1. For the following production functions calculate the marginal products, the technical rates of substitution and indicate whether they have increasing, decreasing or constant returns to scale:
   a. \( x_1^{1/4} x_2^{3/4} \)
   b. \( x_1 + (x_2)^{1/2} \)
   c. \( (x_1^{1/3} + x_2^{1/3})^3 \)

2. Suppose a particular company has the production function \( Y = \min\{L, 2K\} \).
   a. Does this production function exhibit decreasing, increasing or constant returns to scale? Explain
   b. Draw a few isoquants for this production function. \( L \) on the \( x \)-axis.
   c. If the firm wants to produce ten units, calculate input demands? How do these demands depend on the relative prices of capital and labor in this case? Explain.

3. A firm has two variable factors and a production function \( f(x_1, x_2) = (2x_1 + 4x_2)^{1/2} \).
   a. On a graph with factor 1 on the \( x \)-axis, draw the production isoquants corresponding to an output of 3 and to an output of 4.
   b. If the price of the output good is 4, the price of factor 1 is 2, and the price of factor 2 is 3, find the profit-maximizing amount of factor 1, of factor 2 and the profit-maximizing output.
   Hint: Pay close attention to the shape of the isoquants.

4. A firm produces a soft drink using two ingredients, sugar (\( S \)) and bubbly water (\( B \)) in fixed proportions: 6 tablespoons of sugar per 12 oz. of bubbly water.
   a. What is the production function (Hint: Be careful, maybe a graph can be helpful)
   b. Does this production function exhibit constant, increasing or decreasing returns to scale? Explain.
   c. Write down the firm's cost minimization problem and solve for the conditional factor demands, \( S(w_S, w_B, y) \) and \( B(w_S, w_B, y) \).
   d. Solve for the long run cost function.