1. Define \( x = [1 \ p \ w \ a \ s]' \), \( \beta = [\beta_1 \ \beta_2 \ \beta_3 \ \beta_4 \ \beta_5]' \), \( \theta = [\theta' \ \sigma]' \)

(a) \( L(\theta) = d^* \log \left[ \Phi \left( \frac{x'\beta-f}{\sigma} \right) \right] + (1-d)^* \log \left[ 1 - \Phi \left( \frac{x'\beta-f}{\sigma} \right) \right] \)

(b) Define \( y_i = \text{Ind}(i \ \in \ \text{sample of actual purchases}) \). The LR statistic below is asymptotically distributed as an \( \chi^2 \) under the null:

\[
LR = 2 \left[ \max_{\beta_1, \sigma, \sigma_h} \sum_{i=1}^{N+M} \left( \Phi \left( \frac{x_i'\beta-f}{\sigma} \right) + (1-d_i)^* \log \left[ 1 - \Phi \left( \frac{x_i'\beta-f}{\sigma} \right) \right] \right) \right] \\
- \max_{\beta, \sigma} \left[ \sum_{i=1}^{N+M} \left( \Phi \left( \frac{x_i'\beta-f}{\sigma} \right) + (1-d_i)^* \log \left[ 1 - \Phi \left( \frac{x_i'\beta-f}{\sigma} \right) \right] \right) \right]
\]

(c) \( LR = 2 \left[ \max_{\beta_1, \sigma, \sigma_h} \sum_{i=1}^{N+M} \left( \Phi \left( \frac{x_i'\beta-f}{\sigma} \right) + (1-d_i)^* \log \left[ 1 - \Phi \left( \frac{x_i'\beta-f}{\sigma} \right) \right] \right) \right] \rightarrow \chi^2

2. Using WESML approach:

\[
\hat{\mu} = \sum_{y_i = \text{log}(20k)}^{+} \frac{y_i}{9} \sum_{y_i = \text{log}(20k)}^{\text{log}(20k)} \frac{y_i}{.6} \\
\sum_{y_i = \text{log}(20k)}^{+} \frac{1}{9} \sum_{y_i = \text{log}(20k)}^{\text{log}(20k)} \frac{1}{.6}
\]

3. (a) It will not be consistent as long as \( z \) is not clean. This condition does not depend on the true value of \( y \).

(b) Do the omitted variable version of the Hausman Test: Regress \( y \) on \( 1, x, z \) and \( z \# \) and test whether the coefficient of \( z \# \) is zero by means of a Wald test.

4. (a) If \( X_1 = X_2 = X \), then: \( \hat{\epsilon}_i = Q \epsilon_i \), \( \hat{\epsilon}_2 = Q \epsilon_2 \), \( Q = I - X(X'X)^{-1}X' \) and \( \text{tr}(E[\hat{\epsilon}_i \hat{\epsilon}_2']) = \text{tr}(Q) = (T-K) \sigma_{12} \neq 0 \)

(b) \( \hat{\beta}_j = \beta_j - (X_j'X_j)^{-1}X_j' \hat{\epsilon}_j \), \( j = 1, 2 \). If \( X_1, X_2 = 0 \), then \( E[(\hat{\beta}_1 - \beta_1)(\hat{\beta}_2 - \beta_2)] = (X_1'X_1)^{-1}X_1'X_2' \sigma_{12} = 0 \)

5. (a) Let \( z_i \) be any random variable. Then the following must hold:

\[ \text{cov}(z_i, \hat{\epsilon}_i) = \text{cov}(z_i, \epsilon_i) - (\hat{\beta}_2 - \beta_2) \text{cov}(z_i, w_i) \]

And, asymptotically, \( \text{cov}(z_i, \hat{\epsilon}_i) = \text{cov}(z_i, \epsilon_i) \). Thus, for \( z_i = \epsilon_2i \):

\[ \text{cov}(\epsilon_2i, \hat{\epsilon}_i) = \text{cov}(\epsilon_2i, \epsilon_i) = 0 \]

And for \( z_i = w_i \):

\[ \text{cov}(w_i, \hat{\epsilon}_i) = \text{cov}(w_i, \epsilon_i) \neq 0 \]

(b) From (a) we can see that \( \hat{\epsilon}_i \) is a proper instrument for \( w_i \), then supply equation can be estimated by 2SLS using \( 1, m_i \) and \( \hat{\epsilon}_i \) as instruments.