Problem Set 4 – Due Feb. 25

1. Fill in the remaining steps in the proof of the mixture space theorem.
   a. Show that as constructed, $F$ represents $\succ$.
   b. Show that $F$ is affine in the mixing operation, i.e., that
      \[
      F(h_a(\pi, \rho)) = aF(\pi) + (1 - a)F(\rho)
      \]
      for all $\pi, \rho \in \Pi$ and for all $a \in [0, 1]$. Here, it is enough to show at least one case.
   c. Show that $F$ is unique up to positive affine transformations.

2. Kreps Chp. 7 problem 3

3. Kreps Chp. 7 problem 5

4. Problem 1 from PS1, revisited. Let $\Delta_S([0, 1000])$ denote the set of simple lotteries on $[0, 1000]$. All lotteries below will be elements of $\Delta_S([0, 1000])$. If $p^n$ is a sequence in $\Delta_S([0, 1000])$, say that $p^n \to p$ if $E_{p^n}[v] \to E_p[v]$ for any bounded, continuous function $v : [0, 1000] \to \mathbb{R}$ (that is, $p^n \to p$ in the weak topology). Replace Axiom 4 with the following:
   \begin{enumerate}
   \item[Axiom 4'] If $p \succ q$ and $p^n \to p$, then there exists $N$ sufficiently large such that $p^n \succ q$ for all $n \geq N$. Similarly, if $q \succ p$ and $p^n \to p$, there exists $N$ sufficiently large such that $q \succ p^n$ for all $n \geq N$.
   \end{enumerate}
   a. Show that Axiom 4’ implies Axiom 4, but they are not equivalent.
   b. With Axiom 4’ in place of Axiom 4, do problem 1 of PS 1.

   See also Kreps’s discussion pp. 65-68.

5. Tell me a little about your interests. Something between a paragraph and a page is fine.