UC Berkeley
Haas School of Business
Berkeley MBA for Executives Program

Game Theory
(XMBA 296)

Block 5
Social learning
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“Men nearly always follow the tracks made by others and proceed in their affairs by imitation.” Machiavelli (Renaissance philosopher)
Examples

Business strategy

– TV networks make introductions in the same categories as their rivals.

Finance

– The withdrawal behavior of small number of depositors starts a bank run.
Politics

– The solid New Hampshirites (probably) can not be too far wrong.

Crime

– In NYC, individuals are more likely to commit crimes when those around them do.
Why should individuals behave in this way?

Several “theories” explain the existence of uniform social behavior:

– benefits from conformity

– sanctions imposed on deviants

– network / payoff externalities

– social learning

Broad definition: any situation in which individuals learn by observing the behavior of others.
The canonical model of social learning

- Rational (Bayesian) behavior
- Incomplete and asymmetric information
- Pure information externality
- Once-in-a-lifetime decisions
- Exogenous sequencing
- Perfect information / complete history
Coin flip

Urn A
- a
- a
- b

Urn B
- a
- b
- b

1/2 1/2
Bayes’ rule

Let \( n \) be the number of \( a \) signals and \( m \) be the number of \( b \) signals. Then Bayes’ rule can be used to calculate the posterior probability of urn \( A \):

\[
\Pr(A \mid n, m) = \frac{\Pr(A) \Pr(n, m \mid A)}{\Pr(A) \Pr(n, m \mid A) + \Pr(B) \Pr(n, m \mid B)} = \frac{\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^n\left(\frac{1}{3}\right)^m}{\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^n\left(\frac{1}{3}\right)^m + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)^m\left(\frac{2}{3}\right)^n} = \frac{2^n}{2^n + 2^m}.
\]
An example

• There are two decision-relevant events, say $A$ and $B$, equally likely to occur *ex ante* and two corresponding signals $a$ and $b$.

• Signals are informative in the sense that there is a probability higher than $1/2$ that a signal matches the label of the realized event.

• The decision to be made is a prediction of which of the events takes place, basing the forecast on a private signal and the history of past decisions.
• Whenever two consecutive decisions coincide, say both predict \( A \), the subsequent player should also choose \( A \) even if his signal is different \( b \).

• Despite the asymmetry of private information, eventually every player imitates her predecessor.

• Since actions aggregate information poorly, despite the available information, such herds / cascades often adopt a suboptimal action.
What have we learned from Social Learning?

• Finding 1
  – Individuals ’ignore’ their own information and follow a herd.

• Finding 2
  – Herds often adopt a wrong action.

• Finding 3
  – Mass behavior may be idiosyncratic and fragile.
Informational cascades and herd behavior

Two phenomena that have elicited particular interest are informational cascades and herd behavior.

– Cascade: agents 'ignore' their private information when choosing an action.

– Herd: agents choose the same action, not necessarily ignoring their private information.
• While the terms informational cascade and herd behavior are used interchangeably there is a significant difference between them.

• In an informational cascade, an agent considers it optimal to follow the behavior of her predecessors without regard to her private signal.

• When acting in a herd, agents choose the same action, not necessarily ignoring their private information.

• Thus, an informational cascade implies a herd but a herd is not necessarily the result of an informational cascade.
A model of social learning

Signals

– Each player \( n \in \{1, \ldots, N\} \) receives a signal \( \theta_n \) that is private information.

– For simplicity, \( \{\theta_n\} \) are independent and uniformly distributed on \([-1, 1]\).

Actions

– Sequentially, each player \( n \) has to make a binary irreversible decision \( x_n \in \{0, 1\} \).
Payoffs

- $x = 1$ is profitable if and only if $\sum_{n \leq N} \theta_n \geq 0$, and $x = 0$ is profitable otherwise.

Information

- Perfect information

$$\mathcal{I}_n = \{\theta_n, (x_1, x_2, \ldots, x_{n-1})\}$$

- Imperfect information

$$\mathcal{I}_n = \{\theta_n, x_{n-1}\}$$
A three-agent example
A three-agent example

\[ \hat{\theta}_1 \quad \hat{\theta}_2 \quad \hat{\theta}_3 \]

\[ x = 0 \]

\[ x = 1 \]

\[ x = 1/2 \]

\[ x = -1/2 \]
A three-agent example under perfect information
A three-agent example under imperfect information
A sequence of cutoffs under imperfect and perfect information
A sequence of cutoffs under imperfect and perfect information
The decision problem

- The optimal decision rule is given by

\[ x_n = 1 \text{ if and only if } \mathbb{E} \left[ \sum_{i=1}^{N} \theta_i \mid \mathcal{I}_n \right] \geq 0. \]

Since \( \mathcal{I}_n \) does not provide any information about the content of successors’ signals, we obtain

\[ x_n = 1 \text{ if and only if } \mathbb{E} \left[ \sum_{i=1}^{n} \theta_i \mid \mathcal{I}_n \right] \geq 0 \]

Hence,

\[ x_n = 1 \text{ if and only if } \theta_n \geq -\mathbb{E} \left[ \sum_{i=1}^{n-1} \theta_i \mid \mathcal{I}_n \right]. \]
The cutoff process

- For any \( n \), the optimal strategy is the \textit{cutoff strategy}

\[
x_n = \begin{cases} 
1 & \text{if } \theta_n \geq \hat{\theta}_n \\
0 & \text{if } \theta_n < \hat{\theta}_n
\end{cases}
\]

where

\[
\hat{\theta}_n = -\mathbb{E} \left[ \sum_{i=1}^{n-1} \theta_i \mid \mathcal{I}_n \right]
\]

is the optimal history-contingent cutoff.

- \( \hat{\theta}_n \) is sufficient to characterize the individual behavior, and \( \{\hat{\theta}_n\} \) characterizes the social behavior of the economy.
Overview of results

Perfect information

- A cascade need not arise, but herd behavior must arise.

Imperfect information

- Herd behavior is impossible. There are periods of uniform behavior, punctuated by increasingly rare switches.
• The similarity:
  – Agents can, for a long time, make the same (incorrect) choice.

• The difference:
  – Under perfect information, a herd is an absorbing state. Under imperfect information, continued, occasional and sharp shifts in behavior.
• The dynamics of social learning depend crucially on the extensive form of the game.

• The key economic phenomenon that imperfect information captures is a succession of fads starting suddenly, expiring rather easily, each replaced by another fad.

• The kind of episodic instability that is characteristic of socioeconomic behavior in the real world makes more sense in the imperfect-information model.
As such, the imperfect-information model gives insight into phenomena such as manias, fashions, crashes and booms, and better answers such questions as:

– Why do markets move from boom to crash without settling down?

– Why is a technology adopted by a wide range of users more rapidly than expected and then, suddenly, replaced by an alternative?

– What makes a restaurant fashionable over night and equally unexpectedly unfashionable, while another becomes the ‘in place’, and so on?
The case of perfect information

The optimal history-contingent cutoff rule is

\[
\hat{\theta}_n = -\mathbb{E} \left[ \sum_{i=1}^{n-1} \theta_i \mid x_1, \ldots, x_{n-1} \right],
\]

and \( \hat{\theta}_n \) is different from \( \hat{\theta}_{n-1} \) only by the information reveals by the action of agent \((n - 1)\)

\[
\hat{\theta}_n = \hat{\theta}_{n-1} - \mathbb{E} \left[ \theta_{n-1} \mid \hat{\theta}_{n-1}, x_{n-1} \right],
\]

The cutoff dynamics thus follow the cutoff process

\[
\hat{\theta}_n = \begin{cases} 
\frac{-1+\hat{\theta}_{n-1}}{2} & \text{if } x_{n-1} = 1 \\
\frac{1+\hat{\theta}_{n-1}}{2} & \text{if } x_{n-1} = 0
\end{cases}
\]

where \( \hat{\theta}_1 = 0 \).
Informational cascades

- $-1 < \hat{\theta}_n < 1$ for any $n$ so any player takes his private signal into account in a non-trivial way.

Herd behavior

- $\{\hat{\theta}_n\}$ has the martingale property by the Martingale Convergence Theorem a limit-cascade implies a herd.
The case of imperfect information

The optimal history-contingent cutoff rule is

$$\hat{\theta}_n = -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid x_{n-1}\right],$$

which can take two values conditional on $x_{n-1} = 1$ or $x_{n-1} = 0$

$$\overline{\theta}_n = -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid x_{n-1} = 1\right],$$

$$\underline{\theta}_n = -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid x_{n-1} = 1\right].$$

where $\overline{\theta}_n = -\underline{\theta}_n$. 
The law of motion for $\bar{\theta}_n$ is given by

$$\bar{\theta}_n = P(x_{n-2} = 1|x_{n-1} = 1) \left\{ \bar{\theta}_{n-1} - \mathbb{E} [\theta_{n-1} | x_{n-2} = 1] \right\}$$
$$+ P(x_{n-2} = 0|x_{n-1} = 1) \left\{ \theta_{n-1} - \mathbb{E} [\theta_{n-1} | x_{n-2} = 0] \right\},$$

which simplifies to

$$\bar{\theta}_n = \frac{1 - \bar{\theta}_{n-1}}{2} \left[ \bar{\theta}_{n-1} - \frac{1 + \bar{\theta}_{n-1}}{2} \right]$$
$$+ \frac{1 - \theta_{n-1}}{2} \left[ \theta_{n-1} - \frac{1 + \theta_{n-1}}{2} \right].$$
Given that $\bar{\theta}_n = -\bar{\theta}_n$, the cutoff dynamics under imperfect information follow the cutoff process

$$\hat{\theta}_n = \begin{cases} 
\frac{-1 + \hat{\theta}_{n-1}^2}{2} & \text{if } x_{n-1} = 1 \\
\frac{1 + \hat{\theta}_{n-1}^2}{2} & \text{if } x_{n-1} = 0
\end{cases}$$

where $\hat{\theta}_1 = 0$. 
Informational cascades

- $-1 < \hat{\theta}_n < 1$ for any $n$ so any player takes his private signal into account in a non-trivial way.

Herd behavior

- $\{\hat{\theta}_n\}$ is not convergent (proof is hard!) and the divergence of cutoffs implies divergence of actions.

- Behavior exhibits periods of uniform behavior, punctuated by increasingly rare switches.
Sequential social-learning model:
Well heck, if all you smart cookies agree, who am I to dissent?
Imperfect information:
Which way is the wind blowing?!