Problem Set #1  
Economics 240B  
Spring 2010  
Due February 10

Turn in (correct) answers to the following exercises from An Introduction to Classical Econometric Theory, by Paul A. Ruud:

Chapter 13: Exercises 13.5, 13.6, 13.9, 13.11, 13.12.  

Additional Questions:

1. Suppose the coefficients $\beta = (\beta_1, \beta_2)'$ in the linear model $y = X\beta + \varepsilon$ are estimated by classical least squares, where it is assumed that the errors $\varepsilon$ are independent of the matrix $X$ of regressors with scalar covariance matrix $V(\varepsilon) = V(\varepsilon|X) = \sigma^2 I$. An analysis of $N = 347$ observations yields

$$
\hat{\beta} = \begin{pmatrix} 0.25 \\ -0.25 \end{pmatrix}, \quad s^2 = 0.1, \quad X'X = \begin{bmatrix} 40 & 10 \\ 10 & 5 \end{bmatrix}.
$$

Construct an approximate 95% confidence interval for $\gamma \equiv \beta_1 / \beta_2$, under the (possibly heroic) assumption that the sample size is large enough for the usual limit theorems and linear approximations to be applicable. Is $\gamma_0 = 0$ in this interval?

2. Consider the linear regression model

$$
y = \beta x + \alpha z + u,
$$

where $\beta$ and $\alpha$ are unknown scalar parameters, $x$ and $z$ are $n$-dimensional vectors of (jointly) i.i.d. random variables, and $u$ is an $n$-dimensional vector of unobservable i.i.d. random variables (“error terms”) with zero mean and unit variance which is independent of $x$ and $z$. Suppose we are given a preliminary estimator $\hat{\alpha}$ of $\alpha$ that is independent of $u$ and has the asymptotic distribution

$$
\sqrt{n}(\hat{\alpha} - \alpha) \xrightarrow{d} N(0, 1).
$$

Define a “second stage” estimator $\tilde{\beta}$ of $\beta$ as

$$
\tilde{\beta} \equiv (x'x)^{-1}x'(y - \hat{\alpha}z).
$$

Assuming $\text{plim } n^{-1}x'x \equiv c \neq 0$ and $\text{plim } n^{-1}x'z \equiv d \neq 0$ exist, and all random variables have finite fourth moments, obtain the asymptotic distribution of $\tilde{\beta}$. 

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