Less of a Puzzle: A New Look at the Forward Forex Market[1]

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ABSTRACT

The two-country monetary model is extended to include a consumption externality with habit persistence. This is set within a limited participation framework. The model is simulated using the artificial economy methodology. The 'puzzles' in the forward market are re-examined. The model is able to account for (a) the persistence in the forward discount (b) the random walk in spot exchange rates (c) the low volatility of the forward discount (d) the high volatility of expected forward speculative profits and (e) the even higher volatility of spot rate changes. The major innovation is that it is able to replicate the extent of the bias of the forward discount as a predictor of realised spot rate changes.

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Campbell and Cochrane (1999) have proposed a preference specification in which there is both an aggregate consumption externality[2] and utility is time-inseparable because of habit-persistence. They apply this to the ‘equity premium’ puzzle and show that their model can replicate the relevant ‘stylised facts’ for US data. In this paper, we extend the Campbell-Cochrane preferences to both a monetary and an international setting. The ‘puzzles’ associated with the forward foreign exchange market are revisited in the light of the new theoretical framework[3].

Numerous writers (a good example is Backus, Gregory and Telmer, 1993) have shown that the forward discount is a poor predictor of realised future spot exchange depreciation. Indeed, the forward discount typically fails to even forecast the correct direction of spot exchange rate changes. The most challenging task of a good theory is to replicate this ‘bias’. Previous attempts using habit persistence specifications, such as Bekaert (1996) have fallen short of this objective. We argue that Campbell–Cochrane preferences in a limited participation model succeed in doing this.

A related problem is that the standard deviation of the forward discount is smaller than that of expected forward speculative profits[4] and is an order of magnitude less than that of spot rate changes. The standard cash-in-advance (CIA) for goods model is unable to replicate this at plausible levels of risk aversion. Our model is not only able to replicate these relative volatilities but the absolute volatility of spot rate changes also.

It is always possible to solve one problem at the expense of creating another. Moore and Roche (1998) have already shown that the persistence of the forward discount can be satisfactorily explained by persistence in home and foreign money shocks, even in the standard CIA model. The theory presented here succeeds in maintaining this explanation.

The model we present is a general equilibrium one so that it can address issues other than the forward market ‘puzzles’. It is easy to show[5] that the empirical persistence in home and foreign money shocks implies persistence in spot rate changes in the standard CIA model. This is obviously counterfactual though Moore and Roche (1998) show that limited participation monetary models go some distance towards replicating the observed lack of autocorrelation in spot rate changes. A pleasing feature of the new models presented here is that they successfully predict the lack of persistence in spot rate changes for virtually all parameter values considered.

The plan of the paper is as follows. Section I provides a brief introduction to Campbell-Cochrane preferences and develops the theoretical framework. Section II summarises the results of simulating an artificial economy. The conclusion is in section III.

I. A Theoretical framework

I.1 Background

Campbell and Cochrane (1999) have proposed a novel solution to the ‘equity premium’ puzzle in closed economies. They introduce a simple and tractable modification to the utility function in the standard model. An *external* habit is introduced which has the following features:

i) The real risk-free rate is constant when real shocks are i.i.d. Even if these shocks are serially correlated, it does not depend directly on their variance, in contrast to the standard model. This is achieved by ensuring that precautionary savings effects and intertemporal substitution effects cancel each other out. This removes the ‘risk-free rate’ puzzle. (See Weil, 1989).

ii) Habit persistence is a highly non-linear function of past consumption. By this means, Campbell and Cochrane overcome the irritating problem with existing habit persistence models that marginal utilities can sometimes take on negative values.

iii) The habit adjusts slowly in response to consumption. This differs from the usual habit persistence specifications that model the habit as proportional to past consumption choices.

iv) Habit persistence takes the form of an aggregate consumption externality i.e. ‘Keeping Up with the Jones’s’ effects along the lines of Abel (1990) and Duesenberry (1949). It is this feature which makes the model so simple.

Habit persistence preferences have already been proposed before as a solution to the puzzles in the forward market. In an overlapping generations model with ordinal-certainty-equivalent preferences, incorporating habit persistence, Sibert (1996) finds that habit persistence has almost no impact on the properties of expected forward speculative profits. Its main effect is to tilt consumption intertemporally. Her results are relatively easy to explain. In an economy where there are no opportunities for intertemporal substitution, the usual linear habit persistence specification leads to wild gyrations in the real risk free rate. Whenever consumption shocks can be spread over time, as in an overlapping generations model or in a model with production, this does not occur. This is because intertemporal substitution in consumption can take place. Jermann (1998) provides an excellent discussion of this point.

The other recent attempt to explain forward market puzzles, using habit persistence, is contained in Bekaert (1996). His model is of the conventional Lucas two-country variety where equilibrium consumption equals (half of) the current endowment. This ensures that habit persistence is effective. He is able to match the observed volatilities of spot rate changes, forward premia and expected forward speculative profits but never at the same configuration of parameters. Bekaert argues that the standard for judging a successful theory of the forward market is its ability to replicate the stylised fact that the volatility of expected forward speculative profits exceeds that of the forward premium. His model can only achieve this when the absolute volatilities are implausibly high.

The reason why we believe that the Campbell-Cochrane framework can help to solve the forward market puzzles is as follows. The standard cash in advance model is able to match the forward premium relatively well. However, it is unable to account for the observed high volatility in spot exchange rate changes. It is even more unsuccessful at explaining the volatility of expected forward speculative profits. The trouble is that existing attempts to improve performance with the latter leads to greatly increased volatility in forward premia. This is an international analogue to the equity premium and risk-free rate puzzles in closed economy stock markets. Campbell-Cochrane preferences provide the opportunity to ‘anchor’ the volatility of the forward premium because it forces real risk-free rates to display low volatility. The parameters of the model can then be varied to improve performance in other respects.

Of course, the forward premium consists of nominal and not just real interest rate differentials. This blunts the potential effectiveness of the Campbell-Cochrane approach. However, Moore and Roche (1998) have shown that limited participation household monetary models[6] are superior to standard monetary models in explaining exchange rate behaviour. Consequently, our modelling strategy is to develop two new models. The first is an application of Campbell-Cochrane preferences to an otherwise standard two-country, two-money cash-in-advance (CIA) model. The second incorporates Campbell-Cochrane preferences into the limited participation framework.

I.2 Preferences

The basic structure of the model is the well-known Lucas two-country, two-good, two-money representative agent story. The representative consumer in each country has an instantaneous utility function:

$$U = \sum_{t=0}^{\infty} \beta^t \left[\alpha \frac{(C_{it}^1 - H_{it}^1)^{1-\gamma}}{1-\gamma} + (1-\alpha) \frac{(C_{it}^2 - H_{it}^2)^{1-\gamma}}{1-\gamma} \right], \quad i = 1,2, \tag{1}$$

where α, β and γ are the share parameter, subjective discount rate and curvature parameter[7] respectively. C_{it}^j is country i's consumption of the jth country's good and H_{it}^j is country i's habit level with respect to good j. In this way, we are following Bekaert (1996) by specifying a separate habit for each good and for each country. The habits are re-parameterised by defining the *surplus consumption ratios* as follows:

$$X_{it}^j = \frac{C_{it}^j - H_{it}^j}{C_{it}^j}, \quad i = 1,2 \quad j = 1,2, \tag{2}$$

The log of the surplus consumption ratios are assumed to evolve as follows[8]:

$$x_{t+1}^j = (1-\phi)\bar{x}^j + \phi x_t^j + \lambda(x_t^j)\left(v_{t+1}^j\right), \quad j = 1,2, \tag{3}$$

where $\phi < 1$, is the habit persistence parameter, $\bar{x}^j$ is the steady state value for the surplus consumption ratio for good j, v_t^j is the innovation to the stochastic process for the endowment of good j. $\lambda(x_t^j)$ is a function, which describes the sensitivity of the future surplus consumption ratio to endowment innovations: it depends non-linearly on the current surplus consumption ratio. For a fuller discussion of the properties of $\lambda(x_t^j)$, see Appendix A.

I.3 The Standard Model with Habit Persistence

In this section, the conventional Lucas two-country two-money equilibrium is reworked using the preferences in equation (1). At this stage, money is introduced with the simplest cash-in-advance for goods specification: home goods are paid for with home money while foreign goods are paid for with foreign money.

$$P_t^i = \frac{M_t^i}{C_{jt}^i + C_{it}^i} = \frac{M_t^i}{Y_t^i} \quad i = 1,2$$

where P_t^i is the own currency price of the ith country's good, M_t^i is the ith country's money stock and Y_t^i is the ith country's endowment.

The stochastic processes for the exogenous variables are:

$$Log\left[\frac{Y_{t+1}^i}{Y_t^i}\right] = \mu^i + v_{t+1}^i \quad i = 1,2 \quad v_{t+1}^i \sim N(0, \sigma_y^2), \tag{4}$$

and

$$Log\left[\frac{M_{t+1}^i}{M_t^i}\right] = \pi_t^i + u_{t+1}^i \quad i = 1,2 \quad u_{t+1}^i \sim N(0, \sigma_m^2). \tag{5}$$

Equation (4) incorporates the Campbell-Cochrane assumption that endowments (and therefore consumption) follow a random walk with drift[9]. Equations (4) and (5) incorporate the assumption that the variances of home and foreign shocks are the same. More significantly, it will be assumed that covariances between all shocks are zero. Empirical justification is provided in Moore and Roche (1998). Engel (1992) also argues that the empirical covariance between real and nominal shocks is low or zero. In the discussion following equation (9), it is shown that these assumptions have the effect of ensuring that the standard model *cannot* generate a risk premium in the forward market. Consequently, the novel results from the new models are attributable only to the preference specification and to the monetary framework.

Using equations (1)-(5) and the usual expression for a one period nominal bond[10], nominal interest rates (i_t^j) are:

$$i_t^j = r^j + (\pi_t^j - \mu^j) - \frac{\sigma_m^2 + \sigma_y^2(1 - 2\gamma(1 + \lambda(x_t^j)))}{2} \quad j = 1,2, \tag{6}$$

The interpretation of equation (6) is conventional. The nominal rate of interest consists of three elements: r^j, a constant own-good real interest rate (See equation A3 in Appendix A); $\pi_t^j - \mu^j$, the expected rate of inflation and a risk premium.

Equation (6) differs from the standard model in two ways. Firstly, each own real rate of interest has the Campbell-Cochrane properties of being constant *and* of not being directly dependent on the variance of endowments. Secondly, the risk premium includes the sensitivity function $\lambda(x_t^j)$. In fact, the risk premium in the standard model is obtained by setting $\lambda(x_t^j) = 0$ in the third expression in equation (6).

Using covered interest parity and equation (6), it is easy to obtain an expression for the forward discount, $f_t - s_t$ where f_t and s_t are the logs of the forward and spot rates respectively.

$$f_t - s_t = [(1-\gamma)(\mu^1 - \mu^2) + (\pi_t^1 - \pi_t^2)] + \sigma_y^2 \gamma(\lambda(x_t^1) - \lambda(x_t^2)). \tag{7}$$

The expression in square brackets in equation (7) is the forward discount in the standard model. It is clear that, in the standard case, the volatility and the degree of persistence in money shocks maps onto the volatility and persistence of the forward discount. In the habit model, the additional feature is the final term in equation (7). This depends on the difference between the two sensitivity functions $\lambda(x_t^j)$. Moore and Roche (1998) show that in the standard and limited participation household monetary models the least 'puzzling' feature of the forward market is the persistence of the forward discount. This is because it can be explained by persistence in money shocks. It is not at all clear from equation (7) if this explanation can be retained with habit preferences.

Under the habit preferences, the expression for spot exchange rate changes is complicated and far from intuitive[11]:

$$\begin{aligned}\Delta s_{t+1} = {} & [(1-\gamma)(\mu^1-\mu^2)+(\pi_t^1-\pi_t^2)+u_{t+1}^1-u_{t+1}^2]+ \\ & [\gamma(1-\phi)(x_t^2-x_t^1)+v_{t+1}^2\{1-\gamma(\lambda(x_t^2)+1)\}-v_{t+1}^1\{1-\gamma(\lambda(x_t^1)+1)\}],\end{aligned} \tag{8}$$

The first expression in square brackets provides us with spot rate changes under standard preferences. Comparing it with the square-bracketed expression in equation (7), we can see why the standard model is so puzzling with respect to the spot rate. It implies that persistence in the forward discount and spot rate changes should be the same. Since, empirically, spot rate changes are white and the forward discount is highly persistent, this is clearly an inadequate theory. Comparing equations (7) and (8) as a whole shows that habit persistence drives a wedge between the two variables. However, the non-linearity is such that only simulations can help us to assess the importance of habits.

Backus, Gregory and Telmer (1993) and Bekaert (1996) argue that the forward discount ‘bias’ can be summarised by the inability of the standard model to explain the high volatility of expected forward speculative profits:

$$E_t[f_t - s_{t+1}] = \gamma(1-\phi)(x_t^2-x_t^1)-\sigma_y^2\gamma(\lambda(x_t^1)-\lambda(x_t^2)). \tag{9}$$

In the standard model without habits, this expectation is zero. To see this, recall that forward speculative profit is obtained by subtracting spot rate changes from the forward discount. In the standard case, this is found by subtracting the first square bracketed term in equation (8) from that in equation (7). This yields $u_{t+1}^1 - u_{t+1}^2$: the difference between home and foreign money shocks. The expectation of this expression is, of course, zero. This is a consequence of the restrictions placed on the forcing processes. In the absence of such restrictions, it can be shown that expected forward speculative profit in the standard model equals:

$$-\tfrac{1}{2}\left[(1-\gamma)^2\left(\sigma^2_{y^1}-\sigma^2_{y^2}\right)+\left(\sigma^2_{m^1}-\sigma^2_{m^2}\right)-2(1-\gamma)\left(\sigma^2_{m^1y^1}-\sigma^2_{m^2y^2}\right)\right]$$

where $\sigma^2_{y^i}$, i=1,2 are home and foreign endowment shock variances; money shock variances are analogously defined and $\sigma^2_{m^iy^i}$ is the covariance between endowment and money shocks within country i. This can be decomposed into a ‘true’ (See Engel, 1998) risk premium and a Jensen’s Inequality term. The risk premium is $-\tfrac{1}{2}\left[\left(\gamma^2-2\gamma\right)\left(\sigma^2_{y^1}-\sigma^2_{y^2}\right)+2\gamma\left(\sigma^2_{m^1y^1}-\sigma^2_{m^2y^2}\right)\right]$. A sufficient condition for the risk premium to be zero is that the two covariances are zero and that home and foreign endowment variances are equal. The Jensen’s inequality term is $-\tfrac{1}{2}\left[\left(\sigma^2_{y^1}-\sigma^2_{y^2}\right)+\left(\sigma^2_{m^1}-\sigma^2_{m^2}\right)-2\left(\sigma^2_{m^1y^1}-\sigma^2_{m^2y^2}\right)\right]$. This is also zero if, in addition, home and foreign money shock variances are equal. In the discussion following equations (4) and (5), all of these conditions were assumed.

Equation (9) reveals that the model indeed provides us with a time-varying risk premium that depends both directly and indirectly on the surplus consumption ratios. As with the forward discount and spot rate changes, it is only possible to assess the effects of these new terms on absolute and relative volatilities through simulations.

I.4 A Limited Participation Model with Habit Persistence

Moore and Roche (1998) analyse forward market puzzles using a limited participation model with standard preferences along the lines of Lucas (1990), Fuerst (1992), Grilli and Roubini (1992) and Christiano, Eichenbaum and Evans (1997). Limited participation models differ from the usual cash-in-advance two-country models as follows. Asset portfolios cannot be adjusted costlessly. This idea is implemented by specifying that all portfolio decisions are made before the realisation of money shocks, both foreign and domestic. This is significant because assets, as well as goods, must be purchased with cash that must be accumulated in advance.

Moore and Roche (1998) exploit the fact that this sluggish portfolio behaviour does not affect forward contracts. Since there are no margin requirements, the model drives an additional liquidity 'wedge' between spot rate changes and the forward discount. Their model is simulated using the artificial economy methodology. It improves on the standard two-country cash-in-advance model in a number of ways. It gets closer to the observed lack of autocorrelation in spot exchange rate changes. It maintains the explanation that persistence in the forward discount is caused by persistence in money shocks. It can mimic the volatility of spot exchange rate changes though only at the expense of implying implausibly high volatility in the forward discount. Finally, the model goes some distance in explaining the forward discount bias puzzle but falls short of resolving it.

We are making an additional theoretical contribution in this section by setting the habit preferences of Section I.2 within the limited participation framework. The formal specification of the model is outlined in Appendix B. Here, we crystallise its main features into four 'efficiency' conditions.

Firstly, purchasing power parity does not hold in any conventional form. The spot exchange rate is determined as follows:

$$S_t = \frac{E_t\left[(1-\alpha)(C_{2t+1}^2 X_{2t+1}^2)^{-\gamma} \big/ P_{t+1}^2 \right]}{E_t\left[\alpha(C_{1t+1}^1 X_{1t+1}^1)^{-\gamma} \big/ P_{t+1}^1 \right]} \frac{q_t^1}{q_t^2}, \tag{10}$$

where $q_t^j = 1/1 + i_t^j$, $j = 1,2$ are the prices of one period home and foreign nominal bonds. In the standard CIA model, the real exchange is the instantaneous marginal rate of substitution between home and foreign goods. Equation (10) differs from this in that the price-weighted marginal utilities are expected future values. This reflects the fact that all decisions are made before shocks are known. The appearance of the bond price ratio in equation (10) is its most important feature. It follows, firstly, from the fact that goods arbitrage can only be mediated through money. However unlike the standard CIA model, money has another opportunity cost because of its use in purchasing bonds. Goods arbitrage diverts monetary resources away from asset markets and this effect appears as the bond price ratio in equation (10).

The second and third efficiency conditions are pricing conditions for home and foreign bonds:

$$E_{t-1}\left[\beta\frac{(C_{1t+1}^{i}X_{1t+1}^{i})^{-\gamma}}{q_t^i P_{t+1}^i}-\frac{(C_{1t}^{i}X_{1t}^{i})^{-\gamma}}{P_t^i}\right]=0,\quad i=1,2. \tag{11}$$

The two equations in (11) embody the sluggish portfolio assumption of the model. If expectations were taken at time t instead of t-1, closed form expressions for bond prices would emerge. Because portfolios are set before shocks are known, the Fisher equation, even allowing for a risk premium, only holds on average. Unlike in the standard CIA model, the bond prices remain implicit. The additional source of non-linearity can be interpreted as giving rise to a ‘liquidity’ premium. This adds further volatility to bond prices and through equation (10) to the spot exchange rate.

The final efficiency condition is simply the usual covered interest parity condition. It would be pleasing if the approximations, which we applied in Section I.3 to the standard CIA model with habits, could be extended here. However, the non-linearity of equations (10)-(11) makes this almost completely unrewarding. To clarify the model any further, we need to conduct numerical simulations.

II Empirical and Model Evidence

II.1 Calibration

We calibrate the model discussed in Section I and compare the moments generated from the model with those in quarterly data. There are ten parameters to choose. We present the baseline parameterisation in Table I.

The discount rate, β, is assumed to be $(1.03)^{-.25}$ which is based on an annual real rate of interest of 3 percent; a value commonly used in the literature (see for example Christiano, 1991). The share of consumption of home produced goods in the domestic agent's utility function, α, is set equal to 0.5; a value estimated in Stockman and Tesar (1995). These parameters remain constant in the various experiments we simulate. The elasticity of intertemporal substitution, $1/\gamma$, and the AR(1) coefficient of the log of the surplus consumption ratio, ϕ, have major effects in the habit persistence models. In the baseline parameterisation we set the elasticity of intertemporal substitution equal to 1/0.7. A value of 0.7 for γ is used by Campbell and Cochrane (1999) to model the historical price-dividend ratio for the US.

Table I
Baseline parameterisation

β	0.9926	γ	0.70
α	0.50	ϕ	0.97
	Endowment Growth	Money Growth	
Mean	0.4725%	3.12%	
Standard Deviation	0.75%	0.54%	
AR(1) Coefficient	0.00	0.33	

The AR(1) coefficient of log surplus consumption equals to 0.97 a value also used in Campbell and Cochrane (1999). Later in this section we perform sensitivity analysis and examine how the results change when the elasticity of intertemporal substitution varies between 2.5 and 0.5 and when we change the AR(1) coefficient of the log of the surplus consumption ratio from 0.83 to 0.97.

We need to specify parameters of the exogenous endowment and money growth rate processes.[12] We hypothesise that at the very most a second-order four-variable vector autoregression should capture the basic features of these growth processes. We estimate six vector autoregressions: each VAR consisted of four variables. They are endowment and money growth from the US and each of the G7 countries. We compare the Schwarz Information Criterion for the general VAR(2) model and various restricted versions of that process. We were unable to reject the hypothesis that endowment and money growth are both described by scalar AR(1) processes for the country pairs considered. In the light of this, we decided to use the same parameters for the forcing processes for both the home and foreign countries.

In the baseline parameterisation we assume the following values for parameters of the quarterly AR(1) endowment growth process; the unconditional mean is set equal to 0.4725 percent, the standard error of the AR(1) process is assumed to be 0.75 percent and the first-order autocorrelation coefficient is set equal to 0.0. These are the values used in Campbell and Cochrane (1999) and are in the range commonly found in the data. We assume the following values for parameters of the quarterly AR(1) money growth process; the unconditional mean is set equal to 3.12 percent, the standard error of the AR(1) process is assumed to be 0.54 percent and first-order autocorrelation coefficient is set equal to 0.33. All of these numbers are in the range found in G7 countries over the period 1973-1998. In the next sub-section, the sensitivity of our results to some of the assumptions about the forcing processes will be explored.

II.2 Results and Sensitivity Analysis

The results from the baseline parameterisation for the moments of interest are presented in Table II. We call the standard CIA model, A, the Moore and Roche (1998) limited participation household monetary model, B, the CIA model with habit preferences, C, and the limited participation household monetary model with habit preferences, D. We also report typical values of exchange rate moments found in quarterly data.

Table II
Moments in Quarterly Data and in the Theoretical Economy

The results presented in the last four columns are the mean of 10000 replications using the baseline parameterisation. The coefficient b is from the following regression for predicting the returns from currency speculation $E_t(F_t - S_{t+1})/S_t = a + b(F_t - S_t)/S_t$.

	Data	Model A	Model B	Model C	Model D
Standard Deviation of $(S_{t+1} - S_t)/S_t$	6%	0.84%	0.80%	19.92%	6.19%
Standard Deviation of $E_t(F_t - S_{t+1})/S_t$	1-3%	0.07%	1.12%	2.77%	1.58%
Standard Deviation of $(F_t - S_t)/S_t$	0.7-1.3%	0.26%	1.25%	1.62%	1.11%
AR(1) Coefficient of $(S_{t+1} - S_t)/S_t$	0	0.26	0.02	-0.02	0.003
AR(1) Coefficient of $(F_t - S_t)/S_t$	0.5-0.9	0.31	0.88	0.85	0.84
Mean of b	> 1	0.06	0.88	-2.06	1.43
Percentage of rejections of 'speculative efficiency hypothesis' H_0:a=0,b=0 (size of 5%)	High	5.02%	99.99%	24.60%	97.20%
Percentage of b>1	High	0.30%	13.16%	3.09%	84.67%
Percentage of $\|b\|$>1	High	0.30%	13.16%	71.11%	84.88%

A telling indicator of the success of a model in explaining the forward market is its ability to account for the volatility, persistence and forward bias anomalies simultaneously. We discuss each of these anomalies in turn. We perform sensitivity analysis to determine whether the results change if we vary baseline parameter settings.

II.2.1 Volatility anomaly

The volatility anomaly concerns the ranking in standard deviations of relevant exchange rate variables commonly found in the data. The standard deviation of spot changes (6%) is higher than that of the expected profit from currency speculation (1-3%), which in turn is higher than that of the forward discount (0.7-1.3%). While Models A and B fail to produce either the ranking or the magnitude of the standard deviations, the habit models perform remarkably well. However Model C tends to produce too much volatility generally and particularly in spot exchange rate changes. This is consistent with the results of Bekaert (1996). Model D produces values that are usually found in the data. As far we can determine, this is the first model that replicates both absolute and relative volatilities at plausible parameter values.

To see why this occurs, consider the effect of varying the habit persistence parameter in Models C and D. As ϕ increases, the variability of the *change* in the surplus consumption ratio declines. Consequently, the variability of the intertemporal marginal rate of substitution, and therefore the three exchange rate volatilities decline as ϕ approaches unity[13]. This is illustrated for Model C in Figure 1 in which ϕ, the AR(1) coefficient of the log of the surplus consumption ratio, is allowed to vary from .83 to .97. Another way of thinking about it is to note from equation (A2) in Appendix A that the steady state surplus consumption ratio rises as ϕ rises. The local curvature of the utility function is given by

$$-C_i^j \frac{\partial^2 U / \partial (C_i^j)^2}{\partial U / \partial C_i^j} = \frac{\gamma}{\overline{X}} \quad i, j = 1,2 \tag{12}$$

Campbell and Cochrane (1999) show that the expression in (12) is positively related to the coefficient of relative risk aversion for the one good case[14]. As $\overline{X}$, the steady state surplus consumption ratio rises, risk aversion and therefore exchange rate volatilities decline.

Figure 2 graphs the results of the same experiment for Model D. By contrast, the three volatilities are much less sensitive[15] to changes in ϕ. The effect of the cash-in-advance-for-assets constraint is to raise the volatilities of the forward and risk premia *anyway* (See the results for Model B in Table 1). This dominates the effect of variations in the persistence of habits with one exception. Moore and Roche (1998) have already noted that the volatility of spot rate changes is only raised in limited participation models above its value in standard CIA model at very low levels of intertemporal substitutability (=high levels of risk aversion in models A and B)[16]. The main impact of introducing habits is to raise the volatility of spot rate changes. However the volatility in the limited participation model increases by less than that in the CIA model. The most pleasing aspect of introducing Campbell-Cochrane habits is that, in both CIA and limited participation models, the volatility of the forward premium is affected less than the volatility of expected forward speculative profits. This is because of the constant real risk rate assumption that is built into Campbell-Cochrane preferences. The consequence is that the *ranking* of the two volatilities is consistent with the data for *both* models C and D.

It is also worthwhile showing the effect of varying γ, the inverse of the intertemporal elasticity of substitution on the exchange rate volatilities. This is illustrated in Figures 3 and 4 where the elasticity varies from 2.5 ($\gamma = 0.4$) to 0.5 ($\gamma = 2$). As the elasticity of intertemporal substitution falls, the lack of intertemporal substitution opportunities means that it is more difficult to diffuse the effect of shocks over different time periods. Consequently, all exchange rate volatilities rise. This is particularly noticeable for model D (Figure 4) for values of γ above unity (log utility).

I.2.2 Persistence anomaly

It is commonly observed that spot changes have low or zero first order autocorrelation coefficients while the forward discount is highly persistent. The persistence of the forward discount and lack of it in spot rate changes has eluded many previous studies with the notable exceptions of Bekaert (1994, 1996) and Moore and Roche (1998). The baseline results for Model A, reported in Table II, define the problem. Both autocorrelations are close to the autocorrelation coefficient of the money growth process (0.33) as one might expect from the discussions following equations (7) and (8). Models B, C and D are very encouraging in addressing the problem of the overprediction of the persistence of the spot depreciation in the standard model. These models also produce AR(1) coefficients of the forward discount that are in the range empirically found.

Since Moore and Roche (1998) found that money growth persistence has major effects on the AR(1) coefficients of spot changes and the forward discount we investigate this and vary the AR(1) coefficient in money growth from 0.2 to 0.8. The summarised results are graphed in Figures 5 and 6. In order to match the persistence of the forward discount that is typically found in the data, its AR(1) coefficient needs to be above 0.5 and as close to 0.8 as possible. By contrast, the AR(1) coefficient in spot rate changes needs to be as close to zero as possible.

For Model A the AR(1) coefficient for both the spot change and forward discount reflects that of the money growth process. The AR(1) coefficient for spot rate changes rises in Model B, particularly when the AR(1) coefficient in money growth exceeds 0.5 and is always close to zero for Models C and D. For Models B, C and D, the AR (1) coefficient for the forward discount declines somewhat as money growth persistence rises but is never far from the empirically observed level. The AR(1) coefficient for the spot change (forward discount) in Model A represents an *upper bound* (*lower bound*) to the value found in the other three models.

II.2.3 Forward bias

The best-known exchange rate anomaly is the forward bias. According to the ‘speculative efficiency hypothesis’, the profit from forward speculation should be unpredictable. However, a regression of the profit from forward speculation on the forward discount typically produces an estimate of the slope, *b*, greater than 1 and a rejection of the efficiency hypothesis (H_0:a=0,b=0). In Table II we report the mean estimate of *b*, the number of rejections of the efficiency hypothesis using a 5% significance level and the percentage of estimates of *b* and |*b*| greater than unity. The standard explanation for this anomaly is that it is accounted for by a 'time-varying risk premium. Unfortunately, this is not good enough as the standard CIA model (model A) fails miserably in replicating this anomaly. Model B does better in that it succeeds in rejecting the 'speculative efficiency hypothesis' almost all of the time. While this goes some way towards what we empirically observe it does not produce too many estimates of b above unity.

Model C produces a perverse result of a negative average estimate of *b* of -2.06. It is interesting that Bekaert (1996) reports that he was unable to generate a single simulation that gave rise to an estimate of *b* greater than unity. Since he was also modelling habit persistence but without the sophistication of the limited participation model, it would be worth revisiting his work to determine if he was also obtaining this perverse result. Backus, Foresi and Telmer (1995) demonstrate that in order to obtain a value of *b* typically found in the data, the covariance of the expected spot change and expected forward speculative profits must be negative and *larger* in absolute value than the variance of the expected spot change. We find in the baseline parameterisation Model's C and D produce negative covariances 90% and 85% of the time, respectively. However the variance of the expected spot change is large in Model C, thus producing the perverse negative slope. Backus, Foresi and Telmer (1998) have already noted the possibility that such a perverse result can occur. Interestingly, they find it in a two-currency version of Cox, Ingersoll and Ross (1985).

At the baseline parameterisation only Model D succeeds in matching the data. This model rejects the efficiency hypothesis 97% of the time and produces the result that 85% of the slope coefficients are greater than unity with a mean of 1.43. We plot a histogram of the estimates of b for Model D in Figure 7. Changing the elasticity of intertemporal substitution produces a striking change to our baseline results. We plot the percentage number of estimates of b greater than unity for Model D in Figure 8. As before, we increase γ from 0.4-2.00. When $\gamma<1.0$ there is a large percentage of estimates of b greater than unity. However, this collapses to almost none when γ exceeds unity. It appears that an essential part of the explanation of the time-varying risk premium is that the intertemporal elasticity of substitution should exceed unity.

III Conclusion

This paper has proposed a modelling strategy that makes substantial progress towards resolving many of the outstanding puzzles in respect of the forward foreign exchange market. A model that combines habit persistence, a constant real rate of interest and a limited participation monetary framework is capable of explaining the volatility, persistence and forward bias puzzles.

Alvarez, Atkeson and Kehoe (1999) also claim to have resolved the forward bias anomaly. Their work is related to limited participation models such as our models B and D as follows. Both obtain liquidity effects by introducing trading frictions in asset markets. Limited participation models retain the assumption of complete markets but the trading friction is exogenous (i.e. *all* agents are required to use cash-in-advance for assets). By contrast, Alvarez, Atkeson and Kehoe introduce trading frictions by relaxing the assumption of complete markets and by allowing the extent of the trading friction to be endogenously determined. Their strategy is as follows. Asset markets and goods markets are segmented in the sense that agents must pay a fixed cost to transfer money between the two markets. This segmentation is endogenous because, in equilibrium, some agents chose to pay the fixed cost while some do not.

They derive sufficient conditions for *one* of Fama's necessary conditions for the forward premium anomaly viz., that the covariance between expected forward speculative profits and expected spot rate changes be negative. The most important feature of our work is that, in model D, we combine the asset market trading friction with external habit persistence. By contrast, Alvarez, Atkeson and Kehoe maintain time inseparable preferences and do not determine the implication of their model for Fama's second necessary condition: that the variance of expected forward speculative profits exceed the variance of expected spot rate changes. They do not simulate their model so that it is difficult to come to any definite conclusion about it at this stage.

Three research directions are immediately suggested. Firstly, it would be useful to compare these results to the implications of the new sticky price general equilibrium models. (See Obstfeld and Rogoff, 1998 and Engel, 1998). Secondly, it would be interesting to apply this approach to *nominally* denominated asset prices in closed economies. Thirdly, there are many issues unresolved in international real business cycle theory, such as the behaviour of real exchange rates. It would be worthwhile to examine whether this model casts any light on them.

APPENDIX A

The form of the sensitivity function $\lambda(x_t^i)$ is:

$$\lambda(x_t^i) = \frac{\sqrt{1-2(x_t^i - \bar{x}^i)} - 1}{\bar{X}^i} \quad for \quad x_t \leq x_{max} \tag{A1}$$

where $x_{max}^i = \bar{x}^i + \frac{1-(\bar{X}^i)^2}{2}$; $\bar{X}^i$ is the steady state value of the surplus consumption ratio for good i; and

$$\bar{X}^i = \sigma_y^i \sqrt{\frac{\gamma}{1-\phi}} \, . \tag{A2}$$

The parameters γ and ϕ are defined in Section I.2 of the text. σ_y^i is the standard deviation of the innovation to the endowment of the i^th good.

The properties of $\lambda(x_t^i)$ are discussed extensively in Campbell and Cochrane (1999). The main feature, which needs emphasising here, is the implication of this specification for risk-free real rates of interest. The own good real rates of interest, r^1 and r^2 are:

$$r^i = -\log\left[\beta E_t \frac{U_{it+1}}{U_{ii}}\right] = -\log\beta + \gamma\mu^i - \frac{\gamma(1-\phi)}{2} \quad i = 1,2, \tag{A3}$$

For comparison the own real interest rates, in the absence of habits, are:

$$r^i = -\log\beta + \gamma\mu^i - \frac{\gamma^2(\sigma_y^i)^2}{2} \quad i = 1,2. \tag{A4}$$

The main difference between (A4) and (A3) is that the real interest rate in the habit model is independent of the variance of real shocks. The similarity is that both are constant over time so long as endowment shocks are i.i.d. i.e. $\mu_t^i = \mu^i \forall t$. In the case of the habit model this risk-free rate constancy is not obtained at the expense of implausible asset price properties.

APPENDIX B

The households in both countries have the same intertemporal utility function:

$$U=\sum_{t=0}^{\infty}\beta^{t}\,\frac{\left[(C_{it}^{1}-H_{it}^{1})^{\alpha}(C_{it}^{2}-H_{it}^{2})^{(1-\alpha)}\right]^{1-\gamma}-1}{1-\gamma},\quad i=1,2, \tag{B1}$$

where the definitions are given in Section I.2. The agent in the goods market faces the following cash-in-advance constraint

$$N_{it}^{j}\geq P_{t}^{j}C_{it}^{j},\quad i=1,2,\quad j=1,2, \tag{B2}$$

where N_{it}^{j} is the amount of money of country j held by the household of country i for transactions in the goods market at time t and P_{t}^{j} is the price of country j goods in terms of country j money. The agent faces the following cash-in-advance constraint

$$Z_{it}^{1}+S_{t}Z_{it}^{2}\geq q_{t}^{1}B_{it}^{1}+S_{t}q_{t}^{2}B_{it}^{2},\quad i=1,2, \tag{B3}$$

where Z_{it}^{j} is the amount of money of country j held by the household of country i for transactions in the asset market at time t, S_t is the domestic price of foreign currency at time t, $\mathrm{q}_{\mathrm{t}}^{\mathrm{j}}$ is the price of country i's discount bonds and B_{it}^{j} is the total amount of bonds of country j held by the household of country i at time t.

If interest rates are positive both cash-in-advance constraints will hold with equality. Thus, at the beginning of period t+1, the domestic households holding of domestic currency

$$M_{1t+1}^{1}\geq P_{t}^{1}C_{t}^{1}+B_{1t}^{1}-F_{t}G_{t}^{1} \tag{B4}$$

is made up of proceeds from the sale of the endowment, the redemption of the discount bonds, and proceeds from forward contracts, F_t, in the previous period. $G_{t}^{1}>0$ constitutes the number of "long" forward contracts. The domestic household's holding of foreign currency is

$$M_{1t+1}^{2}\geq B_{1t}^{2}+G_{t}^{1} \tag{B5}$$

Analogously the foreign households holding of foreign currency is

$$M_{2t+1}^{2} \geq P_t^2 C_t^2 + B_{2t}^2 - G_t^2 \tag{B6}$$

and of domestic currency is

$$M_{2t+1}^{1} \geq B_{2t}^1 + F_t G_t^2 \tag{B7}$$

where $G_t^2 > 0$ constitutes a "short" position in forward foreign exchange for the foreign country.

The only role for the government is to have a central bank that engages in open market operations. In each period the central bank of each country changes the money stock by issuing one-period discount bonds. The bonds are redeemed at the beginning of the next period. Exogenous money growth is given by

$$\frac{M_{t+1}^{j}}{M_t^j} = (1 + \pi_t^j), \quad j = 1,2 \tag{B8}$$

where

$$\begin{aligned} M_{it}^j &= N_{it}^j + Z_{it}^j, \quad i = 1,2, \quad j = 1,2, \\ M_t^j &= M_{1t}^j + M_{2t}^j, \quad i = 1,2, \end{aligned} \tag{B9}$$

where $\mathrm{M}_{\mathrm{it}}^{\mathrm{j}}$ is the total amount of money of country j held by the household of country i at time t. Exogenous endowment growth is given by

$$\frac{C_{t+1}^{j}}{C_t^j} = (1 + \mu_{t+1}^j), \quad j = 1,2. \tag{B10}$$

Equilibrium in the goods market is given by

$$C_t^j = C_{1t}^j + C_{2t}^j \quad j = 1,2. \tag{B11}$$

Equilibrium in the asset market is given by

$$Z_t^j = Z_{1t}^j + Z_{2t}^j = q_t^j (B_{1t}^j + B_{2t}^j) = q_t^j B_t^j, \quad j = 1,2. \tag{B12}$$

Equilibrium in the forward foreign exchange market is given by

$$G_t^1 = G_t^2 . \tag{B13}$$

We can now formulate the domestic household's problem. Let V(•) represent the value function. Assuming that the cash-in-advance constraints are binding, the domestic household solves

$$\begin{aligned} & V\left(M_{1t}^{1}, M_{1t}^{2}\right) = Max \quad E_{t-1}\left(Max\left[U\left(\frac{N_{1t}^{1}}{P_t^1}, \frac{N_{1t}^{2}}{P_t^2} \right) + \beta E_t V\left(M_{1t+1}^{1}, M_{1t+1}^{2}\right)\right]\right) \\ & s.t. \qquad (M_{1t}^{1} - N_{1t}^{1}) + S_t (M_{1t}^{2} - N_{1t}^{2}) \geq q_t^1 B_{1t}^1 + S_t q_t^2 B_{1t}^2 , \end{aligned} \tag{B14}$$

where expectations[17] are taken over the set of four exogenous stochastic state variables $\{ C_{it}^{j} \ \ M_{it}^{j}$ for i=1, 2, j=1, 2$\}$. The first maximisation is with respect to N_{1t}^1 and N_{1t}^2 . The second maximisation is with respect to B_{1t}^1 , B_{1t}^2 and G_t^1 .

There is no closed form solution for this non-linear stochastic rational expectations model. Thus we find an approximate solution using the linear-quadratic methods of Christiano (1991). This yields optimal linearised rules for domestic and foreign bond prices, spot and forward exchange rates as functions of the endowment and money shocks and the log of surplus consumption ratio. A technical appendix containing this solution method is available from the authors upon request.

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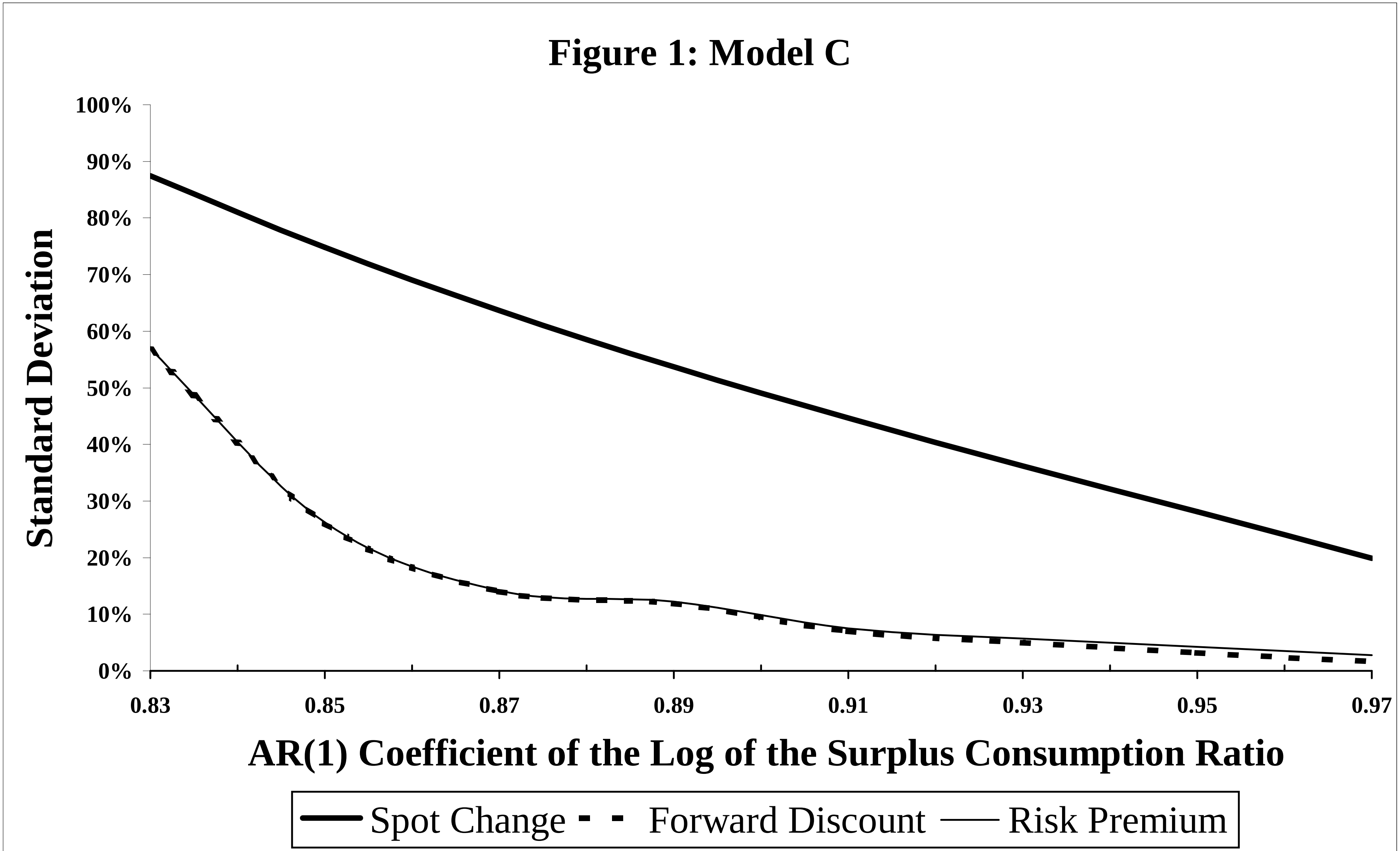

Figure 1: Model C
Standard Deviation
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0.83
0.85
0.87
0.89
0.91
0.93
0.95
0.97
AR(1) Coefficient of the Log of the Surplus Consumption Ratio
Spot Change
Forward Discount
Risk Premium

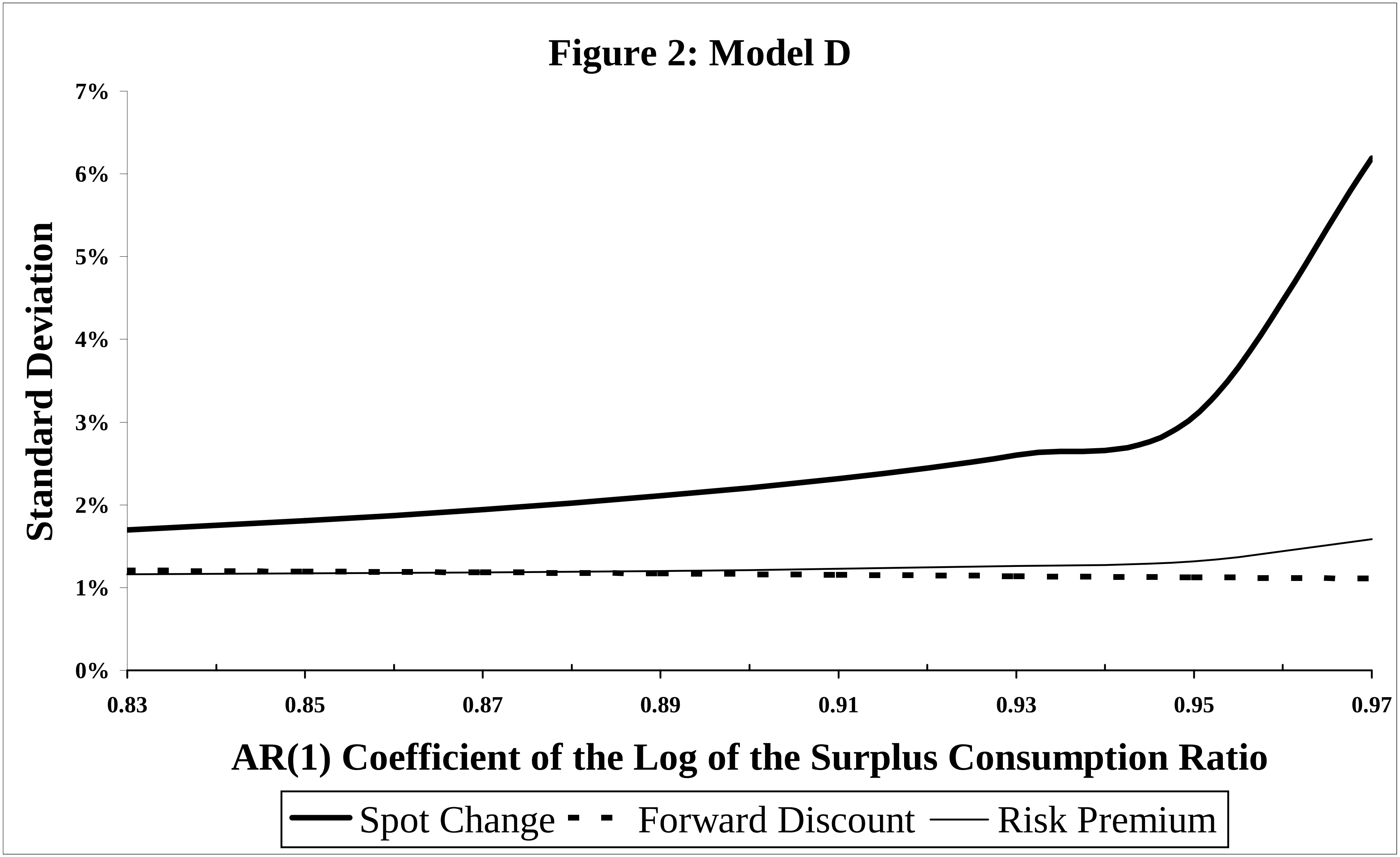
Figure 2: Model D
Standard Deviation
7%
6%
5%
4%
3%
2%
1%
0%
0.83
0.85
0.87
0.89
0.91
0.93
0.95
0.97
AR(1) Coefficient of the Log of the Surplus Consumption Ratio
Spot Change
Forward Discount
Risk Premium

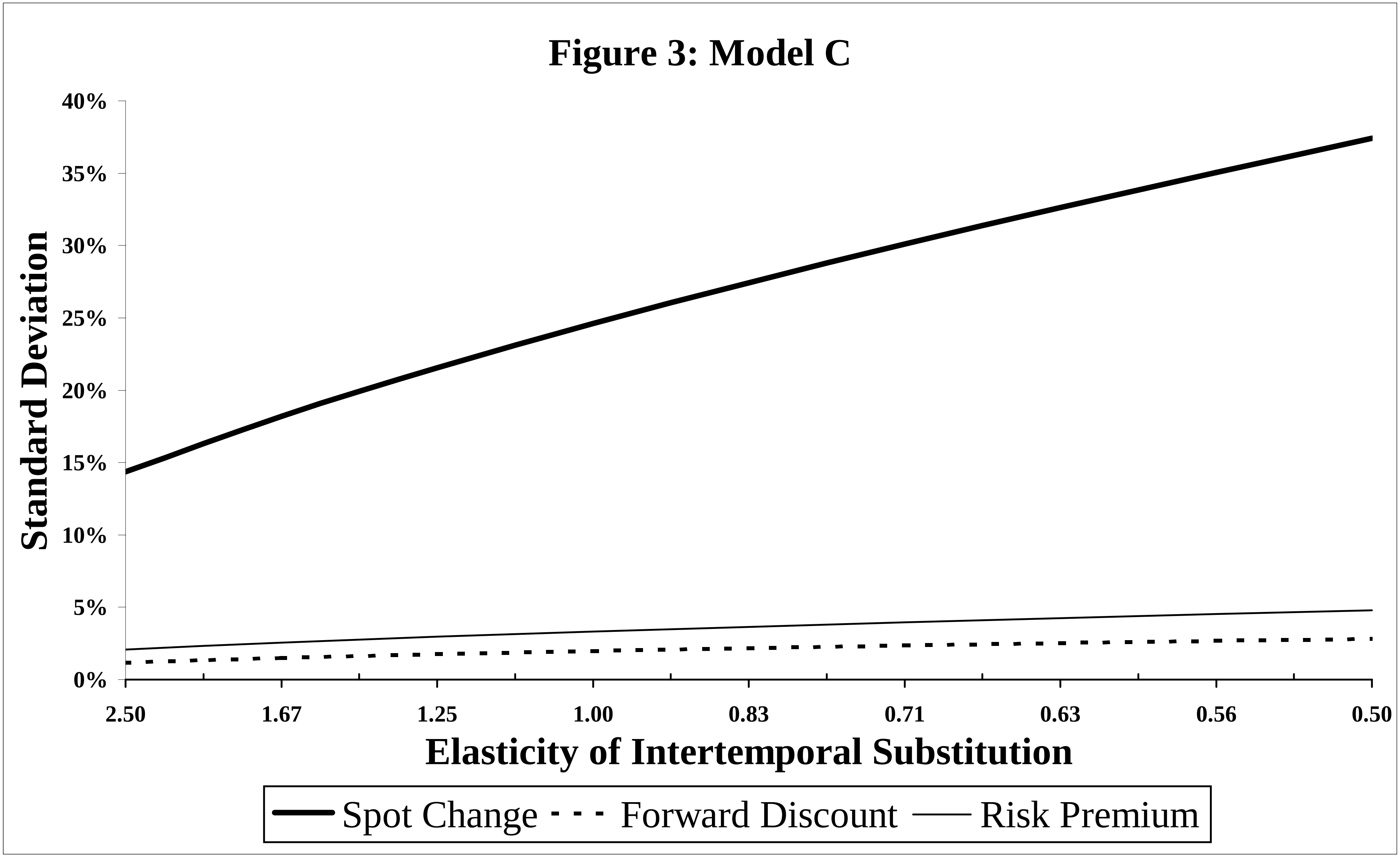
Figure 3: Model C
40%
35%
30%
25%
20%
15%
10%
5%
0%
Standard Deviation
2.50
1.67
1.25
1.00
0.83
0.71
0.63
0.56
0.50
Elasticity of Intertemporal Substitution
Spot Change
Forward Discount
Risk Premium

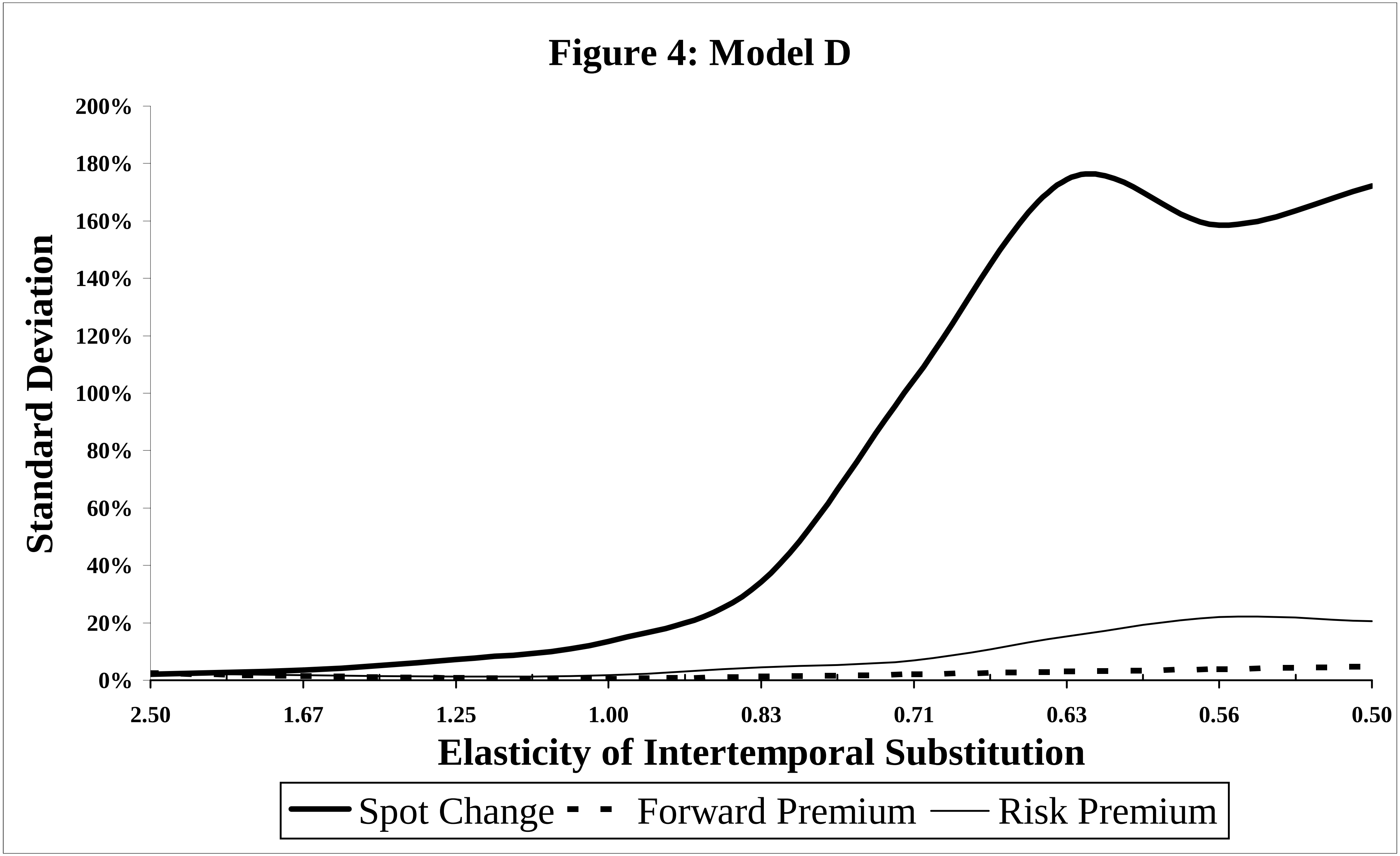

Figure 4: Model D
Standard Deviation
200%
180%
160%
140%
120%
100%
80%
60%
40%
20%
0%
2.50
1.67
1.25
1.00
0.83
0.71
0.63
0.56
0.50
Elasticity of Intertemporal Substitution
Spot Change
Forward Premium
Risk Premium

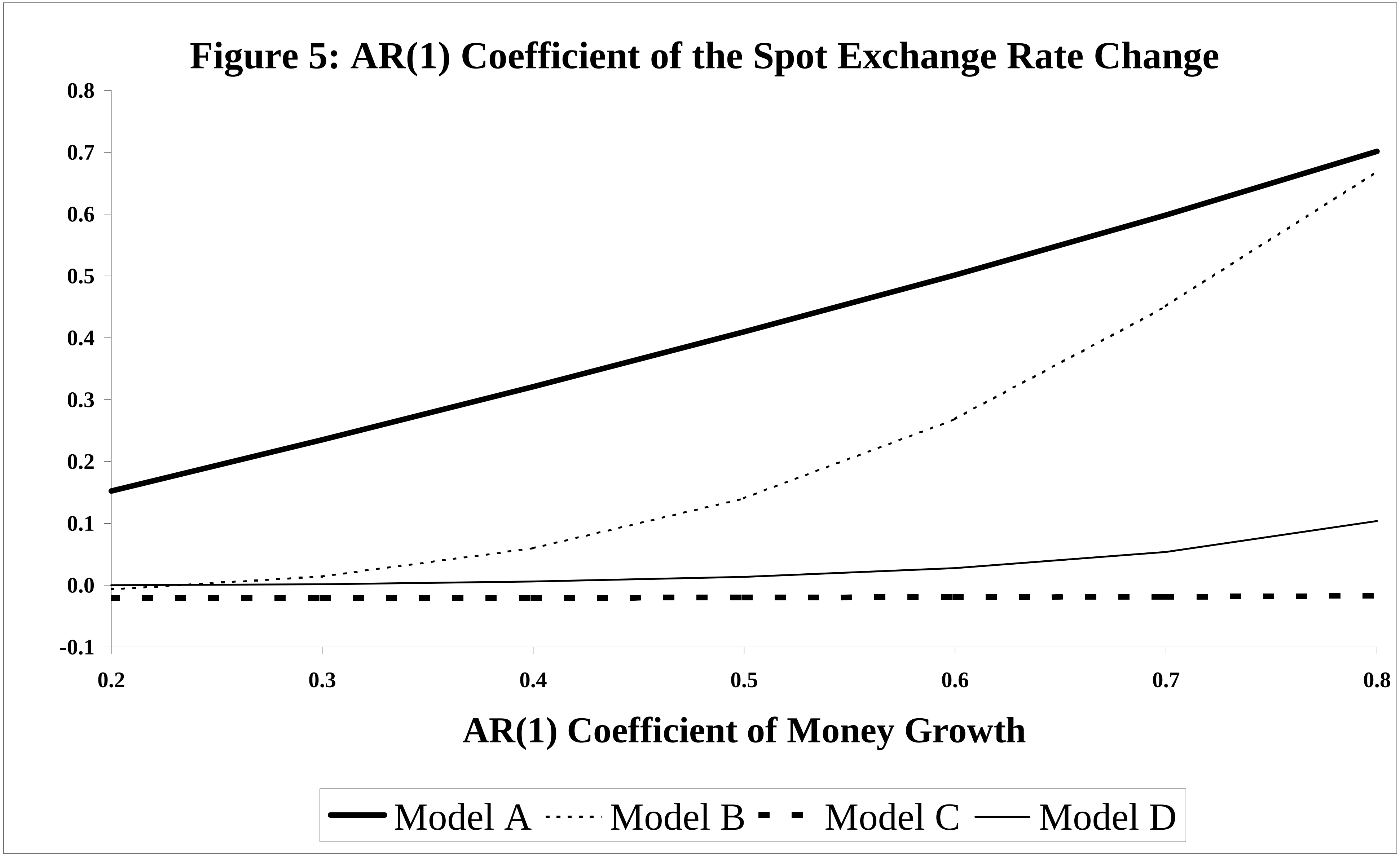
Figure 5: AR(1) Coefficient of the Spot Exchange Rate Change
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
AR(1) Coefficient of Money Growth
Model A
Model B
Model C
Model D

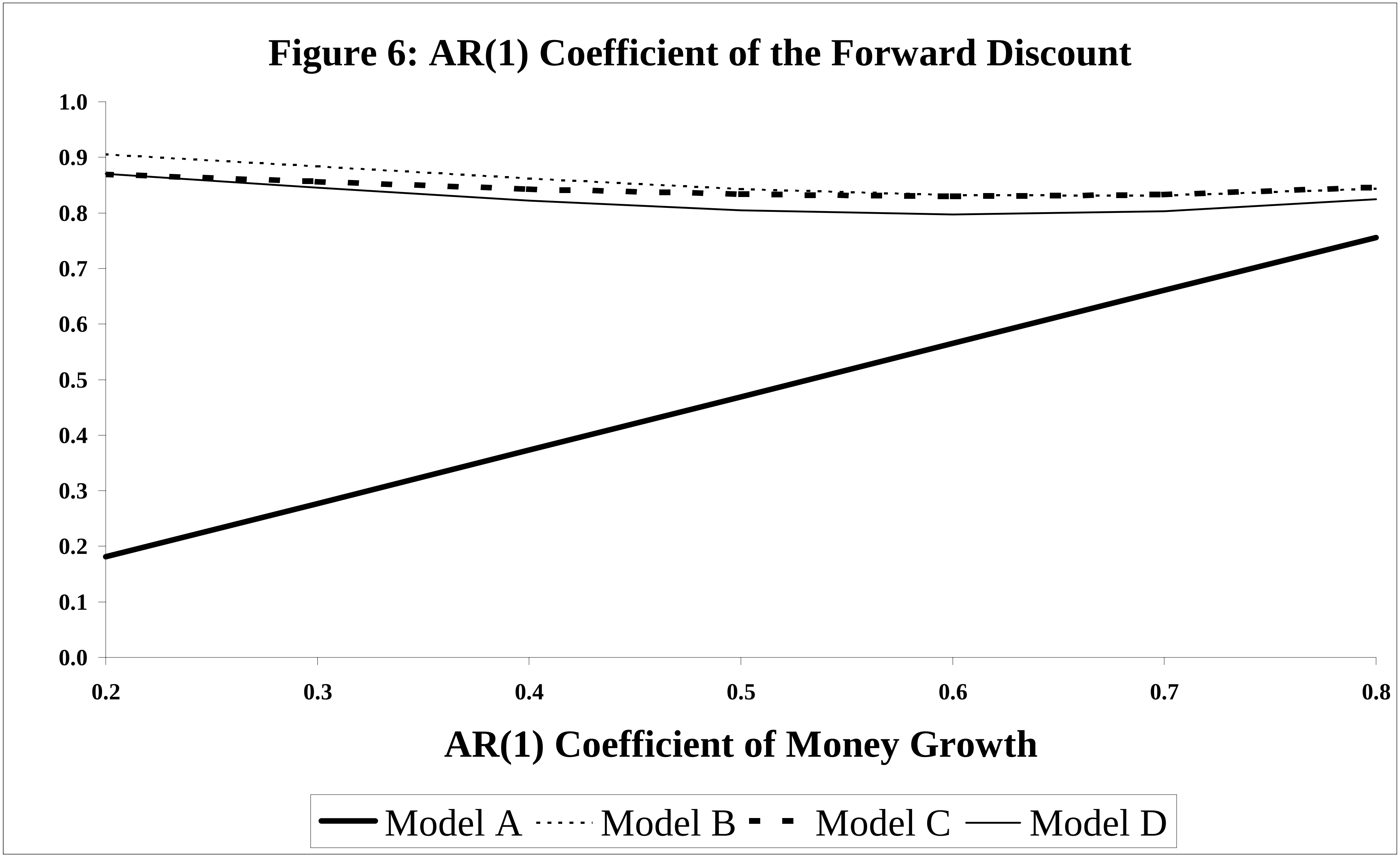
Figure 6: AR(1) Coefficient of the Forward Discount
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
AR(1) Coefficient of Money Growth
Model A
Model B
Model C
Model D

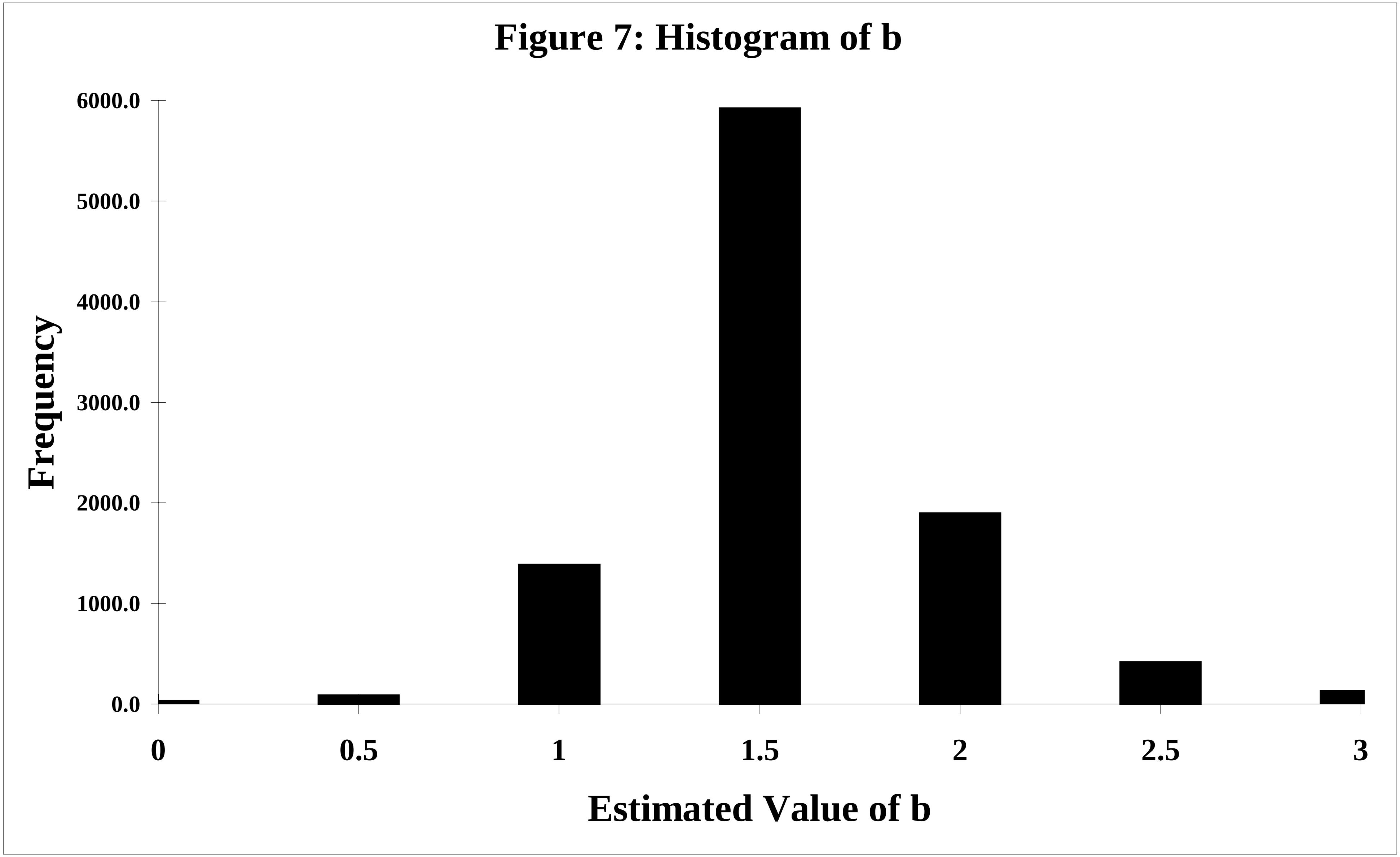
Figure 7: Histogram of b
Frequency
6000.0
5000.0
4000.0
3000.0
2000.0
1000.0
0.0
0
0.5
1
1.5
2
2.5
3
Estimated Value of b

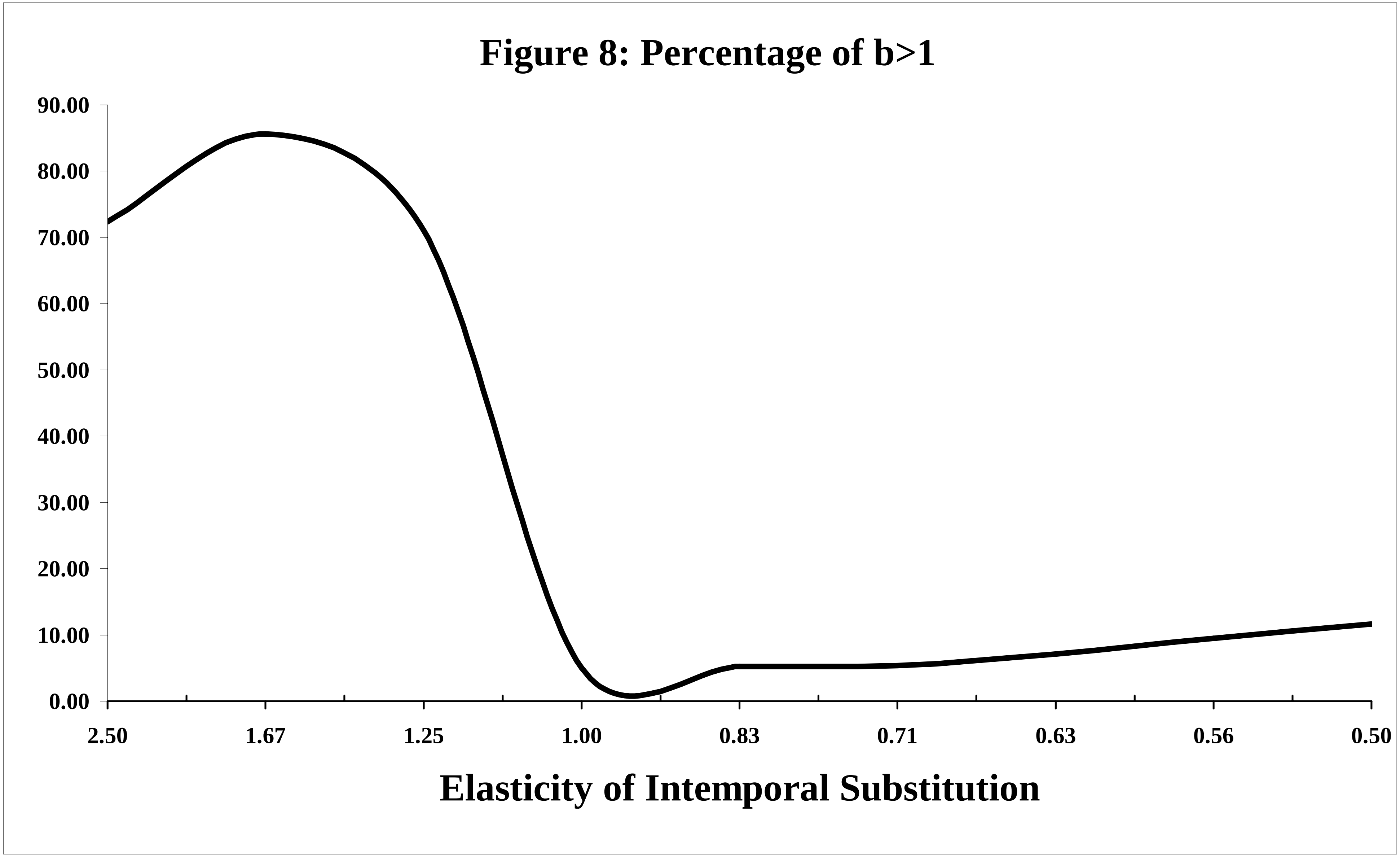
Figure 8: Percentage of b>1
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
2.50
1.67
1.25
1.00
0.83
0.71
0.63
0.56
0.50
Elasticity of Intemporal Substitution

[1] We are grateful to John Cochrane and Richard Friberg for their comments. This paper also benefitted from the contributions of seminar participants at the University of Strathclyde and the University of California at Santa Cruz.
[2] See Abel (1990) and Duesenberry (1949).
[3] For discussions of forward market puzzles, see Bekaert (1996), Engel (1996), Hodrick (1987) and Sibert (1996).
[4] Expected forward speculative profits are often referred to as 'the' risk premium. This is, in general, incorrect because of non-convexities: see Engel (1998). In addition, this paper suggests an additional liquidity premium interpretation for models that incorporate limited participation household behaviour.
[5] See Moore and Roche (1998).
[6] For a general discussion of limited participation models, see Christiano, Eichenbaum and Evans (1997).
[7] Habit persistence makes utility time inseparable so that the curvature parameter cannot be interpreted as the coefficient of relative risk aversion. Its inverse is, however, the intertemporal elasticity of substitution.
[8] From this point the country subscript is suppressed for ease of notation.
[9] It is easy to specify a richer stochastic process for endowments. However the constant risk-free rate result is then lost. We do not feel that this is overly restrictive: see the discussion on calibration in section II.I
[10] In the remainder of this section, the importance of habits is illustrated using linear approximations. In section 2, the models are simulated using linear quadratic methods.
[11] To derive equation (8), we use the property that the two steady state surplus consumption ratios, $\bar{x}^1$ and $\bar{x}^2$, are the same, given the assumption that home and foreign endowment shocks have the same variance. See Appendix A.
[12] We use quarterly G7 data from 1973-1998 to replicate stylised facts about exchange rates and to calibrate parameters in the exogenous shock processes. We use the US$ as the numeraire for exchange rates. We proxy the endowment series as seasonally adjusted real GDP and the money series as seasonally adjusted M3. All series are sourced from Datastream. Our results are similar to many other studies and are available upon request.
[13] At ϕ =1, the steady state surplus consumption ratio is not defined. In effect, model C converges on model A while model D converges on model B.
[14] Defining risk aversion in a multi-good model is not trivial. An intertemporal model has as many goods as time periods. In addition, our model has two goods in each time period.
[15] Compare the scales in Figures 1 and 2.
[16] Schlagenhauf and Wrase (1995) make a related point.
[17] We assume that expectations are such that (B14) is well behaved.