PROBLEM SET 5 (due October 9)

1. In a two-good, two-consumer exchange economy, consumer 1 has utility $u^1 = a^1 + \log b^1$ and endowment vector $(1,0)$, and consumer 2 has utility $u^2 = \log a^2 + b^2$ and endowment vector $(0,1)$.

   (A) Analyze very carefully the Marginal Rate of Substitution ($MRS = -\frac{db}{da}\big|_{u=\text{const}}$) of the first consumer. Show that it is constant along every horizontal line (i.e., fixed $b^1$) in this consumer’s consumption set, with $MRS^1 = b^1$. By symmetry, the MRS of consumer 2 is constant on every vertical line in his consumption set, with $MRS^2 = \frac{1}{a^2}$. Draw an Edgeworth box for this economy. Show that no point in the interior of the box is on the contract curve, but the north and west boundaries are. Show that the core of this economy is the entire contract curve.

   (B) Show that the offer curve of the first consumer from his endowment is his vertical axis. By symmetry, the offer curve of the second consumer from his endowment is his horizontal axis. Verify that the intersection of these offer curves at $(0,1)$ for the first consumer and $(1,0)$ for the second consumer, corresponding to prices $p^1 = p^2$, is a Walrasian equilibrium.

   (CC) Discuss qualitatively and graphically what the contract curve and offer curves would look like in this economy if the endowment of consumer 1 were reduced or increased. (Do a simple graphical analysis, avoid difficult analytics.)

2. JR, Exercise 5.1(a-c).

3. JR, Exercise 5.12.