Problem Set 8

1. True/False: state your reasons clearly and succinctly.
   a) A profit-maximizing firm will always minimize costs.
   b) In the long run a firm always operates at the minimum level of average costs for the optimally sized plant to produce a given amount of output.

2. Assume downward sloping or flat demand and a U-shaped LRAC curve. In each of the following situations, determine graphically and/or verbally:
   a) Does the firm have the cost-minimizing amount of capital given its output level? If not, should the firm increase or decrease its amount of capital given its output?
   b) Does the firm have the profit maximizing level of output given its amount of capital? If not, should the firm increase or decrease its level or output, given its capital?
   If the situation is impossible, state why.
   i) SRAC > LRAC, SRMC > LRMC, MR = SRMC
   ii) SRAC > LRAC, SRMC < LRMC, MR = SRMC
   iii) SRAC < LRAC, SRMC > LRMC, MR = SRMC
   iv) SRAC > LRAC, SRMC > LRMC, MR > SRMC
   v) SRAC > LRAC, SRMC < LRMC, MR > SRMC
   vi) SRAC = LRAC, SRMC = LRMC, MR > SRMC
   vii) SRAC = LRAC, SRMC > LRMC, MR = SRMC
   viii) SRAC = LRAC, SRMC = LRMC, MR = SRMC

3. A firm has a production technology given by: \( Q = L^{0.25} \cdot K^{0.25} \). Initial input prices are given by \( w = 1 \) and \( r = 1 \). Suppose for the moment that the amount of capital is fixed at \( K = 4 \).
   a) Is the marginal product of labor diminishing? Why?
   b) Find the short run total cost function, and then the short-run marginal and average cost curves.
   c) Suppose now that the period is long enough that both inputs can be adjusted.
   d) Find the capital-labor ratio.
   e) Find the returns to scale of this production technology.
   f) If the rental price of capital is doubled, what happens to the profit maximizing output level?

4. Suppose a firm faces a cost function of \( C = 8 + 4q + q^2 \), so that its marginal cost is \( MC = 4 + 2q \).
   a) What is the firm’s fixed cost, \( F \)?
   b) What is the formula for the firm’s variable cost, \( VC \)?
   c) What is the formula for the average cost, \( AC \)?
   d) What is the formula for average variable cost, \( AVC \)?
   e) On a diagram, draw the \( AC \), \( AVC \), and \( MC \) curves.
5. Suppose a firm’s short run cost curves were found to be:

Total Cost = SRTC = 1 + 2Q + Q^2

Marginal Cost = SRMC = 2 + 2Q , where Q is output.

Assume the firm behaves as a price-taker and sells its output at P = $8 per unit (that is, its demand curve is flat at P = $8).

a) If the firm maximizes profits, how much will it produce?
b) What are the marginal, average, and total cost at that point?
c) What is the firm’s profit?