Leftovers, review and takeaways
Lectures 13-14

Oct. 1, 2011
Pricing

While there is some involved analysis required, the important takeaways about optimal pricing are

- At the optimal quantity produced $q^*$, marginal revenue equals marginal cost

\[ MR(q^*) = MC(q^*) \]

- Marginal revenue comes from the underlying demands curve. Demand curves themselves come from consumer preferences.
Simple (nondiscriminatory) pricing

A firm engages in simple pricing for a particular product if that product is sold for the same price per unit no matter who the buyer is or how many units the buyer purchases.

The profit-maximizing quantity for the firm to produce (if it should be in business at all) $q^*$ satisfies:

$$(i) \quad MR(q^*) = MC(q^*)$$
$$(ii) \quad MR(q) > MC(q) \text{ for all } q < q^*$$
$$(iii) \quad MR(q) < MC(q) \text{ for all } q > q^*$$
Note that marginal profit,

\[ MR(q) - MC(q) \]

is positive for all \( q < q^* \), that is, every additional unit in this region contributes positively to total profit.

On the other hand, marginal profit is negative for all \( q > q^* \), that is, every additional unit in this region reduces total profit.

\[ \Rightarrow \] Increasing the total profit in the region \( q < q^* \) and descending the total profit in the region \( q > q^* \).
Profit-maximizing price and quantity

Price/unit ($P$)

$P(q^*)$

$MC(q)$

$MR(q)$

$q^*$

Demand $P(q)$
Total costs, profit, and consumer surplus

Price/unit \( (P) \)

Consumer Surplus

Demand \( P(q) \)

\( P(q^*) \)

Profit

Total cost

\( q^* \)

\( MC(q) \)

\( MR(q) \)

Quantity \( (q) \)
What simple pricing loses?

Price/unit ($P$)

Quantity ($q$)

Demand ($P(q)$)

Marginal Revenue ($MR(q)$)

Marginal Cost ($MC(q)$)

$q^*$

Deadweight loss
The Holy Grail of pricing

• If the firm can capture all the welfare generated from selling $q$ units, then the firm will want to produce $q^{**} > q^*$ such that $P(q^{**}) = MC(q^{**})$.

• Because this outcome is so good, any form of pricing that achieves this Holy Grail is known as perfect price discrimination.

• For historic reasons, perfect price discrimination is also known as first-degree price discrimination.

⇒ Can the firm ever obtain the Holy Grail? Generally, the answer is no!!!
Two-part tariffs

A two-part tariff is pricing with an entry fee and per-unit charge. It can help get a firm closer to the Grail than can simple pricing.

Formally, a two-part tariff consists of an entry fee $F$ and a per-unit charge $p$. A consumer’s expenditure if she buys $q$ units is given by

$$T(q) = \begin{cases} 
0 & \text{if } q = 0 \\
F + pq & \text{if } q > 0 
\end{cases}$$
If there are $N$ homogeneous (have identical demands) consumers, then under the profit-maximizing two-part tariff, the firm

- produces $q^{**}$ units, where $P(q^{**}) = MC(q^{**})$

- sets the per-unit charge $p$ to equal $P(q^{**})$

- sets the entry fee $F$ to equal average consumer surplus $CS/N$.

If consumers are heterogeneous, the firm can still profit from using a two-part tariff, but designing the optimal tariff is much more complicated...
Third- and second-degree price discrimination

Third-degree price discrimination is charging different prices on the basis of observed group membership.

- Examples: Senior-citizen/child/student discounts, and geography-based third-degree price discrimination.

Second-degree price discrimination is price discrimination via induced revelation of preferences.

- Examples: quantity discounts, quality distortions (an adverse selection problem!).
Bertrand’s oligopoly model (1883)

In Cournot’s game, each firm chooses an output, and the price is determined by the market demand in relation to the total output produced.

An alternative model, suggested by Bertrand, assumes that each firm chooses a price, and produces enough output to meet the demand it faces, given the prices chosen by all the firms.

⇒ As we shall see, some of the answers it gives are different from the answers of Cournot.
Suppose again that there are two firms (the industry is a “duopoly”) and that the cost for firm $i = 1, 2$ for producing $q_i$ units of the good is given by $cq_i$ (equal constant “unit cost”).

Assume that the demand function (rather than the inverse demand function as we did for the Cournot’s game) is

$$D(p) = A - p$$

for $A \geq p$ and zero otherwise, and that $A > c$ (the demand function in PR 12.3 is different).
Because the cost of producing each unit is the same, equal to $c$, firm $i$ makes the profit of $p_i - c$ on every unit it sells. Thus its profit is

$$\pi_i = \begin{cases} 
(p_i - c)(A - p_i) & \text{if } p_i < p_j \\
\frac{1}{2}(p_i - c)(A - p_i) & \text{if } p_i = p_j \\
0 & \text{if } p_i > p_j 
\end{cases}$$

where $j$ is the other firm.

In Bertrand’s game we can easily argue as follows: $(p_1, p_2) = (c, c)$ is the unique Nash equilibrium.
Using intuition,

– If one firm charges the price $c$, then the other firm can do no better than charge the price $c$.

– If $p_1 > c$ and $p_2 > c$, then each firm $i$ can increase its profit by lowering its price $p_i$ slightly below $p_j$.

⇒ In Cournot’s game, the market price decreases toward $c$ as the number of firms increases, whereas in Bertrand’s game it is $c$ (so profits are zero) even if there are only two firms (but the price remains $c$ when the number of firm increases).
Markets with asymmetric information

- The traditional theory of markets assumes that market participants have complete information about the underlying economic variables:
  
  - Buyers and sellers are both perfectly informed about the quality of the goods being sold in the market.
  
  - If it is not costly to verify quality, then the prices of the goods will simply adjust to reflect the quality difference.

⇒ This is clearly a drastic simplification!!!
• There are certainly many markets in the real world in which it may be very costly (or even impossible) to gain accurate information:
  – labor markets, financial markets, markets for consumer products, and more.

• If information about quality is costly to obtain, then it is no longer possible that buyers and sellers have the same information.

• The costs of information provide an important source of market friction and can lead to a market breakdown.
Nobel Prize 2001
“for their analyses of markets with asymmetric information”
The Market for Lemons

**Example I**

- Consider a market with 100 people who want to sell their used car and 100 people who want to buy a used car.

- Everyone knows that 50 of the cars are “plums” and 50 are “lemons.”

- Suppose further that

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<thead>
<tr>
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<th>seller</th>
<th>buyer</th>
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<tbody>
<tr>
<td>lemon</td>
<td>$1000</td>
<td>$1200</td>
</tr>
<tr>
<td>plum</td>
<td>$2000</td>
<td>$2400</td>
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– If it is easy to verify the quality of the cars there will be no problem in this market.

– Lemons will sell at some price $1000 − 1200 and plums will sell at $2000 − 2400.

– But happens to the market if buyers cannot observe the quality of the car?
If buyers are risk neutral, then a typical buyer will be willing to pay his expected value of the car

\[ \frac{1}{2} \times 1200 + \frac{1}{2} \times 2400 = $1800. \]

But for this price only owners of lemons would offer their car for sale, and buyers would therefore (correctly) expect to get a lemon.

Market failure – no transactions will take place, although there are possible gains from trade!
Example II

- Suppose we can index the quality of a used car by some number $q$, which is distributed uniformly over $[0, 1]$.

- There is a large number of demanders for used cars who are willing to pay $\frac{3}{2}q$ for a car of quality $q$.

- There is a large number of sellers who are willing to sell a car of quality $q$ for a price of $q$. 
If quality is perfectly observable, each used car of quality $q$ would be soled for some price between $q$ and $\frac{3}{2}q$.

What will be the equilibrium price(s) in this market when quality of any given car cannot be observed?

The unique equilibrium price is zero, and at this price the demand is zero and supply is zero.

The asymmetry of information has destroyed the market for used cars. But the story does not end here!!!
Signaling

• In the used-car market, owners of the good used cars have an incentive to try to convey the fact that they have a good car to the potential purchasers.

• Put differently, they would like choose actions that signal that they are offering a plum rather than a lemon.

• In some case, the presence of a “signal” allows the market to function more effectively than it would otherwise.
Example – educational signaling

– Suppose that a fraction $0 < b < 1$ of workers are competent and a fraction $1 - b$ are incompetent.

– The competent workers have marginal product of $a_2$ and the incompetent have marginal product of $a_1 < a_2$.

– For simplicity we assume a competitive labor market and a linear production function

$$L_1 a_1 + L_2 a_2$$

where $L_1$ and $L_2$ is the number of incompetent and competent workers, respectively.
– If worker quality is observable, then firm would just offer wages

\[ w_1 = a_1 \text{ and } w_2 = a_2 \]

to competent workers, respectively.

– That is, each worker will be paid his marginal product and we would have an efficient equilibrium.

– But what if the firm cannot observe the marginal products so it cannot distinguish the two types of workers?
– If worker quality is unobservable, then the “best” the firm can do is to offer the average wage

\[ w = (1 - b)a_1 + ba_2. \]

– If both types of workers agree to work at this wage, then there is no problem with adverse selection (more below).

– The incompetent (resp. competent) workers are getting paid more (resp. less) than their marginal product.
– The competent workers would like a way to signal that they are more productive than the others.

– Suppose now that there is some signal that the workers can acquire that will distinguish the two types.

– One nice example is education – it is cheaper for the competent workers to acquire education than the incompetent workers.
To be explicit, suppose that the cost (dollar costs, opportunity costs, costs of the effort, etc.) to acquiring $e$ years of education is

$$c_1e \text{ and } c_2e$$

for incompetent and competent workers, respectively, where $c_1 > c_2$.

Suppose that workers conjecture that firms will pay a wage $s(e)$ where $s$ is some increasing function of $e$.

Although education has no effect on productivity (MBA?), firms may still find it profitable to base wage on education – attract a higher-quality work force.
In the educational signaling example, there appear to be several possibilities for equilibrium:

[1] The (representative) firm offers a single contract that attracts both types of workers.

[2] The (representative) firm offers a single contract that attracts only one type of workers.

[3] The (representative) firm offers two contracts, one for each type of workers.
• A **separating equilibrium** involves each type of worker making a choice that separate himself from the other type.

• In a **pooling equilibrium**, in contrast, each type of workers makes the same choice, and all getting paid the wage based on their average ability.

Note that a separating equilibrium is wasteful in a social sense – no social gains from education since it does not change productivity.
Example (cont.)

Let \( e_1 \) and \( e_2 \) be the education level actually chosen by the workers. Then, a separating (signaling) equilibrium has to satisfy:

[1] zero-profit conditions

\[
\begin{align*}
    s(e_1) &= a_1 \\
    s(e_2) &= a_2
\end{align*}
\]

[2] self-selection conditions

\[
\begin{align*}
    s(e_1) - c_1 e_1 &\geq s(e_2) - c_1 e_2 \\
    s(e_2) - c_2 e_2 &\geq s(e_1) - c_2 e_1
\end{align*}
\]
In general, there may be many functions $s(e)$ that satisfy conditions [1] and [2]. One wage profile consistent with separating equilibrium is

$$s(e) = \begin{cases} 
  a_2 & \text{if } e > e^* \\
  a_1 & \text{if } e \leq e^*
\end{cases}$$

and

$$\frac{a_2 - a_1}{c_2} > e^* > \frac{a_2 - a_1}{c_1}$$

$\implies$ Signaling can make things better or worse — each case has to examined on its own merits!
The Sheepskin (diploma) effect

The increase in wages associated with obtaining a higher credential:

– Graduating high school increases earnings by 5 to 6 times as much as does completing a year in high school that does not result in graduation.

– The same discontinuous jump occurs for people who graduate from collage.

– High school graduates produce essentially the same amount of output as non-graduates.
Example – quality choice

– Next we consider a variation of the lemons model where quality may be determined by the producers.

– Suppose that each consumer wants to buy a single unit and that there are two different qualities available:

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<td>high</td>
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</tr>
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<td>low</td>
<td>$800</td>
<td>$1150</td>
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If the industry is perfectly competitive (zero profits), then what we would expect to be the equilibrium quality produced?
– If the fraction of high-quality producers is $q$, then a risk-neutral consumer would be willing to pay

$$p = 1400q + 800(1 - q).$$

– For both qualities to be produced we must have $p \geq 1150$. The lowest value of $q$ that satisfies this inequality is $q = \frac{7}{12}$.

– The equilibrium value of $q$ is between $\frac{7}{12}$ and 1. But these equilibria are not equivalent from the social point of view.