Appendix III
Testing rationality

Let $\{(p^i, x^i)\}_{i=1}^{50}$be the data generated by some individual's choices, where p^i denotes the i-th observation of the price vector and x^i denotes the associated portfolio. A portfolio x^i is *directly revealed preferred* to a portfolio x^j, denoted $x^i R^D x^j$, if $p^i \cdot x^i \geq p^i \cdot x^j$. A portfolio x^i is *revealed preferred* to a portfolio x^j, denoted $x^i R x^j$, if there exists a sequence of portfolios $\{x^k\}_{k=1}^K$ with $x^1 = x^i$ and $x^K = x^j$, such that $x^k R^D x^{k+1}$ for every $k = 1, ..., K-1$. The Generalized Axiom of Revealed Preference (GARP). which requires that if $x^i R x^j$ then $p^j \cdot x^j \leq p^j \cdot x^i$ (i.e. if x^i is revealed preferred to x^j, then x^i must cost at least as much as x^j at the prices prevailing when x^j is chosen). It is clear that if the data are generated by a non-satiated utility function, then they must satisfy GARP. Conversely, the following result due to Afriat (1967) tells us that if a *finite* data set generated by an individual's choices satisfies GARP, then the data can be rationalized by a well-behaved utility function.

Afriat's Theorem If the data set $\{(p^i, x^i)\}$ satisfies GARP, then there exists a piecewise linear, continuous, increasing, concave utility function $u(x)$ such that for each observation (p^i, x^i)

$$u(x) \leq u(x^i) \text{ for any } x \text{ such that } p^i \cdot x \leq p^i \cdot x^i.$$

Hence, in order to show that the data are consistent with utility-maximizing behavior we must check whether it satisfies GARP. Since GARP offers an exact test, it is desirable to measure the *extent* of GARP violations. We report measures of GARP violations based on three indices: Afriat (1972), Varian (1991), and Houtman and Maks (1985).

Afriat (1972) Afriat's *critical cost efficiency index* (CCEI) measures the amount by which each budget constraint must be adjusted in order to remove all violations of GARP. For any number $0 \leq e \leq 1$, define the direct revealed preference relation $R^D(e)$ as $x^i R^D(e) x^j$ if $ep^i \cdot x^i \geq p^i \cdot x^j$, and define $R(e)$ to be the transitive closure of $R^D(e)$. Let e^* be the largest value of e such that the relation $R(e)$ satisfies GARP. Afriat's CCEI is the value of e^* associated with the data set $\{(p^i, x^i)\}$. It is bounded between zero and one and can be interpreted as saying that the consumer is 'wasting' as much as $1 - e^*$ of his income by making inefficient choices. The closer the CCEI is to one, the smaller the perturbation of the budget constraints required to remove all violations and thus the closer the data are to satisfying GARP. Although the CCEI provides a summary statistic of the overall consistency of the data with GARP, it does not give any information about which of the observations (p^i, x^i) are causing the most severe violations. A single large violation may lead to a small value of the index while a large number of small violations may result in a much larger efficiency index.

Varian (1991) Varian refined Afriat's CCEI to provide a measure that reflects the minimum adjustment required to eliminate the violations of GARP associated with each observation (p^i, x^i). In particular, fix an observation (p^i, x^i) and let e^i be the largest value of e such that $R(e)$ has no violations of GARP within the set of portfolios x^j such that $x^i R(e) x^j$. The value e^i measures the efficiency of the choices when compared to the portfolio x^i. Knowing the efficiencies $\{e^i\}$ for the entire set of observations $\{(p^i, x^i)\}$ allows us to say where the inefficiency is greatest or least. These numbers may still overstate the extent of inefficiency, however, because there may be several places in a cycle of observations where an adjustment of the budget constraint would remove a violation of GARP and the above procedure may not choose the 'least costly' adjustment. Varian (1991) provides an algorithm that will select the least costly method of removing all violations by changing each budget set by a different amount. When a single number is desired, as here, one can use $e^* = \min \{e^i\}$. Thus, Varian's (1991) index is a lower bound on the Afriat's CCEI.

Houtman and Maks (1985) (HM) HM find the largest subset of choices that is consistent with GARP. This method has a couple of drawbacks. First, some observations may be discarded even if the associated GARP violations could be removed by small perturbations of the budget constraint. Further, since the algorithm is computationally very intensive, we were unable to compute the HM index for a small number of subjects (ID 211, 324, 325, 406, 504 and 608) with a large number of GARP violations. In those few cases we report upper bounds on the consistent set.

Table AIII1 lists, by subject, the number of violations of the Weak Axiom of Revealed Preference (WARP) and GARP, and also reports the values of the three indices. Subjects are ranked according to (descending) CCEI scores. We allow for small mistakes resulting from the imprecision of a subject's handling of the mouse. The results presented in Table AIII1 allow for a narrow confidence interval of one token (i.e. for any i and $j \neq i$, if $d(x^i, x^j) \leq 1$ then x^i and x^j are treated as the same portfolio).

[Table AIII1 here]

Figure AIII1 compares the distributions of the Varian efficiency index generated by the sample of hypothetical subjects (gray) and the distributions of the scores for the actual subjects (black). The horizontal axis shows the value of the index and the vertical axis measures the percentage of subjects corresponding to each interval. The histograms show that actual subject behavior has high consistency measures compared to the behavior of the hypothetical random subjects. Figure AIII2 shows the distribution of the HM index. Note that we cannot generate a distribution of this index for random subjects because of the computational load.

[Figure AIII1 here]
[Figure AIII2 here]

Table AIII1: WARP and GARP violations and the three indices by subject (sorted according to descending CCEI)

ID	WARP	GARP	Afriat	Varian	HM
205	0	0	1.000	1.000	50
213	0	0	1.000	1.000	50
215	0	0	1.000	1.000	50
216	0	0	1.000	1.000	50
219	0	0	1.000	1.000	50
303	0	0	1.000	1.000	50
304	0	0	1.000	1.000	50
306	0	0	1.000	1.000	50
314	0	0	1.000	1.000	50
316	0	0	1.000	1.000	50
317	0	0	1.000	1.000	50
320	0	0	1.000	1.000	50
326	0	0	1.000	1.000	50
508	0	0	1.000	1.000	50
509	0	0	1.000	1.000	50
604	0	0	1.000	1.000	50
411	2	4	0.999	0.978	48
416	1	1	0.999	0.979	49
405	2	2	0.999	0.933	48
417	1	1	0.998	0.996	49
301	3	11	0.997	0.951	48
505	1	1	0.996	0.995	49
501	2	2	0.995	0.985	48
605	5	5	0.992	0.982	45
323	3	3	0.991	0.978	47
302	2	7	0.990	0.943	48
414	1	1	0.990	0.951	49
413	5	7	0.989	0.979	47
210	1	1	0.988	0.967	49
408	1	1	0.987	0.986	49
415	4	5	0.987	0.934	47
402	5	7	0.987	0.834	47
311	3	3	0.986	0.804	48
313	2	2	0.986	0.970	48
217	7	14	0.986	0.935	46
410	4	4	0.984	0.954	47
515	5	6	0.984	0.973	46

ID	WARP	GARP	Afriat	Varian	HM
407	3	3	0.984	0.972	48
503	2	5	0.982	0.961	49
512	8	8	0.982	0.960	43
207	3	15	0.981	0.941	47
601	1	1	0.981	0.981	49
516	4	4	0.981	0.975	46
520	8	9	0.979	0.907	46
412	7	12	0.976	0.928	46
514	2	3	0.975	0.952	49
204	4	10	0.973	0.970	47
318	4	6	0.972	0.809	48
502	5	17	0.971	0.880	47
609	3	5	0.969	0.880	47
202	6	12	0.968	0.944	46
203	4	14	0.966	0.946	48
319	3	20	0.966	0.727	48
327	2	5	0.965	0.915	49
519	4	5	0.963	0.944	47
315	10	33	0.959	0.795	45
312	4	13	0.957	0.952	47
513	10	37	0.957	0.822	45
309	4	17	0.952	0.890	48
218	5	10	0.951	0.907	48
214	8	21	0.949	0.916	45
206	9	147	0.948	0.855	47
602	6	11	0.947	0.861	45
510	8	13	0.946	0.914	45
409	6	15	0.943	0.935	46
208	8	14	0.942	0.912	45
308	2	6	0.938	0.930	49
511	16	231	0.936	0.472	42
507	16	39	0.929	0.843	44
209	15	94	0.929	0.825	46
307	5	12	0.916	0.914	46
403	8	27	0.916	0.724	46
404	26	117	0.915	0.729	42
517	13	32	0.911	0.845	43
322	8	96	0.905	0.768	47
506	5	294	0.892	0.568	48
401	3	3	0.874	0.838	49

ID	WARP	GARP	Afriat	Varian	HM
607	37	179	0.870	0.712	37
212	5	111	0.866	0.697	47
305	17	182	0.852	0.695	45
608	23	549	0.847	0.570	29
324	18	453	0.840	0.657	29
606	18	241	0.839	0.470	44
518	26	121	0.816	0.732	43
201	16	147	0.797	0.526	42
321	27	375	0.757	0.356	44
325	27	702	0.739	0.398	32
328	21	559	0.705	0.401	33
504	29	794	0.697	0.355	33
310	22	241	0.690	0.366	43
603	12	322	0.686	0.229	47
406	39	881	0.653	0.225	30
211	83	669	0.611	0.361	34

Figure AIII1: The distributions of GARP violations Varian (1991)

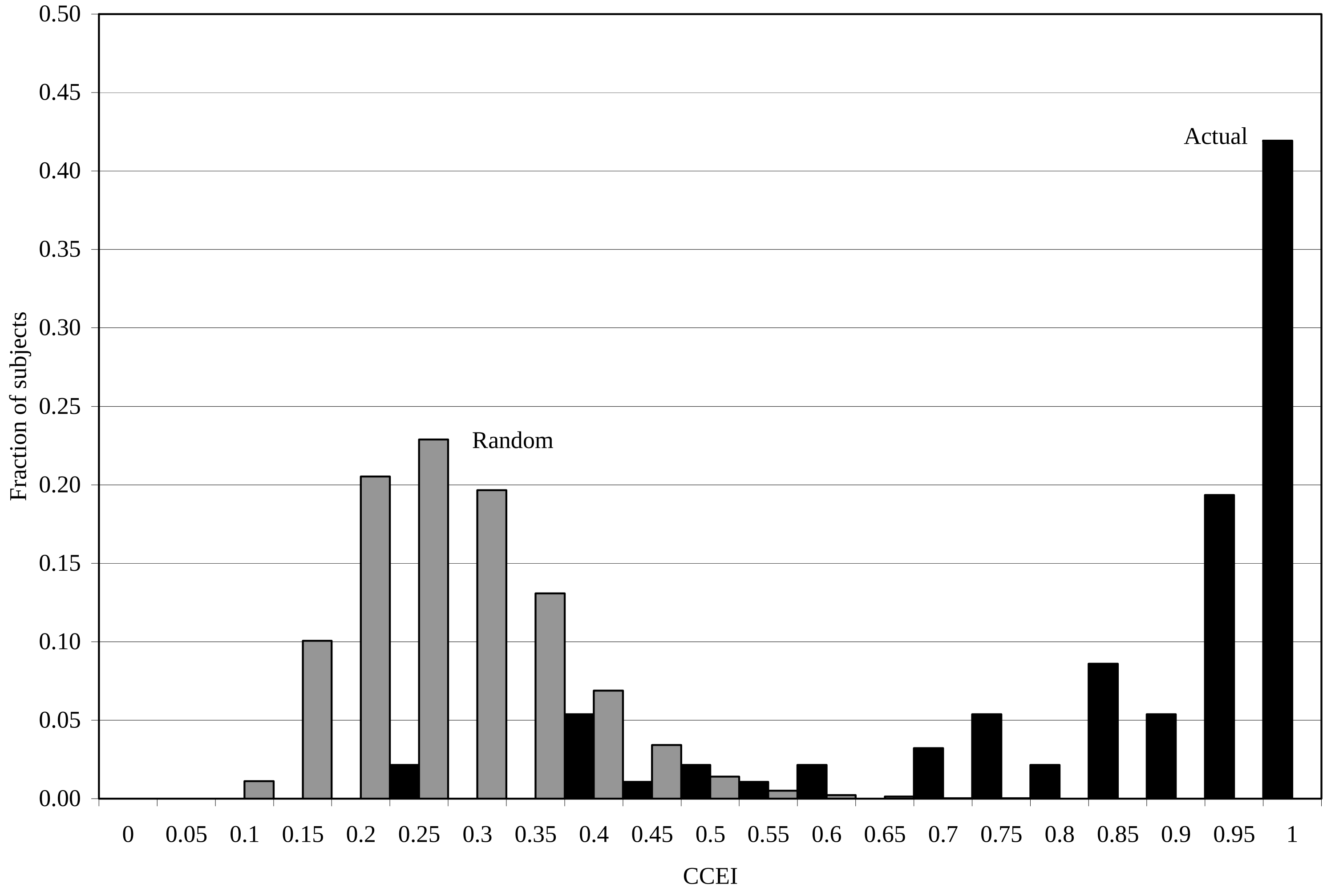

Figure AIII2: The distributions of GARP violations Houtman and Maks (1985)