The Tax (Dis)Advantage Of A Firm Issuing Options On Its Own Stock*

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Abstract

It is common for firms to issue or purchase options on the firm's own stock. Examples include convertible bonds, warrants, call options as employee compensation, and the sale of put options as part of share repurchase programs. This paper shows that option positions with implicit borrowing—such as put sales and call purchases—are tax-disadvantaged relative to the equivalent synthetic option with explicit borrowing. Conversely, option positions with implicit lending—such as compensation calls—are tax-advantaged. We also show that firms are better off from a tax perspective issuing bifurcated convertible bonds—bonds plus warrants—rather than an otherwise equivalent standard convertible. The put option sales which have been popular with some firms are like issuing debt with non-deductible interest and thus have a tax cost. For example, we estimate that in 1999 the tax cost to Microsoft of written puts was about $80m per year.

Keywords: Corporate tax, options, put warrants, convertible bonds, compensation options

JEL Codes: G32, H25

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Tax issues play a central role in discussions of capital structure and security design. However, little consideration has been paid to the tax consequences of a firm issuing options on the firm's own stock, despite the fact that such issues are relatively common. For example, firms issue convertible bonds, warrants, call options as part of employee compensation, and sell put options and sometimes purchase calls as part of share repurchase programs.

This paper shows that the issue of an option on the firm's own stock is not tax-neutral for the firm. I obtain a simple measure of the tax cost (or benefit) of issuing an option, and examine the importance of the tax cost for several common option types, including call and put sales and convertible bonds.

The tax non-neutrality of options on the firm's own stock arises because transactions in the firm's own shares—including options—are tax-free, while interest income or expense is taxed. Any option position has a synthetic equivalent comprised of some position in the stock and either borrowing or lending. By comparing the cash flows of the option position with that of the equivalent synthetic position, it is straightforward to show that option positions with implicit borrowing—put sales and call purchases—are tax-disadvantaged relative to the equivalent position with explicit borrowing. Conversely, option positions with implicit lending—such as warrants—are tax-advantaged.

For European options, the present value of the cash flow difference between the synthetic and explicit position is $\tau r T B$, where τ is the corporate tax rate, r the interest rate, T the maturity of the option, and B the lending implicit in the option. The sign of B determines whether the option position is tax-advantaged or disadvantaged.

This result depends only on the corporate tax rate, and thus seems to ignore the tax treatment of the investor emphasized by Miller (1977). However, I make the comparison between the issuance of an actual option and a synthetic option by using a dealer as a tax intermediary, thereby holding fixed the tax position of investors. Specifically, suppose Firm A issues a synthetic option, which results in investors having some position in shares and bonds. Firm B issues the actual option to a dealer. The dealer in turn hedges the position. The hedging transaction by the dealer results in the ultimate investors having the *same* position in shares and bonds as in the case when the firm issues the synthetic option directly. Personal taxes are held fixed in comparing the two alternatives, so they can be ignored. Dealers do in fact intermediate in this way when firms sell put options, and dealers sometimes buy convertible bonds and sell the components.

For some firms, options can be a significant part of capital structure. To take one prominent example, Microsoft in June 1998 had compensation options outstanding on 446 million shares, against 2.5b outstanding shares. In addition, Microsoft "enhance[d] its repurchase program by selling put warrants,"[1] taking in $538 million in premium from sales of puts on 60 million shares in 1998 alone.[2] For firms which make any use of compensation options, Core and Guay (2001) find that options are 6.9% of shares outstanding. Using a broader sample, Eberhart (2001) finds options to be 12% of shares outstanding.

A firm can issue a tax-advantaged option and earn an arbitrage profit by explicitly replicating the offsetting position, trading bonds and its own stock.[3] However, even if the firm does not offset the option it is still possible to compute the implicit loss or gain due to the tax treatment of the option. If the issue is tax disadvantaged, there might be some non-tax reason for the security to be issued. The tax disadvantage is then a lower bound on the value of to the firm of having issued the particular security.

Section 1 discusses Section 1032 of the tax code, which governs the treatment of transactions in a firm's own stock. Section 1.1 presents a simple one-period binomial numerical example illustrating the effect. Section 2 discusses the effect of taxes in pricing derivatives. The calculation of the tax benefit implicitly assumes that the option is traded in a public market, or priced as if it were. In this case, I assume

[1] Microsoft 1998 10-K

[2] Companies typically describe put sales as a way to reduce the cost and/or risk of share repurchase programs. We discuss possible rationales later in the paper. Gibson and Singh (2000) suggest that put sales serve to signal firm quality, although many such sales are conducted in secrecy and hence cannot be a signal. In the practitioner literature they are often described as a risk-management technique for share repurchase programs. For example, see Thatcher, Flynn, Ehrlinger, and Reel (1994). A number of companies have sold puts; Angel, Gastineau, and Weber (1997) report that option traders believe that over 10% of firms repurchasing their shares sell puts, and according to Gibson and Singh (2000), over 100 firms have issued puts since 1988.

[3] Mozes and Raymar (2000) make this point in the context of executive options. They argue that firms which have issued significant executive options—equivalent to issuing shares and lending—will offset the option position by repurchasing shares and borrowing.

the option price is determined by market-makers who are taxed symmetrically on all forms of income and hence are tax-netural. When the market is bilateral and the firm's counterparty is not tax-neutral—as in the case of compensation options—the net tax advantage can depend on the idiosyncracies of the counterparty.

Section 3 values the tax non-neutrality by comparing the cash flows to a firm selling a put option to one synthetically creating a put by transacting in shares and borrowing. The sale of puts by Dell and Microsoft, and related institutional issues, are examined in Section 4. Section 5 looks at convertibles, and shows that firms are better off issuing straight debt plus warrants (a bifurcated convertible) rather than an otherwise equivalent convertible. I also compare the tax benefit from a straight debt issue with that of a bifurcated issue and show that the comparison is ambiguous. Section 6 discusses compensation options, comparing an explicit option issue to a stock-contingent bonus. Section 7 concludes.

1 Section 1032

Section 1032 of the Internal Revenue Code governs the tax treatment of the corporate exchange of stock for money or property. It reads, in part, as follows:

> (a) Nonrecognition of Gain or Loss.—No gain or loss shall be recognized to a corporation on the receipt of money or other property in exchange for stock (including treasury stock) of such corporation. *No gain or loss shall be recognized by a corporation with respect to any lapse or acquisition of an option to buy or sell its stock (including treasury stock).* [emphasis added]

The non-taxability of option transactions (such as put sales) was added to Section 1032 in 1984.

The following example, taken from Warren (2000), illustrates in the simplest way how the non-taxability of equity transactions can generate tax arbitrage opportunities. (The same observation has also been made by Titman (1985, 2000).) Suppose a firm with a stock price of 100 is taxed at 40% and has $100 in cash. The interest rate is 10%. Consider the following two alternatives:

1. The firm invests the \$100 in bonds.

2. The firm uses the \$100 to repurchase one share, and simultaneously enters into a forward contract to sell a share in one year for \$110.

Under the first alternative, the firm in one year has \$106 ($= 100\times[1+.1\times(1-.4)]$). Under the second alternative, which is completely free of tax under Section 1032, the firm has \$110 in one year, generated by the forward sale of the stock.[4] The higher return from the forward contract occurs because of the asymmetric treatment between stock transactions and bond transactions.

Note that in the forward transaction, the firm has implicitly bought a bond (paying \$100 today, and receiving \$110 in 1 year), only it is not recognized as a bond either for tax or accounting purposes. In this sense, the firm's lending is off-balance-sheet.

This example using a forward contract generalizes to other derivatives. We illustrate this using a simple one-period binomial model to demonstrate the tax non-neutrality of a written put. In this example, for simplicity, we ignore dilution and assume that the option transaction is small relative to outstanding shares. However, we account for dilution in Section 3.

1.1 Pricing and Synthetically Creating a Put

Suppose the stock price today is \$100 and over the next year will take one of the two values depicted in the top panel of Figure 1. Using standard arguments (see for example Cox, Ross, and Rubinstein (1979)), we can price a put option on the stock. Suppose the risk-free rate is 6%. If the strike on the put is 100, the theoretical price of a put with one year to expiration, given the stock price distribution in Figure 1, is \$20.97.

From the top panel of Figure 1, in one year the value of a purchased put is 0 if the stock price is 164.87 and 39.35 if the stock price is 60.65.

\$20.97 is the cost of a portfolio of shares and bonds which replicates the payoff to the put. Specifically, the purchase of a put on one share is equivalent to shorting,

[4] There have been proposals to tax the interest component of a forward sale on the firm's own stock, which would cause both transactions to have the same after-tax return. However, options have not been included.

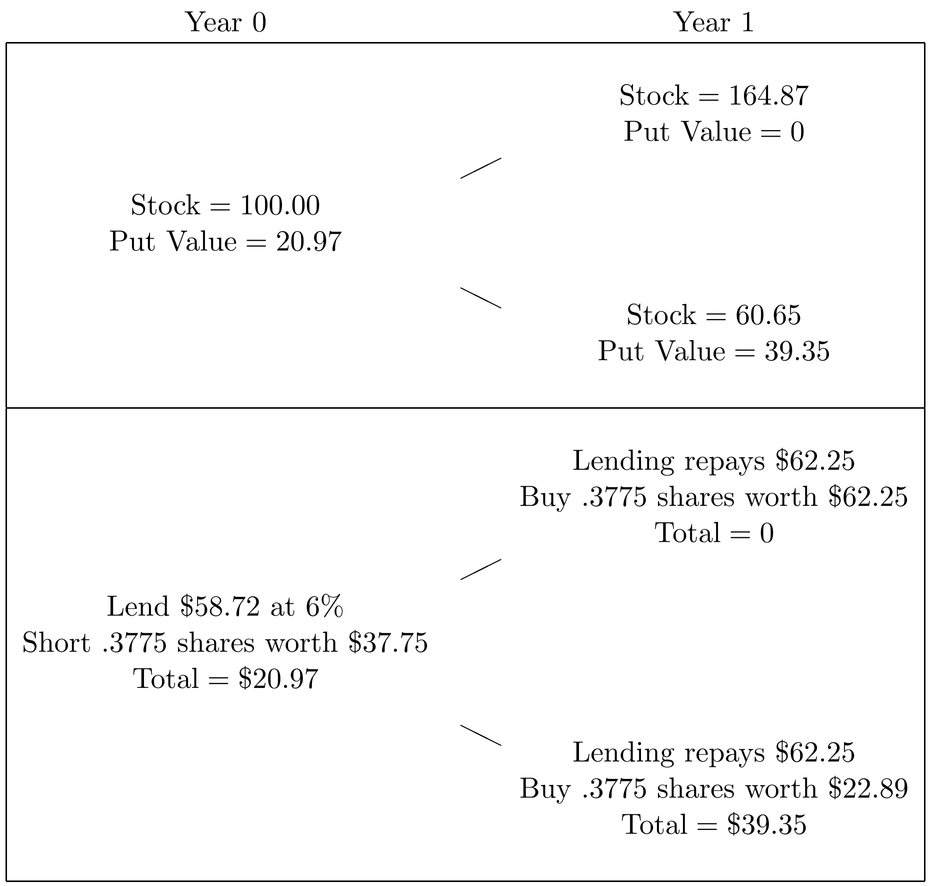

Figure 1: Top panel depicts binomial stock price movements and put option values, assuming current stock price is $100, the stock volatility is 50%, and the risk-free rate is 6%. Bottom panel depicts transactions in the stock and borrowing which replicate a purchased put option.

at the same time as the sale, .3775 shares, and lending \$58.72. The cost of this strategy is

$$.3775 \times \$100 - \$58.72 = -\$20.97 \tag{1}$$

That is, buying the put entails a net cash payment of \$20.97.

The bottom panel of Figure 1 shows how the replicating portfolio duplicates the payoff to the purchased put.

1.2 The Firm as a Put Seller

Now consider the position of a firm which writes the put instead of purchasing it; the cash flows are therefore reversed from those in Figure 1. Assume the firm also plans to repurchase one share in one year (this is not critical for the example) and that the firm's marginal tax rate is 40%.

If the cash inflow from selling the put is invested in taxable T-bills, after one year the firm has

$$\$20.97 \times (1 + .06 \times (1 - .4)) = 21.72.$$

At that time, the written put will be worth 0 (if the stock price is 164.87) or -\$39.35 (if the stock price is \$60.65). In the former case the firm must pay \$164.87 to buy back one share. The net cost is

$$-\$164.87 + \$21.72 = -\$143.15$$

In the latter case the firm is obligated to buy the share for \$100 when it is worth only \$60.65. Using the put sale proceeds to offset this expense gives a net cost of

$$-\$100 + \$21.72 = -\$78.38$$

In both cases, one share has been retired. The entire transaction is untaxed under Section 1032. Table 1 summarizes the cost of buying back one share after one year.

1.3 The Firm as a Synthetic Put Seller

As an alternative to the put sale at time 0, the firm could repurchase .3775 shares and borrow \$58.72. After spending \$37.75 to buy back the fractional share, there

Table 1: Cash flows for a put seller. Cash outflow for put seller after one year assuming that one share is repurchased. If the share price rises, the net cost of buying the share is the price ($164.87) less the future value of the option premium. If the share price falls, the net cost is the strike price on the put ($100) less the option premium.

Stock Price	Cost of share repurchase	FV(Option premium)	Total
164.87	-164.87	21.72	-143.15
60.65	-100.00	21.72	-78.28

Table 2: Cash flow for a synthetic put seller. The total column is the net cost in one year of buying back .3775 shares today and the remaining .6225 shares in one year. The net result is that one share is repurchased but the cost is lower than in Table 1 by one year's tax deduction on interest. The "Gain" column is the difference between the total cash flow in this table and in Table 1.

Stock Price	Cost of .6225 shares	Repayment of borrowing	FV("Option premium")	Total	Gain
164.87	-102.63	-60.83	21.72	-141.74	1.41
60.65	-37.75	-60.83	21.72	-76.86	1.41

would be $20.97 to invest in T-bills, as with the put sale. In order to retire one share, in one year the firm has to buy back the remaining .6225 shares and repay the borrowing, for which the interest cost is tax-deductible. The repayment of borrowing costs

$$\$58.72 \times (1 + .06 \times (1 - .4)) = \$60.83.$$

The firm's situation is depicted in Table 2. Comparing Table 2 to Table 1, the total cash flow is greater in Table 2 by $1.41 whether the stock price rises or falls. This is the amount of interest tax saving due to borrowing explicitly rather than implicitly via the option. The firm borrows $58.72; the tax deduction on one-year's interest on this amount is $.06 \times .4 \times 58.72 = 1.41$.

2 Derivative Pricing With Taxes

The example in Section 1 hinges on the forward price, $F_{0,1}$, being determined by the standard formula

$$F_{0,1} = S_0(1 + r) \tag{2}$$

where S_0 is the current stock price, r the interest rate, and $F_{0,1}$ the time 0 forward price for delivery at time 1. Similarly, the put-writing example in Section 1 depends on the option pricing formula not depending on taxes.

It is natural to ask whether the forward price *should* be affected by the taxation of market participants. In particular, if the forward price in the previous example had been 106 instead of 110, the forward sale would have generated the same return as a bond investment. In this section we review the reason that the standard formulas for forward and option prices (such as equation (2)) include no adjustment for taxes.[5]

2.1 The Dealer's Position

Consider a dealer who, at time t, holds a derivative expiring at time T, with a price of $\phi_t(S_t, T)$. (We will hereafter assume the expiration time is T and not include it in the notation.) The dealer hedges this position by taking an offsetting position in shares and bonds.

We suppose that each form of income for the dealer is taxed at a different rate: interest is taxed at the rate τ_i, capital gains on a stock at the rate τ_g, capital gains on options at the rate τ_O, and dividends at the rate τ_d. We assume initially that all investors are in the same tax bracket, that taxes on all forms of income are paid on an accrual basis, and that there is no limit on the ability to deduct losses, or to offset losses on one form of income against gains on another form of income.

Suppose that, as in Cox, Ross, and Rubinstein (1979), the stock moves binomially, so that at time t the stock price is S_t, and at time $t+h$ can be worth either $S^+_{t+h} = u_h S_t$ or $S^-_{t+h} = d_h S_t$. A dividend worth $D_t = \delta S_t$ is paid just prior to time $t+h$. The effective interest rate from time t to $t+h$ is $r_h \approx rh$. Given h, there are $N = T/h$ binomial periods between 0 and T.

Let $\Delta_t(S_t)$ denote the number of shares and $B_t(S_t)$ the investment in bonds the dealer holds to hedge the derivative. We choose Δ_t and B_t by requiring that the after-tax return on the stock/bond portfolio equal the after-tax return on the option

[5] The impact of taxes on derivative prices was studied by Scholes (1976), Cornell and French (1983), and Cox and Rubinstein (1985, pp. 271-274), who showed how prices depend upon taxes when capital gains, dividends, and interest are taxed at different rates. In Cornell and French (1983), the timing of tax payments differs across sources of income, so there is a non-neutrality even when tax rates are the same. This issue is also discussed in Scholes and Wolfson (1992).

in both the up and down states. Thus we require that

$$\begin{aligned}[S_{t+h} - \tau_g(S_{t+h} - S_t) + \delta S_t(1-\tau_d)]\,\Delta_t + [1 + r_h(1-\tau)]\,B_t \\ = \phi_{t+h}(S_{t+h}) - \tau_O\left(\phi_{t+h}(S_{t+h}) - \phi_t(S_t)\right) \quad (3)\end{aligned}$$

whether $S_{t+h} = S^+_{t+h}$ or $S_{t+h} = S^-_{t+h}$. The initial cost of the derivative is then

$$\phi_t(S_t) = \Delta_t(S_t)S_t + B_t(S_t). \quad (4)$$

Solving for ϕ_t gives[6]

$$\phi_t = \frac{1}{1 + r_h\frac{1-\tau_i}{1-\tau_O}}\left[p^*\phi_{t+h}(S^+_{t+h}) + (1-p^*)\phi_{t+h}(S^-_{t+h})\right] \quad (5)$$

where

$$p^* = \frac{1 + r_h\frac{1-\tau_i}{1-\tau_g} - \delta\frac{1-\tau_d}{1-\tau_O} - d}{u-d} \quad (6)$$

is the tax-adjusted risk-neutral probability that the stock price the next period will be S^+_t.

In the absence of taxes, we obtain the standard expressions for the option price and for the replicating portfolio:

$$\Delta_t(S_t) = \frac{\phi_{t+h}(S^+_{t+h}) - \phi_{t+h}(S^-_{t+h})}{S^+_{t+h} - S^-_{t+h}} \quad (7)$$

$$B_t(S_t) = \frac{1}{1+r_h}\frac{u_h\phi_{t+h}(S^-_{t+h}) - d_h\phi_{t+h}(S^+_{t+h})}{u_h - d_h} \quad (8)$$

The risk-neutral probability that the stock price will go up is

$$p_h = \frac{1 + r_h - \delta_h - d_h}{u_h - d_h} \quad (9)$$

[6]The solutions for Δ and B are

$$\Delta = \frac{1-\tau_O}{1-\tau_g}\;\frac{\phi_1(S^+_1) - \phi_1(S^-_1)}{S^+_1 - S^-_1}$$

$$B = \frac{1}{1 + r_h\frac{1-\tau_i}{1-\tau_O}}\;\left[\frac{u\phi_1(S^-_1) - d\phi_1(S^+_1)}{u-d} - \frac{\Delta}{1-\tau_O}S_0\left(\frac{\tau_g - \tau_O}{1-\tau_g} + \delta(1-\tau_d)\right)\right]$$

Finally, the option price is given by

$$\phi_t = \frac{1}{1+r_h}\left[p_h \phi_{t+h}(S^+_{t+h}) + (1-p_h)\phi_{t+h}(S^-_{t+h})\right] \tag{10}$$

By comparing equation (5) with (10), and equation (6) with (9), we obtain the following proposition.

Proposition 1 *When the marginal investor is taxed, the fair price for an option is obtained by making two substitutions in the standard formula:*

1. *Replacing the dividend yield, δ, with $\delta^* = \delta\frac{1-\tau_d}{1-\tau_O} + r\left(\frac{1-\tau_i}{1-\tau_O} - \frac{1-\tau_i}{1-\tau_g}\right)$*
2. *Replacing the interest rate, r, with $r^* = r\frac{1-\tau_i}{1-\tau_O}$.*

In practice, broker-dealers are taxed at the same rate on all forms of income. It follows from Proposition 1 that the fair price for any derivative for a broker-dealer is independent of taxes. Put differently, dealers are tax-neutral in the sense that they value any security on a pre-tax basis. If V_t is the value of the security at time t and D_t its cash flow, and if the dealer is marked to market for tax purposes, then the value of the security is given recursively by

$$V_t = \frac{D_{t+1}(1-\tau) + V_{t+1} - \tau(V_{t+1} - V_t)}{1 + r(1-\tau)}$$

This can be rewritten

$$V_t = \frac{D_{t+1} + V_{t+1}}{1+r}$$

This result relies on two assumptions: all forms of income are taxed on accrual at the same rate, and taxable income includes changes in fair market value.

Since broker-dealers are likely to be marginal in most derivatives markets, it is plausible that equation (2) would describe the forward price in practice. (Cornell (1985) shows empirically that taxes do not seem to affect the pricing of S&P 500 futures contracts.) An implication is that investors who are not in the same tax bracket with respect to all forms of taxable income will find almost any derivatives positions tax-advantaged or dis-advantaged relative to the tax-neutral treatment of the dealer.

While a dealer values the option without consideration of taxes, the fair price for the firm for a derivative on its own stock reflects the differential tax treatment of debt and equity (including derivatives on its equity). For the firm, because of Section 1032, $\tau_g = 0$ and $\tau_O = 0$. Thus, the fair price is obtained by using the after-tax interest rate as the interest-rate. For futures, this is demonstrated by setting $\phi_T(S_T) = S_T - F_0$ and solving for F_0 in equation (5).

In the remainder of this paper we will assume that market prices of derivatives are determined by tax-neutral dealers, and hence prices are described by standard formulas without any adjustment for taxes. Under these conditions, the replicating portfolio for the dealer is given by the standard expressions, equations (7) and (8). We will refer to Δ and B defined by these equations as the "implicit share" and "implicit debt" in the option.

It will prove useful to understand how $B_t(S_t)$, computed from equation (8), behaves over time. Consider an n-period binomial tree, with implicit debt viewed as a function of time and the stock price. Proofs of the following are in the Appendix.

Proposition 2 *Implicit debt at time t is the present value of expected implicit debt in the next binomial period, at time $t + h$. That is,*

$$B_t(S_t) = \frac{1}{1 + r_h} \left[p_h B_{t+h}(S_{t+h}^+) + (1 - p_h) B_{t+h}(S_{t+h}^-) \right]$$

Corollary 1 *For a put or call, the absolute value of $B_t(S_t)$ never exceeds the strike price on the option.*

3 The Tax (Dis)advantage of Options

We now compute a measure of the tax advantage of a firm writing an option on its own stock. As with the examples in Section 1, we compare the actual derivative with the synthetic equivalent, undertaken by the firm using actual borrowing or lending and transacting in its own shares. It is important to note that this comparison is between two forms of a transaction with different tax consequences *at the level of the firm.* Shareholder-level taxes are the same in the two cases and therefore can be ignored. If the firm creates a synthetic option using its own borrowing and shares, investors hold the securities the firm issues. If instead the firm sells an option to a dealer, the dealer hedges the position by selling a synthetic equivalent. This must be held by investors in the aggregate, and thus imposes the same risk on investors as the firm selling the synthetic directly. Thus, for example, it is irrelevant for this comparison whether debt is tax-advantaged relative to equity in the sense of Miller (1977).

3.1 Analysis of Put Writing

Consider a firm with assets worth A_t, and n shares outstanding. For simplicity, assume that there is no debt currently outstanding. Assets follow a binomial process, where assets in one period, A_{t+h}, can be either $A^+_{t+h} = u_A A_t$ or $A^-_{t+h} = d_A A_t$ with $u_A > d_A$. Corresponding to the process for assets, there is a process for the stock price, S_t.

We focus on the case where a firm writes a European put option on its own stock, although a similar analysis would hold for any derivative on the firm's stock. A written put potentially creates a fixed obligation for the firm, much like debt, so in practice default is possible. However, this is an important issue only with written puts (including—as a special case—long forward contracts). Default will not occur on a purchased put and default should not occur on a purchased call.[7] Default also should not occur on a written call since it is always possible to issue new shares to satisfy the option buyer. We assume that parameters are such that the firm does not default.

3.1.1 Actual Option Sale

Consider first the sale of an actual put requiring the firm to buy m shares for the strike price K if the stock price is less than K. The initial premium is ϕ_0, which is paid out immediately as a dividend (it could also be invested at the risk-free rate). At any point in time, the firm has n shares outstanding and a liability of $\phi_t(S_t)$ per

[7] With a purchased call, a bridge loan with acquired stock as collateral can be used to fund exercise.

share. The value of a share at time t is thus

$$S_t = \frac{A_t - m\phi_t(S_t)}{n} \tag{11}$$

At expiration, the value of the put is zero for $A_T > nK$, in which case

$$S_T = \frac{A_T}{n}. \tag{12}$$

At this point the firm may repurchase m shares at the market price, but this would not affect the share price.

For asset values such that $A_T < nK$, put exercise is optimal. The firm repurchases m shares for K per share, and the value of a share is then

$$S_T = \frac{A_T - mK}{n - m} \tag{13}$$

The no-default assumption ensures that $A_T > mK$. Note that equation (13) can be rewritten

$$S_T = \frac{A_T}{n} - \left(\frac{n}{n-m}\right) m \left(K - \frac{A_T}{n}\right)$$

This is the standard result for warrants: put exercise is optimal as long as $A_T/n < K$, and the value of m put warrants is the value of m ordinary puts, adjusted for dilution in this case by multiplying by $n/(n-m)$.

3.1.2 Synthetic Option Transaction

Now we analyze the synthetic version of a put warrant sale, assuming that the replicating transactions can be undertaken costlessly. For a purchased put, $B > 0$ and $\Delta < 0$.

Note first that, by definition, the replicating portfolio matches the value of the option one period hence. Thus, in addition to equation (4), the value of the option satisfies

$$\phi_{t+h}(S_{t+h}) = \Delta_t(S_t)S_{t+h} + B_t(S_t)(1 + r_h) \tag{14}$$

where $S_{t+h} \in \{u_h S_t, d_h S_t\}$.

For tax reasons the cash flows will not be the same for a firm synthetically creating the warrant as for a firm issuing an actual warrant. Thus in order to keep the stock price for the two firms the same, we imagine that the firm replicating the option issues a special class of equity with price V_t, which at time t pays its holders the difference between the two cash flows, δ_t. At time 0 the proceeds from issuing the security, V_0, are paid to common shareholders.[8] This security keeps the price of the common shares the same as if the firm issued an actual put, and the present value of its cash flows is the difference in the value of the two strategies.

The firm replicates the sale of puts written on m shares by borrowing $mB_0(S_0)$ at time 0 to repurchase $m\Delta_0(S_0)$ shares.[9] This generates a positive cash flow of

$$m\phi_0(S_0) = m\Delta_0(S_0)S_0 + mB_0(S_0)$$

As with the actual put sale, we assume $m\phi_0$ is paid out as a dividend to shareholders.

At time $t+h$, the firm will have $n + m\Delta_t(S_t)$ shares outstanding, and a debt obligation of $B_t(S_t)(1 + r_h(1-\tau))$, with equations (7) and (8) used to compute B and Δ. The share value is thus

$$S_{t+h} = \frac{A_{t+h} - q_{t+h} - mB_t(1 + r_h(1-\tau))}{n + m\Delta_t} \tag{15}$$

From the definition of the replicating porfolio, equation (14), we have

$$\phi_{t+h}(S_{t+h}) = S_{t+h}\Delta_t(S_t) + B_t(S_t)(1+r) \tag{16}$$

We can rewrite equation (15) to more easily compare to the firm which writes an actual put:

$$\begin{aligned} S_{t+h} =& \frac{A_{t+h} - q_{t+h} - m\Delta_t S_{t+h} - mB_t(1 + r(1-\tau))}{n} \\ =& \frac{A_{t+h} - m\phi_{t+h}(S_{t+h}) + mB_t r_h \tau - q_{t+h}}{n} \end{aligned} \tag{17}$$

Setting $q_{t+h} = mB_t r_h \tau$ in equation (17) gives us the same share price as in equation (11).

[8] If the price is negative, common shareholders pay this amount to the owners of special equity.

[9] Note that in replicating the option it is not necessary for the firm to undertake explicit borrowing. Since cash is negative debt, the same effect can be achieved by using the firm's pre-existing cash balance to buy stock.

The strategy of keeping $n + m\Delta$ shares outstanding requires the firm to trade frequently. As the stock price rises, the firm will re-issue shares, and as the stock price falls it will buy additional shares. However, it is well-known that this replicating strategy is self-financing. The only difference arises from the tax benefit to explicit debt, $mB_t r_h \tau$.

Finally, consider the penultimate binomial period, time $T - h$. If $A_T/n > K$ at expiration, we have $\phi_T(S_T) = 0$. As in other periods, we set $q_T = mB_{T-h} r_h \tau$. Equation (16) then implies

$$-\Delta_{T-h} S_T = B(1 + r) \tag{18}$$

The share price is

$$\begin{aligned} S_T &= \frac{A_T - q_T + mB_{T-h} r_h \tau}{n} \\ &= \frac{A_T}{n} \end{aligned} \tag{19}$$

If $S_T < K$ at expiration, we have $\phi_T(S_T) = K - S_T$. Equation (16) implies

$$(1 + \Delta_{T-h}) S_T + B_{T-h}(1 + r_h) = K \tag{20}$$

The share price at time T is

$$S_T = \frac{A_T - q_T - B_{T-h}(1 + r_h(1 - \tau))}{n + m\Delta_{T-h}}$$

Using equation (20), we can rewrite this to get

$$\begin{aligned} S_T &= \frac{A_T - q_T - m\left(S_T(1 + \Delta) - B(1 + r_h(1 - \tau))\right)}{n - m} \\ &= \frac{A_T - q_T + mB_{T-h} r_h \tau - mK}{n - m} \\ &= \frac{A_T - mK}{n - m} \end{aligned} \tag{21}$$

As long as $q_T = mB_{T-h} r_h \tau$, equations (12) and (13), and equations (19) and (21) yield the same value for the shares.

This section has shown that as long as the tax deduction on debt is paid as a dividend to holders of the special stock, the ordinary common shares have the same value whether the firm sells a put or replicates selling a put.

It is interesting to note that the option premium arises from the firm trading against its own shareholders. If the share price falls over time, the effect of the synthetic strategy is to buy back shares gradually rather than all at once. In particular, if at time T, $A_T < nK$, the actual put would have entailed paying mK at time T to buy back m shares. With a synthetic strategy, the firm would have bought $m\Delta_0$ shares at time 0, and would have gradually bought $m(1+\Delta_0)$ additional shares over the life of the option.

If, on the other hand, $A_T > nK$, the put is out of the money at expiration. In this case, the firm would have bought $m\Delta_0$ shares at time 0 and gradually sold them back at a gain as the stock price rose.

3.2 Valuing the Tax (Dis)Advantage

The example in the previous section showed that the synthetic European put generates a cash flow of $mr_hB_t(S_{t-h})\tau$ per binomial period greater than an actual put. What is the value at time 0 of this cash flow?

Let $V_t(S_t)$ denote the present value at time t of the dividend stream of tax deductions beginning at $t+h$, conditional upon the stock price being S_t. In the final period, the value of the dividend is $mr_h\tau B_{T-h}(S_{T-h})$, hence the value of that dividend in period $T-h$ is

$$V_{T-h}(S_{T-h}) = \frac{mr_h\tau B_{T-h}(S_{T-h})}{1+r_h}$$

Discounting occurs at the risk-free rate since this is a traded security priced in the same fashion as any other traded derivative claim on the firm.

Now consider the node at time $T-2h$ at which the stock price is S_{T-2h}. There are two periods worth of dividends to value. First, the debt incurred at time $T-2h$ produces a dividend one period hence of $mr_h\tau B_{T-2h}(S_{T-2h})$. Second, the stock price will either go up to S^+_{T-2h}, with a corresponding dividend value of $V_{T-h}(S^+_{T-2h})$, or down to S^-_{T-2h}, with a dividend value of $V_{T-h}(S^-_{T-2h})$. Thus, at time $T-2h$, the present value of future dividends is

$$V_{T-2h}(S_{T-2h}) = \frac{mr_h\tau B_{T-2h}(S_{T-2h})}{1+r_h} + \frac{1}{1+r_h}\left[V_{T-h}(S^+_{T-2h})p_h + V_{T-h}(S^-_{T-2h})(1-p_h)\right] \tag{22}$$

Since V_{T-h} is proportional to B_{T-h}, this can be rewritten

$$V_{T-2h}(S_{T-2h}) = \frac{mr_h\tau}{1+r_h}\left[B_{T-2h}(S_{T-2h}) + \frac{B_{T-h}(S^+_{T-2h})p_h + B_{T-h}(S^-_{T-2h})(1-p_h)}{1+r_h}\right]$$

By Proposition 2, the second term in square brackets equals $B_{T-2h}(S_{T-2h})$. Thus,

$$V_{T-2h}(S_{T-2h}) = \frac{mr_h\tau}{1+r_h}\ 2B_{T-2h}(S_{T-2h})$$

Proceeding recursively back $N = T/h$ steps, we have

$$V_0(S_0) = \frac{mr_h\tau}{1+r_h} NB_0(S_0)$$

Letting $h \to 0$, we obtain[10]

$$V_0(S_0) = mrT\tau B_0(S_0) \tag{23}$$

Proposition 3 *The value of the tax benefit from issuing a European-style derivative on the firm's own stock is $mrT\tau B_0(S_0)$, where m is the number of shares underlying the derivative, r is the risk-free rate, T is the time to maturity, τ is the coroporate tax rate, and $B_0(S_0)$ is implicit debt for a derivative with one underlying share.*

Corollary 2 *It is tax-advantaged for a firm to buy puts, sell calls, and short forward contracts (all have $B > 0$), and tax-disadvantaged to sell puts, buy calls, and go long forward contracts (all have $B < 0$).*

If equation (23) seems peculiar, note that the formula in Proposition 3 is easy to derive under certainty. The price at time 0 of a security paying the continuous

[10]The calculation can only be done numerically for American options, since the tax advantage stops accruing when the option is early-exercised.

tax deduction on a pure discount bond paying \$1 at time T is

$$\int_0^T r\tau e^{-r(T-s)}e^{-rs}ds = r\tau T e^{-rT} \tag{24}$$

$$= r\tau T B(0) \tag{25}$$

where $B(0) = e^{-rT}$. This is the formula in Proposition 3. The size of the tax deduction is on average growing at r and is discounted at that rate, hence the total tax advantage appears undiscounted.

3.3 Magnitude of the The Tax Benefit

Table 3 computes the absolute value of the tax benefit of forwards, and calls and puts of different maturities, volatilities, and strikes. To interpret this table, consider first the tax benefit on the forward. The forward entails borrowing to buy the stock, so for all maturities in the row labelled "Forward", $B(0) = 100$, and the tax benefit is $.35 \times .06 \times T \times 100$, proportional to T.

We then consider option strike prices which are 80%, 100%, and 120% *of the forward price for a given maturity.* As the maturity changes, the forward price changes so the option strike price changes as well. A 20% out-of-the-money call option with 3 years to maturity and a 6% interest rate is approximately at-the-money in the standard usage.

The sum of the tax benefits as a percentage of the stock price for puts and calls in the $K/F = 100\%$ row equals the tax benefit for the forward contract with the same maturity. For example in the first column, the put and call tax benefits where $K/F = 1$ are 1.18 and 0.92. The sum is 2.10, the same as for the forward. The reason is that buying a call and selling a put with $K = F$ replicates a forward contract. The implicit debt of the two positions is therefore the same. Changes in the volatility simply redistribute the tax advantage between the call and put.

Table 3 also computes the tax benefit as a percentage of the option premium. This can be misleading since the premium for some strategies, such as a forward or costless collar, is zero, but there is still a tax benefit or cost. Nevertheless, for longer-lived options, the tax benefit can exceed 20% of the option premium.

Table 3: Absolute value of tax benefit/cost for forward contracts, puts, and calls written by a firm on its own stock. All entries are computed as $|\tau r T B(0)|$, with $\tau = .35$, $r = .06$, $\delta = 0$, and σ and T given in the table. For a forward contract (the row labelled "Forward", $B(0) = e^{-rT}F$. Option strikes are expressed as a percentage of the forward price for a given maturity, K/F. Implicit debt, $B(0)$, and option prices are computed using the Black-Scholes formula.

T		1	1	3	3	5	5
σ		30.00%	60.00%	30.00%	60.00%	30.00%	60.00%
F		106.18	106.18	119.72	119.72	134.99	134.99
	K/F	Tax Advantage as % of stock price					
Forward		2.10	2.10	6.30	6.30	10.50	10.50
	80%	0.46	0.79	2.18	3.12	4.21	5.82
Put	100%	1.18	1.30	3.80	4.40	6.63	7.86
	120%	1.95	1.83	5.51	5.72	9.17	9.96
	80%	1.22	0.89	2.86	1.92	4.19	2.58
Call	100%	0.92	0.80	2.50	1.90	3.87	2.64
	120%	0.57	0.69	2.05	1.84	3.43	2.64
		Tax Advantage as % of option premium					
	80%	13.13	6.27	21.64	11.81	28.28	16.48
Put	100%	9.86	5.50	18.52	11.09	25.24	15.80
	120%	7.68	4.94	16.27	10.54	22.99	15.27
	80%	5.17	2.72	9.51	4.13	12.01	4.66
Written Call	100%	7.76	3.40	12.22	4.79	14.74	5.30
	120%	10.39	4.03	14.74	5.38	17.20	5.85

Table 4: Percentage of shares outstanding sold forward using puts or similar transactions. Data is from company 10-Ks.

Company	1996	1997	1998	1999
Dell	2.99	4.43	7.76	1.93
Intel	0.91	0.91	0.15	0.05
Maytag	0.00	2.96	4.48	10.4
Microsoft	1.11	.25	2.49	3.30

4 Written Puts

The synthetic equivalent of a written put entails borrowing, hence written puts are tax-disadvantaged relative to the synthetic equivalent. In this section we examine the tax cost of put writing, looking at specific transactions by Microsoft and Dell, and discuss some possible rationales for the practice.

Written puts and related option positions are disclosed in footnotes but are off-the-balance sheet. To gain an idea of the size of these transactions, Table 4 depicts the percentage ofshares sold forward by several firms. In some cases the companies sold puts and received substantial premium. Microsoft, for example, raised $1.3 billion from put sales in 1998 and 1999. In others (Dell in some years) the company both sold puts and bought calls and therefore the premium is presumably small (it is at any rate unreported).

4.1 Microsoft

As of June, 1999, Microsoft had outstanding put warrants for 163m shares, with strike prices ranging from $59 to $65 and expirations ranging from 3 months to 2.75 years.[11] If these puts were to expire in-the-money, the potential liability would be approximately $\$60 \times 163m = \$9.78b$. For an at-the-money put, implicit debt would be approximately 40-50% of the strike, depending on maturity. A conservative estimate of implicit debt is thus $B = \$9.78b \times .4 = \$3.912b$. The per-year measure

[11] It is interesting to compare Microsoft's 10-K's year-to-year. It appears that outstanding put options are restructured periodically. The strikes outstanding in 1998—$72-$77 per share—do not correspond to any strikes outstanding in 1999—$59-$65 per share, following a 2:1 stock split in March 1999.

of the tax cost at issue, assuming it is at-the-money, is thus

$$\$3.912b \times .06 \times .35 = \$82m.$$

As of June 30, 1999, these puts were substantially out-of-the-money, with Microsoft trading at around \$90/share. This substantially lowers implicit debt, creating a tax cost closer to \$20m. Of course, as Microsoft's share price fell subsequently, the tax cost would have risen because implicit debt would also have risen as the puts came more into the money.

4.2 Dell Computer[12]

Dell undertook a more complicated transaction, described this way in its 1998 10-K (obviously some creative interpretation of this passage is necessary):

> The Company utilizes equity instrument contracts to facilitate its repurchase of common stock. At February 1, 1998 and February 2, 1997, the Company held equity instrument contracts that relate to the purchase of 50 million and 36 million shares of common stock, respectively, at an average cost of \$44 and \$9 per share, respectively. Additionally, at February 1, 1998 and February 2, 1997, the Company has sold put obligations covering 55 million and 34 million shares, respectively, at an average exercise price of \$39 and \$8, respectively. The equity instruments are exercisable only at expiration, with the expiration dates ranging from the first quarter of fiscal 1999 through the third quarter of fiscal 2000.

A natural interpretation of this passage is that Dell sold puts on 55 million shares at a strike of \$39 and bought calls on 50 million shares at a strike of \$44. However, the vague language in the quoted passage raises the possibility that the purchased "equity instrument contracts" were not plain vanilla calls.[13]

[12] Details of specific transactions are generally proprietary, and the analysis of Dell Computer is this section is entirely my own.

[13] By contrast, Dell's 1997 10-K stated explicitly that Dell sold puts and bought calls in an earlier transaction.

By comparing the 1997 and 1998 10-Ks, one can verify that the higher-strike contracts were entered into between Feb. 1997 and Feb. 1998. At issue, therefore, the expiration of the higher-strike contracts could have been anywhere between about one and three and one-half years.

Lacking further details, we can try to perform a back-of-the-envelope assessment of this transaction. Suppose that at issue the stock price was \$43, hence the puts were 10% out-of-the-money. Also suppose the calls were capped (i.e. the maximum gain from exercise was limited) to give the transaction a zero premium. Further suppose that Dell's stock volatility was 50% (close to its historical volatility from February, 1996 to February, 1998), the risk-free rate was 5.5%, the options at issue had 3 years to expiration, and Dell has a 35% marginal tax rate.

Given these assumptions, the Black-Scholes price for the put is 8.317. A call with a \$44 strike and a cap at \$88.742 would have a premium of 9.149. Selling 55m puts and buying 50m of these capped calls would generate a zero premium.

The purchased capped call is replicated by borrowing \$4.143 to buy .309 shares, while the written put is replicated by borrowing \$18.241 to buy .23 shares. Given the number of options outstanding, the implicit amount borrowed is

$$55m \times \$4.143 + 50m \times \$18.241 = \$1.21b$$

One year's interest deduction on this borrowing would be

$$\$1.21b \times 5.5\% \times .35 = \$23.30m$$

This back-of-the-envelope calculation suggests that Dell would lose \$23m annually were it to engage in this option transaction to hedge its repurchases, as opposed to borrowing to fund current repurchases.[14] Using Proposition 3, the present value of the tax disadvantage over 3 years would be \$69m. (In comparing these transactions keep in mind that in any case, Dell will have to pay cash in the future, whether to pay option strikes to repurchase shares at that time, or to repay debt used to

[14]For the capped call, implicit borrowing declines, and actually becomes implicit lending—which is tax-advantaged— if the stock price becomes sufficiently great. The reason is that if the capped position is deep-in-the-money, it is equivalent to a fixed receipt of $50m \times (88.74 - 44) = \$2.04b$. In this case Dell effectively holds a zero coupon bond. It is important to keep in mind that this is merely a guess about the structure of the transaction.

repurchase shares at an earlier date.)

If the position were a pure collar, that is if the call were not capped, the loss would be larger because the implicit borrowing is greater. The call would be equivalent to borrowing \$14.697 to hold .725 shares. Net implicit borrowing would be

$$55m \times \$14.697 + 50m \times \$18.241 = \$1.74b$$

One year's interest deduction would be \$33.5m. In this case, the amount of debt implicit in the option position is not very sensitive to the stock price. If the 39 strike puts are in-the-money at expiration, Dell would be required to pay $\$39 \times 55m = \$2.145b$ for shares, while if the 44 strike call is in the money, Dell would be required to pay $\$44 \times 50m = \$2.2b$. The amount \$1.74b reflects the present value of this likely obligation. The implicit debt amount would decline significantly only if Dell's stock price were between \$39 and \$44 with a short time to expiration.

4.3 Why Sell Puts?

In this section we discuss some possible explanations for firms selling put options despite their tax disadvantage. Angel, Gastineau, and Weber (1997) also discuss put sales in the context of a repurchase program, but without discussing taxes. To anticipate the discussion, there is not an obvious rationale for firms to sell puts. Graham (2000) argues that firms leave money on the table by using too little leverage given their tax rate, and it could be that put-writing is another manifestation of failing to firms failing to optimize their tax position.

4.3.1 Private Information about the Stock Price

Written puts pay off when the stock price rises, hence it appears to be an attractive strategy for managers believing the stock price will rise. However, if shares are to be ultimately repurchased, and given the belief that the price will rise, a preferred strategy is repurchasing shares today, financing the repurchase either with cash or by issuing debt. The reason is that when the share price rises, the put is not exercised, and the firm buys shares back at the higher market price rather than the low price at the time the put was issued.

Thinking about private information, however, raises several questions. First, who is the counterparty? Second, is there a sense in which put sales are a better way to trade than open market repurchases?

Who is the Buyer? Put sales are typically not registered and are sold directly to a counterparty. This would seem to make the private information explanation suspect (who would buy in this situation?), except when the counterparty is a broker-dealer who then delta-hedges the issue. According to a practitioner this is common. The transaction is conducted in secret, and the quantities of puts sold on a given day are small enough that trading by the dealer is not expected to have much market impact. Subsequent trading occurs only because the delta of the put changes. From the dealer perspective, the transaction may be attractive not only for fees, but also because owning the puts provides a useful hedge for a typical dealer's portfolio (see below). Since the whole transaction is off-balance-sheet for the firm, the dealer effectively becomes a conduit through which the firm can borrow and trade.

Regulatory Issues Since an exercised written put results in a repurchase, share-repurchase regulations should logically affect the logistics of put-writing. Rule 10b-18, promulgated by the SEC under the Securities Exchange Act of 1934, provides a safe harbor under which a firm can buy its own stock without facing charges of manipulation. One of the key elements is that a firm is permitted to buy up to 25% of its average daily trading volume over the preceding four weeks. In practice, firms reportedly write European puts and stagger their put-writing to stay within this safe harbor.[15] (This also facilitates secrecy, as discussed above.) Dell's average trading volume, for example, was over 5m shares in 1996 and 1997, and over 7m shares in 1997 alone. Thus, Dell should have been able to repurchase over 1m shares daily, and could have issued the options described above over a several month period.

[15]The applicability to put writing of various SEC rules, including 10b-18, was clarified by a Feb. 22, 1991 letter from the SEC, file number TP 90-375. The SEC established a safe harbor, which among other things, required adherence to the 10b-18 volume restrictions and required that puts be issued out-of-the-money. This ruling concerned exchange-traded puts, and there is apparently some uncertainty about the extent to which rules apply to private put transactions. Angel, Gastineau, and Weber (1997) state that Rule 10b-18 applies to the put sale, not the exercise.

4.3.2 Firms Have a Zero Marginal Tax Rate

Firms with a zero marginal tax rate would, in perfect markets, be indifferent about selling puts. Put sales would be less of a puzzle for such firms. As is well-known, it is difficult to infer a firm's marginal tax rate by looking at publicly-available data. All of the firms in Table 4 reported positive incomes taxes paid (on the statement of cash flows) in all years, except Dell in 1999. For example, Microsoft reported incomes taxes paid of \$430m, \$758m, \$1.1b, \$1.1b, \$874m, and \$800m in 1995–2000. However, both firms also report significant option compensation expense. Graham's measure of the marginal tax rate (Graham (1996)) is close to 35% for all companies and years in the Table, except for Maytag in the late 1990's. However, Graham's measure does not account for the tax deduction stemming from option exercise.[16] Thus, for these firms in particular, the Graham tax rate measure may be too high. There are no data to rule out the possibility that firms believe their marginal tax rate is zero.

Table 5 shows reported net income and the tax benefit of compensation expense for Microsoft and Dell for several years. Especially in the later years, the tax benefit of compensation expense is greater than 35% of net income, in which case taxable income would be zero. Perhaps put issuance indicates that companies such as Microsoft and Dell believe their marginal tax rate is zero.

4.3.3 Ratings Agency Arbitrage

According to practitioners, ratings agencies do not treat written puts as debt, even though they create a fixed, debt-like obligation for the firm when the stock price is low. If the firms in Table 4 did issue debt instead, the rise in the debt-equity ratio would, except for Maytag in 1999, be about 1-3%. If this changed the firm's credit rating from AA3 (Microsoft's rating) to BBB2, an extreme change, this would raise the borrowing cost by about 70 basis points (using yields in November 2000). Even

[16] When employees exercise compensation options, the difference between the stock price and strike price is deductible for tax purposes but does not affect reported net income. Companies do report the resulting tax deduction separately in their 10-K. The number reported as tax benefit of compensation expense is described like this in the Microsoft 2000 10-K: "As required by Emerging Issues Task Force (EITF) Issue 00-15, Classification in the Statement of Cash Flows of the Income Tax Benefit Received by a Company upon Exercise of a Nonqualified Employee Stock Option, stock option income tax benefits are classified as cash from operations in the cash flows statement."

	Microsoft		Dell	
Year	Net Income	Tax benefit of compensation expense	Net Income	Tax benefit of compensation expense
1995	1,453	179		
1996	2,195	352		
1997	3,454	796	518	37
1998	4,490	1,553	944	164
1999	7,785	3,107	1,460	444
2000	9,421	5,535	1,666	1,040

Table 5: Compensation expense due to option exercise and taxable income. Source: company 10-Ks.

double this change would be less than the annual value of the tax deduction on that amount of debt. If there are other, indirect costs to a lower credit rating, the lost tax deduction from selling puts would be a lower bound for the magnitude of these costs.

While written puts reduce a firm's "true" credit rating, written calls—equivalent to issuing shares and lending—raise it. Firms which write puts tend to be firms which have sold calls as compensation options. Thus, it may be that the written puts actually serve to bring the firm's true credit rating back into line with its posted credit rating.

4.3.4 The Firm Believes Volatility is High

Selling options would permit the firm to speculate on volatility. However, entering into a collar (as Dell did) does not accomplish this since options are both bought and sold. Moreover, since the firm's counterparty is a delta-hedging broker, who would be hurt by a decline in volatility, this explanation seems unlikely.

4.3.5 Automatic Dynamic Rebalancing

If the firm were to directly mimic the delta-hedging of the broker, transaction costs would be high. This said, it is not clear why the particular dynamic rebalancing associated with a put would be valuable for the firm.

The dynamic rebalancing might, however, be valuable for the broker serving as counterparty. Although there are no public statistics, it is widely believed that dealers on average are option writers. Written option positions are risky and difficult to hedge (Green and Figlewski (1999)). A position where a dealer writes options and hedges the position with stock is said to be delta-hedged and to have negative gamma.[17] A delta-hedged negative gamma position has the characteristic that the dealer can lose substantial money if there is a large move in the stock price, either up or down. Since dealers in the aggregate are thought to be option writers (end-users on average wish to buy options, rather than sell them), put writing by firms provides an opportunity for brokers to add gamma to their market-making portfolio, reducing the consequences of large stock price moves. Brokers could therefore charge lower fees for such a position.

4.3.6 Ignorance

Suppose a firm were to write puts for mistaken reasons, and thus bear the tax cost with no offsetting benefit. Would managers of the firm come to realize they had made a mistake? The arguments we have made are abstract, insofar as they involve comparing dynamic and (seemingly) static capital structures. One way to think about Proposition 3 is that a written put is underpriced given the tax treatment accorded the firm on comparable transactions.

However, it is not obvious how managers and directors would come to understand this. The firm can obtain competitive quotes for puts from different banks, ensuring "fair" pricing. There is no line item on the income statement which reflects the implicit tax loss. Analysts do not appear to criticize firms for these transactions. In fact, the secrecy with which many of these transactions are undertaken may reduce scrutiny.

Perhaps most important, many put-issuing firms made money on the strategy for years because their stock price went up consistently. Given a belief that the stock price would rise, the firm could have issued debt to finance a share repurchases instead of selling puts. Repurchasing shares, however, would have generated only implicit income, while put sales generated explicit gains. Thus, despite the ex-ante tax-inefficiency of put sales, advocates of this strategy might have (mistakenly) looked smarter ex-post.

[17]Gamma is the second derivative of the option price with respect to the stock price.

5 Convertible Bonds

Written calls (warrants) are tax advantaged relative to synthetic warrants, which entail lending. The simplest convertible bond is an ordinary bond coupled with a call option, and hence would seem to be tax-advantaged. However, a firm has a choice of issuing a convertible bond explicitly, or issuing one implicitly by simultaneously issuing a non-convertible bond together with a warrant. These otherwise equivalent transactions have a different tax treatment. We will refer to the separate bond and warrant as a *bifurcated convertible*, meaning that the bond portion is issued separately from the claim on shares.

In this section we ask two questions. First, is bifuraction tax-advantaged relative to the bundling which occurs with an ordinary convertible? To answer this we can compare the tax deduction in the two cases constructed to have identical pre-tax cash flows. We show that, from a tax perspective, the bifurcated convertible is preferred to the ordinary convertible.

Second, we ask whether a convertible is tax-advantaged relative to an ordinary bond without convertibility which raises the same amount of money. This comparison is different from the others in this paper since the pre-tax cash flows are not identical, making the interpretation of the results problematic. Even with this caveat, the answer is ambiguous.

5.1 The Tax Benefit of Bifurcation

A convertible bond is like an ordinary bond plus a call option to convert the bond into shares. Thus, the coupon on a convertible bond is the coupon on an ordinary bond reduced by the amortized option premium. If a T-period bond is convertible into m shares, we can use standard annuity calculations to show that the convertible coupon, ρ, is

$$\rho = r\left(1 - \frac{m\phi}{D(1-e^{-rT})}\right) \tag{26}$$

where D is the bond principal and ϕ the option premium per share. The option has a strike price of D/m per share.

We can analyze the tax issues associated with bifurcation by comparing two firms:

	Firm A	Firm B		
	Convertible	Warrant	Debt	Firm B Total
Time 0	D	$m\phi$	$D - m\phi$	D
Coupon	ρD	0	ρD	ρD
Maturity, $S \leq D/m$	D	0	D	D
Maturity, $S > D/m$	mS	$m \times (S - D/m)$	D	mS

Table 6: Comparison of pre-tax cash flows for firm A, which issues ordinary convertible debt and firm B, which issues an equivalent warrant and discount bond. The cash flows for Firm A's convertible match the total cash flows for Firm B's warrant plus bond.

Firm A issues ordinary T-period convertible debt with par value D and issued for D. The debt is convertible at maturity, at the holder's option, into m shares. The interest rate on the debt is ρ, from equation (26).

Firm B issues m T-period European warrants, each with premium ϕ and strike price D/m. In addition, A issues T-period debt in the amount $D - m\phi$, with a maturity value of D and paying a coupon of ρD.

Both firms A and B raise D. However, firm B bifurcates the convertible bond for tax purposes by issuing separately a warrant and a bond with a lower principal amount. Firm A simply deducts the stated interest on the convertible bond. Table 6 shows that firms A and B have identical pre-tax cash flows.

Although the convertible and bifurcated convertible have the same pre-tax cash flow, their tax treatment is different. For both the convertible and bifurcated convertible, the interest payment ρD is deductible. This is the only tax deduction for the convertible. However, the bond portion of the bifurcated convertible is issued at a discount to par. For such an original issue discount (OID) bond, the amortized appreciation from the price at issue $(D - m\phi)$ to D at maturity is deductible as interest.

Suppose a bond with par value D is issued for $Q(0) < D$. The original issue discount rules call for amortizing the discount, with the firm receiving a tax deduction each year based on the implied increase in the value of the bond. The annualized rate of appreciation is $ln(D/Q(0))/T = \lambda$. The flow of tax deductions is therefore

$$\tau \int_0^T e^{-rs} \lambda Q(s) ds$$

where $Q(s) = Q(0)e^{\lambda s}$. Evaluating this integral, the present value of the OID tax deductions is[18]

$$\tau \frac{\lambda}{r-\lambda} \left(1 - e^{-(r-\lambda)T}\right) Q(0) \tag{27}$$

The tax deduction due to the coupon is simply an annuity, with a present value of

$$\tau \frac{\rho D}{r} \left(1 - e^{-rT}\right). \tag{28}$$

We can summarize this discussion in the following proposition.

Proposition 4 *For non-callable convertible bonds which are convertible at expiration, the present value of the tax deduction is given by equation* (28). *For an otherwise identical bifurcated convertible, the present value of the tax deduction is given by the sum of equations* (27) *and* (28)*, with* $Q(0) = D - m\phi$.

Here are two examples illustrating these calculations. First, suppose a firm issues a \$100 convertible bond with one year to maturity, which converts into one share. For simplicity, consider the discrete equivalent of equation (26). Suppose the interest rate is 6% and the option premium is \$3.774. Using equation (26), interest on the convertible is \$2 ($\$6 - \$3.774 \times (1.06) = \$2$). The firm can thus deduct \$2 in interest payments.

To issue the bifurcated convertible, the firm sells a warrant for \$3.774 and issues a bond for \$96.326 carrying a \$2 coupon. This is an original-issue discount bond, and thus the appreciation from \$96.326 to \$100 is deductible as interest. The firm has a year 1 interest deduction of $\$2 + \$3.774 = \$5.774$. (Note that $5.774/96.326 = .06$.)

The second example also assumes a \$100 convertible and uses continuous compounding, but assumes the bonds are infinitely-lived. Let $r = 6\%$. Suppose the option premium is \$40. From equation (26), the convertible coupon is 3.6%. The

[18] Since a zero-coupon bond has no payouts and thus is a special case of an option, we would expect that for a zero coupon bond, for which $\lambda = r$, equation (27) reduces to $\tau r T Q(0)$. Using L'Hôspital's rule, one can verify that this is correct.

bifurcated convertible consists of a \$40 warrant and a \$60 bond carrying a coupon of \$3.60. Because the bond is a perpetuity, however, *the implied annual appreciation on the bond, λ, is 0.* At a price of \$60 with a \$3.60 coupon, the bond is at par, and the OID tax deduction is zero. Thus, for the limiting case of infinitely-lived bonds, bifurcation has zero value.[19]

Figure 2 illustrates effects of maturity and volatility on the tax benefit of bifurcation. The top panel of Figure 2 computes the present value of the tax benefit from bifurcation, equation (27). For a wide range of parameters, the benefit of bifurcation exceeds 5% of the value of the bond. Because for a given bond the OID tax deduction increases over time, the annual tax deduction changes over time. The bottom panel computes a level annuity with the same present value as in the top panel.

The perpetuity example discussed earlier provides an explanation for the long-run decline in the value of bifurcation. As $T \to \infty$, the present value of the OID tax deduction approaches 0.

This analysis leaves unanswered the question of why firms issue convertibles in the first place, and whether a bifurcated convertible is inferior to a real convertible along dimensions other than tax. It is possible that a bifurcated convertible does not solve a problem which might be solved by a true convertible. For example, one problem with ordinary debt is that managers can shift value from debtholders to equityholders by increasing asset risk. Because they contain an equity component, convertibles are not so easily expropriated. A bifurcated convertible can solve the same problem *only if existing shareholders cannot buy the warrants.* If shareholders do buy the warrants, the straight debt holders can again be expropriated.

Other practical issues are that conversion reduces debt outstanding. Bifurcated convertibles can accomplish this by creating incentives (such as more favorable conversion terms) if the debt is used as payment for exercising the warrant. Another issue, emphasized by Stein (1992) and Strnad (2001), is that convertibles are generally callable, and this characteristic should be related to the problem they are solving. Presumably a warrant/bond package could be made callable by having the warrant callable and again offering incentives if the is used as payment for exercising

[19] I thank Stewart Myers for the observation that the tax benefit of bifurcation must be zero for a perpetuity.

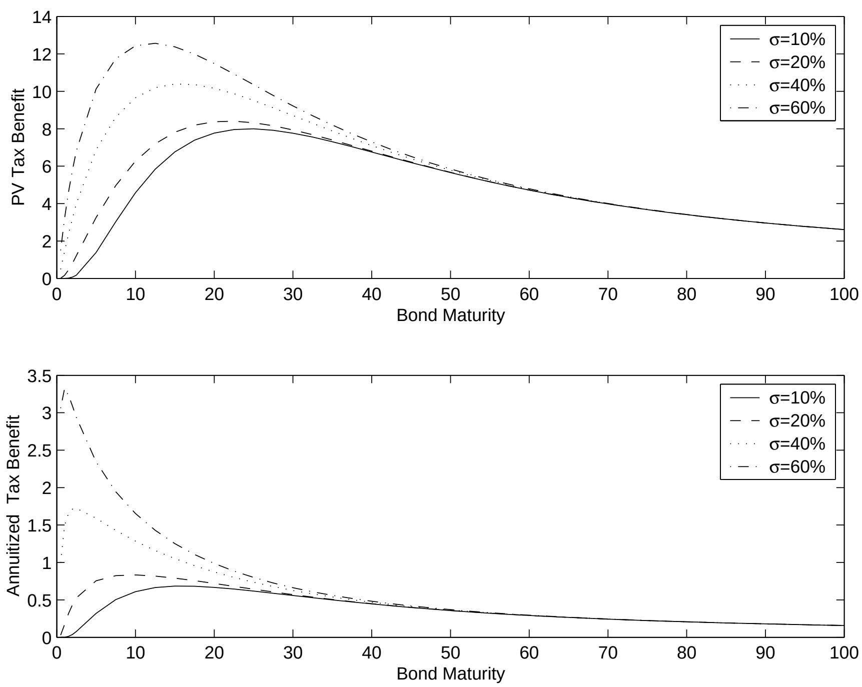

Figure 2: Tax benefit of bifurcation for different times to maturity and different volatilities, σ. The top panel is given by equation (27), with $B(0)$ given by $D - m\phi$. The bottom panel is given by annuitizing equation (27). Calculations assume the tax rate, $\tau = .35$, the stock price is \$70 and the bond maturity value is \$100, with option premia computed using the Black-Scholes formula.

the warrant.

As with written put options, the tax disadvantage of the warrant/bond bundle relative to that of a convertible provides a lower bound on the value of the convertible to the firm.

5.2 Convertibles vs. Ordinary Bonds

The next question we address is whether convertibles and bifurcated convertibles are tax-advantaged relative to ordinary debt. Because the risk of the two positions is different this is something of an apples and oranges comparison. Suppose we compare an issue of D in ordinary debt to D raised with a bifurcated convertible. This is an interesting comparison because the option component of the convertible has an *implicit* tax benefit which, because we were comparing two bonds with the same option, we did not consider so far.

With ordinary debt, the firm receives a full tax deduction on annual interest of rD. The present value of this is

$$\tau D\left(1-e^{-rT}\right) \tag{29}$$

With a bifurcated issue, there are three components to the tax deduction. First there is the deduction on the coupon, $\tau\rho D$. Second is the present value of deduction stemming from appreciation on the OID bond. Third is the implicit tax benefit on the warrant with value $m\phi$, resulting from the warrant's implicit tax-free lending.

The present value of the tax deduction on the coupon, $\tau\rho D$, is

$$\tau Dr\left(1-\frac{m\phi}{D(1-e^{-rT})}\right)\frac{1}{r}\left(1-e^{-rT}\right)=\tau\left(D-m\phi\right)\left(1-e^{-rT}\right)$$

The present value of the OID tax deduction is given by equation (27). Finally, the implicit tax benefit of the option is τrTB, where B is the debt implicit in the option. The sum of these is

$$\tau(D-m\phi)\left(1-e^{-rT}\right)+\tau\frac{\lambda}{r-\lambda}\left(1-e^{-(r-\lambda)T}\right)(D-m\phi)+\tau rTB \tag{30}$$

Subtracting equation (29) from (30) gives

$$-\tau m\phi\left(1-e^{-rT}\right)+\tau\frac{\lambda}{r-\lambda}\left(1-e^{-(r-\lambda)T}\right)(D-m\phi)+\tau rTB \tag{31}$$

This equation makes clear the tradeoff from a convertible. The bifurcated convertible has a smaller coupon, which is offset by the OID deduction and the implicit tax benefit of the option.

Figure 3 plots equation (31) for different times to maturity, volatilities, and two different dividend yields. The comparison between ordinary and bifurcated convertible debt is fundamentally ambiguous. At short horizons the implicit debt in the option makes up for the reduction in the interest tax deduction. However, at long horizons, the OID deduction is relatively less important and implicit debt declines. Thus ordinary debt becomes tax-preferred. When the dividend yield is positive (bottom panel), the option premium declines at long horizons and the convertible becomes more like ordinary debt. The comparison is fundamentally ambiguous.

Proposition 5 *Bifurcated debt may be either tax-advantaged or tax-disadvantaged relative to ordinary debt, depending upon maturity, dividends, and volatility.*

Since a bifurcated convertible is preferred to an ordinary convertible, an ordinary convertible will fare worse in a comparison to plain debt. Figure 4 makes this comparison, showing for the same parameters as in Figure 3 that an ordinary convertible has in almost all cases no tax benefit relative to ordinary debt, although some ambiguity remains.

The analysis in this section has dealt only with European convertible bonds, which convert only at maturity and are non-callable. We have also ignored default and interest rate risk. It seems unlikely that accounting for any of these would change the fundamental ambiguity in a comparison of convertibles to ordinary debt.

6 Compensation Options

Compensation options are written call options. However, compensation options do not fit neatly within the framework of the earlier examples because the counterparty

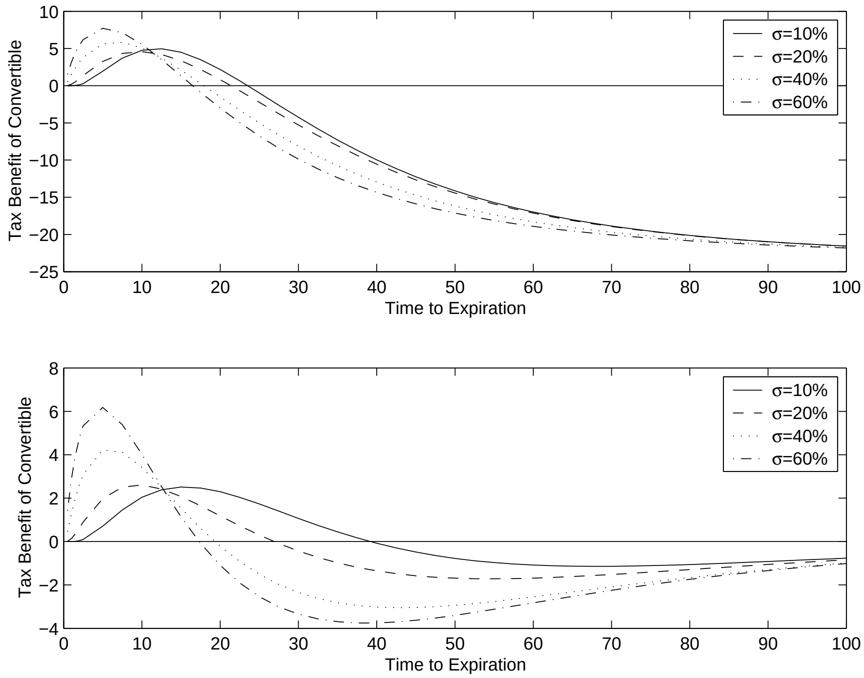

Figure 3: Plot of equation (31), depicting the difference between the present value of the tax deduction for a bifurcated convertible and an ordinary bond for different volatilities, times to maturity, and dividend yields. The dividend yield, δ, is 0 in the top panel and .03 in the bottom panel. Both panels assume $S = 70$, $D = 100$, and $r = .06$.

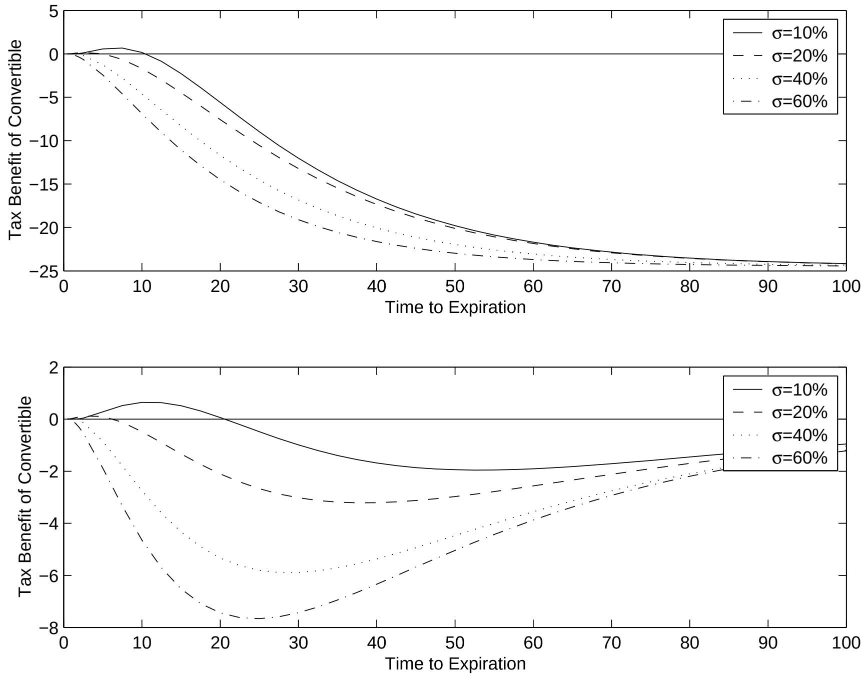

Figure 4: Plot of equation (31) less equation (27), depicting the difference between the present value of the tax deduction for an ordinary convertible and an ordinary bond for different volatilities, times to maturity, and dividend yields. The dividend yield, δ, is 0 in the top panel and .03 in the bottom panel. Both panels assume $S = 70$, $D = 100$, and $r = .06$.

is a taxed employee of the firm rather than a tax-neutral market-maker. Market-makers provide a mechanism for holding fixed the taxation of the investors who serve as the ultimate counterparty. With compensation options, by contrast, the employee's tax position matters, and this is generally altered with changes in the struture of the compensation contract.[20]

The majority of employee options in practice are non-qualified options (NQO's). These have no tax consequences for either party until exercise, at which time the difference between the stock price and strike price is ordinary income for the employee and an ordinary deduction for the firm. The employee can, at the time of exercise, sell the acquired shares with no further tax consequences. Incentive stock options (ISOs) differ in that there is no deduction for the firm. The employee pays capital gains tax on the difference between the stock price and exercise price only when the stock is ultimately sold; there is no tax at the time of exercise.

Taking the deferred, contingent nature of the compensation as given, it is natural to compare three compensation structures: an NQO, an ISO, and a stock-price contingent bonus scheme. With the bonus, we assume the firm ultimately makes the same payment to employees as with the option.

With an NQO and a contingent bonus, employee compensation in both cases is treated as ordinary income for the employee and an ordinary deduction for the firm. The employee receives at time T the after-tax payout $(1-\tau_p)max(S_T-K,0)$, where τ_p is the employee's tax rate on ordinary income. Also in both cases, the firm at time T receives an ordinary deduction of $(1-\tau)max(S_T-K,0)$. The contingent bonus is a written call with the same tax treatment as an NQO. Thus the two are equivalent.

The comparison between an ISO and a bonus is more involved. Suppose the employee's effective capital gains tax rate on the exercise proceeds is τ_g. Then \$1 of ISO income generates $1-\tau_g$ of after-tax income. Similarly, since bonus income is taxed at the rate τ_p, $(1-\tau_g)/(1-\tau_p)$ of bonus income generates $1-\tau_g$ of after-tax

[20] A different kind of comparison which we do not consider is that between the grant of compensation options and immediate cash payments. Scholes and Wolfson (1992), for example, discuss this kind of analysis. Compensation options are non-tradeable, leaving employees with a less diversified porfolio than they would otherwise choose. To compensate for nontradability, the firm must give the employee more than \$1 of options for each \$1 of salary foregone. At the same time, the firm may believe that compensation options make employees more productive. The analysis of issues like this is beyond the scope of the paper.

income.

To compare the ISO and the bonus, we have to compare the cost to the firm of an ISO on one share and a bonus, each of which pays $(1-\tau_g)max(0, S_T - K)$ to the employee. The initial cost to the firm of issuing an ISO on one share is $\phi - \tau rTB$, the option premium less the present value of the tax benefit. Since the bonus is tax deductible, the cost to the firm of the comparable bonus is $(1-\tau)(1-\tau_p)/(1-\tau_g)\phi$.

Thus, the ISO is preferred to the bonus if

$$\phi - \tau rTB < \frac{(1-\tau)(1-\tau_g)}{1-\tau_p}\phi$$

or

$$1 - \frac{\tau rTB}{\phi} < \frac{(1-\tau)(1-\tau_g)}{1-\tau_p} \tag{32}$$

This comparison of an ISO and bonus draws on a similar analysis by Scholes and Wolfson (1992, Chapter 10). The difference is the inclusion of the tax benefit of the actual vs. synthetic option.

We can summarize this discussion in the following proposition.

Proposition 6 *An NQO is identical to an equivalent bonus scheme. The comparison of an ISO with a bonus (or NQO) depends on relative tax rates and is given by equation* (32)*.*

7 Conclusions

A position in options on a firm's own stock is not tax-neutral. This finding is relevant for capital structure decisions, including the decision to issue warrants, compensation options, and convertible bonds. Examples of firms writing put options suggest that the tax implications can be significant, with those firms potentially losing tens of millions of dollars in foregone interest deductions. The possibility that firms are engaging in tax disadvantaged transactions raises an interesting question: how would a firm come to learn that the sale of a put (for example) is tax-disadvantaged? As discussed in Section 4.3, there is not an obvious answer.

The specific analysis in this paper relies heavily on specific U.S. tax rules. A question we do not address is the tax treatment of similar share transactions in other countries, and the extent to which compensation options, warrants, and convertibles are tax advantaged. It would be interesting to carry out a systematic examination of international tax rules and correlate these cross-sectionally with financial policies.

A Appendices

A.1 Proof of Proposition 2

Denote possible option values next period as ϕ_u and ϕ_d, and for the period after that, ϕ_{uu}, ϕ_{ud}, and ϕ_{dd}. From equation (8), possible values of B the next period are

$$\begin{aligned} B_u &= \frac{1}{1+r_h}\frac{u_h\phi_{du} - d_h\phi_{uu}}{u-d} \\ B_d &= \frac{1}{1+r_h}\frac{u_h\phi_{dd} - d_h\phi_{du}}{u-d} \end{aligned} \tag{33}$$

Possible option values next period are

$$\begin{aligned} \phi_u &= \frac{1}{1+r_h}\left[p_h\phi_{uu} + (1-p_h)\phi_{du}\right] \\ \phi_d &= \frac{1}{1+r_h}\left[p_h\phi_{du} + (1-p_h)\phi_{dd}\right] \end{aligned} \tag{34}$$

where $p_h = (1 + r_h - d_h)/(u_h - d_h)$ is the risk-neutral probability of the stock price going up.

Now consider discounted expected next-period B:

$$\frac{1}{1+r_h}\left[p_h B_u + (1-p_h)B_d\right]$$

We can expand this expression and rewrite it, using equations (33) and (34) as

$$\begin{aligned} &\left(\frac{1}{1+r}\right)^2\left[p\left(\frac{u\phi_{du} - d\phi_{uu}}{u-d}\right) + (1-p)\left(\frac{u\phi_{dd} - d\phi_{du}}{u-d}\right)\right] \\ &\quad=\left(\frac{1}{1+r}\right)^2\left[\frac{u}{u-d}\left(p\phi_{du} + (1-p)\phi_{dd}\right) - \frac{d}{u-d}\left(p\phi_{uu} + (1-p)\phi_{ud}\right)\right] \\ &\quad=\frac{1}{1+r}\ \frac{u\phi_d - d\phi_u}{u-d} \\ &\quad=B \end{aligned}$$

This demonstrates in a 2-period setting that B is the expected present value of future B. By recursion it will be true in an n-period setting.

As an aside, in the context of the Black-Scholes model, implicit lending for an option at time t which expires at time T is

$$B_t = \int_K^\infty K e^{-r(T-t)} f(S_T|S_t) dS_T$$

Here it is obvious that $B_t e^{r(T-t)}$ is a random walk.

A.2 Proof of Corollary 1

We need only prove that $|B_{T-h}(S_{T-h})| \leq K$, where $T-h$ is the last binomial period before expiration. The result then follows by recursion from Proposition 2.

If a put option is certain to be in-the-money at expiration, then we have

$$\begin{aligned} B_{T-h}(S_{T-h}) =& \frac{1}{1+r} \quad \frac{u_h(K - d_h S_{T-h}) - d_h(K - u_h S_{T-h})}{u_h - d_h} \\ =& \frac{K}{1+r} \end{aligned}$$

If the put option is only in-the-money in the down state, then $K < u_h S_{T-h}$. This implies that $u_h(K - d_h S_{T-h})/(u_h - d_h) < K$. Thus, we have

$$\begin{aligned} B_{T-h}(S_{T-h}) =& \frac{1}{1+r} \quad \frac{u_h(K - d_h S_{T-h})}{u_h - d_h} \\ <& \frac{K}{1+r} \end{aligned}$$

The demonstration for calls is analogous.

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