Price Setting in the Corsetti-Pesenti Framework

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There are two cases, PCP (producer-currency pricing, price sticky in home currency of the producer) and LCP (local-currency pricing, price sticky in currency of the buyer, so that a producer must set domestic as well as foreign prices). In both cases, prices are set on date $t-1$ to be charged buyers in period $t$.

**PCP case.** Worldwide profits of a Home producer (say, in terms of domestic currency) are given by

$$\Pi = [p(h) - MC] \left[ \frac{p^*(h)}{P_H} \right]^{-\theta} C_H + [E p^*(h) - MC] \left[ \frac{p^*(h)}{P_H^*} \right]^{-\theta} C^*_H,$$

where $p^*(h)$ is the Foreign-currency price at which goods are sold in Foreign. Under PCP, the Foreign price will simply be $p(h)/E$, where $p(h)$ is set a period in advance. Thus, ex post nominal profits under sticky prices (once date $t$ variables including the exchange rate $E$ are realized) will be:

$$\Pi = [p(h) - MC] \left[ \frac{p(h)}{P_H} \right]^{-\theta} C_H + [p(h) - MC] \left[ \frac{p(h)/E}{P_H^*} \right]^{-\theta} C^*_H. \quad (1)$$

Because the firm sets the price a period in advance and asset markets are complete, the payoff to the firm in a given date-$t$ state of nature $s_t$, valued in terms of date $t-1$ money, will be

$$\frac{\pi(s_t)\beta u'[C(s_t)]/P(s_t)}{u'(C_{t-1})/P_{t-1}} \Pi(s_t),$$

where $\pi(s_t)$ is the probability of occurrence of state $s_t$. (Recall that the ratio

$$\frac{\pi(s_t)\beta u'[C(s_t)]/P(s_t)}{u'(C_{t-1})/P_{t-1}}$$

is the value of a unit of money delivered on date $t$ contingent on state $s_t$, measured in terms of money on date $t-1$.) The firm maximizes, with respect to its date $t-1$ choice of $p_t(h)$, the sum of the preceding state-contingent payoffs, and therefore solves the problem

$$\max_{p_t(h)} E_{t-1} \left\{ \frac{\beta u'(C_t)/P_t}{u'(C_{t-1})/P_{t-1}} \Pi_t \right\} \Leftrightarrow \max_{p_t(h)} E_{t-1} \left\{ \frac{\beta P_{t-1} C_{t-1}}{P_t C_t} \Pi_t \right\}.$$

(The equivalence is a consequence of log utility.)

Substituting eq. (1) into the preceding maximization, one expresses the firm’s problem (after dividing by $P_{t-1} C_{t-1}$, which is exogenous to the individual

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producer and known as of date $t-1$, and multiplying by $P_H$, which also is known as of date $t-1$, as

$$\max_{p_t(h)} E_t \left\{ \left[ p_t(h) - MC_t \right] \frac{p_t(h)}{P_{H,t}} - \theta P_{H,t} C_{H,t} \right\}$$

$$+ \left[ p_t(h) - MC_t \right] \frac{\partial p_t(h)/\partial E_t}{P_{H,t}^*} - \theta \left( [p_t(h) - MC_t] \right) \frac{\partial E_t}{P_{H,t}^*}$$

Above, we have used the fact that, under PCP, $P_H$ will always equal $E P_H^*$ (since that relationship holds for each individual Home good $h \in [0,1]$).

Differentiating with respect to $p_t(h)$ yields the first-order condition

$$E_t \left\{ \left[ p_t(h) \right] \frac{\partial p_t(h)/\partial P_{H,t}}{P_{H,t}^*} - \theta \left[ p_t(h) - MC_t \right] \right\} = 0.$$
The first-order condition with respect to $p_t(h)$ is

$$
\mathbb{E}_{t-1} \left\{ \left[ \frac{p_t(h)}{P_{H,t}} \right]^{-\theta} \frac{C_{H,t}}{P_t C_t} - \theta \frac{[p_t(h) - MC_t]}{p_t(h)} \left[ \frac{p_t(h)}{P_{H,t}} \right]^{-\theta} \frac{C_{H,t}}{P_t C_t} \right\} = 0.
$$

Multiplying through by $P_{H,t}$ as above, which is known at date $t - 1$, we get

$$
\mathbb{E}_{t-1} \left\{ 1 - \theta \frac{[p_t(h) - MC_t]}{p_t(h)} \right\} = 0,
$$

or

$$
p_t(h) = \frac{\theta}{\theta - 1} \mathbb{E}_{t-1} \{MC_t\}.
$$

The first-order condition with respect to $p^*_t(h)$ is

$$
\mathbb{E}_{t-1} \left\{ \frac{p^*_t(h)}{P^*_{H,t}} \right\}^{-\theta} \left[ \frac{C^*_{H,t}}{P_t C_t} \right] - \theta \frac{[\mathcal{E}_t p^*_t(h) - MC_t]}{\mathcal{E}_t p^*_t(h)} \left[ \frac{p^*_t(h)}{P^*_{H,t}} \right]^{-\theta} \left[ \frac{C^*_{H,t}}{P_t C_t} \right] = 0.
$$

Now, $P^*_{H,t}$ also is known with certainty as of date $t - 1$, so we may multiply through the expectations operator in the preceding equation and rearrange terms to get

$$
\mathbb{E}_{t-1} \left\{ \left[ \frac{p^*_t(h)}{P^*_{H,t}} \right]^{-\theta} \frac{\mathcal{E}_t P^*_{H,t} C^*_{H,t}}{P_t C_t} - \theta \frac{[\mathcal{E}_t p^*_t(h) - MC_t]}{\mathcal{E}_t p^*_t(h)} \left[ \frac{p^*_t(h)}{P^*_{H,t}} \right]^{-\theta} \frac{\mathcal{E}_t P^*_{H,t} C^*_{H,t}}{P_t C_t} \right\} = 0,
$$

which reduces to

$$
\mathbb{E}_{t-1} \left\{ 1 - \theta \frac{[\mathcal{E}_t p^*_t(h) - MC_t]}{\mathcal{E}_t p^*_t(h)} \right\} = 0
$$

(because $\frac{\mathcal{E}_t P^*_{H,t} C^*_{H,t}}{P_t C_t} = \frac{1}{2}$ under complete markets). We may multiply $p^*_t(h)$ through the expectations operator to yield

$$
\mathbb{E}_{t-1} \left\{ p^*_t(h) - \theta \left[ \frac{p^*_t(h) - MC_t}{\mathcal{E}_t} \right] \right\} = 0
$$

or, solving for $p^*_t(h)$,

$$
p^*_t(h) = \frac{\theta}{\theta - 1} \mathbb{E}_{t-1} \left\{ \frac{MC_t}{\mathcal{E}_t} \right\}.
$$