

In that model the government wishes to peg the price of a resource—call it “gold”—at a price  $p$  measured in terms of a broad commodity basket<sup>(1)</sup>. The private sector’s flow demand curve for gold is

$$D(p_t), D'(p_t) < 0,$$

and there is a “choke price”  $p^c$  such that  $D(p^c) = 0$ . At time  $t = 0$  the total stock of gold in the (world) economy is  $S_0$ ; for simplicity, the marginal cost of extracting gold from the ground for private use is assumed to be zero.

The *laissez-faire* perfect-foresight solution for the price path  $p_t$  is well known from the classic work of Hotelling (1931). The key insight used in deriving this path is that because gold in the ground yields no service flow and costs nothing to extract, its price  $p_t$  must rise at the real rate of interest,  $r$ <sup>(2)</sup>. A rate of price increase greater than  $r$  would lead to an excess flow demand for gold by industry and personal users as gold owners hoard it to earn excess returns; a rate of price increase below  $r$  would lead to an excess supply as owners dump their gold on the market in order to shift into bonds.

The *laissez-faire* gold price can be determined from the above arbitrage argument, which implies a price path from  $t = 0$  of the form

$$p_t = p_0 e^{rt},$$

and the requirement that supply equals demand at each moment. Let  $T$  be the date  $p_t$  following (2), reaches the zero-demand choke price  $p^c$ ,

$$p^c = p_0 e^{rT},$$

or,

$$(1) \quad T = \log(p^c/p_0)/r.$$

Then supply will equal demand on every date if the initial market price  $p_0$  is set to equate intertemporal demand to the total available stock:

$$S_0 = \int_0^{\log(p^c/p_0)/r} D(p_0 e^{rt}) dt.$$

On date  $T = \log(p^c/p_0)/r$ , the economy’s stock of gold is used up and demand is nil. To take a simple concrete example, if  $D(p) = p^{-\sigma}$  (in which case  $p^c = +\infty$ ),

$$(2) \quad p_0 = \tilde{p}(S_0) = (r\sigma S_0)^{-1/\sigma}.$$

Now consider how the equilibrium would look if the government pegs the price of gold at some level  $\bar{p}$  between  $\tilde{p}(S_0)$  and  $p^c$ . Initially gold owners will sell their entire stock  $S_0$  to the government because they can earn a rate of return  $r > 0$  by placing their wealth in bonds instead of gold. For a time, industrial and personal demands therefore will be supplied entirely by the government, which must sell an amount  $D(\bar{p})$  of its reserve each period. It is clear, however, that this situation is unsustainable: eventually the stock  $S_0$  will be depleted and the equilibrium price will have to be at its choke level. The critical problem is to characterize the process through which the government’s price-fixing scheme collapses.

Figure 1 furnishes a simple characterization based on the assumption that  $D(p) = p^{-\sigma}$ . Its two solid graphs show two notional prices of gold. The horizontal line is the natural logarithm of the official price  $\bar{p}$ . The second upward-sloping curve is the natural logarithm of  $\tilde{p}_t$ , defined by the function  $\tilde{p}(S_t)$  in (2), where  $S_t$  is the stock of gold remaining at time  $t$  conditional on the price-fixing scheme remaining in effect until that date:

$$(3) \quad \tilde{p}_t = (r\sigma S_t)^{-1/\sigma} = \{r\sigma[S_0 - D(\bar{p})t]\}^{-1/\sigma}.$$

(1) Monetary models of gold standards, which combine the natural resource aspect of gold with its monetary function, are analyzed by Flood and Garber (1984a), Bordo and Ellson (1985), and Barsky and Summers (1988). Here I do not mean my identification of the resource with gold to be taken too literally.

(2) In more general models, price must rise at a rate of  $r$  less marginal extraction cost. That more general condition allows gold in the ground to coexist with, say, gold jewelry that yields a utility flow. With zero extraction cost, allowing a utility value from holding gold above ground would lead to the immediate extraction of all gold and a rate of price increase somewhat below  $r$ .