities and facets are symmetric.

opportunity, only $\tilde{F}_k = (1-(1-(\hat{f}/F)^{N_0+1}))$ will be patented, where $\hat{f}$ is the equilibrium number of facets chosen by applicants and N_O is the number of firms that have chosen a specific opportunity.[3] Since $\tilde{F}_k$ may be smaller than F_k the total value of patenting in a technology is $V(\tilde{F}_k) \leq V(F_k)$. The probability p_k that a firm obtains a given patent is:

$$p_k(f_{k'}, F_k, N_O(O, o_{k'}, N)) = \sum_{j=0}^{N_O} \frac{1}{j+1} \binom{N_O}{j} \prod_{l=0}^{N_O-j} \left(1 - \frac{f_l}{F_k}\right) \prod_{m=0}^{j} \frac{f_m}{F_k} \tag{1}$$

Then, the expected number of patents a firm owns when it applies for f_i facets is $\gamma_k \equiv p_k f_i$. The properties of p_k are discussed in Appendix C.3.

2.1 Assumptions

Graevenitz *et al.* (2013) assume that the value function $V_k(\tilde{F}_k)$ is convex in covered facets. We show below that this assumption can be relaxed. We generalize the model by introducing a function relating the share of patents the firm holds on an opportunity (s_{ik}) to the proportion of the value V_k the firm can extract through licensing and its own sales: $\Delta(s_{ik})$. This function captures the benefits that a patent portfolio confers in the market for technology. We assume that these portfolio benefits are subject to decreasing returns to scale.

Thus the assumptions we make on the value function and portfolio benefits are:

$$(VF): \quad V(0) = 0, \quad \frac{\partial V}{\partial \tilde{F}_k} > 0 \tag{2}$$

$$(PB): \quad \Delta(0) = 0, \quad \frac{d\Delta(s_{ik})}{ds_{ik}} > 0 \text{ and } \frac{d^2\Delta(s_{ik})}{ds_{ik}^2} < 0 \tag{3}$$

The model contains three types of patenting costs:

- The costs of R&D per opportunity depend on the overall level of R&D activity by all patenting firms: $C_0\left(\sum_j^{N_O} o_j\right)$.
- Per granted patent a firm faces costs of maintaining that patent in force equal to C_a.
- The coordination of R&D on *different* technological opportunities imposes costs $C_c(o_k)$. We assume that

$$\frac{\partial C_c}{\partial o_k} > 0 \tag{4}$$

[3] The properties of N_0 are summarized in Appendix C.

Note that this implies that R&D costs are fixed costs and that there is no technological uncertainty.[4] We do allow for the endogenous determination of the level of R&D fixed costs, which rise as more opportunities are researched simultaneously by rival firms. This reflects competition for inputs into R&D that are fixed in the short run. Coordination costs on different R&D projects also limit the scope of the firm's R&D operations.

Where multiple firms own facets on an opportunity, their legal costs $L(\gamma_{ik}, s_{ik}, h_k)$ depend on the absolute number of patented facets (γ_{ik}), on the share of patents per opportunity that a firm holds (s_{ik}), and on the extent to which they face hold-up (h_k). The first two channels capture the costs of defending a patent portfolio as the number of patents increases, while leaving scope for effects on bargaining costs that derive from the share of patents owned:[5] The hold-up parameter captures contexts in which several firms' core technologies become extremely closely intertwined. Then each firm has to simultaneously negotiate with many others to commercialize its products, which significantly raises costs.

$$(LC):\quad L(\gamma_{ik}, s_{ik}, h_k),\quad \text{where}\quad \begin{aligned}&\frac{\partial L}{\partial \gamma_{ik}} > 0, \frac{\partial^2 L}{\partial \gamma_{ik}^2} \geq 0, \frac{\partial L}{\partial s_{ik}} \leq 0, \frac{\partial^2 L}{\partial s_{ik}^2} \geq 0,\\ &\frac{\partial L}{\partial h_k} > 0, \frac{\partial^2 L}{\partial \gamma_{ik}\partial h_k} > 0, \frac{\partial^2 L}{\partial s_{ik}\partial h_k} > 0\end{aligned} \tag{5}$$

All remaining cross partial derivatives of the legal costs function are zero.

In what follows, we use the following definitions:

$$\omega_k \equiv \frac{o_i}{O_k}, \quad \phi_k \equiv \frac{f_i}{F_k}, \quad \mu_k = \frac{\tilde{F}_k}{V(\tilde{F}_k)}\frac{\partial V(\tilde{F}_k)}{\partial \tilde{F}_k}, \quad \xi_k = \frac{s_k}{\Delta(s_k)}\frac{d\Delta(s_k)}{ds_k}, \quad \text{and} \quad \eta_k = \frac{f_i}{\tilde{F}_k}\frac{\partial \tilde{F}_k}{\partial f_i}.$$

2.2 A Model of Patenting and Entry

Firm *i*'s profits in technology *k*, $\pi_{ik}(o_i, f_i, F_k, O_k, N_k, h_k)$ is a function of the number of opportunities o_i in which the firm invests, the number of facets per opportunity f_i the firm seeks to patent, the total number of patentable facets per opportunity F_k, the number of technological opportunities a technology offers O_k, the number of firms entering the technology N_k, and the degree of hold-up in that technology h_k.

In this section we analyze the following two-stage game G^*:

[4] Introducing technological uncertainty is possible but does not change the main comparative statics results.

[5] Graevenitz et al. (2013) analyse alternative assumptions on legal costs.

Stage 1: Firms enter until $\pi_{ik}(o_i, f_i, F_k, O_k, N_k, h_k) = 0$;[6]

Stage 2: Firms simultaneously choose the number of opportunities, o_i, to invest in and the number of facets per opportunity, f_i, to patent in order to maximize profits π_{ik}.

We solve the game by backward induction and derive local comparative statics results for the symmetric extremal equilibria of the second stage game. For the subsequent analysis it is important to note that all equilibria of this second stage game are symmetric. In case that the second stage game has multiple equilibria we focus on the properties of the extremal equilibria when providing comparative statics results (Milgrom and Roberts, 1994; Amir and Lambson, 2000; Vives, 2005). Equilibrium values of the firms' choices are denoted by a superscript and we drop the firm specific subscripts in what follows, e.g., $\hat{\phi}_k$.

At stage two of the game each firm maximizes the following objective function:

$$\pi_{ik}(o_i, f_i) = o_i\left(V(\tilde{F}_k)\Delta(s_{ik}) - L(\gamma_{ik}, s_{ik}, h_k) - C_0(\textstyle\sum_j^{N_O} o_j) - f_i p_k C_a\right) - C_c(o_i) \quad (6)$$

This expression shows that per opportunity k, the firm derives profits from its share $s_{ik} \equiv p_k f_i / \tilde{F}_k$ of patented facets, while facing legal costs L to appropriate those profits, as well as costs of R&D C_0, costs of maintaining its patent portfolio C_a, and coordination costs across opportunities C_c.

As noted above we generalize the model of Graevenitz et al. (2013) by allowing each firm's proportion of profits to be a non-linear function of the share (s_{ik}) of patents each firm obtains per opportunity. This allows us to analyse effects of entry on patenting, by generalizing the conditions for supermodularity of the second stage of game G* (Cf. Footnote 7). We then draw on this to show when the equilibrium is unique (Cf. Appendix D.2).

2.3 Simultaneous Entry with Multiple Facets

2.3.1 Comparative statics of patenting

We show that the second stage of this game is smooth supermodular:

Proposition 1

The second stage game, defined in particular by assumptions (VF, eq. 2), (PB, eq.3) and (LC, eq. 5) is smooth supermodular if $\mu > \xi_{ik}$ *and if ownership of the technology is expected to be fragmented.*

[6] We treat N_k as a continuous variable here, which is an abstraction that simplifies our analysis.

The second order conditions from which we derive the supermodularity of game G^* contain the following expressions:

$$\left(V\frac{\Delta(\hat{s}_k)}{\hat{s}_k}(\mu_k-\hat{\xi}_k)+\frac{\partial L}{\partial \hat{s}_k}\right)>0 \text{ and } \left(1-2\hat{\eta}_k-\frac{\hat{\phi}_k}{1-\hat{\phi}_k}\right)>0 \qquad (7)$$

These jointly determine the sign of the second order conditions characterizing equilibria of the second stage game. The first and second order conditions for the second stage of game G^* are set out in Appendix D.1.

We discuss each condition in turn:

1. Given our assumptions about the legal cost function *(LC, eq. 5)*, the condition $\left(V\frac{\Delta(\hat{s}_k)}{\hat{s}_k}(\mu_k-\hat{\xi}_k)+\frac{\partial L}{\partial \hat{s}_k}\right)>0 \Leftrightarrow V\frac{\Delta(\hat{s}_k)}{\hat{s}_k}(\mu_k-\hat{\xi}_k)>-\frac{\partial L}{\partial \hat{s}_k}$ implies that $\mu>\hat{\xi}_k$. The elasticity of the value function w.r.t. additional covered patents must exceed the elasticity of the portfolio benefits function w.r.t. the share of patents held by the firm.[7]

2. $\left(1-2\hat{\eta}_k-\frac{\hat{\phi}_k}{1-\hat{\phi}_k}\right)>0 \Leftrightarrow (1-2\hat{\phi}_k)>(1-\hat{\phi}_k)^{(N_0+1)}$. This holds for any $\hat{\phi}_k<\frac{1}{2}$ and N_0 sufficiently large. These restrictions imply a setting in which the ownership of patents that belong to each opportunity is fragmented among many firms. It is more likely to arise if the technology is highly complex, otherwise the condition that $\hat{\phi}_k<\frac{1}{2}$ is less likely to hold.

In Appendix D.2 we derive the conditions under which the equilibrium of game G^* is unique. If there is a unique solution to the optimization problem of the firm at which profits are maximized, then this requires that $\partial^2\pi_k/\partial\hat{f}^2<0$. The restrictions that (i) $\mu_k < 1$ and (ii) the share of overall profits which the firm obtains is decreasing at the margin in the share of patents the firm holds ($\partial^2\pi/\partial\hat{s}_k^2<0$) ensure that there is always such a unique interior solution.

In game G^* the comparative statics of patenting are the same as in the main model analyzed in Graevenitz *et al.* (2013). Specifically, we can show that the following holds in this game:

---

[7] This condition is less restrictive than the assumption in Graevenitz *et al.* (2013) that $\mu_k>1$, since we are allowing for the possibility that $\hat{\xi}_k<1$, see *(PB, eq. 3)*.

$$\frac{\partial^2 \pi}{\partial o_i \partial F_k} > 0, \frac{\partial^2 \pi}{\partial f_i \partial F_k} > 0, \frac{\partial^2 \pi}{\partial o_i \partial O_k} < 0, \frac{\partial^2 \pi}{\partial f_i \partial O_k} < 0 \tag{8}$$

Explicitly considering the effect of hold-up on the incentives to patent, we find that:

Proposition 2

Hold-up in complex technologies reduces patenting incentives.

To see this consider the following cross-partial derivatives for the effects of higher legal costs L due to hold-up:

$$\frac{\partial^2 \pi}{\partial o \partial h_k} = -\frac{\partial L(\hat{\gamma}_k, \hat{s}_k, h_{k)}}{\partial h_k} < 0 \tag{9}$$

$$\frac{\partial^2 \pi}{\partial \hat{f} \partial h_k} = -\frac{\hat{o} p_k}{\tilde{F}_k} \left(\tilde{F}_k \frac{\partial^2 L}{\partial \hat{\gamma}_k \partial h_k} + \frac{\partial^2 L}{\partial \hat{s}_k \partial h_k} (1 - \hat{\eta}_k) \right) < 0 \tag{10}$$

The first of these conditions shows that in equilibrium the expected legal costs of hold-up reduce the number of opportunities, which firms invest in. The second condition shows that firms with larger portfolios are more exposed to hold-up and benefit less from the share of patents they have patented per opportunity. Both effects combine to reduce the number of facets each firm applies for.

2.3.2 Comparative statics of entry

Next we consider the first stage of the game G^*. We derive how the equilibrium number of entrants N_k changes as the complexity and degree of technological opportunity that characterize a technology change. We derive the following propositions:

Proposition 3

There is a free entry equilibrium at which the marginal entrant can just break even, if R&D fixed costs per opportunity (C_0) increase in the number of entrants.

In the model entry benefits firms as long as entrants contribute to the probability that a technological opportunity will be fully developed. This is counteracted by competition between firms on markets for R&D inputs.

This proposition can be derived from the model using the implicit function theorem. Consider how entry affects profits. A free-entry equilibrium exists if the following conditions hold:

$$\pi_k(\hat{o}_k, \hat{f}_k, \hat{N}_k) > 0 \quad \text{and} \quad \pi_k(\hat{o}_k, \hat{f}_k, \hat{N}_k + 1) < 0 \tag{11}$$

The effect of entry on profits at the first stage of game G^* can be shown to be the following:

$$\frac{\partial \pi(\hat{o},\hat{f})}{\partial N_k} = \hat{o}\frac{\partial N_O}{\partial N_k}\left(\frac{\hat{s}_k}{\tilde{F}_k}\frac{\partial \tilde{F}_k}{\partial N_O}\left[V(\tilde{F}_k)\mu\frac{\Delta(\hat{s}_k)}{\hat{s}_k} - \left(V(\tilde{F})\frac{d\Delta(\hat{s}_k)}{d\hat{s}_k} - \frac{\partial L}{\partial \hat{s}_k}\right)\right]\right.$$
$$\left. + \frac{\partial p_k}{\partial N_O}\frac{\hat{f}}{\tilde{F}_k}\left[\left(V(\tilde{F}_k)\frac{d\Delta(\hat{s}_k)}{d\hat{s}_k} - \frac{\partial L}{\partial \hat{s}_k}\right) - \tilde{F}_k\left(\frac{\partial L}{\partial \hat{\gamma}_k} + C_a\right)\right] - \frac{\partial C_0}{\partial N_O \hat{o}}\hat{o}\right) \tag{12}$$

This expression can be further simplified using some results from Appendix C:

$$\frac{\partial \pi(\hat{o},\hat{f})}{\partial N_k} = \hat{o}\varepsilon_{N_O,N}\frac{\hat{s}_k}{N}\left((\varepsilon_{\tilde{F}_k,N_O} - \varepsilon_{p_k,N_O}\hat{\eta}_k)\left[V(\tilde{F})\frac{\Delta(\hat{s}_k)}{\hat{s}_k}(\mu_k - \hat{\xi}_k) + \frac{\partial L}{\partial \hat{s}_k}\right] - \frac{\partial C_0}{\partial N_O \hat{o}}\frac{N_O \hat{o}}{\hat{s}_k}\right) \tag{13}$$

The first two terms in brackets in this derivative are positive and so is the third term. We can show that the limits of $\varepsilon_{\tilde{F}_k,N_0}$ and $\hat{\eta}_k$ in N_0 are both zero. Therefore the above derivative is negative as long as the R&D fixed costs per opportunity are increasing in N_0. This is the condition set out in Proposition 3.

Proposition 4

Under free entry greater complexity of a technology increases entry.

In the model, complexity has countervailing effects: first of all it increases profits, because it is less likely that duplicative R&D arises making each opportunity more valuable, this clearly increases incentives to enter. Next, given the level of patent applications ($\hat{f}_k$), complexity reduces the probability that each facet is patented, which reduces profits and entry incentives. Finally, complexity reduces competition for each facet, which increases the probability of patenting and increases innovation incentives. Overall we show that the positive effects outweigh the negative effects and incentives for entry rise with complexity of a technology.

To derive Proposition 4, continuing directly from Proposition 3, consider how equilibrium profits are affected by the complexity of the technology F_k, the degree of technological opportunity O_k, and the potential for hold-up h_k:

$$\frac{\partial \pi(\hat{o},\hat{f})}{\partial F_k} = \hat{o}\frac{s_k}{F_k}\left((\varepsilon_{\tilde{F}_k,F_k} - \varepsilon_{p_k,F_k}\hat{\eta}_k)\left[V(\tilde{F}_k)\frac{\Delta(\hat{s}_k)}{\hat{s}_k}(\mu_k - \hat{\xi}_k) + \frac{\partial L}{\partial \hat{s}_k}\right]\right) > 0 \tag{14}$$

$$\frac{\partial \pi(\hat{o},\hat{f})}{\partial O_k} = \hat{o}\frac{\partial N_O}{\partial O_k}\frac{\hat{s}_k}{N_O}\left((\varepsilon_{\tilde{F}_k,N_O} - \varepsilon_{p_k,N_O}\hat{\eta}_k)\left[V(\tilde{F}_k)\frac{\Delta(\hat{s}_k)}{\hat{s}_k}(\mu_k - \hat{\xi}_k) + \frac{\partial L}{\partial \hat{s}_k}\right] - \frac{\partial C_O}{\partial N_O \hat{o}}\frac{N_O \hat{o}}{\hat{s}_k}\right) > 0 \tag{15}$$

$$\frac{\partial \pi(\hat{o},\hat{f})}{\partial h_k} = -\hat{o}\frac{\partial L}{\partial h_k} < 0 \tag{16}$$

Propositions 4 follows from the Implicit Function theorem once we know the sign of the derivative of profits w.r.t. F. Under free entry firms' profits decrease with entry:

$$\frac{\partial N}{\partial F_k} = -\frac{\partial \pi}{\partial F_k} \bigg/ \frac{\partial \pi}{\partial N_k} \qquad (17)$$

Therefore, the Implicit Function theorem implies that the sign of the effect of complexity *F* on entry depends on the sign of the effect of complexity on profits.

Equation (14) shows that the effect of complexity on profits depends on the difference between the elasticities $\varepsilon_{\tilde{F}_k, F_k}$ and $\hat{\eta}_k$. The elasticity $\varepsilon_{p_k, F}$ is derived in Appendix C.3:

$$\varepsilon_{p_k, F_k} = N_O^2 \frac{\hat{\phi}_k - \frac{1}{2}\left(1 + \frac{1}{N_O}\right)}{1 - \hat{\phi}_k} \qquad (18)$$

This elasticity is negative for $\hat{\phi}_k < \frac{1}{2}$. The result implies that the first term in brackets in equation (14) is positive. The second term is positive when game G* is supermodular. Overall this implies that greater complexity induces entry. $\hat{\phi}_k < \frac{1}{2}$ is one of two restrictions required for supermodularity of game *G**. This demonstrates that complexity increases entry in settings in which firms are playing a supermodular game and in which complexity also induces more patenting.

Proposition 5

Under free entry greater technological opportunity increases entry.

For any given number of entrants an increase in technological opportunity reduces competition between firms for patents. This increases firms' expected profits and increases entry.

Continuing from the proof of Proposition 4 above, by the Implicit Function theorem the sign of the derivative of profits w.r.t. technological opportunity determines the effect of technological opportunity on entry:

$$\frac{\partial N}{\partial O_k} = -\frac{\partial \pi}{\partial O_k} \bigg/ \frac{\partial \pi}{\partial N_k} \qquad (19)$$

Equation (15) shows that the effect of technological opportunity on profits has exactly the opposite sign to the effect of additional competition on profits, because the expression in brackets is the same as for equation (13). The sign is reversed here because more opportunity reduces the number of firms active in each opportunity, given *N*.

Proposition 6

Under free entry the potential for hold-up reduces entry.

An increase in the potential for hold-up raises firms' expected legal costs. This reduces expected profits and lowers potential for entry.

To derive this prediction, note that by the Implicit Function theorem the sign of the derivative of profits w.r.t. the level of hold-up in a technology area determines the effect of hold-up on entry:

$$\frac{\partial N}{\partial h_k} = -\frac{\partial \pi}{\partial h_k} \Big/ \frac{\partial \pi}{\partial N_k} \tag{20}$$

Hence, equation (16) shows that the effect of hold-up on entry derives from the increased legal costs that the possibility of hold-up imposes on affected firms.

2.4 Entry and Incumbency

The previous section sets out a model in which all firms entered and then invested in patents. At both stages firms' decisions were simultaneous. Here we extend the model to a setting in which some firms, the incumbents, face lower costs ($C_O - \Psi$, where $\Psi > 0$) of entering opportunities. This captures the fact that incumbents have previous experience of doing R&D in a technology area. We analyze how this affects all firms' incentives to patent.

We assume that a fraction λ (where $0 < \lambda < 1$) of the previously active N^P firms remain as incumbents. The remaining firms enter until the marginal profit from entry is reduced to zero.

Objective Functions

First, consider the objective functions of incumbents and entrants and the patenting game in which they are involved. Given symmetry of technological opportunities the expected value of patenting for entrant and incumbent firms in a specific technology area k is:

$$\pi_{ik}^{I}(o_i^I, f_i^I) = o_i^I \left(V(\tilde{F}_k)\Delta(s_{ik}^I) - L(\gamma_{ik}^I, s_{ik}^I) - \left(C_0\left(\sum_{j=1}^{N^P\lambda-1+N^E} o_j \right) - \Psi \right) - f_i^I p_k C_a \right) - C_c(o_i^I) \tag{21}$$

$$\pi_{ik}^{E}(o_i^E, f_i^E) = o_i^E \left(V(\tilde{F}_k)\Delta(s_{ik}^E) - L(\gamma_{ik}^E, s_{ik}^E) - \left(C_0\left(\sum_{j=1}^{N^P\lambda-1+N^E} o_j \right) \right) - f_i^E p_k C_a \right) - C_c(o_i^E) \tag{22}$$

Define a game G^E in which:

- There are λN^P incumbent firms and the number of entrants (N^E) is determined by free entry.

- Entrants and incumbents simultaneously choose the number of technological opportunities $o_i^I, o_i^E \in \left[0, O^n\right]$ and the number of facets applied for per opportunity $f_i^I, \mathrm{f}_i^E \in \left[0, \mathrm{F}^n\right]$. Firms' strategy sets S_n are elements of R^4.

- Firms' payoff functions π_{ik}, defined by (21) and (22), are twice continuously differentiable and depend only on rivals' aggregate strategies.

- Assumptions (*VF, eq. 2*) and (*LC, eq. 5*) describe how the expected value and the expected cost of patenting depend on the number of facets owned per opportunity.

Firms' payoffs depend on their rivals' aggregate strategies because the probability of obtaining a patent on a given facet is a function of all rivals' patent applications. Note that the game is symmetric as it is exchangeable in permutations of the players. This implies that if the game can be shown to be supermodular, symmetric equilibria exist (Vives, 2005).[8]

First order conditions for game G^E:

$$\frac{\partial \pi_{ik}^I}{\partial o_i^I} = V(\tilde{F}_k)\Delta(s_{ik}) - \mathrm{L}(\gamma_{ik}, s_{ik}) - \left(C_O\left(\sum_{j=0}^{\lambda N^P + N^E - 1} o_j \right) - \Psi \right) - \gamma_{ik} C_a - \frac{\partial C_c}{\partial o_i^I} = 0 \qquad (23)$$

$$\frac{\partial \pi_{ik}^I}{\partial f_i^I} = \frac{o_i^I p_k}{\tilde{F}_k}\left(\left[V(\tilde{F}_k)\mu_k \eta_{ik} \frac{\Delta(s_{ik})}{s_{ik}} - \tilde{\mathrm{F}}_k\left(\frac{\partial L}{\partial \gamma_{ik}} + C_a \right) \right] + \left(V\frac{d\Delta(s_{ik})}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}} \right)(1-\eta_{ik}) \right) = 0 \qquad (24)$$

$$\frac{\partial \pi_{ki}^E}{\partial o_i^E} = V(\tilde{F}_k)\Delta(s_{ik}) - \mathrm{L}(\gamma_{ik}, s_{ik}) - \left(C_O\left(\sum_{j=0}^{\lambda N^P + N^E - 1} o_j \right) - \Psi \right) - \gamma_{ik} C_a - \frac{\partial C_c}{\partial o_i^E} = 0 \qquad (25)$$

$$\frac{\partial \pi_{ik}^E}{\partial f_i^E} = \frac{o_i^E p_k}{\tilde{F}_k}\left(\left[V(\tilde{F}_k)\mu_k \eta_{ik} \frac{\Delta(s_{ik})}{s_{ik}} - \tilde{\mathrm{F}}_k\left(\frac{\partial L}{\partial \gamma_{ik}} + C_a \right) \right] + \left(V\frac{d\Delta(s_{ik})}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}} \right)(1-\eta_{ik}) \right) = 0 \qquad (26)$$

Proposition 7

In game G^E the equilibrium number of facets chosen by incumbents and entrants is the same: $\hat{f}^I = \hat{f}^E$.

We show in Appendix C.1 that in the game with incumbents the number of rivals per opportunity $\hat{N}_O$ becomes a function of both $\hat{o}^I$ and $\hat{o}^E$. The first order conditions determining $\hat{f}^I$ and $\hat{f}^E$ both depend on the total number of entrants per technological

[8]Note also that only symmetric equilibria exist as the strategy spaces of players are completely ordered.

opportunity $\hat{N}_O$ and so on both $\hat{o}^I$ and $\hat{o}^E$. This is the only way in which rivals' choices of the number of opportunities to pursue enter these first order conditions.[9] Therefore the two conditions are identical and Proposition 7 holds.

Proposition 8

The second stage of game G^E is smooth supermodular under the same conditions as the game without incumbents. Comparative statics results for game G also apply to game G^E.*

The first order conditions characterizing the game with incumbents and entrants are identical to those for the game without incumbents as long as *Ψ=0*. As this variable is a constant it does not enter into the second order conditions which we need to analyze to establish supermodularity and which underpin the comparative statics predictions in propositions 4-6.

Proposition 9

In the second stage of game G^E incumbents enter more technological opportunities, if they have a cost advantage in undertaking R&D (Ψ > 0).

The first order conditions determining the equilibrium number of opportunities chosen by incumbents and entrants are identical if firms R&D fixed costs per opportunity are the same (*Ψ = 0*). Therefore $\hat{o}^I_{|\Psi=0} = \hat{o}^E$. As the cost advantage of incumbents in undertaking R&D grows, this increases the number of opportunities chosen by incumbents:

$$\frac{\partial^2 \pi^I_{ik}}{\partial o^I_i \partial \Psi} = 1 > 0 \tag{27}$$

Proposition10

In the second stage of game G^E the number of entrants decreases as the cost advantage of incumbents increases.

Due to the supermodularity of the second stage game, increases in incumbents' choices of the number of opportunities in which to invest will raise the number of opportunities entrants invest in as well as the number of facets both entrants and incumbents will seek to patent in equilibrium. The increases in $\hat{o}^I$ and $\hat{o}^E$ will increase the fixed costs of entry into new opportunities C_O, which then reduces entry.

---

[9]Clearly the factors outside the brackets in equations (24) and (26) also depend on these variables, but these do not affect the equilibrium values of f^I_i, f^E_j .

2.5 Predictions of the Model

The model discussed above provides predictions for the level of patent applications and for the probability of entry into patenting which is defined as patenting for the first time in a given technology area. For our empirical analysis, we focus on the predictions for entry that follow from the propositions derived above:[10]

Prediction 1

Greater technological opportunity increases the probability of entry.

Greater technological opportunity reduces competition for facets in each opportunity, which raises expected profits and thereby attracts entry. See Propositions 5 and 8.

Prediction 2

Greater complexity of a technology increases the probability of entry.

Greater complexity has countervailing effects: it reduces competition per facet as well as duplicative R&D, attracting entry. It also increases the likelihood that some of a technology remains unpatented, reducing its overall value and entry. Our model shows that overall complexity increases entry. See Proposition 4 for more detail.

Prediction 3

Greater potential for hold-up reduces the probability of entry.

Hold-up potential increases expected costs of entry, reducing it. See Proposition 6.

Prediction 4

More experienced incumbents are more likely to enter technological opportunities new to them.

See Propositions 9 and 10. Proposition 9 shows that incumbency advantage raises the number of opportunities that incumbents enter. This implies that they also enter new opportunities, which they have not previously been active in. This expansion of activity by incumbents crowds out entry by new entrants (Proposition 10).

3 Data and Empirical Model

This section of the paper describes the data we use in the empirical test of our theoretical predictions. In particular, we discuss how we measure entry, how the set of potential entrants is identified, and which measures and covariates are used.

Our empirical model is a hazard rate model of firm entry into patenting in a technology

[10] Graevenitz *et al.* (2013) tested predictions from a more restrictive version of the model on the level of patent applications using data from the European Patent Office.

area as a function of technological opportunity, technological complexity, hold-up potential that characterize a technology area. We test the predictions set out at the end of the previous section for these variables and for the effect of a firm's prior experience in patenting. Additional firm level covariates include the age and size of firms. The models we estimate are stratified at the industry level. That is, the unit of observation for each entry hazard is a firm-technology area, but the hazard shapes and levels are allowed to vary by the industry that the firm is in. This approach recognizes that patenting propensities vary across industries for reasons that may not be technological (e.g., strategic reasons, or reasons arising from the historical development of the sector).

We use a combination of firm level data for the entire population of UK firms registered with Companies House and data on patenting at the European Patent Office and at the Intellectual Property Office for the UK. The firm data comes from the data held at Companies House provided by Bureau van Dijk in their FAME database. European patent registers do not include reference numbers from company registers, nor does Bureau van Dijk provide the identification numbers used by patent offices in Europe. Linking the data from patent registers to firm register data requires matching of applicant names in patent documents and firm names in firm registers. In our work both a firm's current and previous name(s) were used for matching in order to account for changes in firm names. For more details on the matching of firm- and patent-level data see Appendix A.

Economic studies of entry are frequently hampered by the problem of identifying the correct set of potential entrants (Bresnahan and Reiss, 1991; Berry, 1992). In our case this problem is slightly mitigated by the fact that one set of potential entrants into patenting in a specific technology area consists of all those firms that currently patent in other technology areas. We complement this group of firms with a set of comparable firms from the population of UK firms that have not patented previously.

To construct the sample we deleted all firms from the data for which we have no size measure, because of missing data on assets. We select previously non-patenting firms from the population of all UK firms in two steps: 1) we delete all firms in industrial sectors with little patenting (amounting to less than 2 per cent of all patenting); and 2) we choose a sample of non-patenting firms that matches our sample of patenting firms by industry, size class, and age class. In principle, this approach will result in an endogenous (choice-based) sample. However our focus is on industry and technology area level effects rather than firm-level effects. Therefore we do not expect the sampling approach we adopt to introduce systematic biases into the estimates we report. We provide a number of robustness checks to ensure that our results are stable. These reveal that sample composition does not affect the key results we present below. All estimates are based on data weighted by the probability that a firm is in our sample.[11]

The sample that results from our selection criteria is a set of firms with non-missing assets in manufacturing, oil and gas extraction and quarrying, construction, utilities, trade, and selected business services including financial services that includes all (approximately 10,000) firms applying for a patent at the EPO or UKIPO during the 2001-2009 period and another 10,000 firms that did not apply for a patent.

The definition of technology areas that we use is based on the 2008 version of the ISI-OST-INPI technology classification (denoted TF34 classes). The list is shown in Table 1, along with the number of EPO and UKIPO patents applied for by UK firms with priority dates between 2002 and 2009. A comparison of the frequency distribution of patenting across the technology areas from the two patent offices shows that firms are more likely to apply for patents in Chemicals at the EPO, while Electrical and Mechanical Engineering predominate in the national patent data (see the bottom panel in Table 1).

We treat entry into each technology area as a separate decision made by firms. More than half of firms we observe patent in more than one area and 10 per cent patent in more than four. From the 20,000 firms observed, each of which can potentially enter into each one of the 34 technology areas, we obtain about 700,000 observations at risk. We cluster the standard errors by firm, so our models are effectively firm random effects models for entry into 34 technology areas. Allowing firm choices to vary by technology area is sensible under the assumption that firms' patenting strategies are contingent upon technology and industry level factors and are not homogeneous across technology areas. We confirmed the validity of this assumption through interviews with leading UK patent attorneys.

There are some technology-industry combinations that do not occur, e.g. audio-visual technology and the paper industry, telecommunications technology and the pharmaceutical industry. In order to reduce the size of the sample, we drop all technology-industry combinations for which Lybbert and Zolas (2012) find no patenting in their data and for which there was no patenting by any UK firm from the relevant industry in the corresponding technology category. This removes about 30 per cent of observations from the data. We provide a robustness check for this procedure in Appendix B.

[Table 1 here]

[11] To check this, we estimated the model with and without weights based on our sampling methodology and find little difference in the results.

3.1 Variables

Dependent Variable - Entry

The dependent variable is a dichotomous variable taking the value one if a firm has entered a technology area k at time t and otherwise the value zero. Entry into a technology area is measured by the first time a firm applies for a patent that is classified in that technology area, dated by the priority year of the patent.

Technological opportunity

Our first prediction from the theoretical model is that there will be more entry in technology areas with greater technological opportunity. Additional reasons that a sector may have more or less patenting include sector "size" or "breadth" and the propensity of firms to patent in particular technologies for strategic reasons or because of varying patent effectiveness in protecting inventions. To control for both technological opportunity and these other factors, we include the logarithm of the aggregate EPO patent applications in the technology sector during the year. To capture opportunity more specifically we also include the past 5-year growth rate in the non-patent (scientific publication) references cited in patents in that technology class at the EPO.[12] We have found that the growth rate in non-patent references is a better predictor of entry than the level of non-patent references, which has been used previously. Presumably the growth rate is a better indicator because it capture new or expanded technological opportunity.

Technology complexity

The second prediction of the theoretical model is that technological complexity increases entry, other things equal. Our interpretation of complexity is that it implies many interconnections between inventions in a particular field, rather than a series of fairly isolated inventions that do not connect to each other. To construct such a measure, we use the concept of network density applied to citations among all the patents that have issued in the particular technology area during the 10 years prior to the date of potential entry. We use citations at the U.S. patent office, both because these are richer (averaging 7 or so cites per patent during this period versus 3 for the EPO) and also to minimize correlation with the thickets measure, which is based on EPO data.[13]

The network density measure is computed as follows: in any year t, there are N_{kt} patents that have been applied for in technology area k between 1975 and year t. Each of these patents can cite any of the patents that were applied for earlier, which implies that the

[12] See Graevenitz *et al.* (2013) for a more extensive discussion of this variable in the literature.

[13] It is important to emphasize that although patent offices cooperate and share search reports citations listed on U.S. patents are largely proposed by the applicant, whilst the citations listed on EPO and IPO patents are inserted by the examiner. This explains why the two measures are not highly correlated.

maximum number of citations within the technology area is given by $N_{kt}(N_{kt}\text{-}1)/2$. We count the actual number of citations made and normalize them by this quantity, scaling the measure by one million for visibility, given its small size.

Patent Thickets

The third prediction of our model is that greater potential for hold-up reduces entry. We measure the potential for hold-up in patent thickets using the triples count proposed by von Graevenitz *et al.* (2011). This is a narrower interpretation of this measure than in several previous papers, where it has been used as a proxy for complexity of a technology. In those papers complexity and hold-up potential have the same effect. In contrast, our model provides opposite predictions for the effects of complexity of a technology and potential for hold-up.

The triples measure corresponds to a count of the number of fully connected triads on the set of firms' critical patent citations. At time *t* each unidirectional link between two firms *A* and *B* corresponds to one or more critical references to firm *A*'s patents in the set of patents applied for by firm *B* in the years *t, t-1* and *t-2*. We use the same measure of triples as Harhoff *et al.* (2015), which contains all triples in each technology area. The citation data used is extracted from PATSTAT (October 2011 edition).[14] We normalize the count of triples by aggregate patenting in the same sector, so that the triples variable represents the intensity with which firms potentially hold blocking patents on each other relative to aggregate patenting activity in the technology.

The triples measure has been used in a number of papers since it was suggested by Graevenitz *et al.* (2011). They show that counts of triples by technical area are significantly higher for technologies classified as complex than for areas classified as discrete by Cohen *et al.* (2000). Fischer and Henkel (2012) find that the measure predicts patent acquisitions by Non-Practicing Entities. Graevenitz *et al.* (2013) use the measure to study patenting incentives in patent thickets and Harhoff *et al.* (2015) show that opposition to patent applications falls in patent thickets, particularly for patents of those firms that are caught up in the thickets.

As a robustness check, we have also explored the use of duples, i.e. the count of mutual blocking relationships, to measure hold-up potential. Combining both measures in one regression leads to thorny problems of interpretation. Taken alone the measure has similar effects as the triples measure in this context.

[Table 2 here]

[14] Triples data was kindly provided by Harhoff et al. (2015).

Covariates

It is well known that firm size and industry are important predictors of whether a firm patents at all (Bound *et al.* 1984 for U.S. data). Hall *et al.* (2012b) show this for UK patenting during the period studied here. Therefore, in all of our regressions we control for firm size, industrial sector, and year of observation. We include the logarithm of the firm's reported assets and a set of year dummies in all the regressions.[15] To control for industrial sector, we stratify by industry, which effectively means that each industry has its own hazard function, which is shifted up or down by the other regressors.

We also expect the likelihood that a firm will enter a particular technology area to depend on its prior patenting experience overall, as well as its age. Long-established firms are less likely to be exploring new technology areas in which to compete. Thus we include the logarithm of firm age and the logarithm of the stock of prior patents applied for in any technology by the firm, lagged one year to avoid any endogeneity concerns. The variables on firm size and patent stock also allow us to test *Prediction 4* about the effect of incumbency advantage on entry.

3.2 Descriptive Statistics

Our estimation sample contains about 20,000 firms and 700,000 firm-TF34 sector combinations. During the 2002-2009 period there are about 10,000 entries into patenting for the first time in a technology area by these firms. Table A-2 shows the distribution of the number of entries per firm: 2,531 enter one class, and the rest enter more than one. Table A-2 shows the population of UK firms obtained from FAME in our industries, together with the shares in each industry that have applied for a UK or European patent during the 2001-2009 period. These shares range from over 10 per cent in Pharmaceuticals and R&D Services to less than 0.1 per cent in Construction, Oil and Gas Services, Real Estate, Law, and Accounting.

Empirical Model

We use hazard models to estimate the probability of entry into a technology area. The models express the probability that a firm enters into patenting in a certain area conditional on not having entered yet as a function of the firm's characteristics and the time since the firm was "at risk," which is the time since the founding of the firm. In some cases, our data do not go back as far as the founding date of the firm, and in these cases the data are "left-censored." When we do not observe the entry of the firm into a particular technology sector by the last year (2009), the data is referred to as "right-censored."

In Appendix B, we discuss the choice of the survival models that we use for analysis,

[15] The choice of assets as a size measure reflects the fact that it is the only size variable available for the majority of the firms in the FAME dataset.

how to interpret the results, and present some robustness checks. We estimate two classes of failure or survival models: 1) proportional hazard, where the hazard of failure over time has the same shape for all firms, but the overall level is proportional to an index that depends on firm characteristics; and 2) accelerated failure time, where the survival rate is accelerated or decelerated by the characteristics of the firm. In the body of the paper we present results using the well-known Cox proportional hazards model stratified by industry. Results from the accelerated failure time models were similar but the estimated effects are somewhat larger (shown in Appendix B).

As indicated earlier, our data for estimation are for the 2002-2009 period, but many firms have been at risk of patenting for many years prior to that. The oldest firm in our dataset was founded in 1856 and the average founding year was 1992. Because the EPO was only founded in 1978, we chose to use that year as the earliest date any of our firms is at risk of entering into patenting. That is, we defined the initial year as the maximum of the founding year and 1978. Table B-2 in the appendix presents estimates of our model using 1900 instead of 1978 as the earliest at risk year and finds little difference in the estimates.[16] We conclude that the precise assumption of the initial period is innocuous. Our assumption amounts to assuming that the shape of the hazard for firms founded between 1856 and 1978 but otherwise identical is the same during the 2002-2009 period.

Appendix Table B-1 shows exploratory regressions made using various survival models. None of the choices made large differences to the coefficients of interest, so that we focus here on the results from the Cox proportional hazards model, estimated with stratification by two-digit industry. The effect of the stratification is that we allow firms in each of the industries to have a different distribution of the time until entry into patenting conditional on the regressors. That is, each industry has its own "failure" time distribution, where failure is defined as entry into patenting in a technology area, but the level of this distribution is also modified by the firm's size, aggregate patenting in the technology, network density, and the triples density.

4 Results

Our estimates of the model for entry into patenting are shown in Table 3. All regressions control for size, age, and industry. Both size and age are strongly positively associated with entry into patenting in a new technological area. Our indicator of technological opportunity and technology class size, the log of current patent

[16] The main difference is in the firm age coefficient. Because the models are nonlinear, this coefficient is identified even in the presence of year dummies and vintage/cohort (which is implied by the survival model formulation). However it will be highly sensitive to the assumptions about vintage due to the age-year-cohort identity.

applications in the technology class, is also positively associated with entry into that class, as predicted by our model.

Column 3 of Table 3 contains the basic result from our data and estimation, which is fully consistent with the predictions of our model: greater complexity as measured by citation network density increases the probability of entry into a technology area, as does technological opportunity, measured both as prior patenting in the class and as growth in the relevant science literature. Controlling for both technological opportunity and complexity, firms are discouraged from entry into areas with a greater density of triple relationships among existing firms. We interpret this latter result as an indicator of the discouraging effect of holdup possibilities or the legal costs associated with negotiation of rights or defense in the case of litigation.

We were concerned that our network density (complexity) and triples density (hold-up potential) measures might be too closely related to convey separate information, but we found that the raw correlation between these two variables was -0.001. To check for the impact of potential correlation conditional on year, industry, and the other variables, in columns 1 and 2 of Table 3 we included these two measures of complexity/thickets separately and found that although the coefficients were very slightly lower in absolute value, the results still hold, although it is clear that the aggregate class size is correlated negatively with the triples density via the denominator of the density (compare the change in the log (patents in class) coefficient between columns 1 and 2).

As we show in Appendix B, the estimated coefficients in the table are estimates of the elasticity of the yearly hazard rate with respect to the variable, and do not depend on the industry specific proportional hazard. A one standard deviation increase in the log of network density is associated with a 32 percent increase in the hazard of entry (0.13*2.78), while a one standard deviation in the log of triples density is associated with a 20 percent decrease in the hazard of entry (0.14*1.44). Thus the differences across these technology areas in the willingness of firms to enter them is substantial, bearing in mind that the average probability of entry is only about 1.5 per cent in this sample.

[Table 3 here]

There are fixed costs to patenting, and a firm may be more likely to enter into patenting in a new area if it already patents in another area. To test this idea, in the fourth column of Table 3, we add the logarithm of past patenting by the firm. Firms with a greater prior patenting history are indeed more likely to enter a new technology area – doubling a firm's past patents leads to an almost 100% higher hazard of entry.

In the last column we interact the log of assets with the log of patents, the log of network density, the growth of non-patent literature, and the log of triples density to see whether these effects vary by firm size. The results show that the network density and technological opportunity effects decline slightly with firm size. The triples density effect does not show any size relationship, suggesting that hold-up concerns affect firms of all sizes proportionately. We show this graphically in Figure 1, which overlays the coefficients as a function of firm size on the actual size distribution of our firms. From the graph one can see that the impact of aggregate patenting in a sector is higher and more variable than the impact of the network density, and that both fall to zero for the largest firms. Growth in non-patent literature is positively associated with technology entry for small firms, but negatively for large firms, suggesting the role played by the smaller firms in newer technologies based on science. Large firms seem not to be as active in these areas. Controlling for all these features of a technology, the impact of triples density is uniformly negative across firm size, which contradicts the view that the potential for hold-up discourages entry by smaller firms more than by larger firms.

4.1 Robustness

Table B-2 in the appendix explores some variations of the sample used for estimation in Table 3. Column 1 of Table B-2 is the same as column 4 of Table 3 for comparison. The first change (column 2) was to add back all the technology-industry combinations where Lybbert and Zolas (2012) find no patenting in their data and where there was no entry by any UK firm from the relevant industry into that technology category. These observations are about 20 per cent of the sample. The impact of network density on entry is weaker, but the impact of triples density and the technological opportunity variables is considerably stronger. That is, technology area-industry combinations with no patenting are also those where the technology area displays low technological opportunity.

Next we removed all the firms with assets greater than 12.5 million pounds, to check whether large firms were responsible for our findings.[17] This removed about 2 per cent of the 20,000 firms. Column 3 of Table B-2 shows that the results do not change a great deal, although they are somewhat stronger. In column 4, we removed the telecommunications technology sector from the estimation, because it is such a large triples outlier. Once again, there was little change to the estimates. The last column of Table B-2 shows the results of defining the minimum entry year as 1900. With the exception of firm age, the coefficients are nearly identical to those in column 1 of the table.

[17] 12.5 million pounds is a cutoff based on the definition of Small and Medium-sized Enterprises (SMEs) as firms with fewer than 250 employees. We do not have employment for all our firms, so we assume that assets are approximately 50 thousand pounds per employee in order to compute this measure. For small firms only, this yields an assets cutoff of 2.5 million pounds.

5 Conclusion

Patent thickets arise for a multitude of reasons; they are mainly driven by an increase in the number of patent filings and concomitant reductions in patent quality (that is, the extent to which the patent satisfies the requirements of patentability) as well as increased technological complexity and interdependence of technological components. The theoretical analysis of patent thickets (Shapiro, 2001) and the qualitative evidence provided by the FTC in a number of reports (FTC, 2003; 2011) suggest that thickets impose significant costs on some firms. The subsequent literature has focused on the measurement of thickets (e.g. Graevenitz et al. 2011; Ziedonis, 2004) and has linked thickets to changes in firms' IP strategies in a number of dimensions. There is still a lack of evidence on the effect of patent thickets as well as their welfare implications at the aggregate level.

The empirical analysis of the effects of patent thickets must contend with two challenges: first, patent thickets have to be measured and secondly, effects of thickets must be separated from effects of other factors that are correlated with the growth of thickets, in particular technological complexity.

This paper confronts both challenges. We show that our empirical measure for the density of thickets captures effects of patent thickets predicted by theory. This supports results by von Graevenitz et al. (2011, 2013) and Harhoff et al. (2015) showing that the coefficients on the triples measure capture predicted effects of patent thickets on patenting and opposition. The paper also separates the impact of patent thickets on entry from effects of technological opportunity and complexity and shows that the hold-up potential created by thickets reduces entry into patenting. Controlling for technological opportunity and complexity is important because both are correlated with entry into patenting and the presence of thickets. It is also worth emphasizing that our measure of thickets is purged of effects that are driven by patenting trends in particular technologies. That is, our results are not due to the level of invention and technological progress within a technology field.

Our results demonstrate that patent thickets significantly reduce entry into those technology areas in which growing complexity and growing opportunity increase the underlying demand for patent protection. These are the technology areas, which are associated most with productivity growth in the knowledge economy. However, the welfare consequences of our finding are unclear. Reduced entry into new technology areas could be welfare-enhancing: As is well known from the industrial organization literature, entry into a market may be excessive if entry creates negative externalities for active firms, for instance due to business stealing. This is likely to be true of patenting too. Furthermore, Arora et al. (2008) show that the patent premium does not cover the costs of patenting for the average patent (except for pharmaceuticals). These and related facts might lead one to conclude that lower entry into patenting is likely to increase welfare and that thickets raise welfare by reducing entry.

In contrast, reduced entry into patenting in new technology areas may also be welfare-reducing, for at least two reasons. First, there is the obvious argument that the benefits from more innovation may exceed any business stealing costs (as has been shown empirically in the past by others, e.g., Bloom et al. 2013), so that some desirable innovation may be deterred by high entry costs. Even if this were not true, there is no reason to believe that firms that do not enter into patenting due to thickets are those we wish to deter. Given the incumbency advantage, it is likely that the failure to enter into patenting in these areas reflects less innovation by those who bring the most original ideas, that is, by those who are inventing "outside the box."

References

Amir R, and V. E. Lambson, 2000. On the effects of entry in Cournot markets. *The Review of Economic Studies* 67(2), 235–254.

Arora, A., Ceccagnoli, M., & Cohen, W. M. (2008). R&D and the Patent Premium. *International Journal of Industrial Organization*, 26(5), 1153-1179.

Arundel, A., Patel, P., 2003. Strategic Patenting, Background Report for the Trend Chart Policy Benchmarking Workshop New Trends. Available at http://proinno.intrasoft.be/reports/documents/TCW15_background_paper.pdf.

Berneman, L.P., Cockburn, I., Agrawal, A., Shankar, I., 2009. U.S./Canadian Licensing In 2007-08: Survey Results. *les Nouvelles* 1–8.

Berry, S., 1992. Estimation of a model of entry in the airline industry, Econometrica, 60, 889-917.

Bessen, J., Meurer, M., 2008. *Patent failure: how judges, bureaucrats, and lawyers put innovators at risk*. Princeton, NJ: Princeton University Press.

Bessen, J., Maskin, E., 2007. Sequential Innovation, Patents, and Imitation. *RAND Journal of Economics* 40, 611-635.

Bloom, N., M. Schankerman, and J. van Reenen, 2013. Idenitfying technology spillovers and product market rivalry, *Econometrica* 81 (4), 1347–1393.

Bound, J., Cummins, C., Griliches, Z., Hall, B., Jaffe, A., 1984. Who does R&D and who patents? R&D Patents and productivity, Z. Griliches (ed.). Chicago, University of Chicago Press.

Bresnahan T., Reiss P., 1991. Entry and competition in concentrated markets, *Journal of Political Economy*, 99, 977-1009.

Cockburn, I.M., MacGarvie, M.J., 2011. Entry and Patenting in the Software Industry. *Management Science* 57, 915–933.

Cockburn, I.M., MacGarvie, M.J., Muller, E., 2010. Patent thickets, licensing and innovative performance. *Industrial and Corporate Change* 19, 899–925.

Cohen, W.M., Nelson, R.R., Walsh, J.P., 2000. Protecting their Intellectual Assets: Appropriability Conditions and Why U.S. Manufacturing Firms Patent (or Not). Cambridge, MA: NBER Working Paper No. 7552.

Farrell, J. and Merges, R. P. (2004). Incentives to Challenge and to Defend Patents: Why Litigation Won't Reliably Fix Patent Office Errors and Why Administrative Patent Review Might Help, *Berkeley Technology Law Journal*, 19 (3), 943-970.

Farrell, J., Shapiro, C., 2008. How strong are weak patents? *American Economic Review* 98, 1347–1369.

Federal Trade Commission, 2003. *To Promote Innovation - The Proper Balance of*

Competition and Patent Law and Policy. Washington, DC: GPO. Available at http://www.ftc.gov/reports/innovation/P040101PromotingInnovationandCompetitionrpt0704.pdf

Federal Trade Commission, 2011. The Evolving IP Marketplace:. Aligning Patent Notice and Remedies With Competition. Washington, DC. Available at http://www.ftc.gov/os/2011/03/110307patentreport.pdf

Fischer, T., & Henkel, J. , 2012. Patent trolls on markets for technology–An empirical analysis of NPEs' patent acquisitions. *Research Policy*, 41(9), 1519-1533.

Graevenitz, von G., Wagner, S., Harhoff, D., 2011. How to measure patent thickets—A novel approach. *Economics Letters* 111, 1–4.

Graevenitz, von G., Wagner, S., Harhoff, D., 2013. Incidence and growth of patent thickets: the impact of technological opportunities and complexity. The Journal of Industrial Economics, 61(3), 521-563.

Grindley, P.C., Teece, D.J., 1997. Managing Intellectual Capital: Licensing and Cross-Licensing in Semiconductors and Electronics. *California Management Review* 39, 8–41.

Hall, B.H., 2005. Exploring the Patent Explosion. *Journal of Technology Transfer* 30, 35–48.

Hall, B.H., Harhoff, D., 2004. Post Grant Review Systems at the U.S. Patent Office - Design Parameters and Expected Impact. *Berkeley Technology Law Journal* 19, 981–1015.

Hall, B.H., Helmers, C., von Graevenitz, G., Rosazza-Bondibene, C., 2012a. A study of patent thickets. Draft Report to the UK IPO, 1–66.

Hall, B.H., Helmers, C., Rogers, M., Sena, V., 2012b. The importance (or not) of patents to UK firms. Draft Report to the UK IPO, 1–42.

Hall, B.H., Ziedonis, R.H., 2001. The patent paradox revisited: an empirical study of patenting in the US semiconductor industry, 1979-1995. *RAND Journal of Economics* 101–128.

Harhoff, D. and M. Reitzig (2004). Determinants of Opposition against EPO Patent Grants – The Case of Biotechnology and Pharmaceuticals, *International Journal of Industrial Organization*, 22 (4), 443-480.

Harhoff, D., 2006. Patent Quantity and Quality in Europe - Trends and Policy Implications Collection. In B. Kahin and F. Foray (eds), *Advancing Knowledge and the Knowledge Economy*. Cambridge, MA: MIT Press, 331-350.

Harhoff, D., Graevenitz, von, G., Wagner, S., 2012. Conflict Resolution, Public Goods and Patent Thickets. Munich, Germany: LMU. Mimeo 1–40.

Harhoff, D., Hall, B., Graevenitz, G.V., Hoisl, K., Wagner, S., Gambardella, A., Giuri, P., 2007. *The Strategic Use of Patents and its Implications for Enterprise and Competition*

Policies. Final report for EC Tender No ENTR/05/82, 1–307.

Hegde, D., Mowery, D.C., Graham, S.J.H., 2009. Pioneering Inventors or Thicket Builders: Which U.S. Firms Use Continuations in Patenting? *Management Science* 55, 1214–1226.

Helmers, C., Rogers M., Schautschick P., 2011. Intellectual Property at the Firm-Level in the UK: The Oxford Firm-Level Intellectual Property Database. University of Oxford, Department of Economics Discussion Paper No. 546.

Lemley, M., Shapiro, C., 2005. Probabilistic patents. *Journal of Economic Perspectives* 19, 75–98.

Lemley, M., Shapiro, C., 2007. Patent Holdup and Royalty Stacking. *Texas Law Review* 85:1991.

Lerner, J., 1995. Patenting in the shadow of competitors 38, 463-495.

Lerner, J. and J. Tirole, 2013. Standard-Essential Patents, NBER Working Paper 19664.

Lewis, J.I.D. and R.M. Mott, 2013. The sky is not falling: Navigating the smartphone patent thicket, WIPO Magazine February 2013, http://www.wipo.int/wipo_magazine/en/2013/01/article_0002.html

Lybbert, T. J., Zolas, N. J. 2012. Getting Patents & Economic Data to Speak to Each Other: An 'Algorithmic Links with Probabilities' Approach for Joint Analyses of Patenting & Economic Activity. Geneva, Switzerland: WIPO Working Paper.

Meurer, M. and J. Bessen, 2005. The Patent Litigation Explosion, American Law & Economics Association Annual Meetings, Paper 57.

Milgrom P. and J. Roberts, 1994. Comparing equilibria. *American Economic Review* 84(3), 441–459.

Palmer, J. A. and K. Kreutz-Delgado, 2003. A general framework for component estimation. *Proceedings of the 4th International Symposium on Independent Component Analysis*.

Reitzig, M., Henkel, J., Heath, C., 2007. On sharks, trolls, and their patent prey—Unrealistic damage awards and firms' strategies of "being infringed." Research Policy 36, 134–154.

Schmoch, U., 2009. Document IPC/CE/41/5, Annex, Conception of a Technology Classification for Country Comparisons, 41th session of the IPC Committee of Experts. 1–15.

Shapiro, C., 2000. Navigating the patent thicket: Cross licenses, patent pools, and standard setting. Innovation policy and the economy 119–150.

Somaya, D., 2003. Strategic determinants of decisions not to settle patent litigation. Strategic Management Journal 24, 17–38.

Teece, D., E. Sherry, and P. Grindley, 2014. Patents and "Patent Wars" in Wireless Communications: An Economic Assessment, Digiworld Economic Journal 95, 85-98.

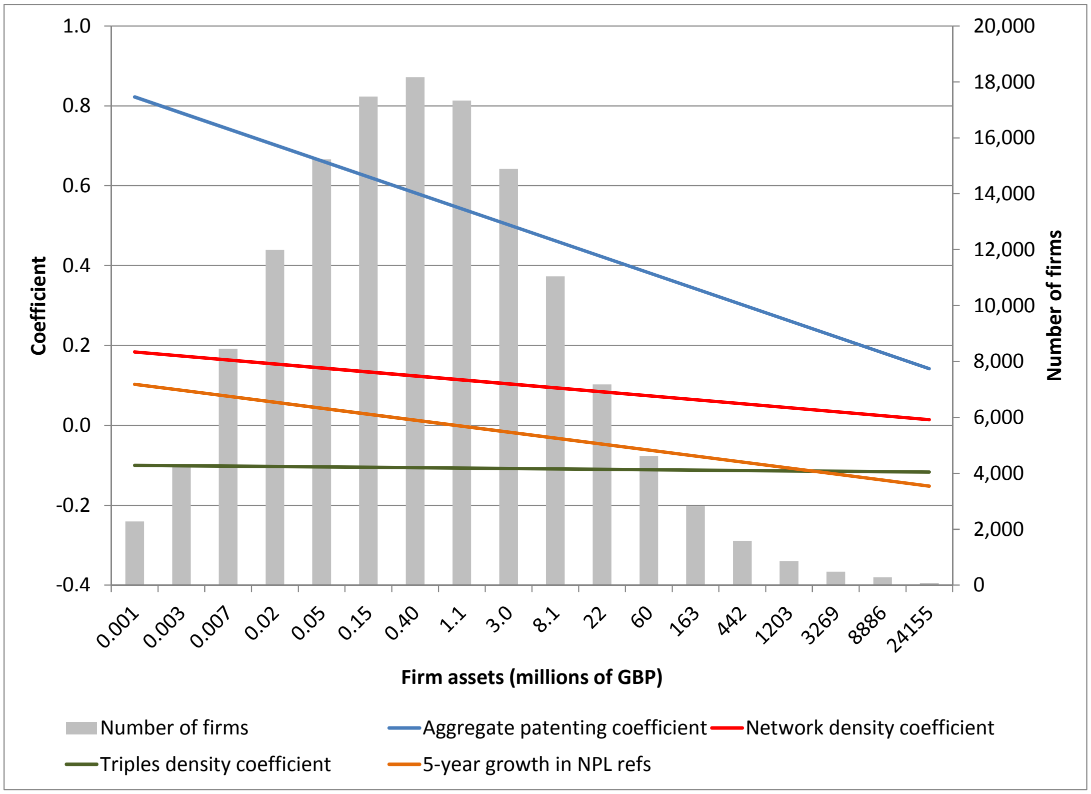
U.S. Department of Justice, Federal Trade Commission, 2007. Antitrust Enforcement and Intellectual Property Rights:. Promoting Innovation and Competition 1–217.

Vives, X., 2005. Complementarities and games: New developments. *Journal of Economic Literature* XLIII(2), 437–479.

WIPO, 2011. The Surge in Worldwide Patent Applications (No. 4/4). WIPO.

Ziedonis, R.H., 2004. Don't Fence Me In: Fragmented Markets for Technology and the Patent Acquisition Strategies of Firms. Management Science 50, 804–820.

Figure 1



Coefficient
Number of firms
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
20,000
18,000
16,000
14,000
12,000
10,000
8,000
6,000
4,000
2,000
0
0.001
0.003
0.007
0.02
0.05
0.15
0.40
1.1
3.0
8.1
22
60
163
442
1203
3269
8886
24155
Firm assets (millions of GBP)
Number of firms
Aggregate patenting coefficient
Network density coefficient
Triples density coefficient
5-year growth in NPL refs

Appendix A: Data

Our analysis relies on an updated version of the Oxford-Firm-Level-Database, which combines information on patents (UK and EPO) with firm-level information obtained from Bureau van Dijk's Financial Analysis Made Easy (FAME) database (for more details see Helmers *et al.* (2011) from which the data description in this section draws).

The integrated database consists of two components: a firm-level data set and IP data. The firm-level data is the FAME database that covers the entire population of registered UK firms.[18] The original version of the database, which formed the basis for the update carried out by the UKIPO, relied on two versions of the FAME database: FAME October 2005 and March 2009. The main motivation for using two different versions of FAME is that FAME keeps details of "inactive" firms (see below) for a period of four years. If only the 2009 version of FAME were used, intellectual property could not be allocated to any firm that has exited the market before 2005, which would bias the matching results. FAME is available since 2000, which defines the earliest year for which the integrated data set can be constructed consistently. The update undertaken by the UKIPO used the April 2011 version of FAME. However, since there are significant reporting delays by companies, even using the FAME 2011 version means that the latest year for which firm-level data can be used reliably is 2009.

FAME contains basic information on all firms, such as name, registered address, firm type, industry code, as well as entry and exit dates. Availability of financial information varies substantially across firms. In the UK, the smallest firms are legally required to report only very basic balance sheet information (shareholders' funds and total assets). The largest firms provide a much broader range of profit and loss information, as well as detailed balance sheet data including overseas turnover. Lack of these kinds of data for small and medium-sized firms means that our study focuses on total assets as a measure of firm size and growth.

The patent data come from the EPO Worldwide Patent Statistical Database (PATSTAT). Data on UK and EPO patent publications by British entities were downloaded from PATSTAT version April 2011. Due to the average 18 months delay between the filing and publication date of a patent, using the April 2011 version means that the patent data are presumably only complete up to the third quarter in 2009. This effectively means that we can use the patent data only up to 2009 under the caveat that it might be somewhat incomplete for 2009. Patent data are allocated to firms by the year in which a firm applied for the patent.

---

[18] FAME downloads data from Companies House records where all limited companies in the UK are registered.

Since patent records do not include any kind of registered number of a company, it is not possible to merge data sets using a unique firm identifier; instead, applicant names in the IP documents and firm names in FAME have to be matched. Both a firm's current and previous name(s) were used for matching in order to account for changes in firm names. Matching on the basis of company names requires names in both data sets to be `standardized' prior to the matching process in order to ensure that small (but often systematic) differences in the way names are recorded in the two data sets do not impede the correct matching. For more details on the matching see Helmers *et al.* (2011).

[Tables A-1, A-2, and A-3 here]

Appendix B: Estimating survival models

This appendix gives some further information about the various survival models we estimated and the robustness checks that were performed. We estimated two general classes of failure or survival models: 1) *proportional hazard*, where the hazard of failure over time has the same shape for all firms, but the overall level is proportional to an index that depends on firm characteristics; and 2) *accelerated failure time*, where the survival rate is accelerated or decelerated by the characteristics of the firm. We transform (2) to a hazard rate model for comparison with (1), using the usual identity between the probability of survival to time *t* and the probability of failure at *t* given survival to *t-1*.

The first model has the following form:

$$Pr\left(i \text{ first patents in } j \text{ at } t \mid i \text{ has no patents in } j\, \forall s < t,\ X_i\right)$$

$$h(X_i, t) = h\left(t\right) exp(X_i, \beta)$$

where *i* denotes a firm, *j* denotes a technology sector, and *t* denotes the time since entry into the sample. *h(t)* is the baseline hazard, which is either a non-parametric or a parametric function of time since entry into the sample. The impact of any characteristic *x* on the hazard can be computed as follows:

$$\frac{\partial h\left(X_i, t\right)}{\partial x_i} = h\left(t\right) exp\left(X_i, \beta\right) \beta \quad \text{or} \quad \frac{\partial h\left(X_i, t\right)}{\partial x_i} \frac{1}{\left(X_i, t\right)} = \beta$$

Thus if *x* is measured in logs, β measures the elasticity of the hazard rate with respect to *x*. Note that this quantity does not depend on the baseline hazard *h(t)*, but is the same for any *t*. We use two choices for *h(t)*: the semi-parametric Cox estimate and the Weibull distribution pt^{p-1}. By allowing the Cox *h(t)* or *p* to vary freely across the industrial sectors, we can allow the shape of the hazard function to be different for different industries while retaining the proportionality assumption.

In order to allow even more flexibility across the different industrial sectors, we also use two accelerated failure time models, the log-normal model and the log-logistic model. These have the following basic form:

$$\text{log-normal:} \quad S(t) = 1 - \Phi\left[\frac{\log(\lambda_i t)}{\sigma_j}\right]$$

$$\text{log-logistic:} \quad S(t) = \left[1 + (\lambda_i t)^{1/\gamma_j}\right]^{-1}$$

where *S(t)* is the survival function and $\lambda_i = exp(X_i\beta)$. We allow the parameters σ (log-normal) or γ (log-logistic) to vary freely across industries (*j*). That is, for these models, both the mean and the variance of the survival distribution are specific to the 2-digit industry. In the case of these two models, the elasticity of the hazard with respect to a characteristic *x* depends on time and on the industry-specific parameter (*σ* or *γ),* yielding a more flexible model. For example, the hazard rate for the log-logistic model is given by the following expression:

$$h(t) = \frac{-d \log S(t)}{dt} = \frac{\lambda_i^{1/\gamma_j} t^{-1+1/\gamma_j}}{\gamma_j \left(1+(\lambda_i t)^{1/\gamma_j}\right)}$$

From this we can derive the elasticity of the hazard rate with respect to a regressor *x*:[19]

$$\frac{\partial \log h_{ij}(t)}{\partial x_i} = \frac{-\beta}{(1+\lambda_i t)^{1/\gamma_j}}$$

One implication of this model is therefore that both the hazard and the elasticity of the hazard with respect to the regressors depend on *t*, the time since the firm was at risk of patenting. We sample the firms during a single decade, the 2000s, but some of the firms have been in existence since the 19th century. This fact creates a bit of a problem for estimation, because there is no reason to think that the patenting environment has remained stable during that period. We explored variations in the assumed first date at risk in Table B-2, finding that the choice made little difference. Accordingly, we have used a minimum at risk year of 1978 for estimation in the main table in the text.

[Tables B-1 and B-2 here]

[19] We assume that *x* is in logarithms, as is true for our key variables, so this can be interepreted as an elasticity.

C Previous Results

To help the reader this appendix summarizes a number of results derived by Graevenitz et al. (2013) as well as some additional results that are useful.

C.1 The Expected Number of Rival Investors

The expected number of rival firms N_O that undertake R&D on the same technology opportunity as firm i can be expressed as a sum of products:

$$N_O = \sum_{j=0}^{N} j \binom{N}{j} \prod_{l=0}^{N-j} (1-\omega_l) \prod_{m=N-i}^{N} \omega_m. \tag{C.1}$$

Graevenitz et al. (2013) show that N_O is increasing in ω_n, where $n \in \{l, m\}$.

In the second stage equilibrium N_O can be rewritten as:

$$N_O = \sum_{j=0}^{N} j \binom{N}{j} (1-\hat{\omega}_k)^{(N-j)} \hat{\omega}_k^j. \tag{C.2}$$

Incumbency Advantage

In the case in which there are incumbents and entrants the expected number of rival firms N_O has to rewritten slightly. To do this define:

$$\omega_n^I \equiv o_i^I / O \qquad \omega_n^E \equiv o_i^E / O \tag{C.3}$$

We assume that in a previous period N^p firms entered and of these a fraction λ are still active. Then the expected number of rival firms $\tilde{N}_O$ that undertake R&D on the same technology opportunity as firm i is:

$$\tilde{N}_O = \sum_{j=0}^{\lambda N^P} j \binom{\lambda N^P}{j} (1-\hat{\omega}_k^J)^{(N-j)} (\hat{\omega}_k^J)^j + \sum_{j=0}^{N} j \binom{N}{j} (1-\hat{\omega}_k^E)^{(N-j)} (\hat{\omega}_k^E)^j. \tag{C.4}$$

C.2 The Expected Number of Facets Covered

In the second stage equilibrium the expected number of facets covered through the joint efforts of all firms investing in a technological opportunity is:

$$\tilde{F}_k = F\Big[1 - (1-\hat{\phi}_k)^{(N_O+1)}\Big] \tag{C.5}$$

The derivative of this expression with respect to F is positive:

$$\frac{\partial \tilde{F}_k}{\partial F_k} = 1 - \Big(1-\hat{\phi}_k\Big)^{N_O} \Big(1+\hat{\phi}_k N_O\Big) \geq 0\ . \tag{C.6}$$

The elasticities of $\tilde{F}_k$ with respect to f_k and F are:

$$\hat{\eta}_k = \frac{\hat{\phi}_k(1-\hat{\phi}_k)^{N_O}}{1-(1-\hat{\phi}_k)^{(N_O+1)}} \tag{C.7}$$

$$\hat{\epsilon}_{\tilde{F}_k,F_k} = \frac{1-(1-\hat{\phi}_k)^{(N_O)}(1+\hat{\phi}_j N_O)}{1-(1-\hat{\phi}_k)^{(N_O+1)}}. \tag{C.8}$$

which shows that $1 \geq \epsilon_{\tilde{F}_k F_k} \geq 0$ as the denominator in the fraction is always greater than the numerator. It is useful to observe that the upper bound of the elasticity $\hat{\eta}_k$ is decreasing in N_O. To see this note that the elasticity can be expressed as:

$$\hat{\eta}_k = \frac{(1-\hat{\phi}_k)^{N_O}}{(N_o+1)\left(1-\hat{\phi}_k\frac{N_o}{2!}+\hat{\phi}_k^2\frac{N_o(N_O-1)}{3!}\ldots\right)} \; . \tag{C.9}$$

This shows that the upper bound of the elasticity decreases in N_O: $\lim_{\hat{\phi}_k \to 0} \eta_k = 1/(N_O+1) \leq 1$. Here we use the binomial expansion of $(1-\hat{\phi}_k)^{N_o+1}$. The expression also shows that the lower bound of $\eta_{k|\hat{\phi}_k=1}$ is zero.

C.3 The Probability of Patenting a Facet

Now turn to the probability of obtaining a patent on a facet given N_O. In the equilibrium of the second stage game we can dispense with the firm specific subscript and denote this as:

$$\begin{aligned} p_k =& (1-\hat{\phi}_k)^{N_O} + \frac{N_O}{2}\cdot\hat{\phi}_k(1-\hat{\phi}_k)^{N_O-1} + \frac{(N_O)(N_O-1)}{6}(1-\hat{\phi}_k)^{N_O-2}\hat{\phi}_k^2\ldots\,, \\ =& \sum_{i=0}^{N_O}\frac{1}{i+1}\binom{N_O}{i}(1-\hat{\phi}_k)^{N_O-i}\hat{\phi}_k^i \end{aligned} \tag{C.10}$$

For the comparative statics of the entry stage it is useful to know that the elasticity of p_k w.r.t. F is negative if $\hat{\phi}_k < \frac{1}{2}$:

$$\frac{\partial p_k}{\partial F_k} = \sum_{i=0}^{N_O}\frac{1}{i+1}\binom{N_O}{i}(1-\hat{\phi}_k)^{N_O-i}\hat{\phi}_k^i(-1)\left(\frac{N_O}{F_k}-\frac{N_O-i}{F-\hat{f}}\right) \tag{C.11}$$

Then the elasticity ϵ_{p_k,F_k} is:

$$\epsilon_{p_k,F_k} = N_O^2\frac{\left(\hat{\phi}_k-\frac{1}{2}(1+\frac{1}{N_O})\right)}{1-\hat{\phi}_k} \tag{C.12}$$

D Results

This appendix contains derivations for the propositions set out in Section 2 of the paper.

D.1 Supermodularity of the Second Stage Game

This section sets out the main results needed to show that the second stage of game G* is supermodular.

Consider the first order conditions that determine the equilibrium number of facets ($\hat{f}$) and technological opportunities ($\hat{o}$):

$$\frac{\partial \pi_{ik}}{\partial o_i} = V\Delta(s_{ik}) - L(\gamma_{ik}, s_{ik}) - C_o(\Sigma_{j=1}^{N_o} o_j) - \gamma_{ik} C_a - \frac{\partial C_c}{\partial o_i} = 0 \quad , \tag{D.13}$$

$$\frac{\partial \pi_{ik}}{\partial f_i} = \frac{o_i p_k}{\tilde{F}_k}\left(\left[V\mu_k\eta_{ik}\frac{\Delta(s_{ik})}{s_{ik}} - \tilde{F}_k\left(\frac{\partial L}{\partial \gamma_{ik}} + C_a\right)\right] + \left[V\frac{d\Delta}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}}\right](1-\eta_{ik})\right) = 0 \quad . \tag{D.14}$$

Now, consider the cross-partial derivatives which must be positive, if the second stage game is supermodular. First, we derive the cross partial derivative with respect to firms' own actions:

$$\frac{\partial^2 \pi_{ik}}{\partial o_i \partial f_i} = \frac{p_k}{\tilde{F}_k}\left(\left[V\mu_k\eta_{ik}\frac{\Delta(s_{ik})}{s_{ik}} - \tilde{F}_k\left(\frac{\partial L}{\partial \gamma_{ik}} + C_a\right)\right] + \left[V\frac{d\Delta}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}}\right](1-\eta_{ik})\right) = 0 \quad . \tag{D.15}$$

This expression corresponds to the first order condition (D.14) for the optimal number of facets.

Now consider effects of rivals' actions on firms' own actions:

$$\frac{\partial^2 \pi_k}{\partial o_i \partial o_m} = \frac{\partial \tilde{F}_k}{\partial o_m}\frac{s_{ik}}{\tilde{F}_k}\left[V\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right] + \frac{\partial p_k}{\partial o_m}\frac{f_i}{\tilde{F}_k}\left[\left(V\frac{d\Delta}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}}\right) - \tilde{F}_k\left(\frac{\partial L}{\partial \gamma_{ik}} + C_a\right)\right] \tag{D.16}$$

$$- \frac{\partial C_o}{\partial \Sigma_{j=1}^{N_o} o_j},$$

$$\frac{\partial^2 \pi_k}{\partial o_i \partial f_m} = \frac{\partial \tilde{F}_k}{\partial f_m}\frac{s_{ik}}{\tilde{F}_k}\left[V\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right] + \frac{\partial p_k}{\partial f_m}\frac{f_i}{\tilde{F}_k}\left[\left(V\frac{d\Delta}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}}\right) - \tilde{F}_k\left(\frac{\partial L}{\partial \gamma_{ik}} + C_a\right)\right], \tag{D.17}$$

$$\begin{aligned}\frac{\partial^2 \pi_k}{\partial f_i \partial o_m} &= \frac{\partial \tilde{F}_k}{\partial o_m}\left[\frac{\partial V}{\partial \tilde{F}_k} + \frac{\partial^2 V}{\partial \tilde{F}_k{}^2}\tilde{F}_k\eta_{ik} - \frac{\partial L}{\partial \gamma_{ik}} - C_a + \frac{\partial^2 L}{\partial s_{ik}{}^2}\frac{s_{ik}}{\tilde{F}_k}(1-\eta_{ik})\right] \\ &+ \frac{\partial \eta_{ik}}{\partial o_m}\left(V\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right) - \frac{\partial p_k}{\partial o_m}\left[\frac{\partial^2 L}{\partial \gamma_{ik}{}^2}f_i + \frac{\partial^2 L}{\partial s_{ik}{}^2}\frac{f_i}{\tilde{F}_k}(1-\eta_{ik})\right],\end{aligned} \tag{D.18}$$

$$\begin{aligned}\frac{\partial^2 \pi_k}{\partial f_i \partial f_m} &= \frac{\partial \tilde{F}_k}{\partial f_m}\left[\frac{\partial V}{\partial \tilde{F}_k}\frac{\Delta}{s_{ik}}\left(\xi_{ik}(1-\epsilon_{\tilde{F}_k,f}) + \epsilon_{\tilde{F}_k,f}\right) + \frac{\partial^2 V}{\partial \tilde{F}_k{}^2}\tilde{F}_k\eta_{ik}\frac{\Delta}{s_{ik}} - \frac{\partial L}{\partial \gamma_{ik}} - C_a + \frac{\partial^2 L}{\partial s_{ik}{}^2}\frac{s_{ik}}{\tilde{F}_k}(1-\eta_{ik})\right] \\ &+ \frac{\partial \eta_{ik}}{\partial f_m}\left(V\frac{\Delta}{s_{ik}}(\mu_k - \xi_{ik}) + \frac{\partial L}{\partial s_{ik}}\right) - \frac{\partial p_k}{\partial f_m}\left[\frac{\partial^2 L}{\partial \gamma_{ik}{}^2}f_i + \frac{\partial^2 L}{\partial s_{ik}{}^2}\frac{f_i}{\tilde{F}_k}(1-\eta_{ik})\right] \quad .\end{aligned} \tag{D.19}$$

The second stage game is supermodular, if the equations (D.16)-(D.19) are non-negative. The following results show that the conditions noted in Section 2 above must hold simultaneously if the game is supermodular.

Using the first order condition (D.14), which will hold for any interior equilibrium, it can be shown that:

$$\left[\left(V\frac{d\Delta}{ds_{ik}}-\frac{\partial L}{\partial s_{ik}}\right)-\tilde{F}_k\left(\frac{\partial L}{\partial \gamma_{ik}}+C_a\right)\right]=-\eta_{ik}\left(V\frac{\Delta}{s_{ik}}(\mu_k-\xi_{ik})+\frac{\partial L}{\partial s_{ik}}\right). \tag{D.20}$$

If $\left(V\frac{\Delta}{s_{ik}}(\mu_k-\xi_{ik})+\frac{\partial L}{\partial s_{ik}}\right)>0$, then the second term in the cross-partial derivatives (D.16) and (D.17) is the product of two negative expressions, and then equation (D.17) is positive. Equation (D.16) is also positive in a free entry equilibrium: the negative term at the end is less than the negative term in the derivative of profits w.r.t. N_k in Section 2, which is otherwise the same as equation (D.16): $\frac{\partial C_o}{\partial N_o\hat{o}}\hat{o}>\frac{\partial C_o}{\partial N_o\hat{o}}$.

Turning to equations (D.18) and (D.19) we can show that:

$$\frac{\partial \eta_{ik}}{\partial o_m}=\frac{\partial^2\tilde{F}_k}{\partial f_i\partial o_m}\frac{f_i}{\tilde{F}_k}-\frac{\partial\tilde{F}_k}{\partial f_i}\frac{\partial\tilde{F}_k}{\partial o_m}\frac{f_i}{\tilde{F}_k{}^2}=-\tilde{F}_k{}^{-1}\frac{\partial\tilde{F}_k}{\partial o_m}\left(\frac{\phi_k}{1-\phi_k}+\eta_{ik}\right) \tag{D.21}$$

$$\frac{\partial \eta_{ik}}{\partial f_m}=\frac{\partial^2\tilde{F}_k}{\partial f_i\partial f_m}\frac{f_i}{\tilde{F}_k}-\frac{\partial\tilde{F}_k}{\partial f_i}\frac{\partial\tilde{F}_k}{\partial f_m}\frac{f_i}{\tilde{F}_k{}^2}=-\tilde{F}_k{}^{-1}\frac{\partial\tilde{F}_k}{\partial f_m}\left(\frac{\phi_k}{1-\phi_k}+\eta_{ik}\right) \tag{D.22}$$

This result allows us to rewrite equations (D.18) and (D.19) as follows:

$$\begin{aligned}\frac{\partial^2\pi_k}{\partial f_i\partial o_m}=&\frac{1}{\tilde{F}_k}\frac{\partial\tilde{F}_k}{\partial o_m}\left[\left(V\frac{\Delta}{s_{ik}}(\mu_k-\xi_{ik})+\frac{\partial L}{\partial s_{ik}}\right)\left(1-2\eta_{ik}-\frac{\phi}{1-\phi}\right)+\frac{\partial^2 V}{\partial\tilde{F}_k{}^2}\tilde{F}_k\eta_{ik}+\frac{\partial^2 L}{\partial s_{ik}{}^2}\frac{s_{ik}}{\tilde{F}_k}(1-\eta_{ik})\right]\\&-\frac{\partial p_k}{\partial o_m}\left[\frac{\partial^2 L}{\partial\gamma_{ik}{}^2}f_i+\frac{\partial^2 L}{\partial s_{ik}{}^2}\frac{f_i}{\tilde{F}_k}(1-\eta_{ik})\right],\end{aligned} \tag{D.23}$$

$$\begin{aligned}\frac{\partial^2\pi_k}{\partial f_i\partial f_m}=&\frac{1}{\tilde{F}_k}\frac{\partial\tilde{F}_k}{\partial f_m}\left[\left(V\frac{\Delta}{s_{ik}}(\mu_k-\xi_{ik})+\frac{\partial L}{\partial s_{ik}}\right)\left(1-2\eta_{ik}-\frac{\phi}{1-\phi}\right)+\frac{\partial^2 V}{\partial\tilde{F}_k{}^2}\tilde{F}_k\eta_{ik}+\frac{\partial^2 L}{\partial s_{ik}{}^2}\frac{s_{ik}}{\tilde{F}_k}(1-\eta_{ik})\right]\\&-\frac{\partial p_k}{\partial f_m}\left[\frac{\partial^2 L}{\partial\gamma_{ik}{}^2}f_i+\frac{\partial^2 L}{\partial s_{ik}{}^2}\frac{f_i}{\tilde{F}_k}(1-\eta_{ik})\right].\end{aligned} \tag{D.24}$$

Given assumptions (VF) and (LC) these two equations will be positive if $\left(V\frac{\Delta}{s_{ik}}(\mu_k-\xi_{ik})+\frac{\partial L}{\partial s_{ik}}\right)>0$ and $\left(1-2\eta_{ik}-\frac{\phi}{1-\phi}\right)>0$.

D.2 Uniqueness of the second stage equilibrium

We show that stage 2 of game G* is supermodular. This implies that there exists at least one equilibrium of the stage game. An alternative way of deriving existence of the second stage equilibrium for game G* is to analyze the conditions under which the Hessian of second derivatives of the profit function (H_π) is negative semidefinite. This matrix consists of four derivatives of which only one leads to additional restrictions on the model.

It is easy to see that $\frac{\partial^2\pi}{\partial o_i{}^2}<0$ due to the coordination costs $C_c(o_i)$ and the restrictions we impose with assumption (FVC). The two cross-partial derivatives are both zero in equilibrium - refer to equation D.15.

Therefore, the only expression that remains to analyze is $\frac{\partial^2 \pi}{\partial {f_i}^2}$.

$$\frac{\partial^2 \pi}{\partial {f_i}^2} = \frac{o_i p_k}{\tilde{F}_k}\left[\frac{\partial^2 V}{\partial \tilde{F}_k^{\,2}}\left(\frac{\partial \tilde{F}_k}{\partial f_i}\right)^2 \frac{\Delta \tilde{F}_k}{p_k} + 2\frac{\partial V}{\partial \tilde{F}_k}\frac{\partial \tilde{F}_k}{\partial f_i}\frac{d\Delta}{ds_{ik}}\left(1-\eta_{ik}\right) + V\frac{d^2\Delta}{s_{ik}^2}\frac{p_k}{\tilde{F}_k}\left(1-\eta_{ik}\right)^2 \right.$$
$$\left. - \frac{\partial^2 L}{\partial {\gamma_{ik}}^2}p_k^2 - \frac{\partial^2 L}{\partial {s_{ik}}^2}\left(\frac{\partial s_{ik}}{\partial f_i}\right)^2 - 2\left(V\frac{d\Delta}{ds_{ik}} - \frac{\partial L}{\partial s_{ik}}\right)\frac{\left(1-\eta_{ik}\right)\eta_{ik}}{f_i}\right] \quad .$$

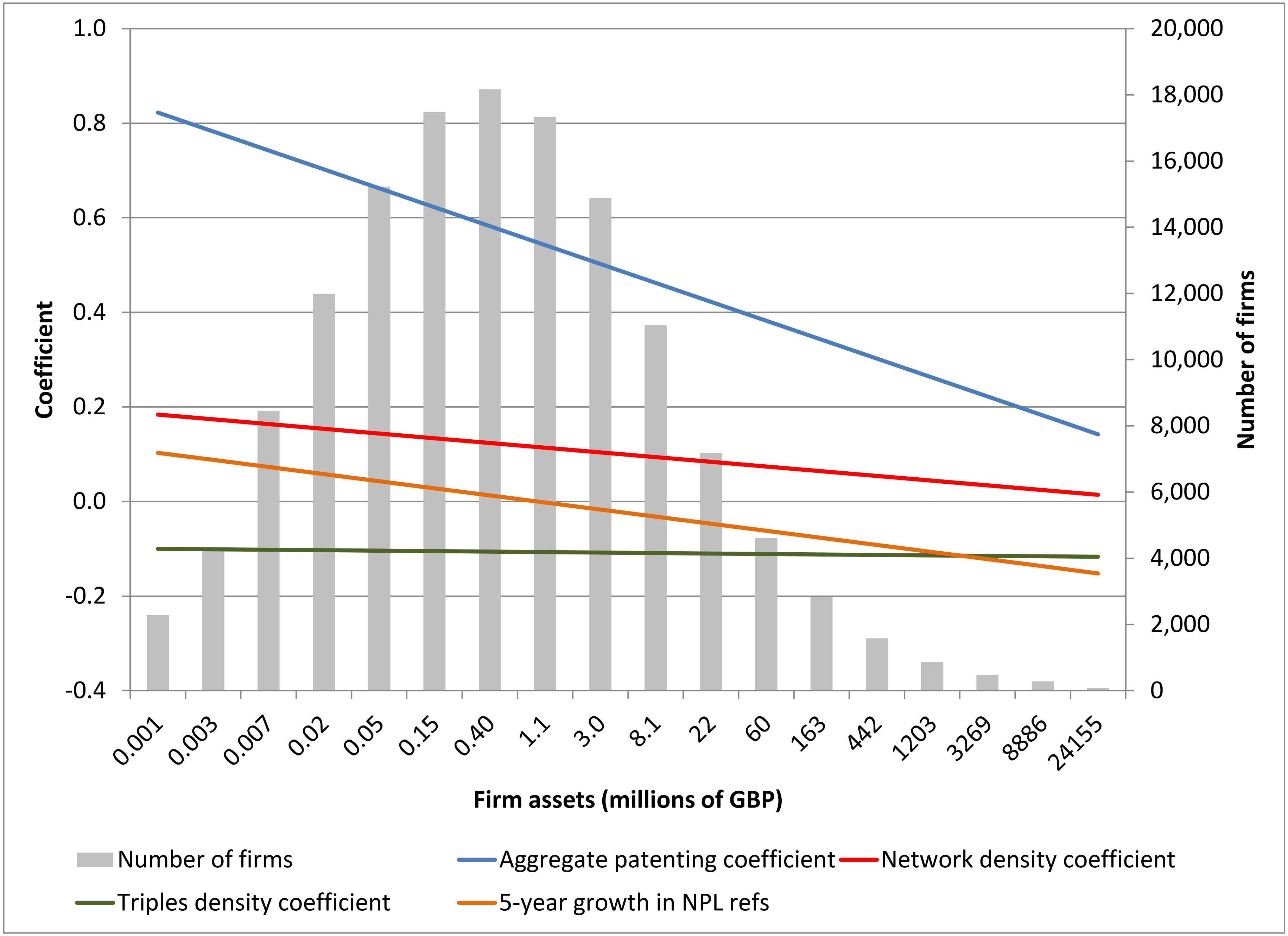
This can be further simplified:

$$\frac{\partial^2 \pi}{\partial {f_i}^2} = \frac{o_i p_k}{\tilde{F}_k}\left[\frac{\partial^2 V}{\partial \tilde{F}_k^{\,2}}\left(\frac{\partial \tilde{F}_k}{\partial f_i}\right)^2 \frac{\Delta \tilde{F}_k}{p_k} + V\frac{d^2\Delta}{s_{ik}^2}\frac{p_k}{\tilde{F}_k}\left(1-\eta_{ik}\right)^2 - \frac{\partial^2 L}{\partial {\gamma_{ik}}^2}p_k^2 \right.$$
$$\left. - \frac{\partial^2 L}{\partial {s_{ik}}^2}\frac{p_k}{\tilde{F}_k}\left(1-\eta_{ik}\right)^2 - 2\left(V\frac{\Delta}{s_{ik}}\xi_{ik}(1-\mu_k) - \frac{\partial L}{\partial s_{ik}}\right)\frac{\left(1-\eta_{ik}\right)\eta_{ik}}{f_i}\right] \quad . \qquad \text{(D.25)}$$

If we impose the restriction that the second derivative of the value function is negative and that the elasticity of the value function, $\mu_k < 1$, then the first and the last terms in the above expression are negative. The sign of the second term in the expression depends on $sign\{\frac{\partial^2 \Delta}{\partial {s_{ik}}^2}\}$, which we will assume is negative. The third and fourth terms in the above expression are negative given the conditions imposed on the legal cost function above.

References

GRAEVENITZ, G., S. WAGNER, AND D. HARHOFF (2013): "Incidence and Growth of Patent Thickets: The Impact of Technological Opportunities and Complexity," *The Journal of Industrial Economics*, 61, 521–563.



1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
Coefficient
20,000
18,000
16,000
14,000
12,000
10,000
8,000
6,000
4,000
2,000
0
Number of firms
0.001
0.003
0.007
0.02
0.05
0.15
0.40
1.1
3.0
8.1
22
60
163
442
1203
3269
8886
24155
Firm assets (millions of GBP)
Number of firms
Aggregate patenting coefficient
Network density coefficient
Triples density coefficient
5-year growth in NPL refs

Table 1
Patenting by Fame firms on Patstat (priority years 2002-2009)

	*Weighted by #owners & #classes**			*Sector shares*	
Technology categories	*GB pats*	*EP pats*	*Total*	*GB pats*	*EP pats*
Elec machinery, energy	1,321	1,101	2,422	6.1%	4.4%
Audio-visual tech	633	549	1,182	2.9%	2.2%
Telecommunications	1,181	1,206	2,386	5.5%	4.8%
Digital communication	590	732	1,323	2.7%	2.9%
Basic comm processes	302	146	447	1.4%	0.6%
Computer technology	1,481	1,302	2,783	6.8%	5.2%
IT methods for mgt	256	224	480	1.2%	0.9%
Semiconductors	269	248	518	1.2%	1.0%
Optics	392	481	873	1.8%	1.9%
Measurement	1,216	1,458	2,674	5.6%	5.8%
Analysis bio materials	132	426	557	0.6%	1.7%
Control	592	542	1,134	2.7%	2.2%
Medical technology	996	1,561	2,558	4.6%	6.3%
Organic fine chemistry	182	1,538	1,720	0.8%	6.2%
Biotechnology	193	950	1,143	0.9%	3.8%
Pharmaceuticals	277	1,876	2,153	1.3%	7.5%
Polymers	114	280	394	0.5%	1.1%
Food chemistry	88	458	547	0.4%	1.8%
Basic materials chemistry	314	1,050	1,363	1.5%	4.2%
Materials metallurgy	161	318	479	0.7%	1.3%
Surface tech coating	287	284	571	1.3%	1.1%
Chemical engineering	507	724	1,231	2.3%	2.9%
Environmental tech	296	344	640	1.4%	1.4%
Handling	996	813	1,809	4.6%	3.3%
Machine tools	428	356	784	2.0%	1.4%
Engines,pumps,turbine	887	942	1,829	4.1%	3.8%
Textile and paper mach	235	304	539	1.1%	1.2%
Other spec machines	742	623	1,365	3.4%	2.5%
Thermal process and app	410	261	671	1.9%	1.0%
Mechanical elements	1,149	854	2,002	5.3%	3.4%
Transport	1,063	930	1,993	4.9%	3.7%
Furniture, games	1,064	612	1,675	4.9%	2.5%
Other consumer goods	630	507	1,137	2.9%	2.0%
Civil engineering	2,237	960	3,196	10.3%	3.8%
Total	**21,619**	**24,959**	**46,578**		
Electrical engineering	6,032	5,508	11,540	27.9%	22.1%
Instruments	3,328	4,468	7,796	15.4%	17.9%
Chemistry	2,418	7,822	10,240	11.2%	31.3%
Mechanical engineering	5,910	5,083	10,993	27.3%	20.4%
Other Fields	3,930	2,079	6,009	18.2%	8.3%

* Weighting by owners does not affect the numbers, since they all get added back into the same cell.
Weighting by classes means that a patent in multiple TF34 sectors is downweighted in each of the sectors.

Table 2

UKIPO and EPO patents: numbers, triples and network density 2002-2009

Technology categories	*Aggregate EPO patents*	*Number of EPO triples@*	*Triples per 1000 patents*	*US Citation network density#*	*Average non-patent references*
Elec machinery, energy	56,714	7751	136.7	39.4	0.420
Audio-visual tech	34,131	13268	388.7	63.5	0.449
Telecommunications	62,288	27049	434.3	79.1	1.235
Digital communication	36,975	16529	447.0	178.5	1.397
Basic comm processes	10,035	2289	228.1	110.8	1.162
Computer technology	60,577	21956	362.4	54.3	1.529
IT methods for mgt	9,312	34	3.7	144.2	0.920
Semiconductors	24,544	9974	406.4	94.0	1.070
Optics	28,458	7767	272.9	58.0	0.806
Measurement	44,320	2503	56.5	45.6	1.000
Analysis bio materials	11,787	26	2.2	319.4	7.040
Control	17,612	308	17.5	112.1	0.445
Medical technology	66,062	4411	66.8	206.2	0.614
Organic fine chemistry	41,137	3993	97.1	32.9	5.253
Biotechnology	33,192	365	11.0	89.6	17.332
Pharmaceuticals	52,671	11222	213.1	76.6	6.391
Macromolecular chemistry	21,307	3722	174.7	92.7	1.236
Food chemistry	9,955	140	14.1	326.3	2.701
Basic materials chemistry	27,679	1929	69.7	84.8	1.498
Materials metallurgy	16,935	405	23.9	91.7	1.130
Surface tech coating	17,429	363	20.8	59.4	0.803
Chemical engineering	24,494	443	18.1	66.2	0.797
Environmental tech	12,708	858	67.5	206.9	0.487
Handling	30,343	252	8.3	66.9	0.137
Machine tools	24,040	508	21.1	64.0	0.191
Engines,pumps,turbine	32,602	6678	204.8	85.4	0.210
Textile and paper mach	23,145	2640	114.1	84.9	0.312
Other spec machines	29,826	319	10.7	65.7	0.422
Thermal process and app	15,290	335	21.9	146.3	0.189
Mechanical elements	32,716	1301	39.8	57.8	0.168
Transport	48,875	10929	223.6	68.3	0.203
Furniture, games	19,847	206	10.4	107.6	0.166
Other consumer goods	19,734	301	15.3	105.6	0.194
Civil engineering	28,817	171	5.9	117.1	0.150
Total	**1,025,555**	**160,945**	**156.9**	**100.3**	**0.800**
Electrical engineering	294,575	98,850	335.6	60.4	1.042
Instruments	168,239	15,015	89.2	96.3	1.181
Chemistry	257,507	23,440	91.0	71.1	4.977
Mechanical engineering	236,836	22,962	97.0	70.1	0.227
Other Fields	68,398	678	9.9	110.8	0.167

@ Triples based on all EPO patenting, priority years 2002-2009 (see text for definition and further explanation).

Network density is 1,000,000 times the number of within technology citations between 1976 and the current year divided by the potential number of such citations.

Table 3

Hazard of entry into patenting in a TF34 Class

538,452 firm-TF34 observations with 10,665 entries (20,384 firms)

Variable	*Cox Proportional Hazard Model*				
Log (network density)	0.115***		0.127***	0.107***	0.184***
	(0.024)		(0.023)	(0.021)	(0.052)
Log (triples density		-0.138***	-0.139***	-0.101***	-0.100***
in class)		(0.011)	(0.011)	(0.010)	(0.023)
Log (patents in class)	0.317***	0.506***	0.545***	0.514***	0.822***
	(0.025)	(0.031)	(0.030)	(0.027)	(0.071)
5-year growth of non-	0.060***	0.084***	0.072***	-0.009	0.103*
patent refs in class)	(0.022)	(0.022)	(0.022)	(0.021)	(0.056)
Log assets	0.270***	0.270***	0.270***	0.142***	0.513***
	(0.011)	(0.011)	(0.011)	(0.013)	(0.083)
Log firm age in years	1.135***	1.135***	1.136***	0.773***	0.767***
	(0.104)	(0.104)	(0.104)	(0.130)	(0.131)
Log (pats applied for				0.836***	0.836***
by firm previously)				(0.021)	(0.021)
Log (network density)					-0.010*
* Log assets					(0.006)
Log (triples density)					-0.001
* Log assets					(0.003)
Log (patents in class)					-0.040***
* Log assets					(0.008)
Log (average NPL refs)					-0.015**
* Log assets					(0.006)
Industry dummies	***stratified#***	***stratified#***	***stratified#***	***stratified#***	***stratified#***
Year dummies	***yes***	***yes***	***yes***	***yes***	***yes***
Log likelihood	-65.96	-65.86	-65.84	-58.69	-58.67
Degrees of freedom	12	12	13	14	18
Chi-squared	1270.6	1429.1	1517.2	3465.1	3458.6

The sample is matched on size class, sector, and age class. Estimates are weighted by sampling probability.

Coefficients for the hazard of entry into a patenting class are shown.

Standard errors are clustered on firm. *** (**) denote significance at the 1% (5%) level.

Time period is 2002-2009 and minimum entry year is 1978. Sample is UK firms with nonmissing assets, all patenting firms and a matched sample of non-patenting firms

Estimates are stratified by industry - each industry has its own baseline hazard.

Table A1

Number of TF34 sectors entered between 2002 and 2009

Number of sectors	*Number of firms*	*Number of entries*
1	2,531	2,531
2	1,347	2,694
3	647	1,941
4	271	1,084
5	155	775
6	71	426
7	45	315
8	29	232
9	20	180
10	14	140
11	4	44
12	2	24
13	3	39
14	0	0
15 or more	13	240
Total	**5,152**	**10,665**

Table A-2

Sample population of UK firms, by industry

	Industry	*Number of firms*	*Number of patenters 2001-2009*	*Share patenting 2001-2009*	*Number of patents 2001-2009*
1	Basic metals	2,836	52	1.83%	231
2	Chemicals	3,834	246	6.42%	126
3	Electrical machinery	2,948	281	9.53%	727
4	Electronics & instruments	9,298	561	6.03%	444
5	Fabricated metals	24,681	606	2.46%	70
6	Food, beverage, & tobacco	8331	102	1.22%	29
7	Machinery	9,365	608	6.49%	313
8	Mining, oil&gas	83,491	15	0.02%	96
9	Motor vehicles	2,337	117	5.01%	22
10	Other manufacturing	94,952	1362	1.43%	150
11	Pharmaceuticals	1,008	105	10.42%	551
12	Rubber & plastics	6,094	398	6.53%	590
13	Construction	295596	372	0.13%	59
14	Other transport	3,292	89	2.70%	6274
15	Repairs & retail trade	128,266	251	0.20%	2324
16	Telecommunications	14,348	133	0.93%	2096
17	Transportation	60,837	75	0.12%	621
18	Utilities	12,880	75	0.58%	428
19	Wholesale trade	138,398	728	0.53%	3608
20	Business services	689,942	1639	0.24%	6757
21	Computer services	177,319	716	0.40%	1132
22	Financial services	183,042	219	0.12%	4930
23	Medicalservices	38,424	103	0.27%	1419
24	Personal services	94,791	196	0.21%	1194
25	R&D services	7,915	713	9.01%	168
26	inactive	37,525	271	0.72%	5673
	Total	**2,131,750**	**10,033**	**0.47%**	**40,032**

Table A3

Entry into techology area 2002-2009

	Numbers			*Shares*		
Technology	*Total patenting in sector by GB firms*	*First time patenter*	*Patented previously in another tech*	*Total entry*	*First time patenter*	*Patented previously in another tech*
Elec machinery, energy	1763.7	214	250	26.3%	12.1%	14.2%
Audio-visual tech	788.3	148	192	43.1%	18.8%	24.4%
Telecommunications	1874.4	146	179	17.3%	7.8%	9.5%
Digital communication	1054.0	92	141	22.1%	8.7%	13.4%
Basic comm processes	256.5	21	93	44.4%	8.2%	36.3%
Computer technology	2167.2	291	251	25.0%	13.4%	11.6%
IT methods for mgt	283.2	117	157	96.7%	41.3%	55.4%
Semiconductors	347.2	38	118	44.9%	10.9%	34.0%
Optics	584.7	55	130	31.6%	9.4%	22.2%
Measurement	1765.3	226	269	28.0%	12.8%	15.2%
Analysis bio materials	339.0	39	111	44.3%	11.5%	32.7%
Control	712.1	165	241	57.0%	23.2%	33.8%
Medical technology	1668.4	184	209	23.6%	11.0%	12.5%
Organic fine chemistry	1569.4	36	83	7.6%	2.3%	5.3%
Biotechnology	701.6	41	99	20.0%	5.8%	14.1%
Pharmaceuticals	1700.7	54	80	7.9%	3.2%	4.7%
Polymers	224.4	27	112	61.9%	12.0%	49.9%
Food chemistry	492.7	36	87	25.0%	7.3%	17.7%
Basic materials chemistry	1020.2	85	144	22.4%	8.3%	14.1%
Materials metallurgy	360.8	54	109	45.2%	15.0%	30.2%
Surface tech coating	400.7	77	195	67.9%	19.2%	48.7%
Chemical engineering	842.7	142	213	42.1%	16.9%	25.3%
Environmental tech	446.6	106	166	60.9%	23.7%	37.2%
Handling	1326.3	274	290	42.5%	20.7%	21.9%
Machine tools	577.8	106	180	49.5%	18.3%	31.2%
Engines,pumps,turbine	1443.4	82	160	16.8%	5.7%	11.1%
Textile and paper mach	442.0	77	137	48.4%	17.4%	31.0%
Other spec machines	847.4	180	234	48.9%	21.2%	27.6%
Thermal process and app	455.7	105	159	57.9%	23.0%	34.9%
Mechanical elements	1445.9	223	317	37.3%	15.4%	21.9%
Transport	1288.2	213	236	34.9%	16.5%	18.3%
Furniture, games	1239.2	288	223	41.2%	23.2%	18.0%
Other consumer goods	788.1	194	246	55.8%	24.6%	31.2%
Civil engineering	2045.8	463	255	35.1%	22.6%	12.5%
Total	**33263.6**	**4599**	**6066**	**32.1%**	**13.8%**	**18.2%**
Electrical engineering	8534.5	1067	1381	28.7%	12.5%	16.2%
Instruments	5069.5	669	960	32.1%	13.2%	18.9%
Chemistry	7760.0	658	1288	25.1%	8.5%	16.6%
Mechanical engineering	7826.7	1260	1713	38.0%	16.1%	21.9%
Other Fields	4073.0	945	724	41.0%	23.2%	17.8%

Table B-1

Hazard of entry into patenting in a TF34 Class - Comparing models

538,452 firm-TF34 observations with 10,665 entries (20,384 firms)

	Proportional hazard		*AFT*	
Variable	*Cox PH*	*Weibull*	*Log logistic*	*Log normal*
Log (network density)	0.108***	0.111***	0.308***	0.247***
	(0.021)	(0.021)	(0.096)	(0.040)
Log (triples density in class)	-0.100***	-0.098***	-0.511***	-0.258***
	(0.010)	(0.010)	(0.071)	(0.024)
Log (patents in class)	0.513***	0.513***	2.177***	1.095***
	(0.027)	(0.027)	(0.304)	(0.089)
5-year growth of non-patent refs in class)	(0.013)	-0.001	-0.077	-0.039
	(0.022)	(0.021)	(0.084)	(0.038)
Log assets	0.149***	0.130***	0.529***	0.198***
	(0.013)	(0.013)	(0.082)	(0.024)
Log (pats applied for by firm previously)	0.848***	0.860***	5.685***	3.973***
	(0.021)	(0.019)	(0.917)	(0.368)
Industry dummies	***stratified#***	***stratified#***	***stratified#***	***stratified#***
Year dummies	***yes***	***yes***	***yes***	***yes***
Log likelihood	-58.8	-96,051.0	-114,127.0	-111,662.1
Degrees of freedom	13	38	38	38
Chi-squared	3183.2	4560.1	168.4	300.1

All estimates are weighted estimates, weighted by sampling probability. For the Cox and Weibull models, coefficients shown are elasticities of the hazard w.r.t. the variable. For the log-logistic, -beta is shown.

*** (**) denote significance at the 1% (5%) level.

Time period is 2002-2009 and minimum entry year is 1978. Sample is all UK firms with nonmissing assets.

AFT - Accelerated Failure Time models

Estimates are stratified by industry - each industry has its own baseline hazard.

Table B-2

Hazard of entry into patenting in a TF34 Class - Robustness

Variable	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*
Log (network density)	0.107***	-0.008	0.114***	0.107***	0.108***
	(0.021)	(0.019)	(0.024)	(0.021)	(0.021)
Log (triples density	-0.101***	-0.183***	-0.114***	-0.103***	-0.103***
in class)	(0.010)	(0.008)	(0.011)	(0.009)	(0.009)
Log (patents in class)	0.514***	0.623***	0.605***	0.520***	0.518***
	(0.027)	(0.024)	(0.031)	(0.027)	(0.027)
5-year growth of non-	-0.009	-0.196***	-0.002	-0.012	-0.012
patent refs in class)	(0.021)	(0.020)	(0.025)	(0.021)	(0.021)
Log assets	0.142***	0.138***	0.186***	0.139***	0.146***
	(0.013)	(0.013)	(0.018)	(0.013)	(0.013)
Log firm age in years	0.773***	0.588***	0.739***	0.778***	0.021
	(0.130)	(0.145)	(0.155)	(0.131)	(0.246)
Log (lagged firm-level	0.836***	0.947***	0.954***	0.840***	0.878***
patent stock)	(0.021)	(0.019)	(0.033)	(0.021)	(0.021)
Year dummies	yes	yes	yes	yes	yes
Observations	538,452	692,038	452,313	523,547	538,452
Firms	20,384	20,384	17,993	20,384	20,384
Entries	10,665	10,665	8,149	10,340	10,665
Entry rate	1.98%	1.54%	1.96%	1.97%	1.98%
Log likelihood	-58.69	-54.60	-40.59	-56.94	-53.81
Degrees of freedom	14	14	14	14	14
Chi-squared	3465.1	5065.8	1688.8	3459.1	3137.4

All estimates are weighted estimates, weighted by sampling probability. Coefficients shown are negative of the estimates (larger coefficient increases entry probability).

*** (**) denote significance at the 1% (5%) level.

Time period is 2002-2009 and minimum entry year is 1978. Sample is UK firms with nonmissing assets, all patenting firms and a matched sample of non-patenting firms

Weibull model stratified by industry.

(1) Estimates from Table 3, for comparison.

(2) Observations for tech sectors of firms whose industry has no such patenting (Lybbert-Kolas) and where there is no entry by any UK firm in that industry are included.

(3) SMEs: firms with assets>12.5 million GBP removed.

(4) The Telecom tech sector is removed.

(5) The minimum founding year is 1900 instead of 1978.