University of California – Berkeley
Department of Economics
Game Theory in the Social Sciences
(ECON C110 | POLSCI C135)
Fall 2023

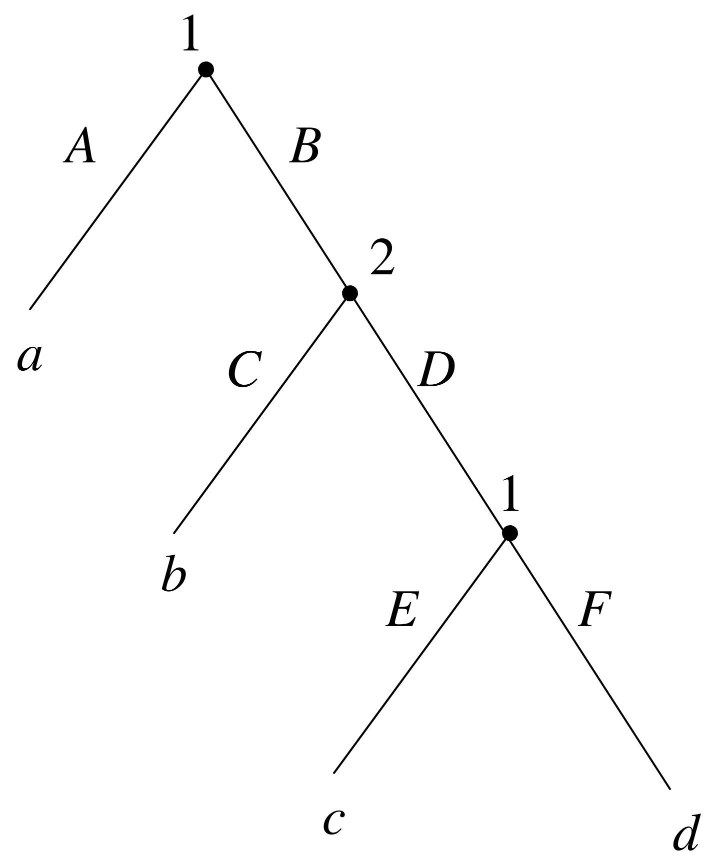
Lecture V
Extensive games with perfect information

Oct 19, 2023

Extensive games with perfect information

- The model of a strategic suppresses the sequential structure of decision making.
  - All players simultaneously choose their plan of action once and for all.

- The model of an extensive game, by contrast, describes the sequential structure of decision-making explicitly.
  - In an extensive game of perfect information all players are fully informed about all previous actions.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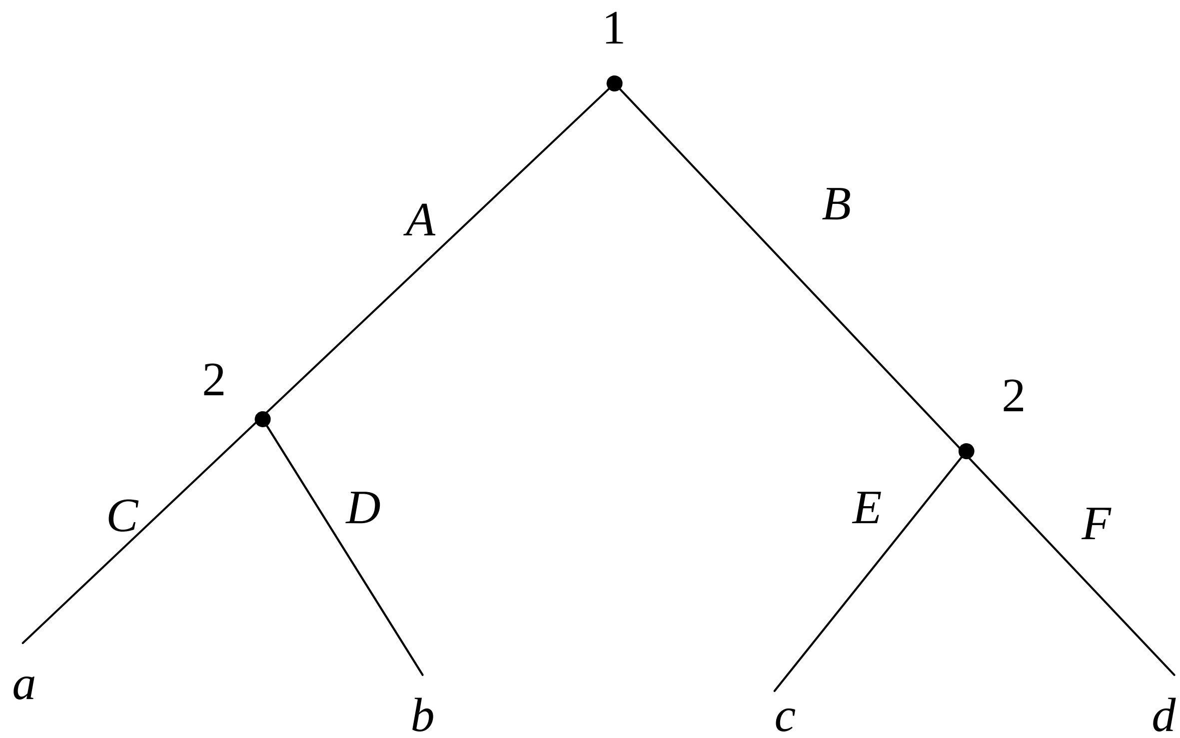
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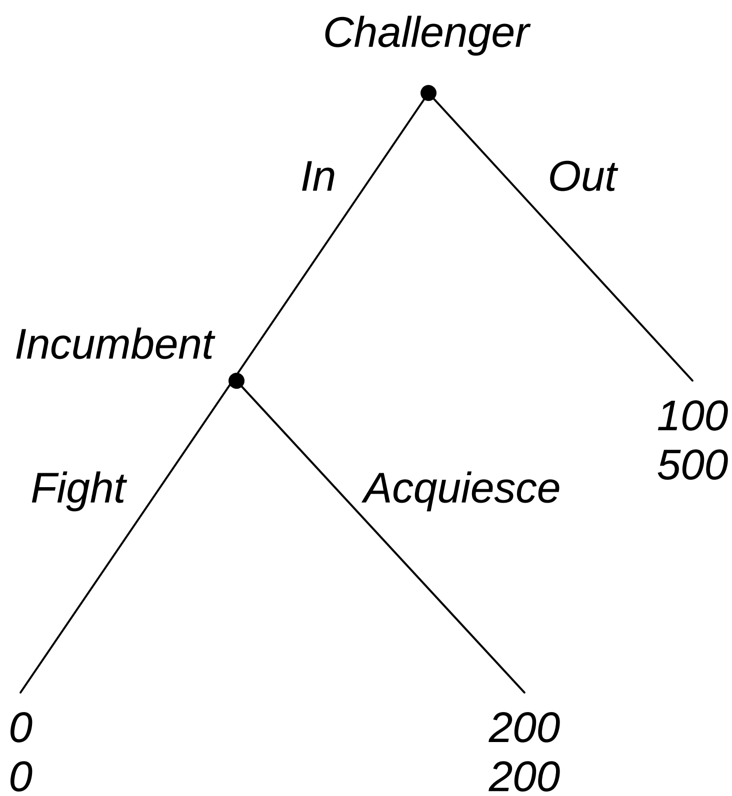
Subgame perfect equilibrium

- The notion of Nash equilibrium ignores the sequential structure of the game.

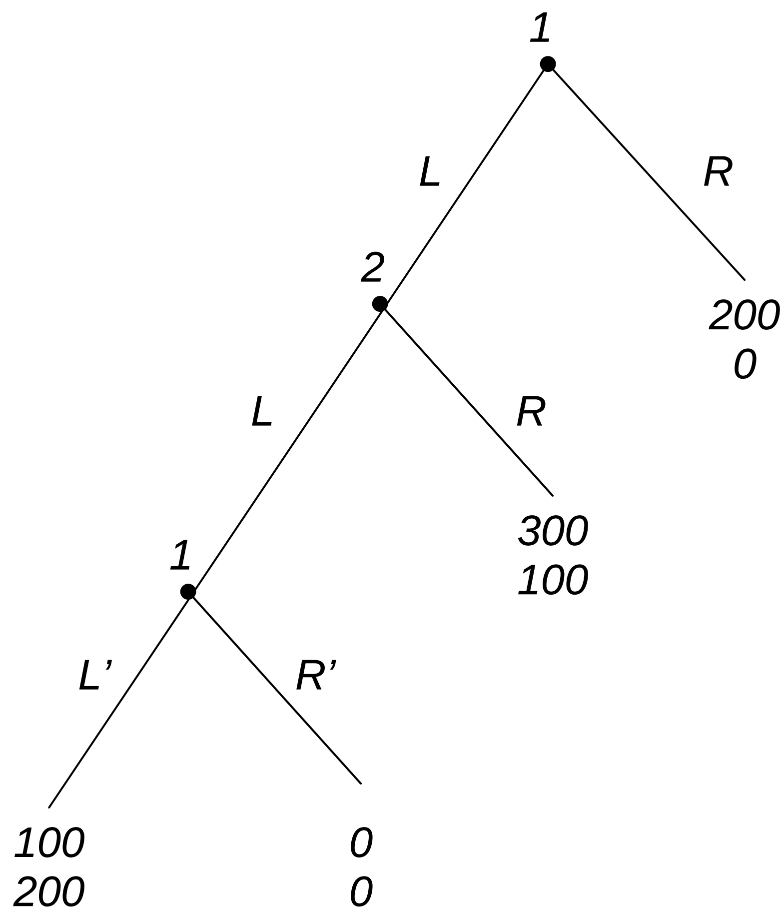
- Consequently, the steady state to which a Nash Equilibrium corresponds may not be robust.

- A *subgame perfect equilibrium* is an action profile that induces a Nash equilibrium in every *subgame* (so every subgame perfect equilibrium is also a Nash equilibrium).

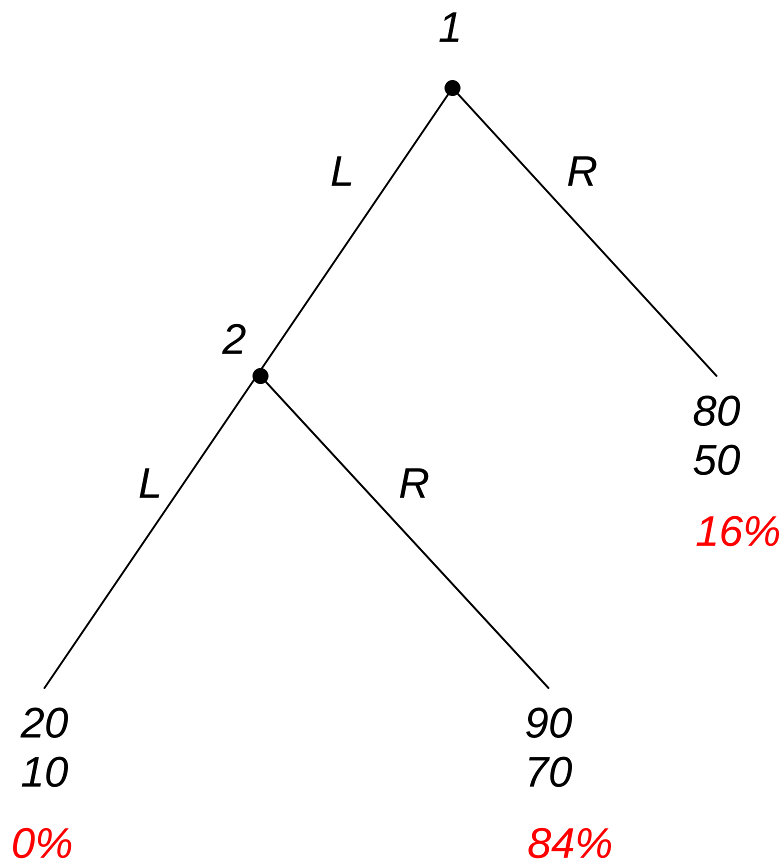
An example: entry game

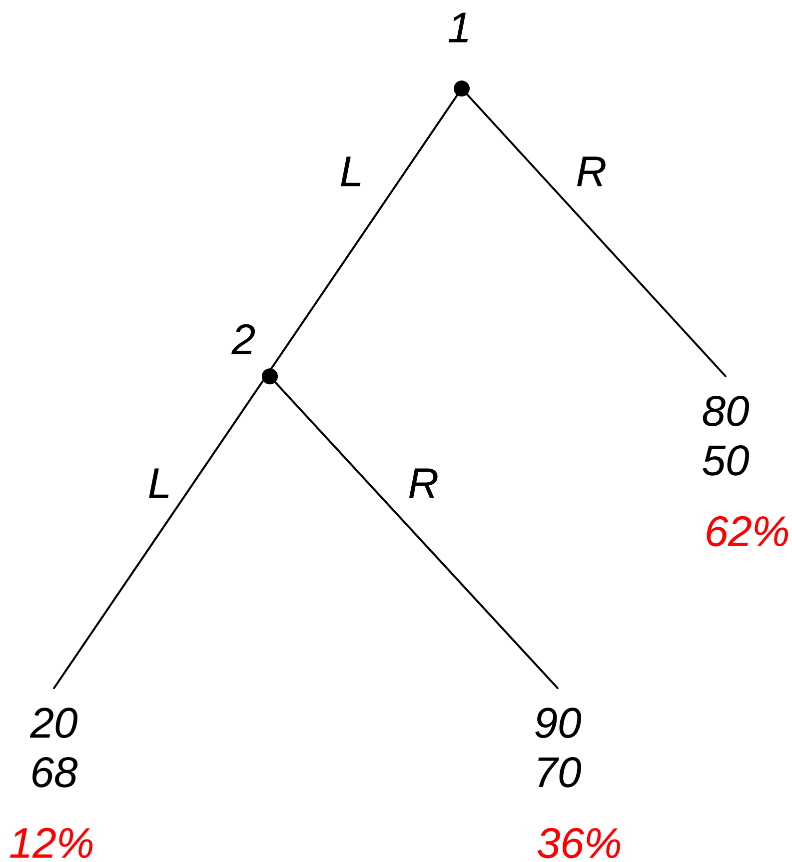


Subgame perfect and backward induction



Two entry games in the laboratory




1
L
R
2
L
R
80
50
62%
20
68
12%
90
70
36%

A review of the main ideas

We study two (out of four) groups of game theoretic models:

[1] Strategic games – all players simultaneously choose their plan of action once and for all.

[2] Extensive games (with perfect information) – players choose sequentially (and fully informed about all previous actions).

A solution (equilibrium) is a systematic description of the outcomes that may emerge in a family of games. We study two solution concepts:

[1] Nash equilibrium – a steady state of the play of a <u>strategic</u> game (no player has a profitable deviation given the actions of the other players).

[1] Subgame equilibrium – a steady state of the play of an <u>extensive</u> game (a Nash equilibrium in every <u>subgame</u> of the extensive game).

$\Longrightarrow$ Every subgame perfect equilibrium is also a Nash equilibrium.

Back to oligopoly...

Stackelberg's duopoly model (1934)

How do the conclusions of the Cournot's duopoly game change when the firms move sequentially? Is a firm better off moving before or after the other firm?

Suppose that $c_1 = c_2 = c$ and that firm 1 moves at the start of the game. We may use backward induction to find the subgame perfect equilibrium.

– First, for *any* output q_1 of firm 1, we find the output q_2 of firm 2 that maximizes its profit. Next, we find the output q_1 of firm 1 that maximizes its profit, *given the strategy* of firm 2.

Firm 2

Since firm 2 moves after firm 1, a strategy of firm 2 is a *function* that associate an output q_2 for firm 2 for each possible output q_1 of firm 1.

We found that under the assumptions of the Cournot's duopoly game Firm 2 has a unique best response to each output q_1 of firm 1, given by

$$q_2 = \frac{1}{2}(A - q_1 - c)$$

(Recall that $c_1 = c_2 = c$).

Firm 1

Firm 1's strategy is the output q_1 the maximizes

$$\pi_1 = (A - q_1 - q_2 - c)q_1 \quad \text{subject to} \quad q_2 = \tfrac{1}{2}(A - q_1 - c)$$

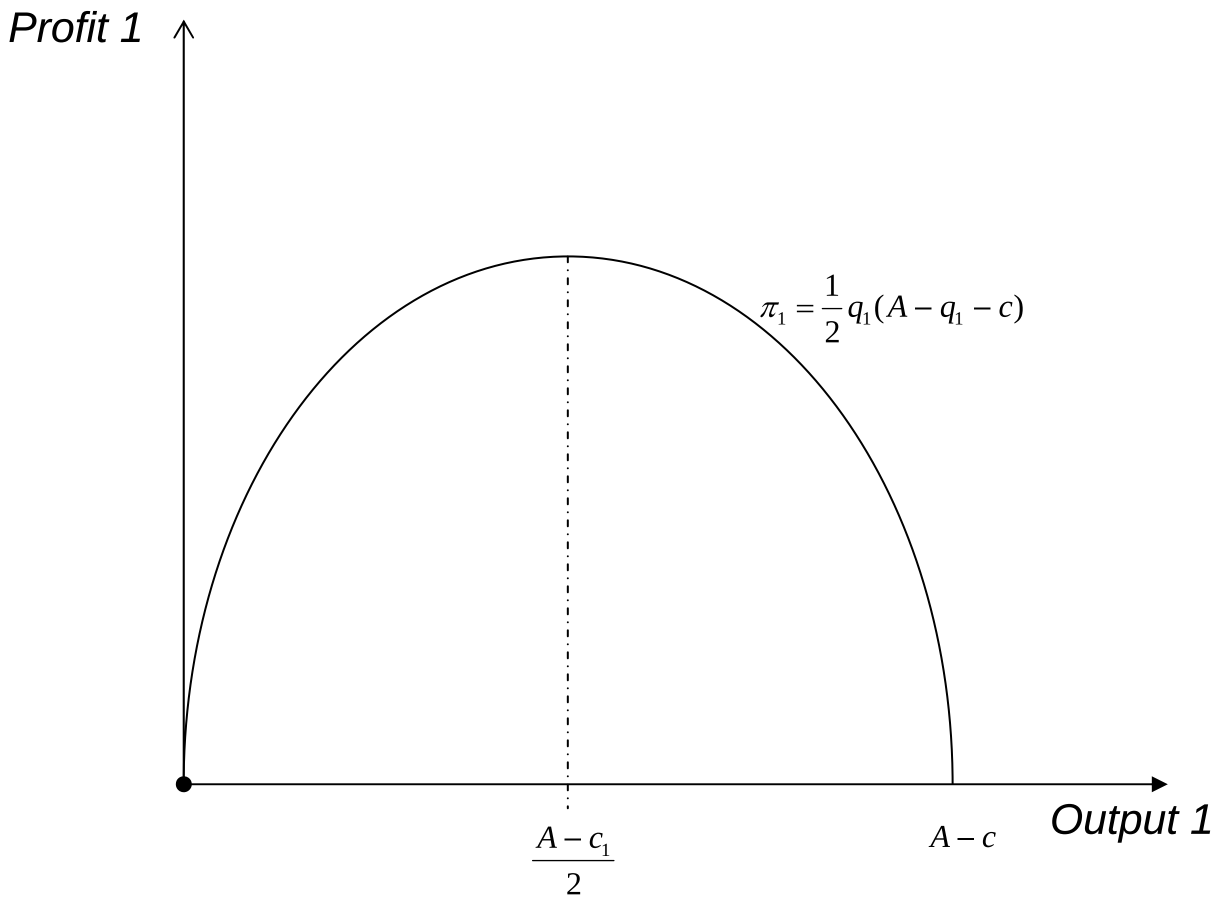
Thus, firm 1 maximizes

$$\pi_1 = (A - q_1 - (\frac{1}{2}(A - q_1 - c)) - c)q_1 = \frac{1}{2}q_1(A - q_1 - c).$$

This function is quadratic in q_1 that is zero when $q_1 = 0$ and when $q_1 = A - c$. Thus its maximizer is

$$q_1^* = \frac{1}{2}(A - c).$$

Firm 1’s (first-mover) profit in Stackelberg's duopoly game



We conclude that Stackelberg's duopoly game has a unique subgame perfect equilibrium, in which firm 1's strategy is the output

$$q_1^* = \frac{1}{2}(A - c)$$

and firm 2's output is

$$\begin{aligned} q_2^* &= \frac{1}{2}(A - q_1^* - c) \\ &= \frac{1}{2}(A - \frac{1}{2}(A - c) - c) \\ &= \frac{1}{4}(A - c). \end{aligned}$$

By contrast, in the unique Nash equilibrium of the Cournot's duopoly game under the same assumptions ($c_1 = c_2 = c$), each firm produces $\frac{1}{3}(A - c)$.

The subgame perfect equilibrium of Stackelberg's duopoly game