Module I
The consumers

- Decision making under certainty (PR 3.1-3.4)
- Decision making under uncertainty (PR 5.1-5.3 and 5.5)

The producers

- Production (PR 6.1-6.4)
- The costs production (PR 7.1-7.2)
The consumers
Objectives

[1] Explain what economists mean by *rationality*, because that term is often misunderstood.

[2] Show that the techniques of economic analysis may be brought to bear on modeling and predicting behavior in many situations.

[3] The economic theory of the consumer can help managers to think *systematically* through their product decisions.
Prologue

Many people think that economists view people as being super-rational and find the material to be highly theoretical and not very “realistic”.

... theories do not have to be realistic to be useful...

Even though the assumptions are pretty unrealistic, the theory predicts behavior well and is quite useful.
A theory can be *useful* in three ways:

A. descriptive (how people actually choose)

B. prescriptive (as a practical aid to choice)

C. normative (how people ought to choose)
Decision making under certainty and uncertainty

The “standard” theory of the economic agent (consumer, manager, policy maker) is best understood as follows:

Preferences \[\rightarrow\] Choice \[\rightarrow\] Constraints

Information \[\rightarrow\] Choice \[\rightarrow\] Beliefs
Behavioral economics incorporate more “realistic” assumptions about decision making based on findings in psychology and related fields:

Preferences ←→ Constraints

Information ←→ Beliefs

↑
↑
↑

Choice

←→
Foundations of Economic Analysis (1947)

Paul A. Samuelson (1915-2009) – the first American Nobel laureate in economics and the foremost (academic) economist of the 20th century (and the uncle of Larry Summers...).
Consider some (finite) set of alternatives or consumption bundles or baskets \((x, y, z, \ldots)\).

- For our purposes it is convenient to consider only the case of two goods, since we can then depict the consumer’s choice behavior graphically.

- We denote a bundle by a single symbol like \(x\), where \(x\) is simply an abbreviation for a list of two numbers \((x_1, x_2)\).

**Bear in mind**: the two-good assumption is more general than you might think it first. Why?!
Formally, we represent the consumer’s preferences by a binary relation \( \succeq \) defined on the set of consumption bundles.

For any pair of bundles \( x \) and \( y \), if the consumer says that \( x \) is at least as good as \( y \), we write

\[
x \succeq y
\]

and say that \( x \) is weakly preferred to \( y \).

**Bear in mind**: economic theory often seeks to convince you with simple examples and then gets you to extrapolate. This simple construction works in wider (and wilder circumstances).
From the weak preference relation $\preceq$ we derive two other relations on the set of alternatives:

- **Strict performance relation**

  $x \succ y$ if and only if $x \succeq y$ and not $y \succeq x$.

  The phrase $x \succ y$ is read $x$ is *strictly preferred* to $y$.

- **Indifference relation**

  $x \sim y$ if and only if $x \preceq y$ and $y \preceq x$.

  The phrase $x \sim y$ is read $x$ is *indifferent* to $y$. 
The basic assumptions about preferences

The theory begins with three assumptions about preferences. These assumptions are so fundamental that we can refer to them as “axioms” of decision theory.

[1] Completeness

\[ x \succeq y \text{ or } y \succeq x \]

for any pair of bundles \( x \) and \( y \).

[2] Transitivity

if \( x \succeq y \) and \( y \succeq z \) then \( x \succeq z \)

for any three bundles \( x, y \) and \( z \).
Together, completeness and transitivity constitute the formal definition of *rationality* as the term is used in economics. Rational economic agents are ones who

have the ability to make choices [1], and whose choices display a logical consistency [2].

(Only) the preferences of a rational agent can be represented, or summarized, by a *utility function* (more later).
The third axiom about consumer’s preferences for one bundle versus another is that “more is better” (goods are desirable).


if $x_1 \geq y_1$ and $x_2 \geq y_2$ then $x \succeq y$

for any pair of bundles $x$ and $y$. 
Indifference curves

We next represent a consumer’s preferences graphically with the use of *indifference curves*.

The consumer is indifferent among all consumption bundles represented by the points graphed on the curve.

The set of indifference curves for all consumption bundles is called the *indifference map*.

– PR Figures 3.1-3.4 here –
The marginal rate of substitution (MRS)

The maximum amount of a good that a consumer is willing to give up in order to obtain one additional unit of another good.

The $MRS$ at any point is equal to the slope of the indifference curve at the point.

If indifference curves are “convex” (bowed inwards), then the $MRS$ falls as we move down the indifference curve, that is it diminishes along the curve.

– PR Figures 3.5-3.6 here –
Utility

A numerical score representing the satisfaction (or happiness?) that a consumer has from a given bundle.

An **ordinal** utility function replicates the consumer’s **ranking** of bundles – from most to least preferred.

\[ x \succeq y \text{ if and only if } u(x_1, x_2) \geq u(y_1, y_2). \]
The Cobb-Douglas utility function (and production function) is widely used to represent preferences

\[ u(x) = x_1^\alpha x_2^\beta \]

where \( \alpha, \beta > 0 \). (Can you draw the Cobb-Douglas indifference curves?)

Paul Douglas (1892-1976) – a University of Chicago economist and a Democratic U.S. Senator from Illinois who earned two Purple Heart medals in WWII (at the age of 50).
Budget sets (PR 3.2)

The budget set includes all bundles on which the total amount of money spent given the market prices $p_1$ and $p_2$ is less or equal to income $I$

$$p_1 x_1 + p_2 x_2 \leq I.$$ 

Rearranging,

$$x_2 \leq \frac{I}{p_2} - \frac{p_1}{p_2} x_1.$$ 

The slope of the budget line $-p_1/p_2$ is the negative of the ratio of the two prices.

– PR Figures 3.10-3.12 here –
Consumer choice (PR 3.3)

The optimal consumption bundle is at the point where an indifference curve is tangent to the budget line, that is

$$MRS = \frac{p_1}{p_2}.$$ 

But maximization is sometimes achieved at a so-called corner solution in which the equality above does not hold.

This is an important result that helps us understand and predict (using econometric tools) consumers’ purchasing decisions.

– PR Figures 3.13 and 3.15 here –
Revealed preferences (PR 3.4)

Economists test for consistency with maximization using revealed preference axioms.

Revealed preference techniques can be used to “recover” the underlying preferences and to forecast behavior in new situations.

The revealed preference barouche was first suggested by Paul Samuelson in his remarkable *Foundations of Economic Analysis* (1947).

– PR Figures 3.18-3.19 here –
Takeaways

• We explained what economists mean by rationality, because that term is often misunderstood.

• The techniques of economic analysis may be brought to bear on modeling and predicting behavior in many situations.

• Consumer theory can help managers to think systemically through their product decisions.
Decision making under uncertainty

- Uncertainty is a fact of life so people’s attitudes towards risk enter every realm of economic decision-making.

- We *must* study individual behavior with respect to choice involving uncertainty.

- Models of decision making under uncertainty play a key role in every field of economics.
Objectives

• Illustrate that agents (consumers and managers) frequently make decisions with uncertain consequences.

• Facing uncertain choices, maximizing the Expected Utility is how agents ought to choose.

• Individual behavior is often contrary to the assumptions of Expected Utility Theory.
Life is full of lotteries :-(

\[
\begin{align*}
x & : = & \frac{p}{1-p} & A \\
& & \frac{1-p}{1-p-q} & B
\end{align*}
\]

\[
\begin{align*}
y & : = & \frac{p}{q} & A \\
& & \frac{1-p-q}{1-p-q} & B \\
& & & C
\end{align*}
\]
A risky lottery (left) and an ambiguous lottery (right)

\[ x := \begin{cases} \frac{1}{2} & \Rightarrow \ A \\ \frac{1}{2} & \Rightarrow \ B \end{cases} \quad \begin{cases} ? & \Rightarrow \ A \\ 1-? & \Rightarrow \ B \end{cases} \]
A compounded lottery

\[ x := \begin{cases} q & \text{$A$} \\ p & \downarrow 1-q \\ 1-p & \downarrow l \\ 1-l & \downarrow $C$ \\ & \downarrow $D$ \end{cases} \]
The reduction of a compounded lottery

\[ x := \frac{q}{p} \frac{1}{1-q} \]

\[ \frac{p}{q} \frac{A}{p q} \]

\[ \frac{1-q}{B} \frac{p(1-q)}{1-p} \]

\[ \frac{l}{C} \frac{(1-p)l}{1-p} \]

\[ \frac{1-p}{D} \frac{(1-p)(1-l)}{1-l} \]
Risk (known probabilities) (PR 5.1)

Probability is the likelihood that a given outcome will occur. If there are two possible outcomes having payoffs $x_1$ and $x_2$ and the probabilities of these outcomes are $\pi_1$ and $\pi_2$, then the expected value is

$$E(x) = \pi_1 x_1 + \pi_2 x_2$$

where $\pi_1 + \pi_2 = 1$ (a probability distribution). More generally, when there are $n$ outcomes the expected value is

$$E(x) = \pi_1 x_1 + \pi_2 x_2 + \cdots + \pi_n x_n$$

where $\pi_1 + \pi_2 + \cdots + \pi_n = 1$. 
When there are two outcomes $x_1$ and $x_2$ occurring with probabilities $\pi_1$ and $\pi_2$ the variance is given by

$$\sigma^2 = \pi_1[x_1 - E(x)]^2 + \pi_2[x_2 - E(x)]^2,$$

and when there are $n$ outcomes $x_1, x_2, \ldots, x_n$ occurring with probabilities $\pi_1, \pi_2, \ldots, \pi_n$ the variance is given by

$$\sigma^2 = \pi_1[x_1 - E(x)]^2 + \pi_2[x_2 - E(x)]^2 + \cdots + \pi_n[x_n - E(x)]^2.$$

The *standard deviation* is the square root of the variance $\sigma$ and it is a standard measure of variability.
An example (PR Tables 5.1-5.3)

<table>
<thead>
<tr>
<th></th>
<th>Outcome 1</th>
<th></th>
<th>Outcome 2</th>
<th></th>
<th>$E(x)$</th>
</tr>
</thead>
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<tr>
<td></td>
<td>$p_1$</td>
<td>$x_1$</td>
<td>$p_2$</td>
<td>$x_2$</td>
<td></td>
</tr>
<tr>
<td>Job 1</td>
<td>.5</td>
<td>2000</td>
<td>.5</td>
<td>1000</td>
<td>1500</td>
</tr>
<tr>
<td>Job 2</td>
<td>.99</td>
<td>1510</td>
<td>.01</td>
<td>510</td>
<td>1500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Outcome 1</th>
<th></th>
<th>Outcome 2</th>
<th></th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_1$</td>
<td>$[x_1 - E(x)]^2$</td>
<td>$x_2$</td>
<td>$[x_2 - E(x)]^2$</td>
<td></td>
</tr>
<tr>
<td>Job 1</td>
<td>2000</td>
<td>250,000</td>
<td>1000</td>
<td>250,000</td>
<td>500</td>
</tr>
<tr>
<td>Job 2</td>
<td>1510</td>
<td>100</td>
<td>510</td>
<td>9,900</td>
<td>14</td>
</tr>
</tbody>
</table>
The paternity of decision theory and game theory (1944)
Preferences toward risk (PR 5.2)

The standard model of decisions under risk (known probabilities) is based on von Neumann and Morgenstern Expected Utility Theory.

Consider a set of lotteries, or gambles, (outcomes and probabilities). A fundamental axiom about preferences toward risk is independence:

For any lotteries $x, y, z$ and $0 < \alpha < 1$

$$x > y \text{ implies } \alpha x + (1 - \alpha)z > \alpha y + (1 - \alpha)z.$$
\[ x := \begin{cases} \frac{p_x}{1 - p_x} \end{cases} \quad \triangleright \quad y := \begin{cases} \frac{p_y}{1 - p_y} \end{cases} \]
\[ x + z := \]

\[
\begin{array}{c}
\begin{array}{c}
p \downarrow \hspace{2cm} \downarrow \hspace{2cm} 1-p_x \\
p_x \uparrow \hspace{2cm} \uparrow \hspace{2cm} A \\
1-p \downarrow \hspace{2cm} \uparrow \hspace{2cm} z \uparrow \hspace{2cm} E \\
1-p \downarrow \hspace{2cm} \uparrow \hspace{2cm} F \\
1-p_z \downarrow \hspace{2cm} \uparrow \hspace{2cm} 1-p_z \\
\end{array}
\end{array}
\]

\[ \Rightarrow \]

\[ y + z := \]

\[
\begin{array}{c}
\begin{array}{c}
p \downarrow \hspace{2cm} \downarrow \hspace{2cm} 1-p_y \\
p_y \uparrow \hspace{2cm} \uparrow \hspace{2cm} C \\
1-p \downarrow \hspace{2cm} \uparrow \hspace{2cm} z \uparrow \hspace{2cm} E \\
1-p \downarrow \hspace{2cm} \uparrow \hspace{2cm} F \\
1-p_z \downarrow \hspace{2cm} \uparrow \hspace{2cm} 1-p_z \\
\end{array}
\end{array}
\]
Expected Utility Theory has some very convenient properties for analyzing choice under uncertainty.

To clarify, we will consider the *utility* that a consumer gets from her or his income.

More precisely, from the consumption bundle that the consumer’s income can buy.
Expected utility is the sum of utilities associated with all possible outcomes, weighted by the probability that each outcome will occur.

In the job example above the expected utility from job 1 is given by

\[ E(u) = .5u($2000) + .5u($1000), \]

and the expected utility from job 1 is given by

\[ E(u) = .99u($1510) + .01u($510). \]

– PR Figures 5.3-5.4 here –
Behavioral economics (PR 5.5)

Allais (1953) I

- Choose between the two gambles:

$$x := \frac{33}{100} \rightarrow \$24,000$$

$$y := \frac{1}{100} \rightarrow \$24,000$$

$$0$$
Allais (1953) II

Choose between the two gambles:

\[ z := \begin{array}{c}
.33 \\
.67 \\
$0
\end{array} \quad \begin{array}{c}
.34 \\
.66 \\
$0
\end{array} \]

\[ \begin{array}{c}
$25,000 \\
$24,000
\end{array} \]
Ambiguity (unknown probabilities)

Ellsberg (1961)

An urn contains 300 marbles; 100 of the marbles are red, and 200 are some mixture of blue and green. We will reach into this urn and select a marble at random:

- You receive $25,000 if the marble selected is of a specified color. Would you rather the color be red or blue?

- You receive $25,000 if the marble selected is not of a specified color. Would you rather the color be red or blue?
Takeaways

- Consumers and managers frequently make decisions with uncertain consequences.

- Facing uncertain choices, Expected Utility consumers maximize the average expected utility associated with each outcome.

- Individual behavior is often contrary to the assumptions of Expected Utility Theory (an important frontier of choice theory).
The producers
Production (PR 6.1)

- In the production process, firms turn inputs (labor, capital, materials) into output.

- The theory of the firm explains how a firm makes (optimal) production decisions and how its costs vary with its output.

- Like the theory of the consumer, the theory of the firm can be used to predict, postdict (explain), and prescribe.
The production function

The production function indicates the (highest) output $q$ that the firm can produce for a given combination of inputs.

For our purposes it is convenient to consider only the case of two inputs – capital $K$ and labor $L$.

We can write the production function as

$$q = F(K, L).$$
The Cobb-Douglas production function is widely used to represent the technology of production

\[ q = K^\alpha \cdot L^\beta \]

where \( \alpha, \beta > 0 \).

The parameters of the Cobb-Douglas function, \( \alpha \) and \( \beta \), determine the marginal product of capital and labor.
The law of diminishing marginal returns (PR 6.4)

The law of diminishing marginal returns $\alpha, \beta < 1$ holds for most production processes:

\[
F'_{K} = \alpha K^{\alpha - 1} L^\beta > 0
\]
\[
F''_{K} = (\alpha - 1)\alpha K^{\alpha - 2} L^\beta < 0
\]

and

\[
F'_{L} = \beta L^{\beta - 1} K^{\alpha} > 0
\]
\[
F''_{L} = (\beta - 1)\beta L^{\beta - 2} K^{\alpha} < 0.
\]

In competitive markets $F'_{K}$ is the price of capital and $F'_{L}$ is the market wage (more in two weeks).
Returns to scale

Returns to scale is the rate at which output $q$ increases as inputs, $K$ and $L$, are increased:

- **constant** – output doubles when inputs are doubled
- **increasing** / **decreasing** – output *more* / *less* than doubles when inputs are doubled.

Returns to scale vary considerably across and within industries. Other things being equal, the greater the return to scale in an industry, the larger the firms are likely to be (a natural monopoly).
The Cobb-Douglas production function exhibits increasing, constant, or decreasing returns to scale if

\[ \alpha + \beta > 1 \]
\[ \alpha + \beta = 1 \]
\[ \alpha + \beta < 1 \]

respectively.

– PR Figures 6.6-6.10 and 7.3-7.8 here –
The short run vs. the long run (PR 6.1)

John Keynes: “The long run is a misleading guide to current affairs. In the long run we are all dead...”
• The “short” run is the period of time in which the quantities of one or more inputs cannot be changed.

• The “long” run is the period of time in which the quantities of all inputs can be changed.

• Typically, we assume that in the short run labor is variable and capital is fixed (more later!)
Costs (PR 7.1)

Question: What is the cost of an activity (producing a good or service)?

– The obvious answer might seem “the money spent on that activity.”

Like so many “obvious” answers, this is not (necessarily) the correct answer.

– The correct way to consider the cost to be the value of the most highly valued forgone activity the value of the best alternative decision).

⇒ Note: Economists describe this way of viewing cost as considering the opportunity cost of an activity or decision.
Accounting cost

- Actual expenses *plus* depreciation charges to capital (this naive view, focusing on expenditures can lead one to make bad decisions).

Economic cost

- Cost of utilizing inputs in production, including opportunity cost (for example, the imputed value of the forgone rent on owned office space).

⇒ **Note:** A sunk expenditure (cannot be recovered or avoided over the relevant decision-making horizon) is not an economic cost.
Example (PR 7.1)

Joe quits his computer programming job, where he was earning a salary of $50,000 per year, to start his own computer software business in a building that he owns and was previously renting out for $24,000 per year. In his first year of business he has the following expenses:

- $40,000 – salary (paid to himself).
- $0 – rent
- $25,000 – other expenses.

What are the accounting cost and the economic cost associated with Joe’s computer software business?
Cost concepts

Fixed cost \((FC)\) does not vary with the level of production and can be eliminated \textit{only} by shutting down (not sunk cost). Variable cost \((VC)\) varies as output varies. Total cost \((TC \text{ or } C)\) is the total economic cost of production

\[
TC(q) = FC + VC(q).
\]

Marginal cost \((MC)\) is the increase in cost resulting from the production of one extra unit of output

\[
MC = \frac{\Delta TC}{\Delta q} = \frac{\Delta VC}{\Delta q}.
\]
Relations among costs in the short run (PR 7.2)

We next explore the relations among total cost, average cost, and marginal cost from an algebraic and graphical perspectives.

Note that

\[ MC(q) = VC(q) - VC(q - 1) \]

and observe that we can express variable cost as follows

\[ VC(q) = MC(1) + \cdots + MC(q) = \sum_{j=1}^{q} MC(j). \]

The variable total of producing \( q \) units is the sum of the marginal costs of producing the first \( q \) units.
The average variable cost ($AVC$) of producing $q$ units is

$$AVC(q) = \frac{VC(q)}{q} = \frac{\sum_{j=1}^{q} MC(j)}{q}$$

and the average total cost ($ATC$) of producing $q$ units is

$$ATC(q) = \frac{TC(q)}{q} = \frac{VC(q)}{q} + \frac{FC}{q}.$$
Result  Marginal cost and average cost

(i) If average variable cost ($AVC$) is decreasing, then marginal cost ($MC$) is less than average variable cost.

(ii) If average variable cost ($AVC$) is increasing, then marginal cost ($MC$) is greater than average average variable cost.
Another way to state this conclusion is

\[ MC(q + 1) < AVC(q) \] then \[ AVC(q + 1) < AVC(q) \]

and

\[ MC(q + 1) > AVC(q) \] then \[ AVC(q + 1) > AVC(q) \]

– PR Figure 7.1 here –
Proof:

\[ AVC(q + 1) = \frac{\sum_{j=1}^{q+1} MC(j)}{q + 1} \]

\[ = \frac{MC(q + 1)}{q + 1} + \frac{\sum_{j=1}^{q} MC(j)}{q + 1} \]

\[ = \frac{MC(q + 1)}{q + 1} + \frac{q}{q + 1} AVC(q). \]

If \( MC(q + 1) \geq AVC(q) \) then \( AVC(q + 1) \geq AVC(q) \), respectively.
Costs in the short run and the long run

The relationship between short-run and long-run cost can be fairly complicated.

In the long run, the firm can change the quantity of any of its input, whereas in the short run the quantity of at least one input is fixed.

As a result, the long-run average cost curve never lies above any of the short-run average cost curves (an envelope).

– PR Figure 7.10 here –