PRODUCTION ECONOMICS:
A DUAL APPROACH TO THEORY AND APPLICATIONS
VOLUME 2
CONTRIBUTIONS
TO
ECONOMIC ANALYSIS

111

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INTRODUCTION TO THE SERIES

This series consists of a number of hitherto unpublished studies, which are introduced by the editors in the belief that they represent fresh contributions to economic science.

The term 'economic analysis' as used in the title of the series has been adopted because it covers both the activities of the theoretical economist and the research worker.

Although the analytical methods used by the various contributors are not the same, they are nevertheless conditioned by the common origin of their studies, namely theoretical problems encountered in practical research. Since for this reason, business cycle research and national accounting, research work on behalf of economic policy, and problems of planning are the main sources of the subjects dealt with, they necessarily determine the manner of approach adopted by the authors. Their methods tend to be 'practical' in the sense of not being too far remote from application to actual economic conditions. In addition they are quantitative rather than qualitative.

It is the hope of the editors that the publication of these studies will help to stimulate the exchange of scientific information and to reinforce international cooperation in the field of economics.

The Editors
The traditional starting point of production theory is a set of physical technological possibilities, often described by a production or transformation function. The development of the theory then parallels the process of firm operation, with the firm seeking to achieve its goals subject to the limitation of its technology and of the economic environment. The results are constructed input demands and output supplies, expressed as functions of the technology and the economic environment.

An alternative approach to production theory is to start directly from observed economic data—supplies, demands, prices, costs, and profits. The advantage of such an attack is that the theory can be formulated directly in terms of the causal economic relationships that are presumed to hold, without the intervening constructive steps required in the traditional theory. Because this approach is not bound by computational tractability in the step from production technology to economic observations, the prospect is opened for more satisfactory models of complex production problems.

It would at first appear that a theory of production couched in terms of economic observables would be less fundamental than one based on the physical technology, and that one could never be sure in an economic theory of consistency with a physical model. However, the theory of production duality establishes that the two approaches are equivalent and equally fundamental. Using duality, the technology underlying an economic model can be reconstructed and tested for compatibility with physical laws, as necessary. Then, the main thrust of analysis can be devoted to developing the structure and relationships of observed economic variables.

The purpose of these volumes is to develop the theory of production from the standpoint of the “dual”—the relationships between economic observables which are dual to the physical technology. The spirit of our treatment is the view that the end purpose of production theory is econometric study of economic problems involving technological limitations. The volumes emphasized the empirical implications of the theory, and therefore the development of the theoretical concepts proceeds with
an eye towards the econometric framework inherent in empirical applications. We hold the view that there is an intimate, symbiotic relationship between theory and econometrics, and that development of a fully successful economic analysis of production requires an integration of theoretical and econometric ideas in a unified approach. The papers in the two volumes of *Production Economics* represent an attempt to achieve this ideal.

The theory of production duality had its beginnings in the work of Hotelling (1932), Hicks (1946), Roy (1942), and Samuelson (1947). A pioneering book by Shephard (1953) provided the first comprehensive treatment of the subject and proof of the basic duality of cost and production. Extensions of the formal theory of duality were later made by McFadden (1962), Uzawa (1964), Shephard (1970), and Diewert (1971). Many of the basic duality results were also obtained by Gorman (1970), working independently. In a paper on the estimation of returns to scale, Nerlove (1963) utilized a cost function to derive econometric estimating equations. Subsequent work by McFadden (1964), Diewert (1969a,b), Christensen, Jorgenson and Lau (1971), and others have established the use of dual cost and profit functions as a basic tool in econometric production analysis.

It is possible to trace the origins of the present volumes back to 1961 when D. McFadden worked as a research assistant to M. Nerlove and H. Uzawa at Stanford University. The contributions of Uzawa (1962, 1964), McFadden (1962, 1963), and Nerlove (1963) date from that period. The empirical implications of duality theory were developed in McFadden (1964 and 1966). The first explicit empirical application of dual flexible functional forms appeared in Diewert's (1969a) study of labor demand functions for the Canadian Department of Manpower and Immigration. The generalized Leontief function [Diewert (1971)] was introduced in that study. The subsequent generation and empirical application of flexible functional forms received their major impetus from McFadden (1966) and Diewert (1969a,b).

Applications of the basic duality concepts continued to evolve at the University of California, Berkeley, during the years 1968–1970 under the auspices of the Project for the Optimization and Evaluation of Economic Growth. The introduction of the translog function by Christensen, Jorgenson, and Lau (1971, 1973), the nested generalized Leontief form by Fuss (1970, 1977b), the hybrid generalized Leontief form by Hall (1973), and the generalized CES form by Denny (1974a) all result from research begun at that time. A. Belinfante, T. Cowing, and P. Frenger also
were associated with the Economic Growth Project at various times. M. Bruno was a visiting scholar at M.I.T., together with D. McFadden, in 1971 when his chapter was written.

The idea of collecting a group of studies in duality under a common cover grew out of a seminar series held at the Economic Growth Project during the summer of 1969. A tentative title, *An Econometric Approach to Production Theory*, was chosen at that time. A number of the papers which appear in this volume have been referenced under that title. Since that time, the contents of the volumes evolved through several additions and deletions and M. Fuss joined D. McFadden as a co-editor. We feel that the current title more accurately reflects the spirit and content of the books.

*Production Economics* is divided into two main parts. *Volume 1* contains the basic theoretical analysis of the duality of cost, profit, and production and a number of investigations of specific functional forms. *Volume 2* contains the empirical applications. In keeping with the spirit of this work, these applications draw heavily on the analysis of *Volume 1*. Details of the contents of both volumes can be found in the two introductions.

The editors have been unable to standardize notation throughout the volumes; however, the notation in each chapter is self-contained. In almost all cases, upper case boldface letters denote sets, lower case boldface letters denote vectors. Upper and lower case Roman and Greek letters are used variously to denote scalars and functions. Derivatives are denoted variously by subscripts (the symbol for the variable with respect to which derivatives are being taken, or the ordinal position of this variable among the arguments), primes, the $\nabla$ operator, or the usual notation $\partial f/\partial x$.

The editors wish to acknowledge the contributions that many individuals have made to the preparation of *Production Economics*. Dale Jorgenson and Zvi Griliches have provided encouragement and ideas. A large intellectual debt is owed to K.J. Arrow, W.M. Gorman, L. Hurwicz, M. Nerlove, and H. Uzawa, whose work provided the background for most of the developments in these books. We thank the contributors, who have displayed stoic patience and goodwill in the lengthy process of refereeing and publication. We also wish to acknowledge the help of several scholars who participated in the early planning, and who have published related work elsewhere: T. Cowing (1974), W. E. Diewert (1971, 1974a), R.E. Hall (1973), C.K. Liew (1976), and M. Ohta (1975).
To G. Katagiri and N. Katagiri goes the credit for careful typing and editing of the manuscript.

The editors accept responsibility for all errors not allocatable to individual contributors. Finally, we thank our wives, Beverlee and Susan, for tolerance and encouragement through the lengthy process of bringing these volumes to completion.
CONTENTS OF VOLUME 2

Introduction to the Series v
Preface vii
Introduction xix

Part III. An Analysis of the Concept of Real Value-Added 1

III.1 Michael Bruno: Duality, Intermediate Inputs and Value-Added 3
1. Introduction 3
2. Duality and value-added 4
3. Value-added in constant prices 7
4. The measurement of total productivity 11
5. Introducing imperfect competition 15
6. Conclusions 16

III.2 W. Erwin Dievert: Hicks' Aggregation Theorem and the Existence of a Real Value-Added Function 17
1. Introduction 17
2. Hicks' aggregation theorem in the consumer context 22
3. Duality between direct and conditional indirect utility functions 24
4. Hicks' aggregation theorem in the producer context 34
5. The duality between value-added "production" and cost functions 37
6. Concluding remarks 40
7. Proofs of theorems 43

III.3 Michael Denny and J. Douglas May: Homotheticity and Real Value-Added in Canadian Manufacturing 53
1. Introduction 53
2. Real value-added and the production structure 54
3. The approximate translog cost function 56
4. Homotheticity and homogeneity 58
5. The Canadian manufacturing data 59
6. The estimation of the production structure 61
7. An alternative real value-added formulation 63
8. Weak homotheticity 65
9. Double deflation and real value-added 68
10. Conclusion 70

Part IV. Empirical Applications of Production Theory: Micro-Economic Data 71

IV.1 Daniel McFadden: Estimation Techniques for the Elasticity of Substitution and Other Production Parameters 73

1. Introduction 73
2. Cost functions 75
3. The elasticity of substitution 78
4. The specification of the ACMS model 80
5. Properties of the elasticity of substitution 82
6. An econometric model of the electricity generation industry 95
7. Appendix 118

IV.2 Peter Diamond, Daniel McFadden, and Miguel Rodriguez: Measurement of the Elasticity of Factor Substitution and Bias of Technical Change 125

1. Introduction 125
2. Notation 127
3. Growth equations 129
4. Production relations 131
5. A non-identification theorem 132
6. Necessary conditions for factor augmenting technical change 135
7. Sufficient conditions for factor augmenting technical change 137
8. Identification for capital augmentation 141
9. Identification for finite parameter families of augmentation functions 143
10. Extensions and conclusions 145
11. Appendix: Generalization to non-constant returns 145

IV.3 Alexander Belinfante: The Identification of Technical Change in the Electricity Generating Industry 149
1. Introduction 149
2. Notation 151
3. Identification problems in the measurement of technical change 152
4. The industry and the data 157
5. The measurement of disembodied technical change 162
6. The measurement of embodied technical change 172
7. The measurement of total technical change 177
8. The bias of technical change 181
9. Concluding remarks 185

IV.4 Melvyn Fuss: Factor Substitution in Electricity Generation: A Test of the Putty-Clay Hypothesis 187
1. Introduction 187
2. The basic model 189
3. The model applied to electricity generation 190
4. Tests of the structure of technology 200
5. Estimation of the putty-clay model 208
6. Conclusion 210
7. Appendices 210

IV.5 Thomas G. Cowing: The Effectiveness of Rate-of-Return Regulation: An Empirical Test Using Profit Functions 215
1. Introduction 215
2. The Averch-Johnson model of a regulated firm 217
3. The profit function for a regulated firm 222
4. Statistical tests of regulatory effectiveness 225
5. Comparison with previous studies 237
6. Conclusions 243
7. Appendix 245
Part V. Empirical Applications of Production Theory: Macroeconomic Data

V.1 Michael Denny and Cheryl Pinto: An Aggregate Model with Multi-Product Technologies

1. Introduction 249
2. Multi-product technologies and final demand 250
3. Jointness and separability of the cost function 253
4. The approximate translog cost function 253
5. Approximate separability and non-jointness of the translog cost function 255
6. An aggregate Canadian production model 258
7. Estimation and hypothesis testing 260
8. Appendix: Data 266

V.2 Petter Frenger: Factor Substitution in the Interindustry Model and the Use of Inconsistent Aggregation

1. Introduction 269
2. The model 271
3. The data 273
4. Estimation 275
5. Empirical results 277
6. 1961 predictions 283
7. Inconsistent shadow elasticities of substitution 287
8. A directional shadow elasticity of substitution 291
9. Inconsistent directional shadow elasticity of substitution 296
10. Appendix 300

Bibliography 311

Author Index 327

Subject Index 331
CONTENTS OF VOLUME 1

Introduction to the Series v
Preface vii
Introduction xix

Part I. Duality of Production, Cost, and Profit Functions 1

I.1 Daniel McFadden: Cost, Revenue, and Profit Functions 3

1. Introduction 3
Part I. Cost Functions 5
2. History 5
3. Production technologies 5
4. The cost function 10
5. The derivative property 14
6. Duality 19
7. Distance functions and economic transformation functions 24
8. Extensions of duality 29
9. Cobb–Douglas and C.E.S. cost functions 38
10. The geometry of two-input cost functions 39
11. Comparative statics for the cost minimizing firm 46
12. Composition of distance and cost functions 49
Part II. Restricted Profit Functions 60
13. The general representation of production possibilities 60
14. The general restricted profit function 66
15. The derivative property of the restricted profit function 74
16. The gauge function for production possibilities 76
17. Duality for the restricted profit function 81
18. Examples of restricted profit functions 93
19. Composition rules for profit functions 95
20. Profit saddle-functions 101
I.2 Giora Hanoch: Symmetric Duality and Polar Production Functions 111

1. Introduction 111
2. A symmetric formulation of cost and production functions 112
3. Polar production and cost functions 118
4. A symmetric formulation of profit functions and production frontiers 121
5. The polar profit and production functions 126
6. Some extensions and an application 129

I.3 Lawrence J. Lau: Applications of Profit Functions 133

1. The profit function – An alternative derivation 133
2. The structure of normalized profit functions 151
3. Extensions to multiple outputs 169
4. Examples of normalized profit functions 190
5. Applications of the normalized profit function 197
6. Summary and conclusions 215

Part II. Functional Forms in Production Theory 217

II.1 Melvyn Fuss, Daniel McFadden, and Yair Mundlak: A Survey of Functional Forms in the Economic Analysis of Production 219

1. The context and objectives of production analysis 219
2. Criteria for the design of functional forms 222
3. Dual transformation, cost and profit functions – Maintained hypotheses on the technology and its representations 225
4. A general approach – Forms linear-in-parameters 230
5. Special non-linear forms 240
6. Separability: Functional implications and tests 244
7. Econometric estimation of production parameters 249
8. Overview of empirical analysis 267
9. Conclusion 268
Contents of Volume I

II.2 Daniel McFadden: The General Linear Profit Function 269
   1. Introduction 269
   2. The basic model 270
   3. General linear profit functions 272
   4. The dual technology of the general linear profit function 281
   5. Applications of the linear profit function 283

II.3 Giora Hanoch: Polar Functions with Constant Two Factors – One Price Elasticities 287
   1. Introduction 287
   2. Elasticities of substitution and transformation 289
   3. A summary of DRES and CDE (implicitly additive) models 294
   4. Multiproduct production frontiers with constant TOES or TOET 301

II.4 Melvyn Fuss and Daniel McFadden: Flexibility versus Efficiency in Ex Ante Plant Design 311
   1. Introduction 311
   2. Historical background 313
   3. A model of the firm with an ex ante–ex post technology 320
   4. Functional forms for the ex ante–ex post production structures 330
   5. Derivation of the two-level structure of technology 345
   6. Separable technology across states 356
   7. A general linear-in-parameters ex ante–ex post technology 359
   8. Concluding remarks 363

Appendices 365

A.1 Daniel McFadden: Definite Quadratic Forms Subject to Constraints 365
   1. Conditions for a matrix to be positive definite 365
   2. Conditions for a matrix to be positive definite subject to constraint 368
Contents of Volume 1

A.2 Daniel McFadden: Necessary and Sufficient Conditions for the Classical Programming Problem 375

A.3 Daniel McFadden: Convex Analysis 383

1. Introduction 383
2. Notation 383
3. Hyperplanes 384
4. Convex sets 384
5. Affine subspaces 384
6. Separation of sets 384
7. Cones 384
8. Polar and normal cones 385
9. Convex functions 385
10. Properties of convex sets 385
11. Semi-bounded sets 387
12. Properties of convex functions 390
13. Properties of maximand correspondences 392
14. Exposed sets 396
15. Conjugate correspondences 397
16. Differential properties 401

A.4 Lawrence J. Lau: Testing and Imposing Monotonicity, Convexity and Quasi-Convexity Constraints 409

1. Introduction 409
2. Hessian matrices of convex and quasi-convex functions 414
3. The Cholesky factorizability of semidefinite and indefinite matrices 421
4. Estimation 437
5. Testing of hypotheses 447
6. Conclusion 452

Bibliography 455

Author Index 471

Subject Index 475
INTRODUCTION

This volume contains applications of the theoretical concepts developed in Volume 1. Several of the applications are themselves theoretical in nature, e.g., the chapters by Bruno, Diewert, and Diamond–McFadden–Rodriguez. However, the majority of the chapters constitute empirical studies designed to illustrate the advantages in estimation of the dual approach to production economics.

One of the most direct applications of duality theory is to the concept of value-added. This application is developed in Part III. The fact that value-added is a function of primary inputs and the prices of outputs and intermediate inputs implies that the value-added function is a member of the class of restricted profit functions analyzed extensively by McFadden in Chapter I.1. Bruno and Diewert in Chapters III.1 and III.2 utilize the properties of restricted profit functions to develop corresponding ones for value-added functions. They also analyze the biases inherent in using single- and double-deflated value-added in place of the correct gross output in production function and factor productivity studies. Functional separability, fixed intermediate input proportions, and constant relative intermediate goods prices emerge as three possible hypotheses, which if satisfied, may lead to a justification of the use of value-added. In Chapter III.3, Denny and May test these hypotheses for Canadian manufacturing using a translog cost function specification for the underlying technology. They reject each of the three hypotheses, thus casting doubt on the validity of the extensive use of the value-added specification.

The majority of papers in Part IV (those by McFadden, Belinfante, Fuss, and Cowing) represent empirical investigations using data drawn from the same source: the fossil-fuel electricity generation industry. This very detailed data set provides pooled cross-section time-series observations on individual plants and firms permitting a correspondingly detailed investigation of the characteristics of technology. In Chapter IV.1, McFadden applies variants of the CES cost function in order to analyze and estimate substitution elasticities. Belinfante, in Chapter
Introduction

IV.3, uses an approach closely related to the Divisia indexing procedure to measure the extent and bias of disembodied and embodied technical change. In Chapter IV.4, Fuss implements empirically a nested generalized Leontief cost structure developed in detail elsewhere [Fuss (1970 and 1977b)], which is a member of the class of nested models explored in Chapter II.4. This model, which provides a detailed specification of the structure of technology, is used to test the "putty-clay" hypothesis for electricity generation. This hypothesis is not rejected on the basis of the evidence presented, but a more restrictive structure which assumes fixed proportions both ex ante and ex post is rejected. Cowing applies the duality approach to an investigation of the Averch–Johnson (1962) model of regulatory behavior in Chapter IV.5. In this chapter a modified Hotelling's lemma is developed and used to generate a system of net supply functions from the quadratic approximation to an arbitrary profit function (see Chapter II.1). This derivation explicitly takes into account the rate of return constraint through an endogenously determined Lagrangian multiplier. Cowing finds that his results are in general agreement with other recent empirical investigations which indicate that the Averch–Johnson thesis has empirical relevance. However, in addition Cowing's model allows inter-firm and inter-temporal comparisons of the effects of regulation.

The chapters by McFadden, Belinfante, Fuss, and Cowing can be seen as an historical evolution in the use of a data set with common antecedents. Electricity generation data were continuously collected and revised during the period 1963–71 at the University of California, Berkeley, and later at the State University of New York at Binghamton. Original versions of these chapters appeared in 1964 (McFadden), 1969 (Belinfante), 1971 (Fuss), and 1975 (Cowing). Also part of the historical evolution are studies by Cowing (1970 and 1974) and Ohta (1975).

Chapter IV.2 by Diamond, McFadden, and Rodriguez is the one non-empirical chapter in Part IV. However, the concepts developed in this chapter have empirical relevance (and are utilized extensively in Belinfante's study). Among these concepts is the Diamond–McFadden Impossibility Theorem, now well-known from Nerlove's (1967) survey of the estimation of CES production functions. This theorem states that there is a non-identifiability of the elasticity of substitution and the bias of technical change in the absence of a priori hypotheses on the structure of technical change. Belinfante's study can be viewed as an example of how such a structure might be imposed.

Production characteristics have most often been estimated using
much more aggregate data than that used in Part IV. Part V presents two studies which apply dual forms to aggregate data. Denny and Pinto's study in Chapter V.1 applies the translog cost function to economy-wide Canadian data using aggregates of consumption, investment, imports, capital, and labor. They estimate a production structure which has consumption and investment as outputs, and capital, labor, and imports as inputs. Tests for separability and non-jointness indicate that while the separability hypothesis should be rejected, the non-joint hypothesis cannot be. The marginal rates of transformation between outputs are shown to be particularly sensitive to the imposition of separability or non-jointness as maintained hypotheses. In Chapter V.2, Frenger applies the generalized Leontief cost function to an analysis of the production structure implied by the Norwegian input–output tables. He tests the Leontief fixed coefficient model (which underlies most uses of input–output tables) for three industrial sectors of the Norwegian economy (textiles, construction, and metals). The two models' predictive abilities are also compared. In general, the fixed coefficient model is rejected both with respect to the hypothesis tests and the prediction comparisons. This empirical investigation is followed by an analysis of possible biases inherent in the use of inconsistent price aggregates in the estimation of separable cost functions, with specific emphasis on the effects this might have on the estimated substitution parameters.

Most of the empirical chapters in Volume 2 present estimates of production structures obtained by estimating a complete system of net supply functions with theoretical constraints such as linear homogeneity and symmetry imposed. This procedure provides many more degrees of freedom in estimation than the more common ones of estimating the production function directly or utilizing the first-order condition for a single input. As long as the implied behavioral and exogeneity of prices assumptions are reasonable, we can expect to obtain more accurate estimates of the parameters of the production structure that are of interest. The force of the dual approach to empirical production economics is the ease with which the required net supply systems are generated and interpreted. The chapters of this volume provide documentation for this assertion.