Abstract

Firms play a pivotal role in international trade, shaping the comparative advantage of the countries. We propose a ‘granular’ multi-sector model of trade, which combines together fundamental Ricardian comparative advantage across sectors with granular comparative advantage due to outstanding productivity draws of individual firms. We develop a SMM-based estimation procedure, which takes full account of the general equilibrium of the model, and jointly estimate the fundamental and the granular forces using French micro-level data with information on firm domestic and export sales across manufacturing industries. The estimated granular model captures the salient features of micro-level heterogeneity across firms and industries, without relying on variation in model parameters across sectors. The estimated model implies that one third of trade flows is explained by granular forces, and that sectors with the extreme export shares are more likely to be of ‘granular’ origin than sectors with average export shares. Failure of a single large firm in a granular sector has dramatic effects on the relative export standing of the sector. We further show that empirically measurable proxies of granularity have a substantial predictive ability for trade flows in the estimated model, even after controlling for fundamental comparative advantage of the sectors. Lastly, extending the model to allow for firm-level productivity dynamics explains the majority of mean reversion in country’s comparative advantage over time.

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1 Introduction

Firms play a pivotal role in international trade. Much of exports is done by a small number of very large firms which enjoy substantial market shares in their markets across destination countries. One may, thus, conjecture that countries export not only the goods they are fundamentally good at producing, but also the goods that their individual firms happened to master to produce in an idiosyncratic way. As a consequence, if some of these individual firms disappeared, the country’s comparative advantage would be dramatically altered. This paper contrasts such granular comparative advantage with the fundamental comparative advantage of a country, which we take in the neoclassical sense of either Ricardian technological advantage or Heckscher-Ohlin factor endowment differences across countries.

By doing so, we revisit the fundamental questions in international trade: What goods do countries trade? What is the source of countries’ comparative advantage? The answers to these questions are interesting in themselves, even if the source of comparative advantage were not consequential for trade flows and welfare gains from trade. Furthermore, the knowledge of the specific source of comparative advantage is instrumental for our ability to predict changes in trade flows in response to a variety of shocks, such as reductions in trade costs and productivity changes across countries.

Casual empiricism suggests that individual firms play a central role in shaping country-level trade patterns: for example, the fate of Nokia in Finland or Intel in Costa Rica have impacted aggregate export patterns in these countries in an important way. In this paper we are concerned with the role of individual firms in shaping the comparative advantage of a country. To what extent the exports of a country are due to its fundamental comparative advantage, immune to the fate of individual firms, versus granular comparative advantage embodied in individual firms and entrepreneurs? In more formal terms, to what extent the country’s comparative advantage is shaped by the high-mean sectoral productivity versus outstanding productivity draw(s) from an otherwise unremarkable mean-productivity distribution? Furthermore, can one identify which specific export sectors are granular and which ones are Ricardian?

The dominant theoretical framework in international trade has shifted towards modeling individual firm exporting decisions, with a focus on firm heterogeneity and selection of the fittest firms into exporting. At the same time, these models typically maintain the continuum assumption for the productivity draws across firms, under which the law of large numbers (LLN) applies and there is no granularity left upon integration across heterogenous firms within sectors. In other words, aggregate trade in such models is not granular, as it does not depend on the productivity of any individual firm, and instead is fully determined by the fundamental sectoral productivity. We refer to such models as continuous models, which are equivalent at the aggregate with neoclassical Ricardian models. In contrast, a granular model dispenses with the assumption of a continuum of draws, and operates away from the LLN limit. Hence, the realized sectoral productivity (which shapes comparative advantage) is in part due to the specific productivity draws of individual firms, acknowledging the role of these firms in shaping economic aggregates (see also Eaton, Kortum, and Sotelo 2012, for a further discussion).

We start our analysis with a standard continuous model based on a multi-sector extension of Melitz (2003) with sectoral Ricardian comparative advantage. As the within-sector heterogeneity of firms col-
lapses, this model converges to the Ricardian Dornbusch, Fischer, and Samuelson (1977) model, which features full specialization within sectors and only inter-industry trade. We maintain the firm heterogeneity assumption, and instead relax the assumption of the continuum of draws with the resulting LLN for aggregate sector productivity. The firm productivities are drawn from a fat-tailed Pareto distribution consistent with the empirically observed Zipf’s law in firm sales distribution. The finite number of draws from a fat-tailed productivity distribution results in pronounced granular effects. Our model is a multi-sector extension of the Eaton, Kortum, and Sotelo (2012) granular model of trade, which allows us to nest simultaneously fundamental and granular comparative advantage in a unified framework.

We develop a simulated method of moments (SMM)-based estimation procedure for the granular model, consistent with the full general equilibrium of the model. We estimate the model using French firm-level data, which has information on both domestic and export sales of French firms across 120 4-digit manufacturing industries. We show that the estimated model reproduces the empirical cross-sectional distributions, in particular the cross-sectoral variation in the number of firms, in the fatness of tails of the sales distributions, in the market shares of the largest firm, and in the sectoral export intensity, all this without relying on heterogeneity in parameters across sectors. At the same time, the continuous model is unable to account for many of these features of the data without bringing in parametric cross-sectional heterogeneity. We argue, therefore, that the granular model, by means of relaxing the convenient, yet counterfactual LLN assumption, provides a significantly better fit to the data.

In the data, despite the fact that a median manufacturing sector features around 270 firms, the market share of the largest firm is on average around 20% due to the fat-tailed productivity distribution. As a result, our estimated granular model, which reproduces these empirical patterns, acts very differently from the continuous LLN model. In the model, as in the data, the single largest firm in a sector tends to have a mass-point effect on the cumulative sectoral sales and exports. The estimated model exhibits strong granular forces, which play a first order role in shaping sectoral comparative advantage and the resulting trade flows.

Our SMM procedure estimates jointly the Ricardian and the granular forces in the model, combining the data on sectoral trade flows and the within-sector distribution of firm sales. Intuitively, sectors in which the largest firm stands out in terms of sales relative to the median firm, exhibit signs of granularity. Therefore, we use the cross-sectoral correlation between the relative size of the largest firm (in the domestic market) and sectoral exports in the model, along with other moments, in order to discipline the relative roles of fundamental and granular comparative advantage.

Using the estimated model, we show that fundamental comparative advantage accounts for two-thirds and the granular residual for one-third of the variation in the realized export shares across sectors. Therefore, a substantial share of international trade is a granular phenomenon. Furthermore, we show that sectors with the extreme export shares are more likely to be of the ‘granular’ origin, as opposed to sectors with average export shares. We next show that measurable proxies of granularity, such as the market share of the top firm relative to the median firm in the sector, have a substantial predictive ability for sectoral exports, even after controlling for the fundamental comparative advantage of the sector. Finally, we consider a counterfactual in which a top firm in a sector fails for an exogenous
reason and has to exit the industry, and show that this has a dramatic effect on the export standing of the sector, often switching it from a net exporter into a net importer.

In the final section of the paper, we extend our model to allow for industry dynamics driven by exogenous firm productivity process. We discipline the firm-level productivity dynamics with the evolution of firm market shares observed in the data, in order to study the implications of the dynamic granular model for the evolution of comparative advantage over time. We show that the dynamic granular model is consistent simultaneously with the hyper-specialization of countries in a few industries and the relatively fast mean reversion in the comparative advantage of the countries, documented by Hanson, Lind, and Muendler (2015). In particular, a granular model with empirically-disciplined firm-level dynamics, and featuring no dynamics in fundamental comparative advantage, accounts for over 60% of mean reversion over time in country’s realized comparative advantage documented by these authors.

Related literature  Up to date, granularity has been mostly explored in macroeconomics for the purpose of studying aggregate fluctuations. However, the granular forces must be at least as prominent in international trade flows, which are shaped at the country-industry level, where granularity is particularly pronounced in the data.

2 The DFS-Melitz Benchmark

In this section, we review the *continuum* model, which serves as a benchmark in our analysis of the *granular* model in Section 3. As a benchmark continuum economy we consider a two-country multi-sector extension of the Melitz (2003) model, with Ricardian comparative advantage across a unit continuum of sectors \( z \in [0, 1] \), as in Dornbusch, Fischer, and Samuelson (1977). We refer to this benchmark economy as DFS-Melitz. More specifically, within each sector \( z \) we consider the Chaney (2008) version of the Melitz model without entry, in which an exogenous mass of firms \( M(z) \) are present and their productivities are drawn from a Pareto distribution with a sector-specific lower bound \( \varphi_z(z) \) and a shape parameter \( \theta \) common across all sectors. We show below that the overall sectoral productivity is determined by

\[
T(z) = M(z)\varphi_z(z)^\theta.
\]  

(1)

Intuitively, a sector is more productive either if there are more draws, \( M(z) \), or if the average productivity of a draw, given by \( \frac{\theta}{\theta - 1} \varphi_z(z) \), is high. The two countries differ in these sectoral productivity measures, \( \{T(z)\} \) at home and \( \{T^*(z)\} \) in foreign, which is the source of the Ricardian comparative advantage across sectors. Some of the specific assumptions and exposition we adopt here are slightly different from what is conventional in the Melitz literature, and this is meant to mimic the granular model of Section 3 in order to make the comparison between the two frameworks most direct.

**Households** in each country have Cobb-Douglas preferences over the consumption of sectoral goods \( Q(z) \):

\[
Q = \exp \left\{ \int_0^1 \alpha_z \log Q(z) dz \right\},
\]  

(2)

where \( \{\alpha_z\} \) are the preference parameters which determine the shares of household income spent on consumption in sector \( z \). This is the only source of cross-sector heterogeneity that we introduce into the model. The sectoral consumption bundles are CES aggregators of individual varieties \( \omega \):

\[
Q(z) = \left[ \int_{\omega \in \Omega(z)} q_z(\omega) \frac{\sigma - 1}{\sigma} d\omega \right]^\frac{1}{\sigma - 1},
\]  

(3)

where \( \Omega(z) \) is the set of varieties available for consumption in sector \( z \) at home, and \( \sigma > 1 \) is the elasticity of substitution across varieties within sectors, common to all sectors. The foreign demand structure is symmetric, with \( \Omega^*(z) \) replacing \( \Omega(z) \). The households supply labor inelastically, with \( L \) units at home and \( L^* \) units in foreign.

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1This multi-sector extension of the Melitz model is akin to the Costinot, Donaldson, and Komunjer (2012) multi-sector extension of the Eaton and Kortum (2002) model, and is different from Bernard, Redding, and Schott (2007) who also considered an extension of the Melitz model to a multi-sector environment, however one in which sectors differed in their capital-labor intensity, as in the classical Heckscher-Ohlin model. The other papers which considered a multi-sector DFS-Melitz environment, albeit under somewhat different formulation, are Okubo (2009) and Fan, Lai, and Qi (2015).
With this demand structure, the home consumer expenditure on variety $\omega$ in sector $z$ is given by:

$$p_z(\omega)q_z(\omega) = s_z(\omega)\alpha_z Y,$$

where $s_z(\omega)$ is the within-sector market share of the variety, and

$$P(z) = \left[ \int_{\omega \in \Omega(z)} p_z(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}$$

is the sectoral price index. Lastly, $Y$ is the aggregate household income at home which is comprised of labor income and dividends:

$$E = wL + \Pi,$$

where $\Pi = \int_0^1 \int_{\omega \in M(z)} \pi_z(\omega) d\omega,$

where $w$ is the wage rate at home and $\Pi$ is the aggregate profits of all domestic firms. Hence, we have assumed that home households own home firms, and the firms pay out their full profits as dividends.

The foreign households are symmetric and differ only in the aggregate supply of labor $L^*$, the equilibrium wage rate $w^*$, and the dividend from the ownership of the foreign firms $\Pi^*$. We choose the foreign labor as numeraire and thus normalize the foreign wage $w^* = 1$, so that $w = w/w^*$ equals the home's relative wage. Finally, the consumer expenditure on the varieties in the foreign market parallels that in the home market given in (4).

**Firms** in sector $z$ operate their individual varieties $\omega$ and choose whether to become active in the home and in the foreign markets. The entry cost in the home market is $F$ units of home labor and it is $F^*$ units of foreign labor in the foreign market respectively, independently of the origin of the firm. This symmetry of entry costs is an assumption we adopt following EKS, and it proves useful in our granular setup in Section 3.

There are $M(z)$ home firms in sector $z$, each endowed with a linear technology with individual productivity $\varphi = \varphi(\omega)$ drawn from a Pareto distribution with a cumulative distribution function $G_z(\varphi) = 1 - (\varphi/\varphi(z))^{-\theta}$, and we require $\theta > \sigma - 1$ for stability. In order to produce one unit of its variety, a firm with productivity $\varphi$ requires $1/\varphi$ units of labor. Lastly, international shipments involve an iceberg melting cost $\tau > 1$, so that $\tau$ units of a good must be shipped for one unit to reach the foreign market. Therefore, a home firm with productivity $\varphi$ has a marginal cost $w/\varphi$ for domestic shipments and $\tau w/\varphi$ for international shipments.

Upon entry, the firms monopolistically compete in each market, setting a constant markup $\sigma/(\sigma-1)$ over their marginal costs. This implies that the firm's operating profit in each market equals $1/\sigma$ of its profit.
revenues, and the overall profit of the firm can be written as:

\[
\pi^*_z(\omega) = \left[ \left( \frac{\sigma w / \varphi(\omega)}{\sigma - 1} \right)^{1-\sigma} \alpha_z Y - wF \right]^+ + \left[ \left( \frac{\sigma w / \varphi(\omega)}{\sigma - 1} \right)^{1-\sigma} \alpha_z Y^* - w^*F^* \right]^+ , \tag{7}
\]

where we substituted the markup pricing rule over the marginal cost into the expression for revenues (4), and we use the notation \([x]^+ \equiv \max\{0, x\}\).\(^3\)

Firms with sufficiently high productivities profitably enter the home and the foreign markets respectively, as is conventional in the Melitz model. We denote with \(\varphi_h(z)\) and \(\varphi_f(z)\) the productivity cutoffs for a domestic firm to enter the home and foreign markets respectively in sector \(z\), and provide the closed-form expressions for these cutoffs in Appendix A.1. The foreign firms are symmetric, and we denote with \(\pi^*_z(\omega)\) their profits, and with \(\varphi^*_h(z)\) and \(\varphi^*_f(z)\) their productivity cutoffs for entry into the home and foreign markets respectively.

**Sectoral Equilibrium**  Using (5), (7) and the markup pricing rule, we can calculate the price index in sector \(z\) in the home market (see Appendix A.1):

\[
P(z) = \frac{\sigma}{\sigma - 1} w \left[ \frac{\kappa}{\kappa - 1} \frac{T(z)}{1 - \Phi(z)} \right]^{-1/\theta} \left( \frac{\sigma w F}{\alpha_z Y} \right)^{(\kappa-1)/\theta} , \tag{8}
\]

where we denote \(\kappa \equiv \theta/(\sigma - 1)\) and

\[
\Phi(z) = \frac{\left( \tau w^* \right)^{-\theta} T^*(z)}{w^{-\theta} T(z) + \left( \tau w^* \right)^{-\theta} T^*(z)} = \frac{1}{\frac{\tau w^*}{w} \left( \frac{\tau w^*}{w} \right)^{\theta} \Phi(z)} , \tag{9}
\]

is the foreign share, that is the share of foreign firms in the sales in the domestic market in sector \(z\), a conventional object in the EK analysis. The foreign share increases with the Ricardian productivity advantage \(T^*(z)/T(z)\) of the foreign country, and decreases with the relative wage \(w^*/w\) and with the variable trade costs \(\tau\).\(^4\) The sectoral price in (8) increases in the wage rate and in the relative fixed cost of entry \((wF)/(\alpha_z Y)\), and decreases in sectoral productivity. It also decreases in the home share \([1 - \Phi(z)]\), as is familiar from the Arkolakis, Costinot, and Rodríguez-Clare (2012) analysis.

The definition of the foreign share, and its symmetric counterpart in foreign country \(\Phi^*(z)\), makes it straightforward to calculate sectoral exports of home and foreign countries respectively:

\[
X(z) = \alpha_z \Phi^*(z) Y^* \quad \text{and} \quad X^*(z) = \alpha_z \Phi(z) Y . \tag{10}
\]

In the appendix we also characterize the allocation of aggregate labor supply to sector \(z\), which in the

\(^3\)Specifically, a home firm sets \(p_\omega(z) = \frac{\sigma}{\sigma - 1} \frac{w}{w(z)}\) in the home market, which results in revenues \((p_\omega(z)/P(z))^{1-\sigma} \alpha_z Y\), according to (4), and the operating profits equal fraction \(1/\sigma\) of these revenues due to constant markup pricing. Net profits are operating profits net of the fixed entry cost. Symmetric characterization applies to profits in the foreign market, with the difference that the marginal cost of delivering a good abroad is augmented by iceberg trade cost \(\tau\).

\(^4\)We note that the foreign share in (9) does not depend on the fixed costs since both domestic and foreign firms are assumed to face the same fixed costs of entry into the home market. As a result, fixed costs in this framework have little effect on the key variables which characterize equilibrium, apart from the price indexes \(P(z)\) and \(P^*(z)\).
home market satisfies:

$$wL(z) = \alpha_z Y \left( \frac{\sigma \kappa - 1}{\sigma \kappa} \right) + \frac{\kappa - 1}{\sigma \kappa} \Phi(z) + \alpha_z Y^{*} \frac{\sigma - 1}{\sigma} \Phi^{*}(z). \quad (11)$$

The last term is labor used in production of goods for foreign market, while the first two terms is labor used for production and entry costs in the home market.\(^5\) This fully characterizes the sectoral equilibrium in sector \(z\) given aggregate variables \((w, w^{*}, Y, Y^{*})\) and exogenous parameters, including Ricardian productivity \(T(z)\) and \(T^{*}(z)\).

**General Equilibrium** requires balanced trade and labor market clearing in both countries, which (together with our choice of numeraire \(w^{*} = 1\)) allow us to solve for \((w, w^{*}, Y, Y^{*})\). These three conditions also imply country budget balances (6) by Walras Law. Using (10), we aggregate net exports \(X(z) - X^{*}(z)\) across sectors to arrive to the trade balance condition:

$$Y \int_{0}^{1} \alpha_z \Phi(z)\,dz = Y^{*} \int_{0}^{1} \alpha_z \Phi^{*}(z)\,dz. \quad (12)$$

Recall from (9), that \(\Phi(z)\) and \(\Phi^{*}(z)\) can be written as function of relative wages \(w/w^{*}\) and the exogenous parameters of the model. Therefore, (12) links relative wages with relative incomes, \(Y/Y^{*}\). A higher relative income \(Y/Y^{*}\) implies a larger relative demand for foreign goods, and hence must be met by a higher relative foreign wage \(w^{*}/w\) to ensure trade balance, replicating the standard DFS logic.

Next, aggregating sectoral labor demand in (11) across \(z\) and equalizing it with inelastic labor supply \(L\), we obtain aggregate labor market clearing, which we manipulate in Appendix A.1 to arrive at:

$$wL = \frac{\sigma \kappa - 1}{\sigma \kappa} Y \quad \text{and} \quad w^{*}L^{*} = \frac{\sigma \kappa - 1}{\sigma \kappa} Y^{*}. \quad (13)$$

Therefore, total labor income is a constant share of GDP (total income), with the complementary share coming from firm profits. Taking the ratio of the two equations in (13) results in:

$$\frac{Y}{Y^{*}} = \frac{wL}{w^{*}L^{*}},$$

which together with (12) allows to solve for both relative wage \(w/w^{*}\) and relative incomes \(Y/Y^{*}\), again as in the DFS model. A more productive country supplies more goods on the foreign market, and trade balance requires that this is offset by both higher wages and incomes in this country.\(^6\)

With these simple characterization for the aggregate variables, we briefly returns to the cross-

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\(^{5}\)Specifically, we show that \((\sigma - 1)/\sigma\) of revenues goes to cover variable production labor costs (in the country of production) and fraction \((\kappa - 1)/(\sigma \kappa)\) goes to cover entry labor costs (in the country of entry). Note that the first term in (11) can be decomposed as \((\sigma \kappa - 1)/(\sigma \kappa) = (\sigma - 1)/\sigma + (\kappa - 1)/(\sigma \kappa)\).

\(^{6}\)Indeed, we can combine the two conditions as

$$\frac{wL}{w^{*}L^{*}} = \frac{\int_{0}^{1} \alpha_z \Phi^{*}(z)\,dz}{\int_{0}^{1} \alpha_z \Phi(z)\,dz},$$

and a higher productivity at home increases the right-hand side of this equation, while a higher \(w/w^{*}\) increases the left-hand side and reduces the right-hand side.
sectional sectoral outcomes. Specifically, we characterize the number of firm-entrants and the labor allocations across sectors. We denote the number of firm-entrants with \( N(z) = M(z)(\varphi_h(z)/\varphi(z))^{-\theta}, \) and using (13) and the solution for \( \varphi_h(z) \) in the appendix and express it as:

\[
N(z) = T(z)\varphi_h(z)^{-\theta} = \frac{\kappa - 1}{\sigma\kappa} [1 - \Phi(z)] \frac{\alpha_z Y}{wF}. \tag{14}
\]

Larger markets \( \alpha_z Y \) and markets with smaller fixed costs \( wF \) have more domestic firm-entrants; so do the markets with less foreign competition (smaller foreign share \( \Phi(z) \)). The sectoral labor allocation (11) can be simplified using (13):

\[
L(z) = \alpha_z L \left[ 1 + \frac{\sigma - 1}{\sigma} \frac{\Phi'(z)Y - \Phi(z)Y}{wL} \right], \tag{15}
\]

In the closed economy \( \Phi(z) = \Phi^*(z) = 0 \), and therefore each sector gets \( \alpha_z L \) units of labor, due to the Cobb-Douglas sectoral preference aggregator. The allocation of labor in the open economy additionally depends on the exports and imports in sector \( z \); in particular, large foreign share \( \Phi(z) \) in the domestic market crowds out domestic labor from the sector.

**DFS Limit** The DFS-Melitz benchmark admits as a limiting case the classical DFS formulation when \( \theta, \sigma \to \infty, F \to 0 \), holding constant \( \kappa = \theta/(\sigma - 1) \), \( \sigma F \) and also the following productivity transformations \( a(z) \equiv T(z)^{1/\theta} \) and \( a^*(z) \equiv T^*(z)^{1/\theta} \).\(^7\) Note that our choice of notation \( a(z) \) and \( a^*(z) \) parallels the original notation in DFS, as in the limit of \( \theta \to \infty \), \( a(z)/a^*(z) \) indeed becomes the measure of relative sectoral productivity of the countries. In what follows, we plot variation across sectors against \( a(z)/a^*(z) \) as our measure of comparative advantage, rather than \( \frac{T(z)}{T^*(z)} \), as this makes easier the comparison across calibrations with different values of \( \theta \) and \( \sigma \). In the DFS limit, the foreign shares \( \Phi(z) \) and \( \Phi^*(z) \) in (9) become step functions, defined by two cutoffs \( z, \tilde{z} \in [0, 1] \). Specifically, we rank all sectors \( z \in [0, 1] \) such that \( a(z)/a^*(z) \) is a monotonically increasing function of \( z \), and define the cutoffs to satisfy:

\[
\frac{a(z)}{a^*(z)} = \frac{w}{\tau w^*} \quad \text{and} \quad \frac{a(\tilde{z})}{a^*(\tilde{z})} = \frac{\tau w}{w^*}, \tag{16}
\]

which implies \( z \leq \tilde{z} \). For sectors \( z \in [0,z] \), foreign is the only supplier of the good on both domestic and foreign markets, goods \( z \in (\tilde{z}, \tilde{z}) \) are non-traded and produced in both countries, and for goods \( z \in (\tilde{z}, 1] \) home is the only world supplier.\(^8\)

More generally, we can study the behavior of the DFS-Melitz economy as we vary \( \theta \) and \( \sigma \) holding constant \( \kappa \) and \( \sigma F \). We do this in Figure 1 by plotting the foreign share \( \Phi^*(z) \) (of home firms in foreign market) across sectors against the measure of sectoral comparative advantage \( a(z)/a^*(z) \) for various values of parameter \( \sigma \) and \( \theta \). Specifically, we consider \( \sigma \in \{1.5, 3, 5, 10, \infty\} \) and corresponding values

\(^7\)This last requirement can be ensured by simply holding constant both \( M(z) \) and \( \varphi(z) \) since \( a(z) = M(z)^{1/\theta}\varphi(z) \to \varphi(z) \) in this case, and similarly in foreign.

\(^8\)The only remaining equilibrium condition in the limiting case is a version of (12), after substituting in the limiting versions of expression (13) \( wL = Y \) and \( w^* L^* = Y^* \), which reads \( (wL)/(w^* L^*) = \int_\tilde{z}^\infty \alpha_z dz / \int_\tilde{z}^\infty \alpha_z^* dz \), and together with (16) allows to solve for equilibrium \( (w/w^*, z, \tilde{z}) \).
Ricardian CA, \( a(z)/a^*(z) \)

Note: Symmetric countries with \( \kappa = 1.25 \), \( \tau = 1.5 \), \( \alpha_z \equiv 1 \), \( M(z) \equiv 1 \), \( \varphi(z) \) drawn from a uniform distribution on \([1, 5]\) and \( \varphi^*(z) \equiv 6 - \varphi(z) \). The other parameters are inconsequential when countries are symmetric.

of \( \theta \) holding constant \( \kappa = 1.25 \). Recall from (10) that \( \Phi^*(z) \) is exports of home normalized by the foreign expenditure on good \( z \), and hence can be viewed as the direct consequence of the home’s comparative advantage. We see from Figure 1 that foreign share is indeed increasing in the Ricardian comparative advantage of the sector. When \( \sigma \) is small, the foreign share is rather flat across sectors, and it becomes steeper approaching a step function as \( \sigma \) increases towards infinity.

3 The Granular Model

We now develop the granular version of the model outlined in Section 2. This model is a multi-sector version of Eaton, Kortum, and Sotelo (2012; henceforth EKS), nesting together the fundamental Ricardian comparative advantage across sectors and the granular comparative advantage arising from the individual productivity draws of the firms. Much of the modeling environment is the same as in Section 2, with the expection that the number of firms in each sector is now discrete. Specifically, there are still a continuum of sectors, but the number of varieties within each sector is finite. The preference structure is the same, but instead of the sectoral aggregator (3), we now have:

\[
Q(z) = \left[ \sum_{i=1}^{K(z)} \frac{q_{z,i}^{\sigma-1}}{q_{z,i}^\sigma} \right]^{\frac{\sigma}{\sigma-1}},
\]

(17)

where \( i \) indexes the product varieties, \( K(z) \) is the total number of products offered in sector \( z \) in the home market. The demand \( q_{z,i} \) and market share \( s_{z,i} \) of individual product \( i \) are still given by (4), where the price index is now a discrete version of (5):

\[
P(z) = \left[ \sum_{i=1}^{K(z)} p_{z,i}^{1-\sigma} \right]^{1/(1-\sigma)},
\]

(18)
with $p_{z,i}$ denoting the prices of individual products.

In addition, instead of a fixed pool of entrants, we adopt the EKS assumption that the number of home firm-entrants in sector $z$ is a random variable distributed Poisson with a parameter $M(z)$.\(^9\) The productivity of each entrant, as in the DFS-Melitz model, is independently drawn from a Pareto distribution with a sector-specific lower bound $\varphi(z)$ and a common shape parameter $\theta$. We denote this distribution $G_z(\varphi)$, as in Section 2. EKS show that under these circumstances, the combined productivity parameter $T(z) \equiv M(z)\varphi(z)^\theta$ is still a sufficient statistic for the country-sector productivity advantage, but now determines it only in expectation, as under granularity the realized comparative advantage becomes a random variable. Formally, EKS show that the number of home entrants with productivity equal or above any given level $\varphi$ is a Poisson random variable with parameter $T(z)\varphi^{-\theta}$, increasing in $T(z)$ and decreasing in $\varphi$.\(^10\)

The rest of the modeling environment remains unchanged. As in the continuous model, a domestic firm with productivity $\varphi$ possesses a linear technology in labor ($y = \varphi l$) to produce output of its product. The product can be marketed domestically at no additional variable cost, or it can be exported at an iceberg trade cost $\tau > 1$. Therefore, the marginal cost for the firm of delivering a unit of its product locally is $w/\varphi$ and it is $\tau w/\varphi$ internationally. As before, in order to enter the home (foreign) market, the firms need to incur a fixed cost $F(F^*)$ in units of home (foreign) labor. While firms are large within their industries, they are still small at the level of the whole economy, since unlike EKS we have a continuum of sectors. As a result, they are competitive in the aggregate labor market, and the assumption that they take the wage rate $w$ as given is internally consistent.

Granularity, however, leads to two important differences in the economic environment for firms. The first difference concerns the nature of competition and price setting, and the second difference concerns entry. We discuss each in turn below. In order to do so effectively, we adopt the following notational convention: we let $i \in \{1, 2, \ldots, \tilde{M}(z) + \tilde{M}^*(z)\}$ rank all potential entrants in the home market in the order of increasing marginal costs. Specifically, there are $\tilde{M}(z)$ potential domestic entrants with marginal cost of supplying home market $mc_{z,i} = w/\varphi_{z,i}$ and $\tilde{M}^*(z)$ potential foreign entrants with marginal cost of supplying home market $mc_{z,i} = \tau w^*/\varphi_{z,i}^*$. We assign firm-product indexes $i$ in such a way that $mc_{z,i} \leq mc_{z,i+1}$ for $i \geq 1$. We refer to the firms with lower indexes $i$ as more efficient firms, independently of what is the source of their lower marginal costs of supplying the market.\(^11\) We denote with $K(z) \leq \tilde{M}(z) + \tilde{M}^*(z)$ the total number of firms (home and foreign together) that choose to enter the home market. Lastly, we introduce an indicator variable $\iota_{z,i}$, which equals 1 if the firm (ranked) $i$ in the home market of sector $z$ is of domestic origin and 0 if it is foreign. The foreign market is characterized in a symmetric way.

\(^9\)Formally, the realized number of entrants $\tilde{M}(z)$ has the probability distribution function $P\{\tilde{M}(z) = m\} = e^{-M(z)}M(z)^m/m!$ for $m = 0, 1, 2, \ldots$.

\(^10\)To avoid the issue of separately specifying the probability distribution for $\varphi < \varphi$, it is convenient, as in EKS, to work in the limit of $\varphi(z) \to 0$ and $M(z) \to \infty$, which maintains constant the value of $T(z)$.

\(^11\)Note that index $i$ is not a property of a firm, but rather a property of a firm-market pair. A firm is characterized by its origin and productivity draw $\varphi$, and a given firm in general has different indexes $i$ in the two markets. Mapping $\varphi$ into $i$ is not essential for characterizing equilibrium in this model.
Price setting  Since the number of firms in any given market is finite, the competition between them is oligopolistic, rather than monopolistic. As a result, firms charge variable markups $\mu_{z,i} \geq 1$ over their marginal costs:

$$p_{z,i} = \mu_{z,i}mc_{z,i}. \quad (19)$$

This contrasts with the constant-markup pricing under monopolistic competition in Section 2. We assume that the firms compete in prices. That is, the firms that chose to enter a given market, set prices as an equilibrium outcome of a Bertrand-Nash game by maximizing market-specific profits and taking prices of their competitors as given.

Under these circumstances, the firms adopt the following markup policy:

$$\mu_{z,i} = \frac{\varepsilon_{z,i}}{\varepsilon_{z,i} - 1}, \quad \text{where} \quad \varepsilon_{z,i} \equiv \varepsilon(s_{z,i}) = \sigma(1 - s_{z,i}) + s_{z,i}, \quad s_{z,i} = \left( \frac{p_{z,i}}{P(z)} \right)^{1-\sigma}. \quad (20)$$

The variable $\varepsilon_{z,i} \in [1, \sigma]$ can be thought of as the effective elasticity of demand for the product of the firm, which is no longer constant when firms are large. In particular, the effective elasticity $\varepsilon_{z,i}$ decreases in the market share $s_{z,i}$ of the firm, and with this the markup of the firm increases.

Given the number of entrants $K(z)$, equations (19) and (20), together with the definition of the price index (18), can be solved for the market shares, markups and prices of the entrants in the home market. The solution exists and is unique, although the closed-form expressions are not available. More efficient firms (with lower ranks $i$) charge higher markups, but lower prices, and hence command greater market shares. Lastly, we can write the resulting profits of the firms in the home market in the following way:

$$\Pi_{z,i} = \frac{s_{z,i}}{\varepsilon(s_{z,i})} \alpha_z Y - wF. \quad (21)$$

Operating profits are a fraction $\frac{\mu_{z,i} - 1}{\mu_{z,i}} = \frac{1}{\varepsilon_{z,i}}$ of revenues, and revenues are the firm’s share $s_{z,i}$ of the sectoral expenditure $\alpha_z Y$ in the market. In equilibrium, firms with higher market shares command higher profits.

Entry  An equilibrium of the entry game is achieved when for a subset of firms equilibrium profits given by (21) are non-negative, while for any additional entrant the profits upon entry would be negative. With granular firms, there exists a multiplicity of equilibria in the entry game, since entry of a number of small firms can crowd out entry of a larger firm. To avoid this issue, we consider a sequential entry game in which more efficient firms (with lower ranks $i$) have the priority to decide whether

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12Much of the earlier granularity literature (including Carvalho and Grassi 2014, Di Giovanni and Levchenko 2012) adopts, however, an ad hoc assumption that markups are constant. The quantitative pricing-to-market literature following Atkeson and Burstein (2008) studies oligopolistic competition with variable markups, but adopts competition in quantities, which is qualitatively similar but results in greater markup variability (see detailed discussion in Amiti, Itskhoki, and Konings 2015). We adopt oligopolistic competition in prices, following EKS, which we view as a more natural assumption, and under which the difference from the constant markup environment is less pronounced.

13The operating profit of the firm (gross of fixed costs) is given by $(p_{z,i} - mc_{z,i})q_{z,i}$, and demand is given by $q_{z,i} = \sum_{j \neq i} \frac{p_{z,j}}{p_{z,j} - \mu_{z,j}mc_{z,j}} \alpha_z E$, where we substituted the expression for price index $P(z)$ into (4). Maximizing with respect to own price $p_{z,i}$, given the prices of the competitors $p_{z,j}$ for $j \neq i$, results in (20).
to enter the market. Equilibrium in the sequential entry game is unique and has the following cutoff property: if firm \( i \) enters, then all firms with lower ranks enter as well; if firm \( i' \) does not enter, then all firms with higher ranks do not enter as well. The equilibrium of the entry game is achieved when the profits of the last (\( K \)th) firm are non-negative, but if another (\( K + 1 \))th firm were to enter, its profits would be negative.\(^{14}\)

**General equilibrium** Equilibrium in each sector, including entry, can be fully characterized given aggregate incomes and wage rates \((Y, Y^*, w, w^*)\). We maintain our normalization \(w^* = 1\). The equilibrium values of \((Y, Y^*, w, w^*)\) are determined from the definitions of aggregate income (analogous to (6)) and labor market clearing, which we can write for home as:

\[
Y = wL + \int_0^1 \left[ \sum_{i=1}^{K(z)} t_{z,i} \Pi_{z,i} + \sum_{i=1}^{K^*(z)} (1 - t_{z,i}^*) \Pi_{z,i}^* \right] dz, \tag{22}
\]

\[
wL = \int_0^1 \left[ \alpha_z Y \sum_{i=1}^{K(z)} \frac{s_{z,i}}{\mu_{z,i}} + \alpha_z Y^* \sum_{i=1}^{K^*(z)} (1 - t_{z,i}^*) \frac{s_{z,i}^*}{\mu_{z,i}^*} + K(z)wF \right] dz. \tag{23}
\]

Aggregate home income is the sum of labor income and profits of the domestic firms in both the home and the foreign markets in all sectors. Home labor income is the sum of wages paid to production workers, both for home and foreign markets, and wages paid to workers engaged in entry activities in the home market. Note that there are \( K^*(z) \) entrants into the foreign market of sector \( z \) and \( t_{z,i}^* \) is the indicator for whether the \( i \)th entrant is a foreign firm. Further, \( \alpha_z Y s_{z,i} \) equals firm sales in the home market, and therefore \( \alpha_z Y s_{z,i} / \mu_{z,i} \) equals firm operating costs for home market production, which are entirely spent on labor in the country of production (similar logic applies in the foreign market).

In addition to (22)–(23), there are two symmetric equations for foreign, with one of these equations redundant due to the Walras Law. Substituting in the expressions for firm profits (21) into (22) and its foreign counterpart, it is easy to see that this aggregate equilibrium system is linear in \((Y, Y^*, w, w^*)\), and its coefficients depend on parameters of the model and equilibrium values of \{\( K(z), K^*(z) \)\} and \{\( s_{z,i}, s_{z,i}^* \)\} \(z, i\), since both firm profits and markups are determined by the firm’s market share. In Section 4.1 we describe a simple computational iterative procedure, which simultaneously solves for the sectoral and general equilibrium of the model. Lastly, we note that as the expected number of firm draws \( M(z) \) and \( M^*(z) \) increase, and the entry fixed cost \( F \) and \( F^* \) decrease, the granular model converges to the continuous limit of Section 2.\(^{15}\)

\(^{14}\)In order to characterize the equilibrium in the entry game, it is useful to introduce notation \( \Pi_i^K \) for the profit of firm ranked \( i \) in the home market of sector \( z \) (indicator omitted for brevity) when a total of \( K \) firms have entered. From (21), \( \Pi_i^K \) is an increasing function of market share \( s_i^K \), defined in a similar way. We have the following properties: \( s_i^K > s_{i+1}^K \) and \( s_i^K < s_{i+1}^{K+1} \), and similarly for profits \( \Pi_i^K \). Then entry is complete when \( \Pi_i^K > 0 \) and \( \Pi_{i+1}^{K+1} < 0 \), and this is satisfied for a unique value of \( K \). Alternatively, it is convenient to define the minimum market share consistent with non-negative profits: \( s_\$ = \min \{ s_i \} = wF / (\alpha E) \). Then entry is complete when \( s_\$ \geq s_i \) and \( s_{i+1}^{K+1} < s_\$. Note that if entry were not sequential, then it is possible to have equilibria in which inefficient firms enter and crowd out more efficient firms from entry.

\(^{15}\)In order to maintain the entry productivity cutoffs stable, \( FT(z) \) and \( F^*T^*(z) \) must stay constant as \( T(z) \) and \( T^*(z) \) increase together with the number of draws (see (A3)).
Some analytical properties  While a complete closed-form characterization of equilibrium is infeasible, there exists a useful analytical property in this formulation of the granular model, which was first pointed out in EKS. Given the structure of productivity draws (Poisson-Pareto) and market entry (common market-specific fixed costs), the distribution of potential market share realization for home and foreign firms conditional on entry is the same in any given market. This implies that the respective distributions of markups and profits are also the same. The only thing that differs across home and foreign firms is the expected number of entrants. We expect more home firms in markets with greater home comparative advantage, if home has lower relative wages, and in the home market (due to the trade cost advantage). At the same time, conditional on observing a foreign and a home firm, they have the same expected market share, markup and profits. Formally, one can show that the expected share of foreign entrants in the home market of sector $z$, both in terms of firm count and market sales, is given by $\Phi(z)$, as defined in (9). In particular, the foreign share is given by:

$$E \left\{ \frac{X^*(z)}{\alpha_z Y} \right\} = \Phi(z) = \frac{1}{1 + \left( \frac{\tau w^*}{w} \right)^\theta \frac{\tau(z)}{T^*(z)}},$$

(24)

and thus the granular model has the same sectoral trade shares as the continuous model, but in expectations.\footnote{Sketch of a proof: Recall that the number of home firms with productivity above $\varphi$ is distributed Poisson with parameter $\tau(z)\varphi^{-\theta}$, and symmetrically for foreign firms. Since the sum of Poissons is a Poisson with parameters adding up, the total number of firms with marginal cost below $c$ of delivering the good to the home market is Poisson with parameter $\c^0 \left[ T(z)w^{-\theta} + T^*(z)(\tau w^*)^{-\theta} \right]$. Note that a foreign firm has a cost $\tau w^*/\varphi$ below $c$, if its productivity $\varphi > \tau w^*/c$. Additionally, the expected fraction of foreign firms with cost below $c$ equals $T^*(z)(\tau w^*)^{-\theta} / \left[ T(z)w^{-\theta} + T^*(z)(\tau w^*)^{-\theta} \right] = \Phi(z)$, and this is true independently of the value of $c$. This immediately implies that the foreign fraction of overall entrants is $\Phi(z)$, and the distributions of market shares (and hence markups and prices) within the groups of foreign and home entrants are the same.}

Using this result, we can calculate the aggregate foreign share in the aggregate home economy:

$$\Phi \equiv \int_0^1 \frac{X^*(z)dz}{Y} = \int_0^1 \alpha_z \frac{X^*(z)}{\alpha_z Y} dz = \int_0^1 \alpha_z \Phi(z) dz,$$

(25)

where we used the assumption of the continuum of sectors which ensures that granularity cancels out on average across sectors, and we can replace the realized sectoral foreign shares $X^*(z)/(\alpha_z Y)$ with their expectations $\Phi(z)$. Recall that $\Phi(z)$ depends only on comparative advantage $\frac{T(z)}{T^*(z)}$, given parameters and the aggregate wage ratio $w/w^*$. If we assume that $\frac{T(z)}{T^*(z)}$ are drawn across sectors from some distribution, which is independent from $\{\alpha_z\}$, then we can simplify $\Phi = E_z \Phi(z)$, where the expectation is taken across the draws of $\frac{T(z)}{T^*(z)}$. In general, foreign share $\Phi$ is an increasing function of relative wage $w/w^*$ and a decreasing function of $\tau$. $\Phi$ also decreases with neutral increases in relative home productivity (i.e., when all $\frac{T(z)}{T^*(z)}$ are multiplied by a constant greater than 1).

Using these properties, we can conveniently rewrite the general equilibrium conditions (22)–(23)
as follows (see Appendix A.2):

\[ wL = wFK + Y \frac{1 - \Phi}{\mu} + Y^{*} \frac{\Phi^{*}}{\mu^{*}}, \quad (26) \]

\[ Y = wL + (1 - \Phi) \left[ \frac{Y^{*}}{\mu^{*}} - wFK \right] + \Phi^{*} \left[ \frac{Y^{*}}{\mu^{*}} - w^{*}F^{*}K^{*} \right], \quad (27) \]

where \( K = \int_{0}^{1} K(z)dz \) is the aggregate number of firms in the home economy across all sectors and \( \mu = \left[ \int_{0}^{1} \alpha z \left( \sum_{i=1}^{S(z,i)} \frac{s_{z,i}}{\mu(z,i)} \right) dz \right]^{-1} \) is the average markup rate in the home economy. \( K^{*} \) and \( \mu^{*} \) are defined symmetrically for the foreign economy, and two symmetric equations apply in the foreign economy. Note that \( [Y^{*} - wFK] \) in (27) are net profits in the domestic market, with fraction \( (1 - \Phi) \) of these profits accruing to domestic firms, and therefore the right-hand side of (27) characterizes the aggregate income (labor plus profits) equal to GDP \( Y \) of the domestic economy. Further, \( Y(1 - \Phi)/\mu \) equals the variable cost expenditure (on labor) of the domestic firms for production and sales in the domestic market, and therefore the right-hand side of (26) equals the total expenditure of all firms on home labor equal to labor income \( wL \).

Equation (26) is the granular version of (13) in the continuous model. Combining it with (27) to solve out \( wL \), we obtain the trade balance condition:

\[ \Phi \left[ Y - wFK \right] = \Phi^{*} \left[ Y^{*} - w^{*}F^{*}K^{*} \right], \quad (28) \]

which is the granular version of (12) in the continuous model. Note that \( [Y - wFK] \) are the sales in the home market net of the fixed cost of entry, and fraction \( \Phi \) of these net sales constitutes foreign income from sales in the home market. The right-hand side of (28) is the symmetric income of home firms from sales abroad, ensuring trade balance. The equations above form the general equilibrium system and allow us to solve for \( (Y, Y^{*}, w/w^{*}) \). With this we complete the description of the granular economy and turn to characterizing its quantitative properties.

### 4 Quantitative Properties of the Granular Model

In this section we develop an SMM (simulated method of moments) estimation procedure for the granular model and apply it to the French firm-level data. Importantly, the estimation procedure takes into account the general equilibrium restrictions imposed by the model. We then use the estimated model to study the quantitative properties of the granular mechanism, including the model fit and the implied role of granularity for the country’s comparative advantage. We close this section with counterfactuals, in particular those in which we study the role of the individual firms in shaping the comparative advantage of a country.

**Data** We use a dataset of French firms (BRN), which reports information on the balance sheets of the firms declared for tax purposes. All firms with revenues over 730,000 euros are included. It reports in particular information on domestic and export sales, and 4-digit industry classification, at the firm
level. We use 2005 as our reference year for calibration. We match this data with international trade data from Comtrade, to get the aggregate imports and exports of France in each industry. The industry classification used in the French data is the French NAF (based on European NACE classification), whereas the trade data uses ISIC rev3. We convert the French data into the ISIC rev3 classification (using the crosswalk between NACE and ISIC, available from UNstats). This leaves us with 117 4-digit manufacturing sectors.

4.1 Estimation and model fit

Estimation procedure We estimate a two-country model with France as home and the rest-of-the-world (ROW) as foreign. We assume that $F^* = F$, yet home and foreign differ in labor endowments $L$ and $L^*$ and the country-sector productivities $\{T(z)\}$ and $\{T^*(z)\}$, which together determine the relative size of the home and foreign markets. Given the Cobb-Douglas preference structure, all variables of interest in the model scale with the common level of productivity, so we can normalize $T^*(z) \equiv 1$ without loss of generality.\footnote{Note that if productivity in sector $z$ of both countries doubles, the quantity in this sector doubles and the price halves without any effect on market shares within or across sectors.} Furthermore, the model scales with $L$ as long as we keep $L^*$ and $L_F$ constant. In other words, $L$ simply determines the units of labor, and hence we normalize $L = 100$. The model, therefore, has four parameters $\{\sigma, \theta, \tau, F\}$ common across sectors and countries, in addition to sectoral Cobb-Douglas shares $\{\alpha_z\}_z$ common across countries, as well as Ricardian sectoral comparative advantage parameters $\{\frac{T(z)}{T^*(z)}\}_z$ and relative country labor endowment $\frac{L}{L^*}$.

We next parametrize the distribution of sector-level comparative advantage parameters $\frac{T(z)}{T^*(z)}$, to reduce the dimensionality of the problem. Hanson, Lind, and Muendler (2015) show evidence that in the cross section of countries, the distribution of measured comparative advantage is well-approximated by a log-normal distribution. We thus make the parameteric assumption that the distribution of the sector-level comparative advantage parameter is lognormal with parameters $(\mu_T, \sigma_T^2)$, i.e. $\log \left( \frac{T(z)}{T^*(z)} \right) \sim \mathcal{N}(\mu_T, \sigma_T^2)$\footnote{Note here that the comparative advantage distribution analyzed in Hanson, Lind, and Muendler (2015) as being well-approximated by a log-normal lumps together what we aim to distinguish here, i.e. the firm-level granular comparative advantage and the fundamental sector-level one. In our model, the distribution of the observed comparative advantage is an outcome of the combination of the distributions of fundamental and granular comparative advantages. It could therefore be that an alternative distribution for the fundamental comparative advantage provides an even better fit to the data. We will explore alternative distributions in further versions of the draft.}. Therefore, the parameter vector we need to estimate is $\Theta \equiv (\sigma, \theta, \tau, F, \mu_T, \sigma_T)$, along with Cobb-Douglas shares and relative labor endowment.

We proceed in two steps. In the first step, we read the Cobb-Douglas share from the data as:

$$\alpha_z = \frac{D(z) + M(z)}{\sum_{z'} D(z') + M(z')} ,$$

where $D(z)$ is domestic sales in sector $z$ and $M(z)$ is total French imports in sector $z$. We also calibrate $w/w^*$ using the ratio of the wage in France to the average wage of its trading partners weighted by trade volume. We find that French wages are 13% above the average wage of its trade partners, and therefore we set $w/w^* = 1.13$, which provides a general equilibrium restriction on other parameters.

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as we discuss below.

In the second step, we follow a simulated method of moments (SMM) procedure to estimate the remaining six parameters (along with \( \frac{L}{L^*} \)), using the French firm-level data across 117 manufacturing industries.

1. For a given \((\mu_T, \sigma_T)\), we get a vector of values for the relative sectoral productivity \(\frac{T(z)}{T^*(z)}\) corresponding to 117 quantiles of the lognormal distribution with parameters \((\mu_T, \sigma_T)\).

2. We use 4 replications of the French sectors,\(^{19}\) where in each of the 4 replications the Cobb-Douglas shares are reshuffled randomly across the sectoral productivity draws to ensure that sectoral shares and productivities are not correlated.\(^{20}\) The simulated economy has therefore \(117 \times 4 = 468\) sectors.

3. For a given \(\theta\) and \(T(z)\) in each sector \(z\), we draw the productivities of potential entrants in a way consistent with the structural assumptions of the model. We follow EKS in using rank-order statistics for the Pareto distribution to directly draw the productivity of the most productive firm, which follows an extreme value distribution, and each firm thereafter, with spacings following an exponential distribution. We draw enough “shadow firms” in each sector to ensure that the least productive ones never enter the market.\(^{21}\)

4. We implement the following fixed point procedure:

   (i) given calibrated value of \(\frac{w}{w^*}\), take an initial guess for \((Y, Y^*)\), pinning down the initial general equilibrium vector.

   (ii) given general equilibrium variables, solve for sectoral equilibrium in each sector and each country, characterizing \(\{K(z), K^*(z), \Phi(z), \Phi^*(z)\}_z, \{s_{z,i}, s^*_i\}_{z,i}\) and thus \(\Phi, \Phi^*, \mu, \mu^*\), as described in Section 3.

   (iii) use the two general equilibrium equations (26) and (28), along with normalization \(L = 100\) and without the foreign counterpart of (26), to solve for \((Y, Y^*)\) given the values of \(\Phi, \Phi^*, \mu, \mu^*\) calculated in (ii). Note that equations (26) and (28) are linear in \((Y, Y^*)\).

   We also recover the value of \(L^*\) consistent with general equilibrium (in particular, with the calibrated value of \(w/w^*\)) from the foreign counterpart to equations (26).

   (iv) We update the values of \((Y, Y^*)\) taking a half step and loop over until convergence.

   (v) Upon convergence, we compute the moments \(\mathcal{M}(\Theta)\) and compare it with the moments from the data.

Given this mapping from parameters of the model to the moments, we choose \(\Theta\) to minimize the

\(^{19}\)The empirical Cobb-Douglas shares are divided by 4 to ensure that they sum to 1.

\(^{20}\)Without such replication, the model is sensitive to the randomness in the match between sectoral shares and productivities, while with four replications the effect of this randomness is already negligible. In future drafts we will explore further the role of the fat tail in the joint distribution of sectoral productivities and Cobb-Douglas shares.

\(^{21}\)Specifically, we use a set of 20,000 firm draws by sector for France and 80,000 for Foreign. For smaller sectors (in terms of Cobb-Douglas shares), we use 8,000 and 20,000 draws instead, to economize on the computing requirement. We check that these constraints do not bind around estimated parameter values, i.e. that not all shadow firms enter.
distance between $\mathcal{M}(\Theta)$ and the empirical moments, using the unweighted quadratic objective.\textsuperscript{22} We discuss the moment selection below. We estimate the model for the two cases: (a) the constant markup case, in which we replace $\mu(s_{z,i})$ in (20) with $\bar{\mu} \equiv \sigma/(\sigma - 1)$, corresponding to the monopolistic competition markup; and (b) the variable markup case, in which we use (20), corresponding to the oligopolistic in prices (Bertrand). We find these two cases to produce very similar results, as these model predicts little markup variation under Bertrand competition (see discussion in Amiti, Itskhoki, and Konings 2015).\textsuperscript{23} The variable markup estimation is numerically more time-intensive, and we discuss some of the details of simulation in this case in Appendix B.

Estimated parameters  We start with the summary of the estimated parameters, reported in the top panel of Table 1 for both (a) the constant markup and (b) the variable markup versions of the model. Both specifications broadly agree on the parameter values, with a slight difference in the estimated parameters of the Ricardian comparative advantage $(\mu_T, \sigma_T)$. Both specification imply a within-sector elasticity of substitution of about 5, consistent with conventional estimates in the literature at the 4-digit level (see Broda and Weinstein 2006). The two specifications also agree on the value of $\kappa = \theta/(\sigma - 1)$ of around 1, which determines the shape of the within-sector sales distribution, the so-called Zipf’s law (see Gabaix 2009). The estimated iceberg trade costs are around 1.4, again broadly in line with the estimated in the literature (see Anderson and van Wincoop 2004).

<table>
<thead>
<tr>
<th>Parameter/variable</th>
<th>Model (a)</th>
<th>Model (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>4.339</td>
<td>3.751</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>5.298</td>
<td>4.927</td>
</tr>
<tr>
<td>$\kappa = \theta/(\sigma - 1)$</td>
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<tr>
<td>$\tau$</td>
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<td>1.410</td>
</tr>
<tr>
<td>$F$</td>
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<td>0.430 $\cdot 10^{-5}$</td>
</tr>
<tr>
<td>$\sigma_T$</td>
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<td>1.284</td>
</tr>
<tr>
<td>$\mu_T$</td>
<td>-0.581</td>
<td>-0.625</td>
</tr>
<tr>
<td>$w/w^*$</td>
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<td>1.130</td>
</tr>
<tr>
<td>$L^*/L$</td>
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</tr>
<tr>
<td>$Y^*/Y$</td>
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<td>3.808</td>
</tr>
<tr>
<td>$1 - wL/Y$</td>
<td>0.170</td>
<td>0.132</td>
</tr>
</tbody>
</table>

Note: Model (a) refers to constant markup and Model (b) refers to variable markup. $1 - wL/Y$ is the profit share in GDP. Standard errors TBD, to be computed by bootstrap.

Lastly, the estimated mean $\mu_T$ of the relative productivity distribution $\log \frac{T(z)}{T^c(z)}$ is around $-0.6$. This distribution also exhibits substantial dispersion across sectors, with a standard deviation that is twice as high as the absolute value of the mean. This creates a lot of room for fundamental Ricardian

\textsuperscript{22}We first find the global minima of the objective on a sparse grid in the parameter space. Then, we start the local numerical optimization routine from 15 different points on the grid with minimal value of the objective function. The convergence to the same parameter vector from these 15 points suggests that we are indeed finding a global minimum.

\textsuperscript{23}In the future versions of the draft we will also consider the case of Cournot competition, which results in greater markup variation, and has been commonly used in quantitative models of pricing-to-market following Atkeson and Burstein (2008).
comparative advantage across sectors, as we further discuss below. The negative mean of the relative productivity distribution implies that home is on average less productive than foreign. Recall however that $T(z) = M(z)\varphi(z)^0$, so that $T(z)$ reflects both the average productivity and the number of draws. Since France is smaller than the rest of the world, it is natural to expect that ROW benefits from substantially more draws, which may offset a possible productivity advantage of France reflected in greater $\varphi(z)$ across sectors. Therefore, we judge the relevance of this estimated parameter based on its implications for the equilibrium values of labor income and GDP for France and the ROW, which we discuss next.

The lower panel of Table 1 reports the general equilibrium variables consistent with the estimated models. In both cases, we have calibrated the wage in France to be 13% above the wage in the ROW, consistent with the data. The estimated constant-markup model implies that France is nearly 6 times smaller than the ROW in terms of labor endowment and 4.3 times smaller in terms of GDP. These factors in the variable-markup model are 4.2 and 3.8 respectively. In reality, France accounts for only 3.7% of world GDP and less than 1% of world population. Also France is about 5 times smaller that the European Union alone. This is however not the right interpretation to give to the values of these variables in the model. The model is a simplification in that it allows for only one foreign country with a single value of iceberg trade costs $\tau$. One therefore needs to discount the countries in the ROW by their proximity to France as reflected in the trade costs. The countries that are close to France should be discounted less, while the countries that do not trade much with France should be discounted more heavily. Therefore, we view the estimated values of relative GDPs and labor endowments as plausible, and leave the fuller analysis of a multi-country world economy for future work. Lastly, we point out that the estimated model implies a profit share of GDP of about 15%, again in line with conventional values from NIPA. Interestingly, the estimation procedure does not target this moment directly or indirectly.

**Moment fit** Table 2 summarizes the moments used in the SMM estimation procedure, reporting the values of the moments in the French firm-level data, as well as in the two versions of the estimated model: (a) constant markups and (b) variable markups. We have chosen 14 moments to match in estimation, as we discuss in detail below, and overall both versions of the model have a very similar quality of fit.

Moments 1–3 in Table 2 characterize the distribution of the log number of firms across manufacturing sectors in France: average across sectors, standard deviation, and ratio of mean to the median in order to capture skewness in this distribution. In a typical (median) sector in France, there are about 270 firms with a substantial variation across sectors. Both versions of the model are able to match very closely the distribution of number of firms across sectors. We illustrate this in panel (a) of Figure 2, which contrasts the distribution of the log number of firms by sector in the constant-markup model and in the data. Appendix Figure A1 plots the same distributions from the variable-markup model and shows that it results in no noticeable differences across the two specifications. We see from Figure 2a that the model distribution lies on top of the empirical distribution, with both distributions exhibiting a smooth bell-shape. The ability of the model to replicate closely the distribution of the number of firms across sectors is crucial for the analysis of granularity.
<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model (a)</th>
<th>Model (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Log number of French firms, mean</td>
<td>5.601</td>
<td>5.569</td>
<td>5.603</td>
</tr>
<tr>
<td>2 Log number of French firms, std. dev.</td>
<td>1.445</td>
<td>1.351</td>
<td>1.262</td>
</tr>
<tr>
<td>3 Log number of French firms, mean/median</td>
<td>0.948</td>
<td>0.984</td>
<td>0.989</td>
</tr>
<tr>
<td>4 Domestic market share of the largest firm</td>
<td>0.198</td>
<td>0.266</td>
<td>0.265</td>
</tr>
<tr>
<td>5 Cum. dom. market share of three largest firms</td>
<td>0.358</td>
<td>0.413</td>
<td>0.426</td>
</tr>
<tr>
<td>6 Pareto shape of domestic sales, mean</td>
<td>0.897</td>
<td>1.005</td>
<td>0.962</td>
</tr>
<tr>
<td>7 Pareto shape of domestic sales, std. dev.</td>
<td>0.230</td>
<td>0.175</td>
<td>0.154</td>
</tr>
<tr>
<td>8 Exports relative to domestic sales, mean</td>
<td>0.631</td>
<td>0.678</td>
<td>0.684</td>
</tr>
<tr>
<td>9 Exports relative to domestic sales, std. dev.</td>
<td>0.690</td>
<td>0.581</td>
<td>0.574</td>
</tr>
<tr>
<td>10 Exports relative to domestic sales, mean/median</td>
<td>1.393</td>
<td>1.462</td>
<td>1.456</td>
</tr>
<tr>
<td>11 Fraction of sectors with exports &gt; domestic sales</td>
<td>0.154</td>
<td>0.188</td>
<td>0.212</td>
</tr>
<tr>
<td>12 Aggregate Import share</td>
<td>0.320</td>
<td>0.468</td>
<td>0.476</td>
</tr>
<tr>
<td>13 Correlation top market share and export share</td>
<td>0.269</td>
<td>0.365</td>
<td>0.284</td>
</tr>
<tr>
<td>14 Correlation top-3 market share and export share</td>
<td>0.314</td>
<td>0.323</td>
<td>0.257</td>
</tr>
</tbody>
</table>

Note: Model (a) refers to constant markups and Model (b) refers to variable markups. See the description of the moments in the text.

The next two moments (4 and 5) in Table 2 are the market share of the largest firm on the domestic market and the market share of the top three firms on the domestic market, measured relative to the sales of other French firms (that is, not taking into account imports in the total market size). In the data, these statistics are equal to 20% and 36% on average across sectors, while both versions of the model somewhat overpredict these values (equal to 27% and 41% respectively in the constant-markup version of the model). Panel (b) of Figure 2 plots the distribution of the top-French-firm market share across sectors in the constant-markup model and in the data. We see that the fit of the model is rather decent, especially in the right tail of the distribution across sectors (i.e., for sectors with the largest firm capturing 20% or more of the market), while the model underpredicts the number of sectors with the largest firm accounting for less than 20% of sales. In addition, and quite intuitively, both the data and the model exhibit a negative relationship between the size of the largest firm and the number of firms in the market, with the respective elasticity equal to $-0.09$ in the data and $-0.07$ in the model.

The next two moments (6 and 7) characterize the distribution of the within-sector domestic sales across firms. Specifically, they report the cross-sector mean and standard deviation of the estimated Pareto shape parameters of the within-sector domestic sales distributions, estimated using a log (rank$-0.5$) regression for firms above the median size (see Gabaix and Ibragimov 2011). On average across sectors, the distributions of domestic sales exhibit Zipf’s law, i.e. the estimated Pareto shape parameter is close to 1, however there is a substantial variation across sectors, with some sectors having shape parameters as high as 1.5 and others as low as 0.5. Panel (c) of Figure 2 illustrates the model fit of the whole cross-sectoral distribution of Pareto shapes. We observe that the model somewhat overpredicts...
the mean and underpredicts the variance of this distribution.

Despite the lack of a perfect fit, panels (a)–(c) of Figure 2 illustrate that the model is capable of capturing the salient features of the cross-sector variation in the number of firms, top-firm market shares and the fatness of tails of the within-sector sales distributions. This is particularly striking given the tightness of the model parameterization, which features only six parameters common across sectors and countries. The only parameters that are allowed to vary across sectors are the Cobb-Douglas sectoral shares. In Appendix Figure A2 we simulate the estimated model in which we shut down variation in the Cobb-Douglas shares keeping all other parameters constant. We see that the variation in Cobb-Douglas shares contributes very little to variation in the Pareto shapes and market shares of the top firm across sectors, and nearly all of the model-generated variation in these moments comes from the granularity of sectoral draws. A continuous model, therefore, would not be able to reproduce this...
variation without assuming rich heterogeneity in sectoral parameters (e.g., \(\sigma, \theta, F\) or \(\tau\)). For the variation in the number of firms across sectors, the heterogeneous Cobb-Douglas shares and the granularity contribute roughly equally.

The next 7 moments (8 through 15) in Table 2 characterize international trade. Moments 8–11 characterize the ratio of sectoral export sales relative to domestic sales, and its variation across sectors. Export sales in a typical French sector equal 63% of the domestic sales, with substantial variation across sectors, in particular with over 15% of sectors featuring export sales in excess of domestic sales. Panel (d) of Figure 2 shows distribution of export-to-domestic sales ratio across sectors, in the data and in the estimated model. The model predicts accurately the mean of this distribution, yet it slightly underpredicts the variance and overpredicts the fraction of sectors in which this ratio exceeds one. At the same time, the model overpredicts the aggregate import share in French manufacturing (the ratio of imports to total domestic sales), which equals 32% in the data and 47% in the model.

In addition to these moments, we decompose the cross-sectoral variation in export sales into the extensive (number of exporters) and intensive (average exports per exporter) margins, and find that in the model the intensive margin explains about one-fourth of the cross-sectoral variation in exports, against two-thirds in the data. This notable discrepancy between the models and the data has been emphasized by Fernandes, Klenow, Meleshchuk, Pierola, and Rodríguez-Clare (2015). Importantly, however, the continuous model attributes nothing to the intensive margin (i.e., its contribution is nil, while the extensive margin drives all of the variation in export sales), and thus the granular model offers an important improvement. In Appendix C.3 we additionally discuss the decomposition of the variation in domestic sales into intensive and extensive margins.

The last two moments in Table 2 characterize the correlation between sectoral export intensity (i.e., the ratio of exports to domestic sales) and the relative size of the largest firm (resp. three largest firms) on the domestic market. These moments discipline the granularity force versus the fundamental comparative advantage: if sectoral export intensity is strongly correlated with the within-sector skewness of the domestic sales distribution, this is suggestive of the importance of the granular force. The models are broadly in line with the empirical correlations between these variables. Overall, we conclude that the model has a reasonable fit and is admissible for further quantitative explorations.

**Identification**  We close this section with a brief discussion of identification in the estimation procedure. Appendix B.1 provides more formal arguments, while here we keep the discussion at an intuitive level. With 14 moments reported in Table 2, the model with six parameters is overidentified, and variation in any of the parameters tends to affect all moments simultaneously. Nonetheless, we can identify which moments are more directly linked to specific parameters, and thus what ensures identification.

First, it is easy to see that the fixed cost of entry \(F\) largely determines the average number of firms that enter a sector. Second, the average Pareto shape parameter across sectors is pinned down, following the predictions of the theory, by \(\kappa = \theta / (\sigma - 1)\). Third, given relative wages \(w_w / w_\sigma\), the aggregate import share \(\Phi\) is decreasing in both \(\mu_T\) and \(\tau\), as explained in Section 3, and thus this moment provides a joint restriction on these two parameters. Once these two parameters are known, we can calculate the import share in the foreign economy, \(\Phi^*\), and use it together with the trade balance condition (28) to
discipline $Y/Y^*$. Indeed, a lower $\Phi^*$ relative to $\Phi$ requires a higher $Y^*/Y$ in order to balance trade. A similar logic applies to the fraction of French sectors with export sales exceeding domestic sales, which would altogether be impossible if the foreign market were no greater than the home market. The mean and the variance of the export sales distribution (relative to domestic sales) also discipline the values of $\theta$ and $\tau$, as $\theta$ is the elasticity of sectoral trade flows with respect to trade costs (as originally shown by Chaney 2008). Lastly, the value of $\sigma_T$ determines the strength of the fundamental Ricardian comparative advantage: greater $\sigma_T$ reduces the correlation between sectoral exports and the size of the largest home firm, with this correlation turning negative as $\sigma_T$ increases further. Since this correlation is positive in the data, it puts a limit on the value of $\sigma_T$.

4.2 Results and counterfactuals

We now use the estimated model to study the properties of trade flows, focusing in particular on the relative roles of the fundamental and granular comparative advantage. We use here the constant-markup version of the model, given its more modest computational intensity and the fact that the variable-markup version appears to be quantitatively very similar, as discussed above.

**Fundamental versus granular trade**  We denote with $\Lambda^*(z) \equiv X^*(z)/(\alpha_z Y^*)$ the realized export share in sector $z$, that is the ratio of home exports to the total size of the foreign market. Realized trade in the granular model is a random variable with the mean given by the expected foreign share, $\mathbb{E}\{\Lambda^*(z)\} = \Phi^*(z)$, which is increasing in fundamental comparative advantage $T(z)/T^*(z)$ (CA for brief), as we discussed in Section 3. Further analytical properties of the distribution of $\Lambda^*(z)$ are unavailable, and thus we characterize it quantitatively using our estimated model. We are also interested in the properties of gross sectoral exports, $X^*(z) = \alpha_z \Lambda^*(z) Y^*$.

First, to get the quantitative feel for the model, we simulate one realization of the economy with 468 sectors using the estimated values of the parameters (see Section 4.1), and plot the results in Figure 3. Specifically, the left panel of the figure plots realized trade shares $\Lambda^*(z)$ against the sector’s fundamental comparative advantage $T(z)/T^*(z)$, which each blue point corresponding to an individual sector $z$. We see that, indeed, $\Lambda^*(z)$ is distributed around $\Phi^*(z)$, depicted in the figure with a red line, with a substantial variation in $\Lambda^*(z)/\Phi^*(z)$ reflecting the strength of the granular force in the model. In the continuous limit of the model, all blue points in the figure must converge to the red line, corresponding to Figure 1.

It turns out to be interesting to swap the axis of the plot, as we do in the right panel of Figure 3. What we observe is that the sectors with the largest realizations of trade shares $\Lambda^*(z)$ are not necessarily the sectors with the largest fundamental comparative advantage. In other words, if we are trying to predict $T(z)/T^*(z)$ based on observed trade share $\Lambda^*(z)$, very large realizations of $\Lambda^*(z)$ imply rather an extreme realizations of the granular residual $\Lambda^*(z)/\Phi^*(z)$ rather than extreme relations of $T(z)/T^*(z)$ and $\Phi^*(z)$. We further explore this intriguing finding in Figure 4, which describes the results across 200 simulations of the estimated economy.

In the left panel of Figure 4, we plot the moments (mean and the percentiles) of the $\Lambda^*(z)$ distribution conditional on $T(z)/T^*(z)$. The figure confirms our observations for a single realization of the economy:
Figure 3: Comparative advantage and trade flows: one realization of the economy

Note: the left panel plots realized sectoral export shares $\Lambda^*(z) = \frac{X^*(z)}{\alpha_z Y^*}$ against the fundamental comparative advantage $T(z)/T^*(z)$, while the right panel reverses the axes and plots $\log T(z)/T^*(z)$ against $\Lambda^*(z) = \frac{X^*(z)}{\alpha_z Y^*}$. Each blue point corresponds to a sector $z$, and the figure shows one realization (draw) of the estimated economy with 468 sectors. The red line in both plots corresponds to the expected export share of the sector $\Phi^*(z) = E \Lambda^*(z)$ as a function of $T(z)/T^*(z)$, which equals the realized trade flows in the continuous limit of the model.

Figure 4: Comparative advantage and trade flows: distribution across realizations

Note: 200 simulations of the estimated economy. LEFT PANEL x-axis: fundamental comparative advantage $T(z)/T^*(z)$; y-axis: export share, $\Lambda^*(z) = \frac{X^*(z)}{\alpha_z Y^*}$. Blue line plots average export share for a sector across simulations as a function of $T(z)/T^*(z)$ of the sector; Pink (Green) lines plot 25th and 75th percentiles across simulations (5th and 95th percentiles respectively) of export shares for a given sector as a function of $T(z)/T^*(z)$ of the sector. RIGHTS PANEL reverses the axes in the left panel. Blue line plots the average $T(z)/T^*(z)$ corresponding to a given observed value of export share $\Lambda^*(z) = \frac{X^*(z)}{\alpha_z Y^*}$ across simulations; Pink line plots the 10th and 90th percentiles across simulations of $T(z)/T^*(z)$ corresponding to a given observed value of export share. Red lines in both panels plot the same curve as Figure 3: expected export share $\Phi^*(z)$ as a function of $T(z)/T^*(z)$.

for a given value of fundamental comparative advantage and associated expected trade share $\Phi^*(z)$, there exists a substantial variation in the realized trade share accounted for by the granular residual $\Lambda^*(z)/\Phi^*(z)$. In other words, there is a wide distribution of possible export-intensity outcomes, for the same level of fundamental comparative advantage. Next, the right panel shows a more striking pattern,
by plotting the distribution of the associated fundamental comparative advantage $T(z)^*(z)$ conditional on the realized traded share $\Lambda^*(z)$. To facilitate this discussion, we need a small definitional digression:

• Define $\hat{A}(z) \equiv \mathcal{F}^{-1}(\Lambda^*(z))$, where $\mathcal{F}(\cdot)$ is the function that maps $T(z)/T^*(z)$ into the expected trade share $\Phi^*(z)$ given the other parameters of the model as a function of $T(z)/T^*(z)$. That is:

$$\Phi^*(z) = \mathcal{F}\left(\frac{T(z)}{T^*(z)}; \Theta\right), \quad \frac{T(z)}{T^*(z)} = \mathcal{F}^{-1}(\Phi^*(z); \Theta) \quad \text{and} \quad \hat{A}(z) = \mathcal{F}^{-1}(\Lambda^*(z); \Theta).$$

**In the continuous model, $\Lambda^*(z) = \Phi^*(z)$, and therefore $\hat{A}(z) = T(z)/T^*(z)$, or in words the observed trade flows allow to directly recover the fundamental comparative advantage. In the granular model, $\hat{A}(z) \equiv \mathcal{F}^{-1}(\Lambda^*(z))$ corresponds to the red curve in Figures 3 and 4.**

The intriguing pattern in the right panel of Figure 4 is that the distribution of fundamental comparative advantage $T(z)/T^*(z)$ conditional on realized trade share $\Lambda^*(z)$ is not symmetric around $\hat{A}(z)$, and the mean of this distribution (the blue line) diverges from $\hat{A}(z)$ (the red line), as $\Lambda^*(z)$ increases. That is, for large realized values of export intensity in a given sector, the actual fundamental comparative advantage level of the sector is systematically lower than what the continuous model would predict. Furthermore, this discrepancy increases with the export intensity of the sector. More specifically, the level of comparative advantage predicted by the continuous model $\hat{A}(z)$ exceeds the 90th percentile (the upper pink line) of the actual distribution of fundamental comparative advantage for very large realizations of $\Lambda^*(z)$, so that the bias from using a continuous model to infer fundamental comparative advantage can be substantial, especially for the big export sectors. This confirms the intuitive observation derived from the right panel of Figure 4, in which we plotted one realization of the granular economy. The "granular residual" $\Lambda^*(z)/\Phi^*(z)$ becomes particularly important to understand the export behavior of sectors with large realized export intensity $\Lambda^*(z)$.

**Trade flows decomposition** We next decompose, in the estimated model, the variation in the log export share $\lambda^*(z) \equiv \log \Lambda^*(z)$ and in the log exports $\log X^*(z) = \log \alpha_z + \lambda^*(z) + \log Y^*$ across sectors into the contributions of the various components as follows:

$$\var(\lambda^*(z)) = \var(\phi^*(z)) + \var(\lambda^*(z) - \phi^*(z)) + 2\cov(\phi^*(z), \lambda^*(z) - \phi^*(z)), \quad (29)$$
$$\var(\log X^*(z)) = \var(\log \alpha_z) + \var(\lambda^*(z)) + 2\cov(\alpha_z, \lambda^*(z)), \quad (30)$$

where $\phi^*(z) \equiv \log \Phi^*(z)$. The first decomposition captures the relative contributions of the fundamental and granular forces to the variation in trade shares across sectors. If we, however, are interested in the gross trade flows, the second decomposition additionally splits the variation in total exports across sectors into the size of the sector (captured by the Cobb-Douglas share) and the export share in the sector. As an alternative to variance decompositions, we also consider regression-based decompositions,
Table 3: Decomposition of variation in trade flows

<table>
<thead>
<tr>
<th>Decomposition of $\text{var}(\lambda^*(z)) = 2.086$</th>
<th>$\phi^*(z)$</th>
<th>$\lambda^<em>(z) - \phi^</em>(z)$</th>
<th>Covar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance-based</td>
<td>0.658</td>
<td>0.323</td>
<td>0.009</td>
</tr>
<tr>
<td>Regression-based</td>
<td>0.667</td>
<td>0.333</td>
<td>—</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decomposition of $\text{var}(\log X^*(z)) = 3.613$</th>
<th>$\log \alpha_z$</th>
<th>$\lambda^*(z)$</th>
<th>Covar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance-based</td>
<td>0.433</td>
<td>0.577</td>
<td>-0.005</td>
</tr>
<tr>
<td>Regression-based</td>
<td>0.428</td>
<td>0.572</td>
<td>—</td>
</tr>
</tbody>
</table>

Note: in the first column of top panel, the variance-based decomposition report \( \text{var}(\phi^*)/\text{var}(\lambda^*) \) and the regression-based decomposition reports \( \text{cov}(\phi^*, \lambda^*)/\text{var}(\lambda^*) \), and by analogy for other entries in the table, with a residual 'Covar' term.

which conveniently feature no covariance terms.\(^{25}\)

Table 3 reports the results from these decompositions. We observe that almost 60% of variation in gross export flows across sectors is due to the variation in trade shares \( \lambda^*(z) \) and 40% is due to the size of the sectors. In turn, the variation in trade shares is two-thirds due to the fundamental comparative advantage and one-third due to granular residual. The covariance terms in the variance decompositions (29)–(30) end up being negligible, and by consequence the variance-based and the regression based decompositions yield identical results. We conclude that granularity plays an important role in shaping sectoral trade flows in our estimated model, accounting for 20% of gross trade flows (one third of 60%) and 33% of the variation in export shares across sectors, which is a natural scale-free measure of comparative advantage.

**Properties of granular trade flows** We now study the covariation properties of the granular component of trade flows with other variables in the estimated model. The top panel of Table 4 reports regressions with log granular residual, \( \lambda^*(z) - \phi^*(z) = \log \Lambda^*(z) - \log \Phi^*(z) \), as the dependent variable. The first two columns show that \( \lambda^*(z) \) is correlated with neither fundamental comparative advantage measured as \( \log(T(z)/T^*(z)) \), nor with the size of the sector measured as \( \alpha_z \): the coefficients in these regressions are close to zero and the R-squared is virtually zero. Therefore, the estimated model suggests that granularity of trade is not systematically related with the size of the sector or the sector’s fundamental comparative advantage. This in turn implies that granular trade flows are closely correlated with the realized export share \( \lambda^*(z) \), a property that is also apparent from the right panels of Figures 3 and 4.

Columns 3 and 4 show the projections of the granular residual on two candidate measures of within-

\(^{25}\)For concreteness, consider the regression counterpart to variance decomposition (29):

\[
\text{var}(\lambda^*(z)) = \text{cov}(\phi^*(z), \lambda^*(z)) + \text{cov}(\lambda^*(z) - \phi^*(z), \lambda^*(z)),
\]

and it can be conveniently obtained by regressing in turn \( \phi^*(z) \) and \( \lambda^*(z) - \phi^*(z) \) on a constant and \( \lambda^*(z) \). The granular contribution in this case is measured as \( \text{cov}(\lambda^*(z) - \phi^*(z), \lambda^*(z))/\text{var}(\lambda^*(z)) \), as opposed to \( \text{var}(\lambda^*(z) - \phi^*(z))/\text{var}(\lambda^*(z)) \) in the variance decomposition (29). When the covariance term in (29) is close to zero, the two decompositions give similar results, as is the case in Table 3.
sector granularity—the market share of the top French firm in the domestic market (i.e., relative to total domestic sales by domestic firms) and the ratio of the market shares of the top and the median French firms on the domestic market (top-median ratio, for brevity). Both measures are likely to be large in the granular sectors, where a handful of firms accounts for a disproportionate share of sales. We see that indeed both variables correlate strongly with the granular trade residual, explaining 25% and 31% of variation in $\lambda^*(z) - \phi^*(z)$ respectively. The top-median ratio is a better proxy for granularity as it is not sensitive to the size of the sector and the number of entrants.

In the lower panel of Table 4 we report similar regressions for the log trade share, $\lambda^*(z) = \log \Lambda^*(z)$. This variable now has an elasticity of nearly one with respect to log expected trade share $\phi^*(z)$ and R-squared of 68%, consistent with the variance decomposition in Table 3. Our two measures of granularity stay significant for the log export share as well, but the R-squared drops to 5% for the top French firm market share, yet it stays at 31% for the top-median ratio. This suggests, in relationship to the variance decomposition in Table 3, that the top-median ratio is nearly a perfect proxy for granularity, as the granularity’s contribution to the variation in export shares is 33%. When we combine all three variables in one regression in column 4, the resulting R-squared is 81%, implying that most of the variation in export shares is already captured by these variables, parroting respectively for the fundamental and the granular comparative advantage.

**Predicting realized trade flows** Next we study the predictive ability for realized export intensity, defined as $\frac{X(z)}{D(z)}$ with $X(z)$ total sectoral exports and $D(z)$ total domestic sales, of various variables characterizing the within-sector sales distribution across firms, both with and without controls for fundamental comparative advantage (a variable that can only be proxied for in the data; see Chor 2010). The results are summarized in Table 5 and largely replicate our findings in Table 4. In the estimated model, the fundamental comparative advantage variable explains two thirds of the variation in the realized export intensity of sectors, and including additionally the variables characterizing the within-sector firm sales distribution props up the R-squared from 67% to 85%. The top-median ratio alone allows to achieve an 84% R-squared, with other variables such as the number of entrants and the median market share having a modest explanatory power. When fundamental comparative advantage is not controlled for, the overall explanatory power of the within-sector firm-level variables reaches 29%, with the top-median ratio alone accounting for 27% of variation in export shares. These results emphasize the important predictive ability for trade flows of the measures of skewness of within-sector sales distribution in granular models.

**Trade effects of individual firm exit** We consider a counterfactual in which the largest firm in a randomly chosen sector fails for an exogenous reason and has to exit the industry. We study the impact of this shock for the aggregate export performance of the sector, which is absent in a model with continuum of firms. In particular, in Figure 5, we plot the drop of the sector in the percentile of the export intensity distribution as a result of the exit of the largest firm, as a function of the sector’s granular residual $\lambda^*(z) - \phi^*(z)$. We observe that in the non-granular sectors, the effects are small, with the relative standing of the sector in terms of export intensity barely changing. At the same time, a
Table 4: Properties of the granular residual

<table>
<thead>
<tr>
<th>Dep. var.: $\lambda^<em>(z) - \phi^</em>(z)$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log(T(z)/T^*(z))$</td>
<td>0.014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_z \cdot 468$</td>
<td>0.016</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top French firm market share</td>
<td>1.946</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log top-median market share ratio</td>
<td></td>
<td>0.288</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.00</td>
<td>0.00</td>
<td>0.25</td>
<td>0.31</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dep. var.: $\lambda^*(z)$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log(T(z)/T^*(z))$</td>
<td>1.014</td>
<td>0.934</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top French firm market share</td>
<td>1.547</td>
<td>1.266</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log top-median market share ratio</td>
<td>0.505</td>
<td>0.242</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.68</td>
<td>0.05</td>
<td>0.31</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Note: Table reports regression coefficients and R-squared. Dependent variable is log granular residual in the top panel and the log export share in the bottom panel. Coefficient estimates have virtually no standard errors. Top French firm market share is the ratio of the largest French firm domestic sales to the total domestic sales of all French firms. Log top-median market share ratio is the ratio of domestic sales of the largest French firm to the median French firm. The Cobb-Douglas share is multiplied by the number of sectors (468) so that the average value of the variable is equal to one to ease the interpretation of the coefficient.

Table 5: Predicting export intensity

<table>
<thead>
<tr>
<th>Dep. var.: $\frac{X(z)}{D(z)}$</th>
<th>A. Controlling for fundamental comparative advantage</th>
<th>B. Without control for fundamental comparative advantage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\log(\frac{T(z)}{T^*(z)})$</td>
<td>0.494</td>
<td>0.483</td>
</tr>
<tr>
<td>Top French firm market share</td>
<td>1.415</td>
<td></td>
</tr>
<tr>
<td>Top 2–3 French firms market share</td>
<td>$-0.323$</td>
<td></td>
</tr>
<tr>
<td>Log top-median market share ratio</td>
<td>0.241</td>
<td>0.325</td>
</tr>
<tr>
<td>Median French firm market share</td>
<td>0.089</td>
<td>1.410</td>
</tr>
<tr>
<td>Log number of French firms</td>
<td>0.072</td>
<td>$-0.036$</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.67</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Note: Table reports regression coefficients and R-squared of log realized export intensity $\frac{X(z)}{D(z)}$ on variables characterizing within-sector sales distribution across firms and proxying for granularity. $X(z)$ is total sectoral exports and $D(z)$ total domestic sales. The top panel additionally controls for fundamental comparative advantage $\log(\frac{T(z)}{T^*(z)})$, while the bottom panel does not. Coefficient estimates have virtually no standard errors. Top French firm market share is the ratio of the largest French firm domestic sales to the total domestic sales of all French firms. Top 2–3 French firms market share is the cumulative market share of the second and third largest French firms. Log top-median market share ratio is the ratio of domestic sales of the largest French firm to the median French firm.
death of the sector leader in more granular sectors has much more dramatic consequences, with sectors sometimes falling from the top-10% of exporters to the bottom-10%. [TO BE COMPLETED]

**Trade openness and granularity** Does trade openness enhance the granularity effects? Intuitively, in the closed economy granularity has no effect on the cross-sector allocation of labor and sales under Cobb-Douglas preferences. This is no longer the case in an open economy, where granularity can shape the allocation of resources across sectors. The natural question to ask is whether granular forces become more pronounced in the more open economies. [TO BE COMPLETED]

**Identifying granular sectors** We close the section with identification of specific granular sectors in the data. So far our estimation procedure recovered only the parameters of the data generating process in the granular model, yet it was silent about whether export performance of the specific French sectors was shaped by granularity. We develop a statistical test, based on the structure of the model, which allows us to identify specific sectors in which the granular forces were particularly pronounced in shaping the trade flows. [TO BE COMPLETED]

## 5 Firm Dynamics and Comparative Advantage

In this section, we develop a dynamic extension for our granular model of trade in order to study the dynamics of comparative advantage. In a recent paper, **Hanson, Lind, and Muendler (2015; HLM henceforth)** emphasize two striking facts:\footnote{Hanson, Lind, and Muendler (2015) use data which splits all products into 135 sectors.}

(i) the hyper-specialization of exports, where a single top sector accounts on average across countries for 21% of a country’s total exports and three top sectors account on average for 45% of total exports; and
(ii) the high turnover of comparative advantage, where a sector in the top-5% of exporting sectors has only a 41% chance of staying in the top-5% of exporting sectors two decades later.

A granular model is well suited to address these facts. Firm dynamics, which is straightforward to discipline empirically using data on the evolution of firm market shares, alters the granular comparative advantage over time, and therefore can explain, at least partially, the high turnover of exporting sectors, replicating simultaneously the hyper-specialization of countries at any given point in time. This is the issue we now address quantitatively.

We introduce dynamics by assuming that the productivity of each firm evolves over time according to a random growth process. Consistent with our earlier assumptions, there is a given pool of firms in each sector drawn in the beginning of times from a Poisson distribution. These firms never leave and new firms never enter, yet the firms decide each period whether to be active or inactive. The decision to become active involves a per-period fixed cost, but no sunk costs, making this choice static. The productivity of every firm, whether active or inactive, evolves over time. In any given time period, firms play a static entry game, as described in Section 3, given the realized productivity distribution in that period. We choose the productivity process to replicate the same cross-sectional Pareto distribution of firm productivities as in the static model. Therefore, our dynamic economy is a sequence of static economies, with the only intertemporal link through the persistent firm-level productivity process. We discipline this process using our French data on the evolution of firm market shares over time.

The specific firm productivity process we adopt is a geometric random walk with drift and a reflecting barrier at the lower bound for productivity draws. Formally, productivity of firm \( i \) in sector \( z \) evolves according to:

\[
\varphi_{z,i,t} = \varphi_z + |\mu + \varphi_{z,i,t-1} + \nu \varepsilon_{z,i,t} - \varphi_z|,
\]

where \( \varphi_z \) is the reflecting barrier, \( \mu = -\theta \nu^2 / 2 < 0 \) is the negative drift term, \( \nu \) is the standard deviation of innovation, and \( \varepsilon_{z,i,t} \) is iid standard normal. If the initial productivity draws \( \varphi_{z,i,0} \) came from a Pareto distribution with the shape parameter \( \theta \) and the lower bound \( \varphi_z \), the cross-sectional distribution of productivities \( \varphi_{z,i,t} \) stays stable over time (see Gabaix 2009).
We normalize $\varphi_z$ to a small number which we ensure never binds in the entry game,\textsuperscript{27} and otherwise this parameter is inconsequential in the model. Given the restriction on the value of $\mu$, this leaves us with only one parameter to estimate—the size of the productivity innovation $\nu$. Importantly, this parameter does not affect any of the static moments used in the simulation, and therefore can be disciplined in a separable way from the rest of the model. Specifically, we choose to match two moments of the firm market share evolution—the short-run persistence of market share measured (inversely) by $\text{var}(\Delta s_{z,i,t+1})$ and the long-run persistence measured by $\text{corr}(s_{z,i,t+10}, s_{z,i,t})$, where one period correspond to a year. We choose the value $\nu = 0.086$ to match these two moments, as we show in the top panel of Table 6. The model is broadly consistent with both the short-run and long-run persistence of market shares, and we choose to err on the conservative side and adopt the parameter value which results in somewhat greater persistence of market shares than in the data.

With this, we simulate the dynamic equilibrium path of the economy over a long period of time, and use the simulated firm data to calculate the HLM moments, as reported in the bottom panel of Table 6. The model matches the hyper-specialization of the top-1% of sectors and somewhat understates the hyper-specialization of the top-3% of sectors (34% versus 45%). The model also accounts for a large fraction of the turnover in comparative advantage sectors: over 20 years, the probability for a sector to remain among top-5% exporting sectors is 64%, against 41% in the data.\textsuperscript{28} If we were to recalibrate the model to match the short-run (rather than long-run) persistence of market shares by setting $\nu = 0.012$ (last column of Table 6), we find that the probability of staying in top-5% of exporting sectors falls from 64% to 57%, very close to the empirical persistence of comparative advantage for developed countries. Therefore, a simple dynamic granular model accounts jointly for the hyper-specialization of industries and a substantial turnover in comparative advantage across industries over time. The turnover implied by the model is arguably a lower bound, since the model features no evolution over time in the fundamental comparative advantage $T(z) \neq T(z)$. Nonetheless, we conclude that a granular model offers an important quantitatively-disciplined source of dynamics for comparative advantage of countries.

6 Conclusion

[TO BE COMPLETED]
A Theory Appendix

A.1 Derivations and proofs for the continuous model of Section 2

Using (7) to write the zero profit conditions for entry into the home and foreign market, which define \( \varphi_h(z) \) and \( \varphi_f(z) \) respectively, we have:

\[
\begin{align*}
\pi_z(\omega) &= wF \left[ \left( \frac{\varphi(\omega)}{\varphi_h(z)} \right)^{\sigma-1} - 1 \right]^+ + w^*F^* \left[ \left( \frac{\varphi(\omega)}{\varphi_f(z)} \right)^{\sigma-1} - 1 \right]^+ , \\
\varphi_h(z) &= \frac{\sigma}{\sigma-1} \frac{w}{P(z)} \left( \frac{\sigma wF}{\alpha_2 Y} \right)^{1/(\sigma-1)} , \\
\varphi_f(z) &= \frac{\sigma}{\sigma-1} \tau w^* \left( \frac{\sigma w^*F^*}{\alpha_2 Y} \right)^{1/(\sigma-1)} .
\end{align*}
\]

(A1)

Using (5) and the markup pricing rule

\[
p_z(\omega) = \begin{cases} \\
\frac{\sigma}{\sigma - 1} \frac{w}{\varphi_z(\omega)}, & \text{if variety } \omega \text{ is by home firm,} \\
\frac{\sigma}{\sigma - 1} \frac{\tau w^*}{\varphi_z^+(\omega)}, & \text{otherwise,}
\end{cases}
\]

(A2)

we can calculate the price index in sector \( z \) in the home market:

\[
P(z) = \frac{\sigma w}{\sigma - 1} \left( \frac{\kappa T(z)}{\kappa - 1} \right)^{1/\sigma} \varphi_h(z)^{\kappa-1} \left[ 1 + \left( \frac{\tau w^*}{w} \right)^{1-\sigma} \frac{T^*(z)}{T(z)} \left( \frac{\varphi_h^*(z)}{\varphi_h(z)} \right)^{\sigma-1-\theta} \right]^{1/\sigma}
\]

where we denote \( \kappa \equiv \theta / (\sigma - 1) > 1 \). Combining this expression with the cutoff condition in (A1), we have:

\[
\begin{align*}
\varphi_h(z) &= \left( \frac{\kappa T(z)}{\kappa - 1} \right)^{\frac{1}{\sigma}} \left[ 1 + \left( \frac{w}{\tau w^*} \right)^{1-\sigma} \frac{T^*(z)}{T(z)} \right]^{\frac{1}{\sigma}} \left( \frac{\sigma wF}{\alpha_2 Y} \right)^{\frac{1}{\sigma}}, \\
\varphi_h^*(z) &= \tau w^* \varphi_h(z), \\
\varphi_f(z) &= \tau \frac{P(z)}{P^*(z)} \left( \frac{w^*F^*/Y^*}{wF/Y} \right)^{\frac{1}{\sigma-1}} \varphi_h(z), \\
P(z) &= \frac{\sigma}{\sigma - 1} \frac{w}{\varphi_h(z)} \left( \frac{\sigma wF}{\alpha_2 Y} \right)^{\frac{1}{\sigma-1}}
\end{align*}
\]

(A3)

Symmetric equation hold for cutoffs and the price index in the foreign market. Therefore, we have fully solved for industry equilibrium in sector \( z \) given aggregate variables \( (w, w^*, Y, Y^*) \) and exogenous parameters, including Ricardian productivity \( T(z) \) and \( T^*(z) \).

We define \( \Phi(z) = \int_{\omega \in M^*(z)} \eta_z(\omega) \, d\omega \) and use equations (4), (A1) and (A3) to calculate this integral.

In Table A1 we provide income accounting in the domestic market for sector \( z \), i.e. the split of the total sectoral expenditure \( \alpha_2 E \) into the incomes of various factors at home and in foreign. As just discussed, the share \( \Phi(z) \) of the expenditure goes to foreign firms, while the complementary share \( 1 - \Phi(z) \) goes to home firms. The revenues of the firms are split between production labor and operating profits, with the latter in turn split between net profits and fixed costs of entry (payed to labor in the
Table A1: Revenue split in sector $z$ in the home market (shares)

<table>
<thead>
<tr>
<th></th>
<th>Home expenditure on Home varieties</th>
<th>Home expenditure on Foreign varieties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$= [1 - \Phi(z)]\alpha_z Y$</td>
<td>$= \Phi(z)\alpha_z Y$</td>
</tr>
<tr>
<td><strong>Home Income</strong></td>
<td>$\sigma - 1$</td>
<td>$\sigma - 1$</td>
</tr>
<tr>
<td><strong>Foreign Income</strong></td>
<td>$-\frac{1}{\sigma}$</td>
<td>$\frac{1}{\sigma}$</td>
</tr>
<tr>
<td><strong>Production (Labor)</strong></td>
<td>$\frac{\kappa - 1}{\sigma\kappa}$</td>
<td>$\frac{\kappa - 1}{\sigma\kappa}$</td>
</tr>
<tr>
<td><strong>Entry Cost (Labor)</strong></td>
<td>$\frac{1}{\sigma\kappa}$</td>
<td>$\frac{1}{\sigma\kappa}$</td>
</tr>
<tr>
<td><strong>Profits</strong></td>
<td>$\frac{\kappa - 1}{\sigma\kappa}$</td>
<td>$\frac{1}{\sigma\kappa}$</td>
</tr>
</tbody>
</table>

Note: the total expenditure on sector $z$ varieties at home equals $\alpha_z Y$, and the table reports the split of this expenditure into incomes of various factors in home at in foreign.

country of entry). Given constant markup pricing and linear production, fraction $(\sigma - 1)/\sigma$ of sales covers the operating costs (payment to production labor), while the remaining fraction $1/\sigma$ is operating profits. With Pareto productivity distribution and given that the marginal firm just breaks even paying the entry cost $wF$, we calculate that fraction $1/\kappa$ of operating profits goes to cover fixed costs and the remaining share $(\kappa - 1)/\kappa$ is net profits of all firms in the sector, where recall that $\kappa \equiv \theta/(\sigma - 1) > 1$. This applies to both domestic and foreign firms, however for foreign firms the operating costs and net profits are income of the foreign factors, while entry costs go to local labor. These results are summarized in Table A1 for the domestic market, and mirror-image results hold in the foreign market.

Next, we use Table A1 to calculate the aggregate demand for home labor (in value terms):\(^{29}\)

$$wL = E \int_0^1 \alpha_z \left[ \frac{\sigma - 1}{\sigma\kappa} (1 - \Phi(z)) + \frac{\kappa - 1}{\sigma\kappa} \Phi(z) \right] dz + E^* \int_0^1 \alpha_z \frac{\sigma - 1}{\sigma} \Phi^*(z) dz,$$

where the first integral is the aggregate demand for domestic labor used to supply goods in the domestic market (both in production and in entry), while the second integral is the aggregate demand for domestic labor used in production for exports. Since $L$ units of labor are supplied inelastically, this equation can be also viewed as the labor market clearing condition. Manipulating the equation above and using the trade balance condition (12), we can rewrite the labor market clearing condition as:

$$wL = \frac{\sigma\kappa - 1}{\sigma\kappa} E,$$

and a symmetric condition holds for foreign. Condition (13) states that labor income in the model is a constant share of total income, with the complementary share coming from firm profits.

\(^{29}\)Since all workers are homogenous and command the same wage in the labor market, the total value spent on labor expenditure $wL$ is sufficient, given the wage rate $w$, to recover the total demand for units of labor, $L$. In this model, it is more convenient to do the calculations in the value terms, however, one can arrive to the same aggregate labor demand by aggregating the physical demand for labor of individual firms.
A.2 Derivations and proofs for the granular model of Section 3

We can rewrite (22)-(23) and their foreign counterparts as:

\[ wL = wFK + Y \frac{S}{\mu_H} + Y^* \frac{1 - S^*}{\mu_H^*}, \]  \hspace{1cm} (A5)  

\[ w^* L^* = w^* F^* K^* + Y^* \frac{S^*}{\mu_F} + Y \frac{1 - S}{\mu_F^*}, \]  \hspace{1cm} (A6)  

\[ Y = wL + \left[ YS\frac{\mu_H - 1}{\mu_H} - wF\chi K \right] + \left[ Y^*(1 - S^*)\frac{\mu_H^* - 1}{\mu_H^*} - w^* F^*(1 - \chi^*)K^* \right], \]  \hspace{1cm} (A7)  

\[ Y^* = w^* L^* + \left[ Y^* S^* \frac{\mu_F^* - 1}{\mu_F^*} - w^* F^* \chi^* K^* \right] + \left[ Y(1 - S) \frac{\mu_F - 1}{\mu_F} - wF(1 - \chi)K \right], \]  \hspace{1cm} (A8)  

\[ 0 = [Y^*(1 - S^*) - w^* F^*(1 - \chi^*)K^*] - [Y(1 - S) - wF(1 - \chi)K], \]  \hspace{1cm} (A9)  

where (A9) is the sum of (A5) and (A7), which is also the sum of (A6) and (A8), and where we used the following notation:

\[ K = \int_0^1 K_z dz, \]  

\[ \chi = \frac{1}{K} \int_0^1 \left( \sum_{i=1}^{K_z} \zeta_{z,i}^* \right) dz, \]  

\[ S = \int_0^1 \alpha_z \left( \sum_{i=1}^{K_z} \zeta_{z,i} S_{z,i}^* \right) dz, \]  

\[ \mu_H = \left[ \frac{1}{S} \int_0^1 \alpha_z \left( \sum_{i=1}^{K_z} \zeta_{z,i} \frac{\zeta_{z,i}^* s_{z,i}^*}{\mu(z_{z,i}^*)} \right) dz \right]^{-1}, \]  

\[ \mu_H^* = \left[ \frac{1}{S^*} \int_0^1 \alpha_z \left( \sum_{i=1}^{K_z} \zeta_{z,i} \frac{\zeta_{z,i} s_{z,i}^*}{\mu(z_{z,i}^*)} \right) dz \right]^{-1}, \]  

\[ \mu_F = \left[ \frac{1}{S} \int_0^1 \alpha_z \left( \sum_{i=1}^{K_z} \zeta_{z,i} \frac{\zeta_{z,i}^* s_{z,i}^*}{\mu(z_{z,i}^*)} \right) dz \right]^{-1}, \]  

\[ \mu_F^* = \left[ \frac{1}{S^*} \int_0^1 \alpha_z \left( \sum_{i=1}^{K_z} \zeta_{z,i} \frac{\zeta_{z,i} s_{z,i}^*}{\mu(z_{z,i}^*)} \right) dz \right]^{-1}. \]  

Lemma A1 \( \chi = S = 1 - \Phi, \chi^* = S^* = 1 - \Phi^*, \mu_H = \mu_F = \mu \) and \( \mu_H^* = \mu_F^* = \mu^* \).

This lemma follows from the property of EKS model described in footnote 16.

B Estimation Appendix

Variable markup case is identical to the constant markup case except for the computation of markups and prices given general equilibrium variables (see Section 4.1). To make the procedure faster, we use the following approximation:

- We show that the number of firms in the variable markup case is larger than in the constant markup case (higher markups of large firms resulting in more entry by small firms), given the same parameters of the model. Therefore, we start with the set of firms that would enter in the constant markup equilibrium.
- For these firms, we compute the Atkeson-Burstein markup distribution and market share distribution, and the corresponding price index.

For all less productive firms, we approximate their markup at the constant markup level and check whether with this markup and this price index they would enter or not.
• This is the equilibrium. It turns out to overpredict entry somewhat, but yields a good approximation overall. We compare the outcome (moments) under these procedure and under the full procedure, and the difference is negligible.

B.1 Identification

[TO BE COMPLETED]

B.2 Shutting down granularity

1. By increase θ.
2. By reducing σ.
3. By increase T(z) and reducing F, keeping FT(z) constant, to increase the median number of firms per sector from 270 to 2,700.
4. By replacing Pareto with log-normal with the same variance.
C Additional Results

C.1 Variable-markup model

(a) Log number of French firms

(b) Top French-firm market share

(c) Pareto shapes of sales distributions

(d) Exports relative to domestic sales

Figure A1: Distributions across sectors: the variable-markup model

Note: the figure replicates figure 2 adding the results from the variable-markup model.
C.2 Model without variation in the Cobb-Douglas shares

(a) Log number of French firms

(b) Top French-firm market share

(c) Pareto shapes of sales distributions

(d) Exports relative to domestic sales

Figure A2: Distributions across sectors: model without variation in Cobb-Douglas shares

Note: the figure replicates figure 2 adding the results from a model (with constant markups) holding all parameters the same apart from the Cobb-Douglas shares which are made uniform across sectors (and equal to $1/468$ where 468 is the number of simulated sectors).
C.3 Extensive versus intensive margin

(a) Extensive margin in the data

(b) Extensive margin: model vs data

Figure A3: Extensive versus intensive margin of cross-sector variation in sales

Note: the figures plot the extensive margin—variation across sectors in the log number of firms against the total sectoral sales.
References


