Problem Set #4
Economics 240B
Spring 2010
Due April 14

Turn in (correct) answers to the following exercises from Ruud’s text:

Chapter 14: Exercises 14.4, 14.9
Chapter 16: Exercises 16.4, 16.8
Chapter 17: Exercise 17.9

Extra Theoretical Question: A random variable \( U \) is said to have a Pareto distribution with parameter \( \lambda \), denoted \( U \sim \text{Pareto}(\lambda) \), if it is continuously distributed on the interval \((1, \infty)\) with density
\[
f(u; \lambda) = \lambda \cdot u^{-(\lambda+1)}.
\]

Suppose you have a random sample \( \{(y_i, x'_i)\}_{i=1}^n \) where the conditional distribution of \( y_i \) given the vector \( x_i \) is Pareto with parameter \( \exp\{x'_i\beta_0\} \), i.e.,
\[
y_i|x_i \sim \text{Pareto}(\exp\{x'_i\beta_0\}).
\]
Also, suppose the marginal distribution of the \( K \)-dimensional regressors \( x_i \) is unspecified and, as usual, \( \beta_0 \) is unknown.

(i) Derive the average log-likelihood function \( L(\beta) \) for this problem, and show that the first-order condition for the MLE \( \hat{\beta} \) can be rewritten in the form
\[
0 = \frac{1}{n} \sum_{i=1}^n u_i(\hat{\beta}) \cdot x_i
\]
for some “pseudo-residual” function \( u_i(\beta) \) which satisfies \( E[u_i(\beta_0)|x_i] = 0 \).

(ii) Derive an expression for the asymptotic distribution of the ML estimator \( \hat{\beta} \), including an explicit expression for its asymptotic covariance matrix, and give a consistent estimator for that matrix. Also, assuming \( K = 1 \) (that is, \( \beta \) is a scalar), give an expression for an approximate 95% confidence interval for \( \beta_0 \).

(iii) For general \( K \), use the ML \( \hat{\beta} \) to estimate the probability that \( y_i > y_0 \) conditional on \( x_i = x_0 \), for some fixed values of \( y_0 \) (in the interval \((1, \infty)\)) and \( x_0 \), and derive the large-sample distribution of this estimator and an estimator of its asymptotic variance.

(iv) Derive the algebraic form of the Wald, likelihood ratio, and score (“LM”) tests of the null hypothesis \( H_0 : \beta_0 = 0 \), and describe the critical region for the test.

(v) Now, assuming the first component of the regressors is a constant, \( x_{i1} \equiv 1 \), and the true “slope coefficients” on the remaining regressors are denoted \( \beta_0^{(2)} \), derive the Wald, LR, and score tests and critical regions for the null hypothesis \( H_0 : \beta_0^{(2)} = 0 \).