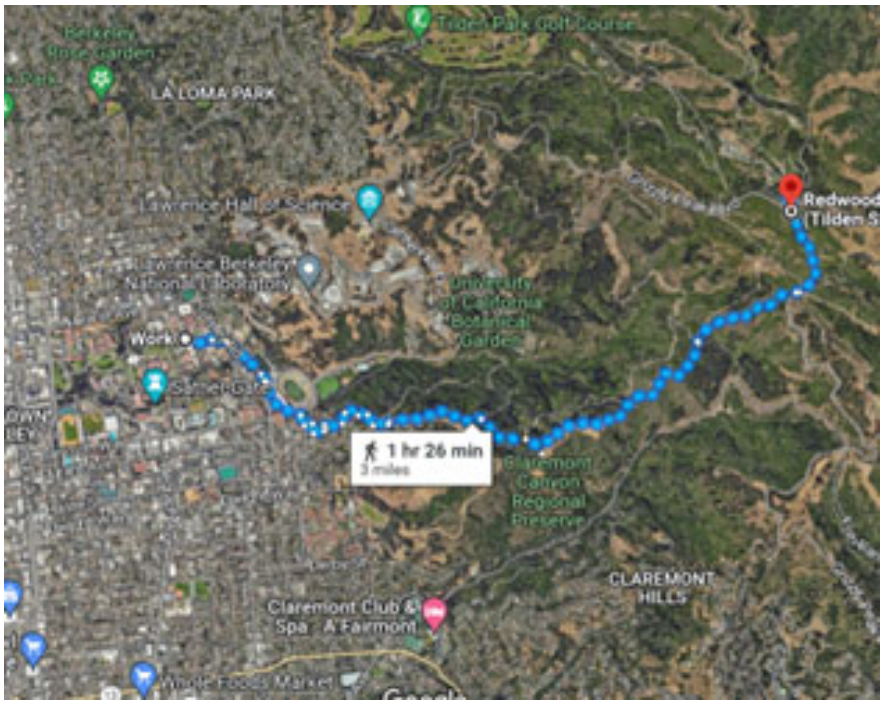
University of California – Berkeley
Department of Economics
Game Theory in the Social Sciences
(ECON C110 | POLSCI C135)
Fall 2023

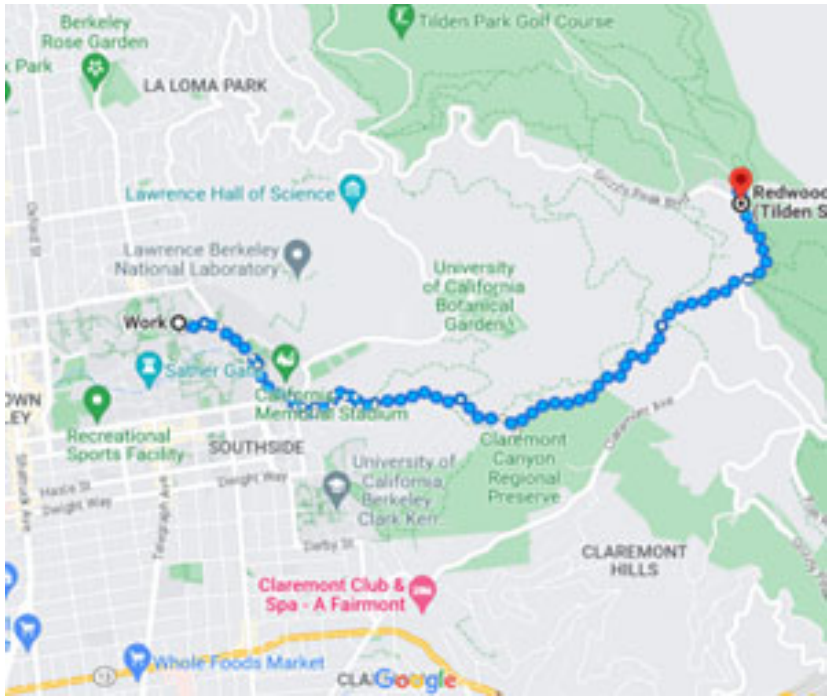
Lecture IX

Games with imperfect (and asymmetric) information

Nov 30, 2023

Game theory in a nutshell…





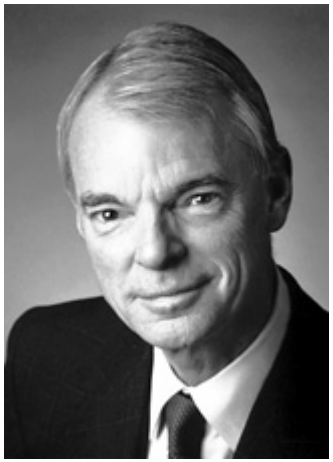
Markets with asymmetric information

- The traditional theory of markets assumes that market participants have complete information about the underlying economic variables:
  - Buyers and sellers are both perfectly informed about the quality of the goods being sold in the market.
  - If it is not costly to verify quality, then the prices of the goods will simply adjust to reflect the quality difference.

$\Longrightarrow$ This is clearly a drastic simplification!!!

- There are certainly many markets in the real world in which it may be very costly (or even impossible) to gain accurate information:
  - labor markets, financial markets, markets for consumer products, and more.
- If information about quality is costly to obtain, then it is no longer possible that buyers and sellers have the same information.
- The costs of information provide an important source of market friction and can lead to a market breakdown.

Nobel Prize 2001
“for their analyses of markets with asymmetric information”



The Market for Lemons

Example I

- Consider a market with 100 people who want to sell their used car and 100 people who want to buy a used car.

- Everyone knows that 50 of the cars are "plums" and 50 are "lemons."

- Suppose further that

	seller	buyer
lemon	\$1000	\$1200
plum	\$2000	\$2400

- If it is easy to verify the quality of the cars there will be no problem in this market.

- Lemons will sell at some price $\$1000 - 1200$ and plums will sell at $\$2000 - 2400$.

- But happens to the market if buyers cannot observe the quality of the car?

– If buyers are risk neutral, then a typical buyer will be willing to pay his expected value of the car

$$\frac{1}{2}1200 + \frac{1}{2}2400 = \$1800.$$

– But for this price only owners of lemons would offer their car for sale, and buyers would therefore (correctly) expect to get a lemon.

– Market failure – no transactions will take place, although there are possible gains from trade!

Example II

- Suppose we can index the quality of a used car by some number q, which is distributed uniformly over $[0, 1]$.

- There is a large number of demanders for used cars who are willing to pay $\frac{3}{2}q$ for a car of quality q.

- There is a large number of sellers who are willing to sell a car of quality q for a price of q.

- If quality is perfectly observable, each used car of quality q would be soled for some price between q and $\frac{3}{2}q$.

- What will be the equilibrium price(s) in this market when quality of any given car cannot be observed?

- The unique equilibrium price is zero, and at this price the demand is zero and supply is zero.

$\Longrightarrow$ The asymmetry of information has destroyed the market for used cars. But the story does not end here!!!

Signaling

- In the used-car market, owners of the good used cars have an incentive to try to convey the fact that they have a good car to the potential purchasers.

- Put differently, they would like choose actions that signal that they are offering a plum rather than a lemon.

- In some case, the presence of a "signal" allows the market to function more effectively than it would otherwise.

Example – educational signaling

- Suppose that a fraction $0 < b < 1$ of workers are *competent* and a fraction $1 - b$ are *incompetent*.

- The competent workers have marginal product of a_2 and the incompetent have marginal product of $a_1 < a_2$.

- For simplicity we assume a competitive labor market and a linear production function

$$L_1 a_1 + L_2 a_2$$

where L_1 and L_2 is the number of incompetent and competent workers, respectively.

– If worker quality is observable, then firm would just offer wages

$$w_1 = a_1 \text{ and } w_2 = a_2$$

to competent workers, respectively.

– That is, each worker will paid his marginal product and we would have an efficient equilibrium.

– But what if the firm cannot observe the marginal products so it cannot distinguish the two types of workers?

– If worker quality is unobservable, then the "best" the firm can do is to offer the average wage

$$w = (1 - b)a_1 + ba_2.$$

– If both types of workers agree to work at this wage, then there is no problem with adverse selection (more below).

– The incompetent (resp. competent) workers are getting paid more (resp. less) than their marginal product.

- The competent workers would like a way to signal that they are more productive than the others.

- Suppose now that there is some signal that the workers can acquire that will distinguish the two types

- One nice example is education – it is cheaper for the competent workers to acquire education than the incompetent workers.

– To be explicit, suppose that the cost (dollar costs, opportunity costs, costs of the effort, etc.) to acquiring e years of education is

$$c_1 e \text{ and } c_2 e$$

for incompetent and competent workers, respectively, where $c_1 > c_2$.

– Suppose that workers conjecture that firms will pay a wage $s(e)$ where s is some increasing function of e.

– Although education has no effect on productivity (MBA?), firms may still find it profitable to base wage on education – attract a higher-quality work force.

Market equilibrium

In the educational signaling example, there appear to be several possibilities for equilibrium:

[1] The (representative) firm offers a single contract that attracts both types of workers.

[2] The (representative) firm offers a single contract that attracts only one type of workers.

[3] The (representative) firm offers two contracts, one for each type of workers.

- A separating equilibrium involves each type of worker making a choice that separate himself from the other type.

- In a pooling equilibrium, in contrast, each type of workers makes the same choice, and all getting paid the wage based on their average ability.

  Note that a separating equilibrium is wasteful in a social sense – no social gains from education since it does not change productivity.

Example (cont.)

– Let e_1 and e_2 be the education level actually chosen by the workers. Then, a separating (signaling) equilibrium has to satisfy:

[1] zero-profit conditions

$$\begin{aligned} s(e_1) &= a_1 \\ s(e_2) &= a_2 \end{aligned}$$

[2] self-selection conditions

$$\begin{aligned} s(e_1) - c_1 e_1 &\geq s(e_2) - c_1 e_2 \\ s(e_2) - c_2 e_2 &\geq s(e_1) - c_2 e_1 \end{aligned}$$

– In general, there may by many functions $s(e)$ that satisfy conditions [1] and [2]. One wage profile consistent with separating equilibrium is

$$s(e) = \begin{cases} a_2 & \text{if} \quad e > e^* \\ a_1 & \text{if} \quad e \leq e^* \end{cases}$$

and

$$\frac{a_2 - a_1}{c_2} > e^* > \frac{a_2 - a_1}{c_1}$$

$\Longrightarrow$ Signaling can make things better or worse – each case has to examined on its own merits!

The Sheepskin (diploma) effect

The increase in wages associated with obtaining a higher credential:

- Graduating high school increases earnings by 5 to 6 times as much as does completing a year in high school that does not result in graduation.
- The same discontinuous jump occurs for people who graduate from collage.
- High school graduates produce essentially the same amount of output as non-graduates.

Social Learning
Herd behavior and informational cascades

"Men nearly always follow the tracks made by others and proceed in their affairs by imitation." Machiavelli (Renaissance philosopher)

Examples

Business strategy

- TV networks make introductions in the same categories as their rivals.

Finance

- The withdrawal behavior of small number of depositors starts a bank run.

Politics

- The solid New Hampshirites (probably) can not be too far wrong.

Crime

- In NYC, individuals are more likely to commit crimes when those around them do.

Why should individuals behave in this way?

Several "theories" explain the existence of uniform social behavior:

- benefits from conformity
- sanctions imposed on deviants
- network / payoff externalities
- social learning

Broad definition: any situation in which individuals learn by observing the behavior of others.

Informational cascades and herd behavior

Two phenomena that have elicited particular interest are *informational cascades* and *herd behavior*.

- Cascade: agents 'ignore' their private information when choosing an action.

- Herd: agents choose the same action, not necessarily ignoring their private information.

- While the terms informational cascade and herd behavior are used interchangeably there is a significant difference between them.

- In an informational cascade, an agent considers it optimal to follow the behavior of her predecessors without regard to her private signal.

- When acting in a herd, agents choose the same action, not necessarily ignoring their private information.

- Thus, an informational cascade implies a herd but a herd is not necessarily the result of an informational cascade.

A model of social learning

Signals

- Each player $n \in \{1, ..., N\}$ receives a signal θ_n that is private information.
- For simplicity, $\{\theta_n\}$ are independent and uniformly distributed on $[-1, 1]$.

Actions

- Sequentially, each player n has to make a binary irreversible decision $x_n \in \{0, 1\}$.

Payoffs

- $x = 1$ is profitable if and only if $\sum_{n \leq N} \theta_n \geq 0$, and $x = 0$ is profitable otherwise.

Information

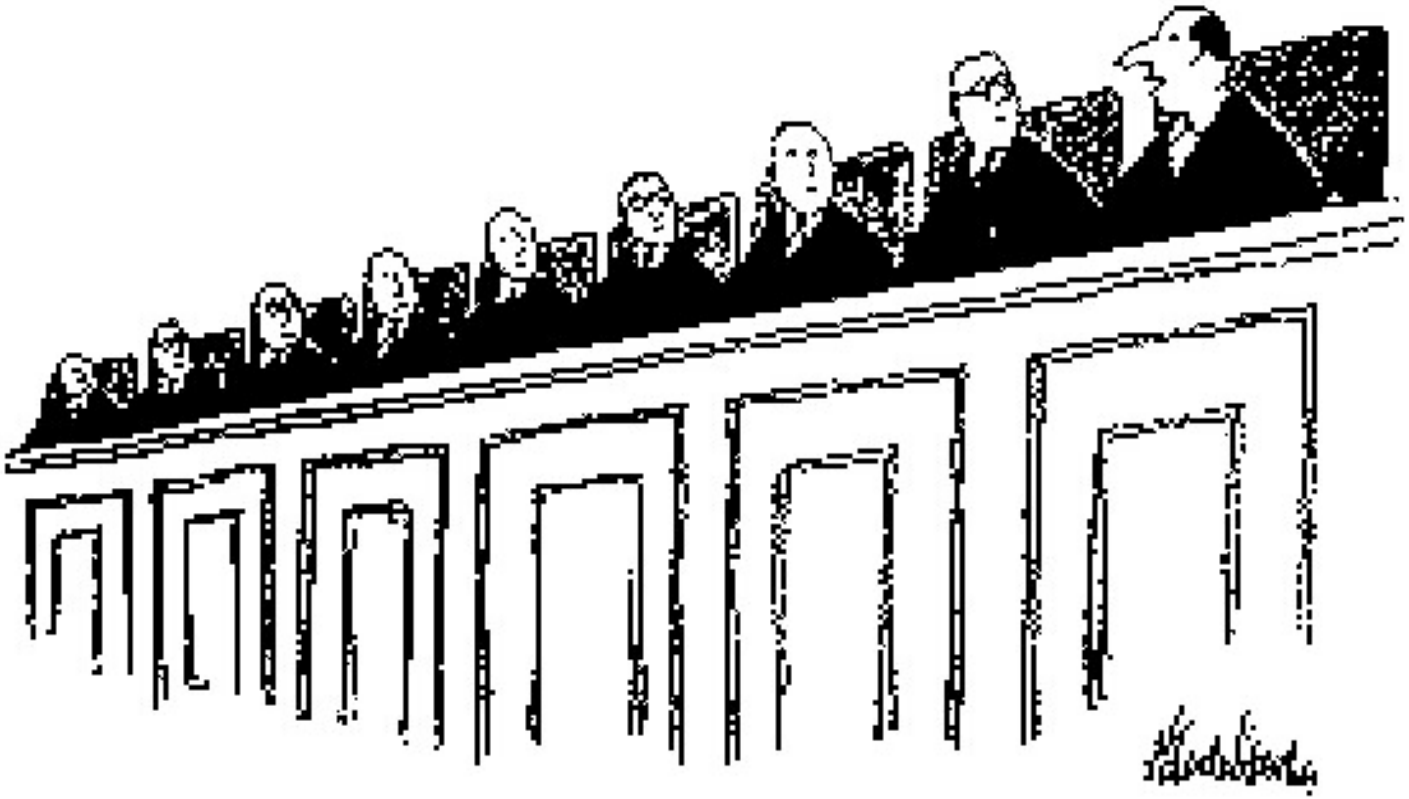
- Perfect information

$$\mathcal{I}_n = \{\theta_n, (x_1, x_2, ..., x_{n-1})\}$$

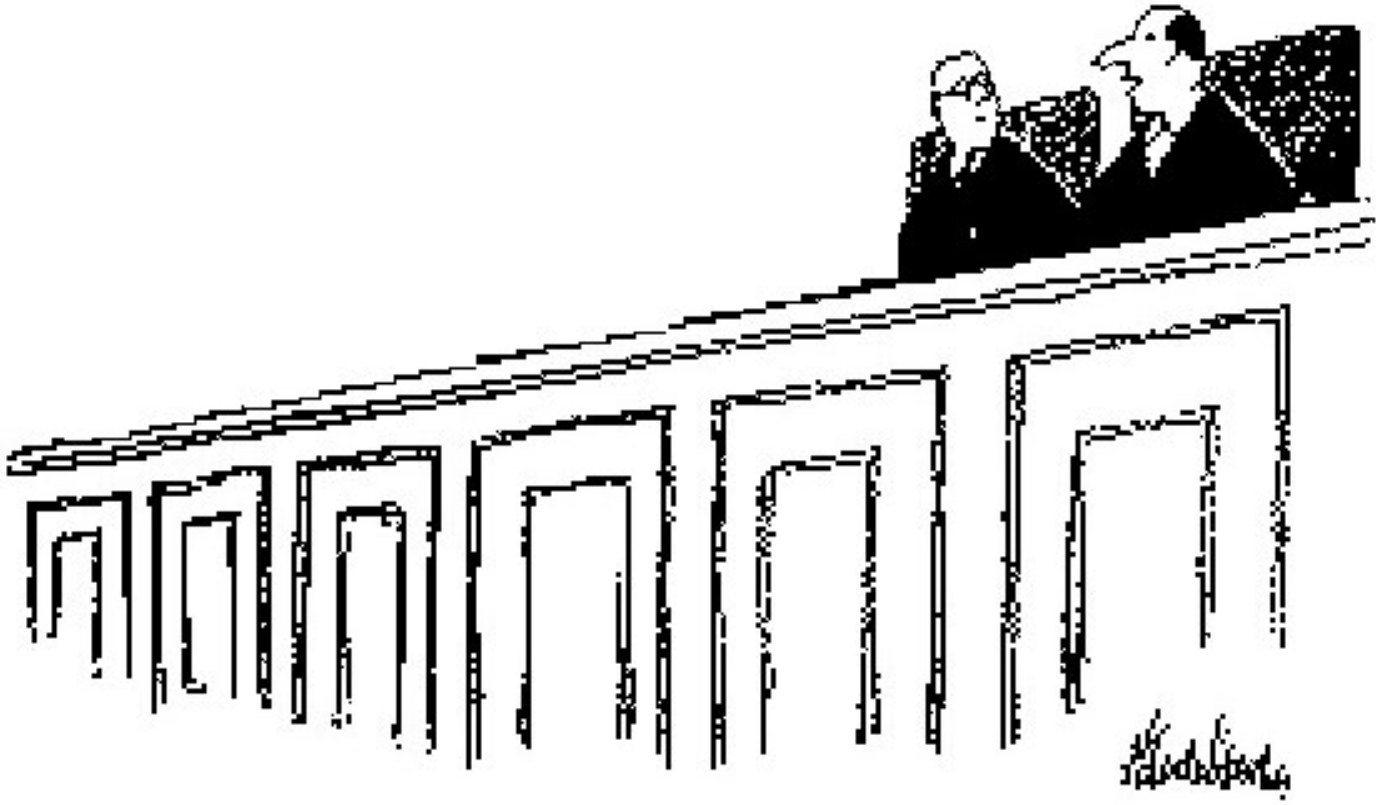
- Imperfect information

$$\mathcal{I}_n = \{\theta_n, x_{n-1}\}$$

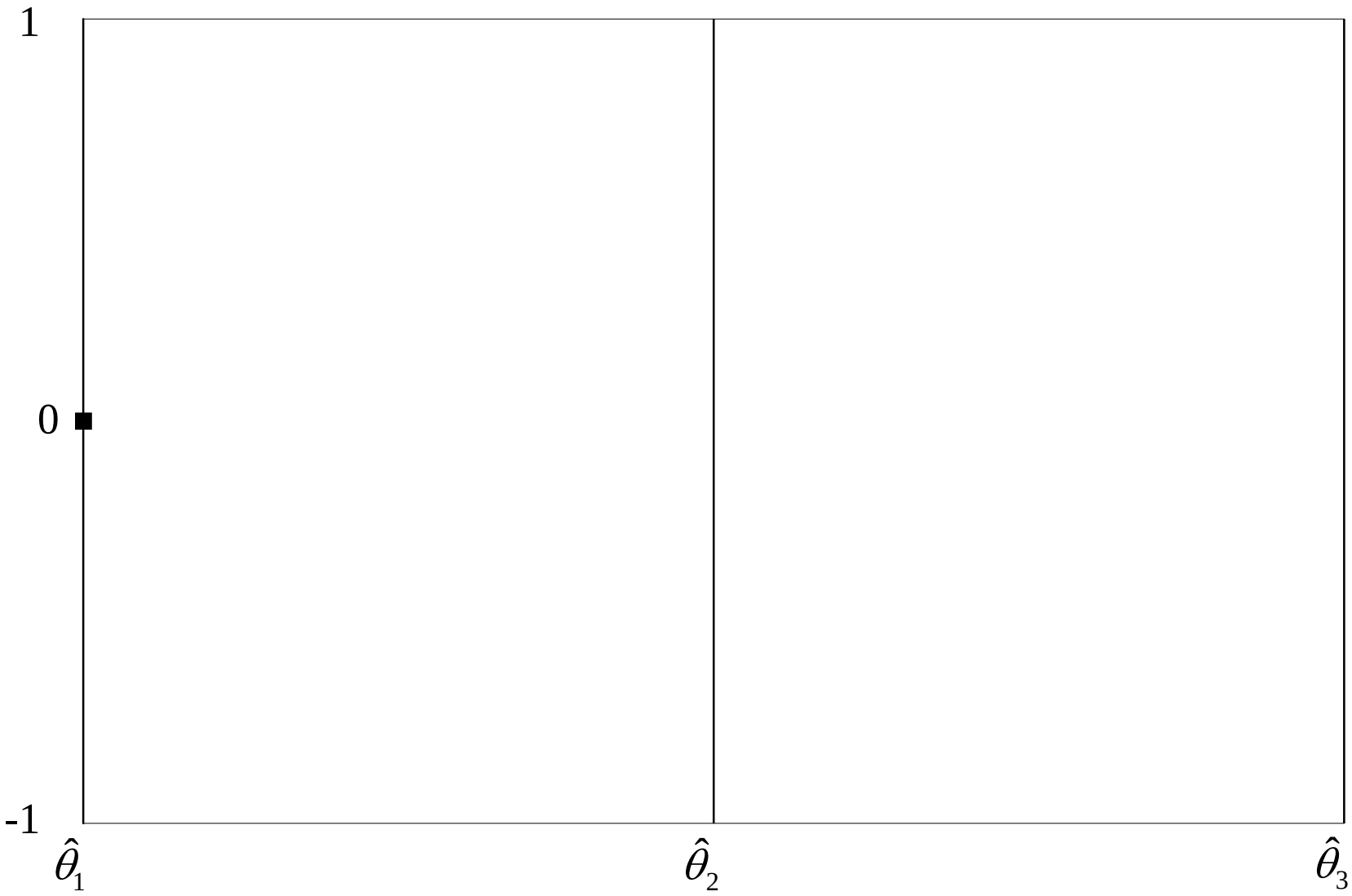
Sequential social-learning model:
Well heck, if all you smart cookies agree, who am I to dissent?



Imperfect information:
Which way is the wind blowing?!



A three-agent example



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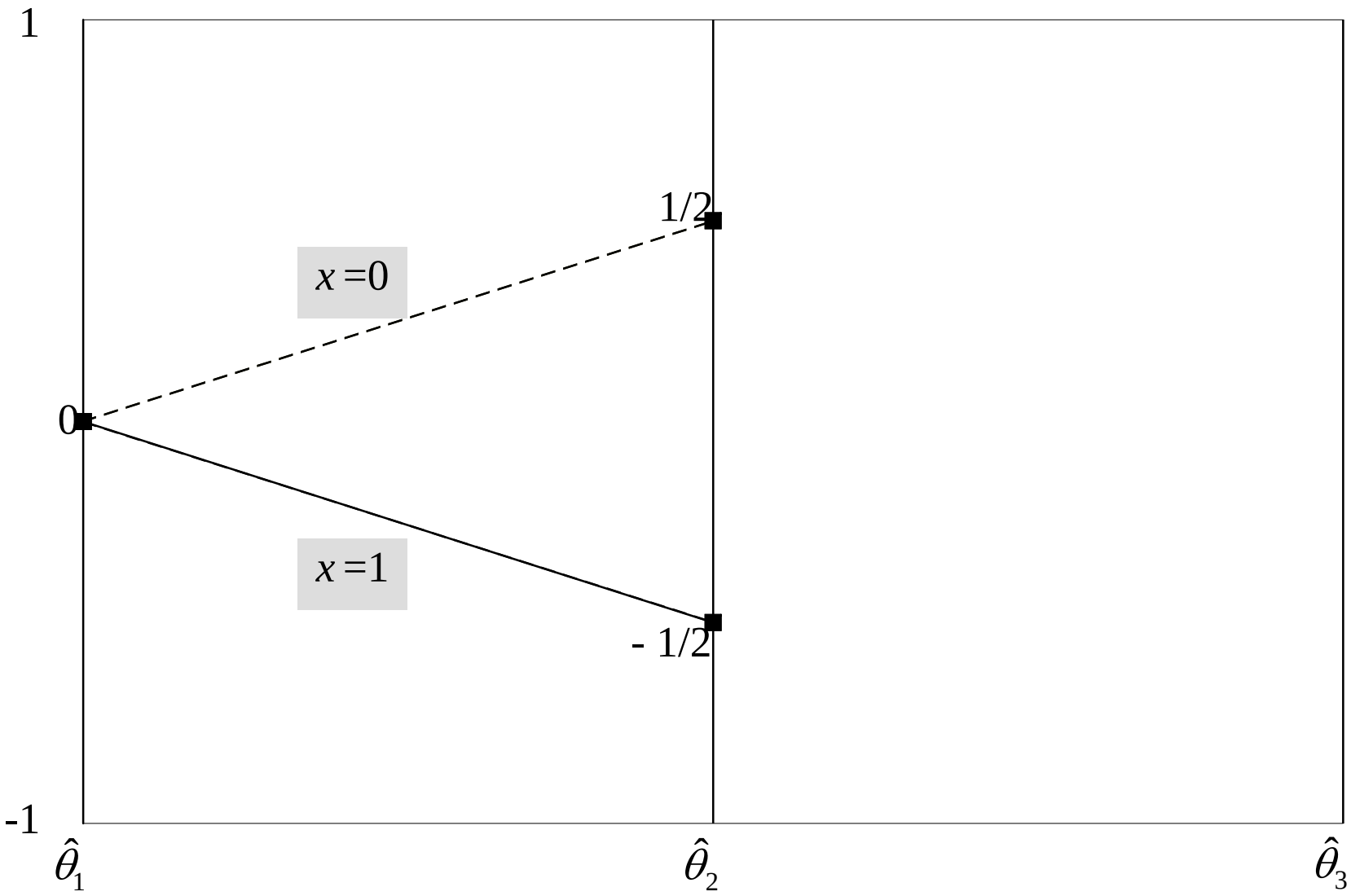
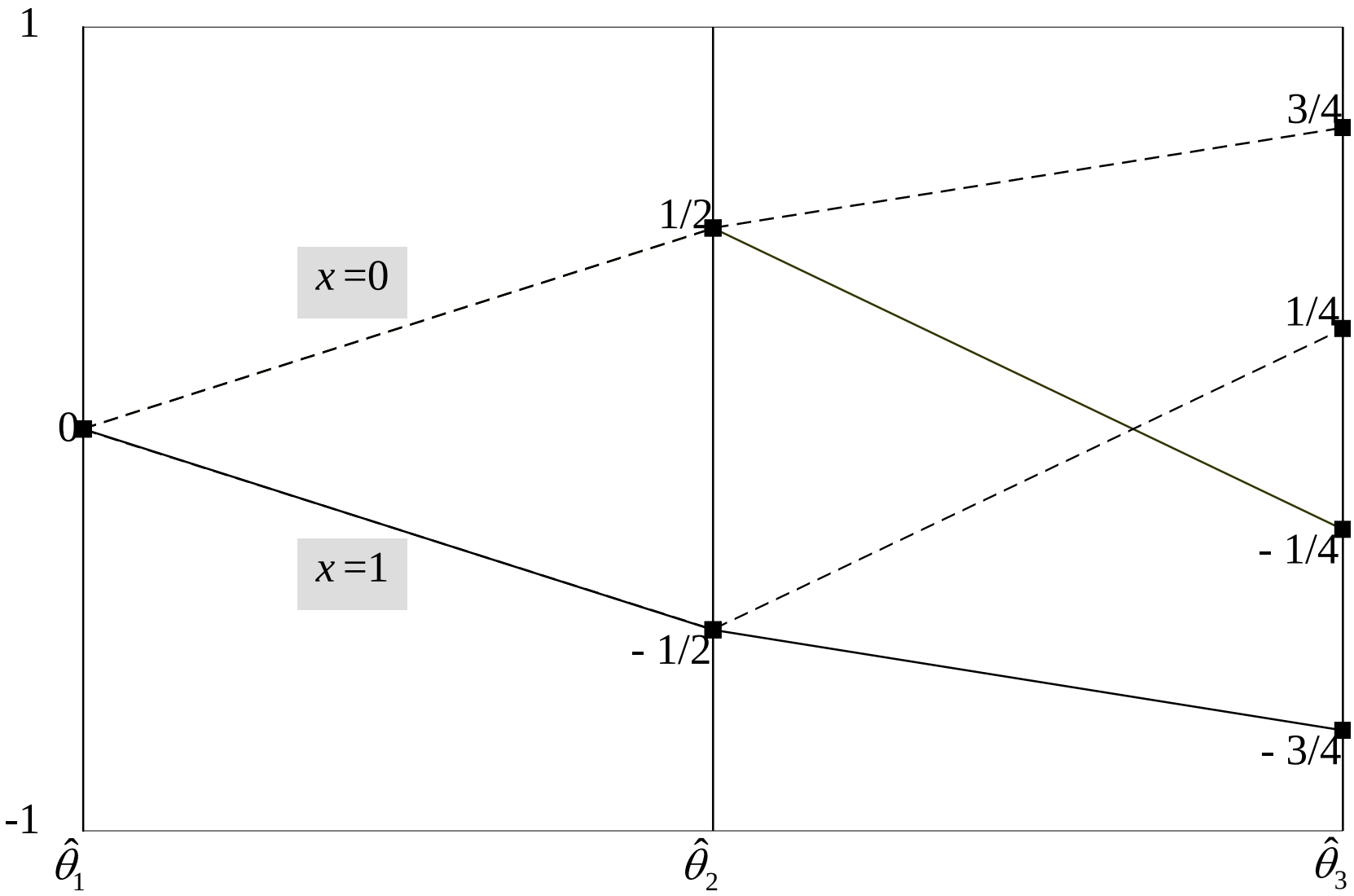
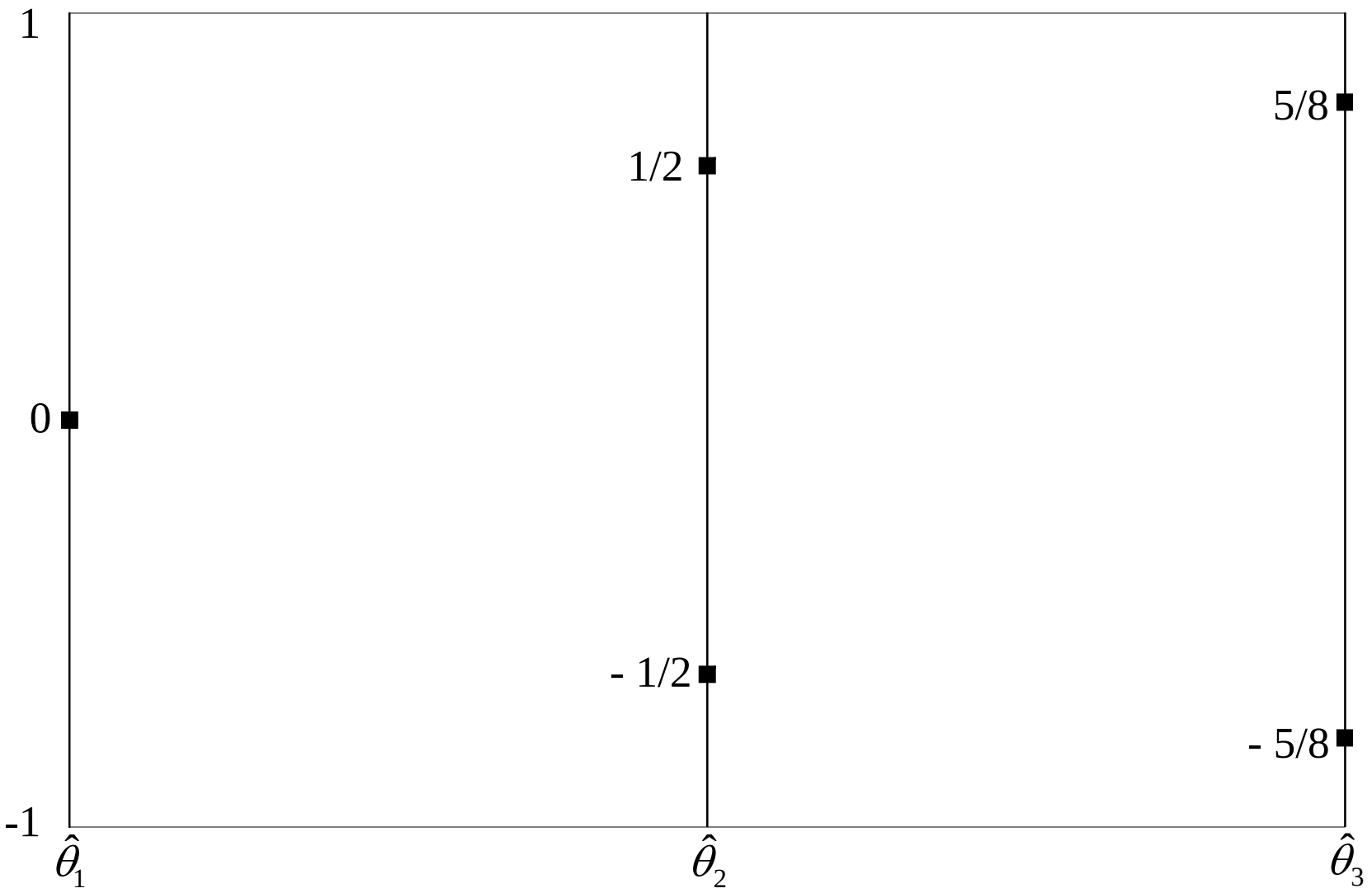


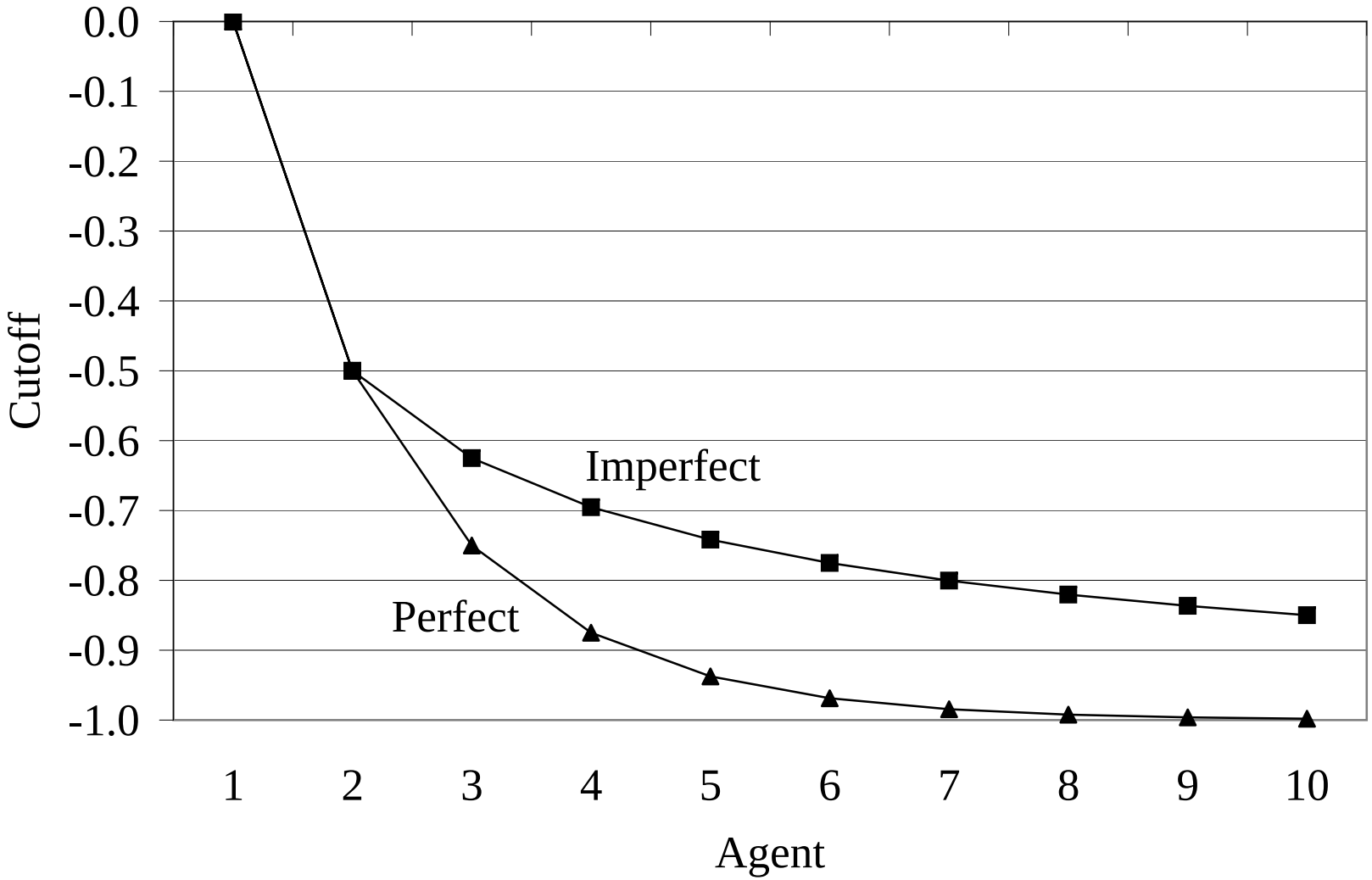
A three-agent example under perfect information



A three-agent example under imperfect information



A sequence of cutoffs under imperfect and perfect information



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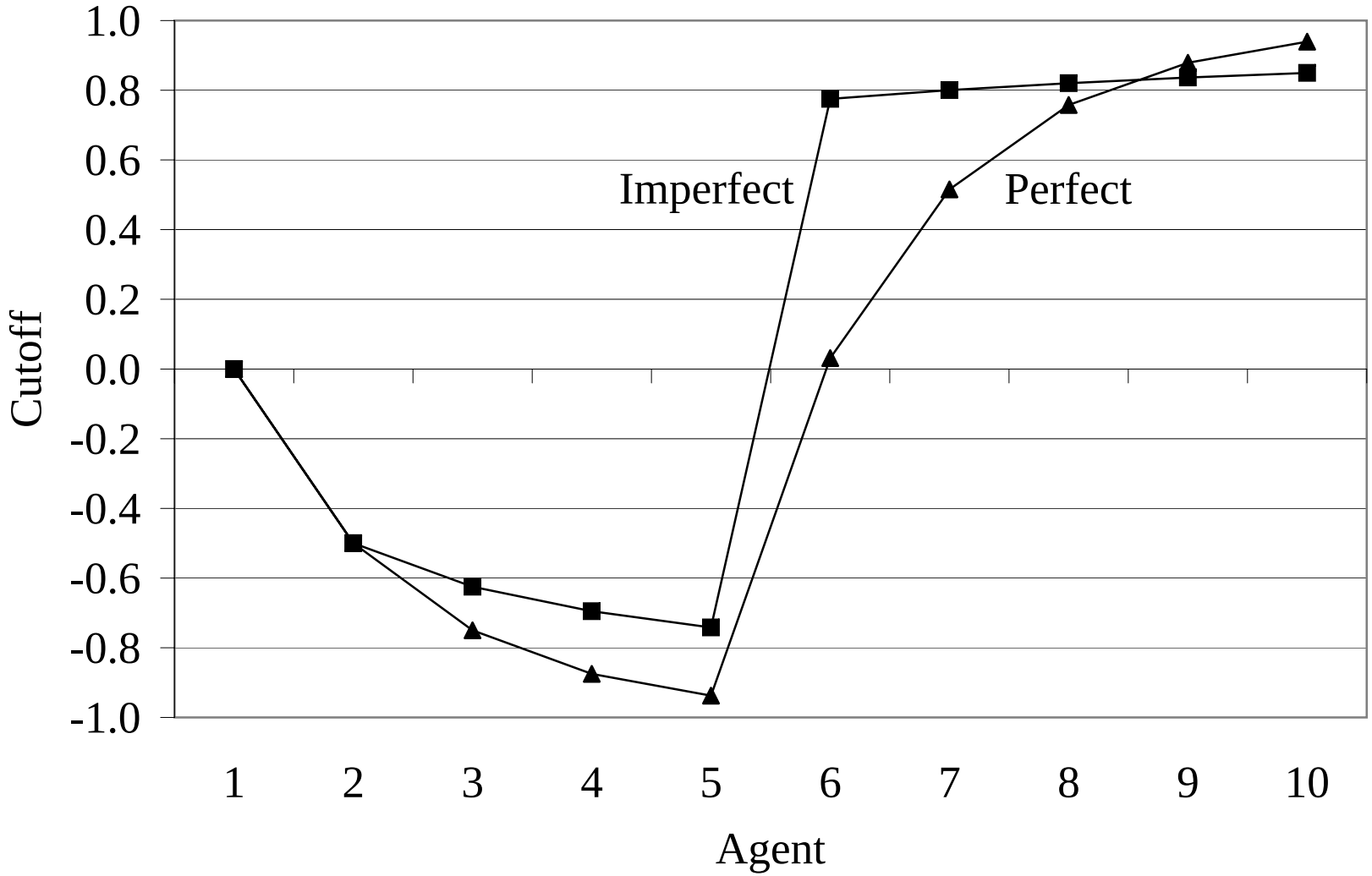

The decision problem

– The optimal decision rule is given by

$$x_n = 1 \text{ if and only if } \mathbb{E}\left[\sum_{i=1}^{N} \theta_i \mid \mathcal{I}_n\right] \geq 0.$$

Since $\mathcal{I}_n$ does not provide any information about the content of successors' signals, we obtain

$$x_n = 1 \text{ if and only if } \mathbb{E}\left[\sum_{i=1}^{n} \theta_i \mid \mathcal{I}_n\right] \geq 0$$

Hence,

$$x_n = 1 \text{ if and only if } \theta_n \geq -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid \mathcal{I}_n\right].$$

The cutoff process

- For any n, the optimal strategy is the *cutoff strategy*

$$x_n = \begin{cases} 1 & if \quad \theta_n \geq \hat{\theta}_n \\ 0 & if \quad \theta_n < \hat{\theta}_n \end{cases}$$

  where

$$\hat{\theta}_n = -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid \mathcal{I}_n\right]$$

  is the optimal history-contingent cutoff.

- $\hat{\theta}_n$ is sufficient to characterize the individual behavior, and $\{\hat{\theta}_n\}$ characterizes the social behavior of the economy.

Overview of results

Perfect information

- A cascade need not arise, but herd behavior must arise.

Imperfect information

- Herd behavior is impossible. There are periods of uniform behavior, punctuated by increasingly rare switches.

- The similarity:
  - Agents can, for a long time, make the same (incorrect) choice.
- The difference:
  - Under perfect information, a herd is an absorbing state. Under imperfect information, continued, occasional and sharp shifts in behavior.

- The dynamics of social learning depend crucially on the extensive form of the game.

- The key economic phenomenon that imperfect information captures is a succession of fads starting suddenly, expiring rather easily, each replaced by another fad.

- The kind of episodic instability that is characteristic of socioeconomic behavior in the real world makes more sense in the imperfect-information model.

As such, the imperfect-information model gives insight into phenomena such as manias, fashions, crashes and booms, and better answers such questions as:

- Why do markets move from boom to crash without settling down?
- Why is a technology adopted by a wide range of users more rapidly than expected and then, suddenly, replaced by an alternative?
- What makes a restaurant fashionable over night and equally unexpectedly unfashionable, while another becomes the 'in place', and so on?