Economics 280C: Answer to Problem #1

(a) As a preliminary step it is useful to calculate the means of variables. Since shocks are i.i.d. here and there are no multiperiod sources of persistence, conditional means are actually unconditional (and will be denoted as such), which simplifies the analysis considerably. It is straightforward to check that, since \( m \) is constant, since \( i^s = p^s = 0 \), and since \( Ev = Ey = Eg = 0 \),

\[
Ee = Ep = w = m, \quad Ey = 0.
\]

By substituting \( Ee = m \) into the interest parity relationship we derive

\[
i_t = m - e_t + \varepsilon_t,
\]

which may be plugged into the money-market equilibrium condition to give

\[
m - p_t = y_t - \lambda (m - e_t) + \varphi_t,
\]

where \( \varphi_t = y_t - \lambda e_t \). Using the aggregate supply equation, which implies \( y_t = \theta (p_t - m) \), to eliminate \( y_t \), we get

\[
(1 + \theta) p_t = -\lambda e_t + (1 + \theta + \lambda) m - u_t. \tag{1}
\]

Now equate aggregate demand and supply to infer that

\[
(\theta + \delta) p_t = \delta e_t + \theta m + g_t. \tag{2}
\]

Equations (1) and (2) are two equations in the unknowns \( e \) and \( p \). Solving, we find that

\[
e_t = m - \frac{1 + \theta}{\delta(1 + \theta) + \lambda(\delta + \theta)} g_t - \frac{\theta + \delta}{\delta(1 + \theta) + \lambda(\delta + \theta)} \varphi_t, \tag{3}
\]

\[
p_t = m + \frac{\lambda}{\delta(1 + \theta) + \lambda(\delta + \theta)} g_t - \frac{\delta}{\delta(1 + \theta) + \lambda(\delta + \theta)} \varphi_t. \tag{4}
\]

From this last equation and the aggregate supply schedule we derive

\[
y_t = \frac{\theta (\lambda g_t - \delta \varphi_t)}{\delta(1 + \theta) + \lambda(\delta + \theta)}. \tag{5}
\]

(b) Since the mutual covariances of all the shocks are zero, \( \sigma^2_{\varphi} = \sigma^2_{\varepsilon} + \lambda^2 \sigma^2_{\varepsilon} \) and, from eq. (5),

\[
\sigma^2_{y_{\text{float}}} = \frac{\theta^2 \lambda^2 \sigma^2_{\varepsilon} + \theta^2 \delta^2 \sigma^2_{\varphi}}{[\delta(1 + \theta) + \lambda(\delta + \theta)]^2}. \tag{6}
\]

(c) Setting aggregate demand and supply equal, one can show that with the exchange rate fixed at \( \bar{e} \) and the money supply endogenous instead,

\[
p = \bar{e} + \frac{g_t}{\theta + \delta},
\]

\[
y = \frac{\theta g_t}{\theta + \delta}. \tag{7}
\]

(d) The variance under a fixed exchange rate is, using eq. (7),
\[
\sigma_{y|\text{fix}}^2 = \frac{\theta^2 \sigma_y^2}{(\theta + \delta)^2}. \quad (8)
\]

(e) From eq. (6), when \( \sigma_{\phi}^2 = 0, \)
\[
\sigma_{y|\text{float}}^2 = \frac{\theta^2 \lambda^2 \sigma_y^2}{[\delta(1 + \theta) + \lambda(\delta + \theta)]^2} < \frac{\theta^2 \sigma_y^2}{(\theta + \delta)^2} = \sigma_{y|\text{fix}}^2.
\]

(f) Under the interest-rate rule, \( E_i = i^* = 0 \) and \( E_p = 0. \) For ex post values, we have
\[
e_t = \frac{\theta + \delta}{\theta + \delta (1 + \psi)} \phi_t - \frac{\psi}{\theta + \delta (1 + \psi)} g_t,
\]
\[
p_t = \frac{\delta}{\theta + \delta (1 + \psi)} \phi_t + \frac{1}{\theta + \delta (1 + \psi)} g_t,
\]
\[
y_t = \frac{\theta (\delta \phi_t + g_t)}{\theta + \delta (1 + \psi)}.
\]

\[
\sigma_{y|\text{float}}^2 = \frac{\theta^2 \delta^2 \phi^2 + \theta^2 \sigma_y^2}{[\theta + \delta (1 + \psi)]^2}.
\]

(g) Under a (credibly) fixed exchange rate, the domestic interest rate is fixed at \( i = i^* = 0. \) Price and output are determined simply by the intersection of aggregate demand and supply, given \( \bar{e}, \) as in part (c) above, so eqs. (7) and (8) still apply. When \( \sigma_{\phi}^2 = 0, \) meaning that there are no interest-rate shocks,
\[
\sigma_{y|\text{float}}^2 = \frac{\theta^2 \sigma_y^2}{[\theta + \delta (1 + \psi)]^2} < \frac{\theta^2 \sigma_y^2}{(\theta + \delta)^2} = \sigma_{y|\text{fix}}^2.
\]

(In general, this could be the case too if the policy shocks \( u \) to the interest rate perfectly offset the portfolio preference shocks \( \bar{e} \) in the interest-parity relation. Of course, that would require an unrealistically high degree of knowledge of market sentiment by the central bank.) If financial shocks dominate (large \( \sigma_{\phi}^2 \) relative to \( \sigma_y^2 \)), the variance inequality would be reversed, and more easily for small values of \( \psi. \) The \( v \) shocks in the LM curve are irrelevant now because the money supply is endogenous and moves to offset automatically shocks to the quantity of money demanded.