Second Midterm Exam
ECONOMICS 241A
Spring 2009

Instructions: This is a 30 point exam, with given weights for each question; all subsections have equal weight. No books, notes, tables, or calculating devices are permitted. You have one hour and 20 minutes to answer the following two questions. Please make your answers elegant – that is, clear, concise, and, above all, correct.

1. (15 points) Consider the linear model
\[ y_i = \beta_0 + \varepsilon_i, \]
and suppose that the unobservable error term \( \varepsilon_i \) satisfies both a zero mean restriction
\[ E[\varepsilon_i] = 0 \]
and a conditional median restriction
\[ E[\text{sgn}(\varepsilon_i)] = 0. \]
Assume that \( \varepsilon_i \) is continuously distributed, with a density \( f(\varepsilon) \) that has lots of derivatives and moments.

(a) Suppose you are given the sample mean \( \bar{y}_N \) and sample median \( \tilde{y}_N \equiv y_{(N+1)/2} \) for these data, along with a consistent estimator \( \hat{V} \) of their joint asymptotic covariance matrix. How could you use these statistics to construct a more efficient estimator of \( \beta_0 \)?

(b) Given a random sample of size \( N \) from this model, derive the asymptotic variance of the (infeasible) efficient GMM estimator of \( \beta_0 \) under these two restrictions. (Assume the relevant stochastic equicontinuity condition holds, so that the order of differentiation and expectation can be interchanged in the calculations.)

(c) Does the estimator you proposed in (a) attain the GMM efficiency bound you derived in (b)? [Hint: you can justify your answer with explicit derivations or a general argument.]
2. (15 points) Suppose you have a sample from a censored regression model with both sample selectivity and endogenous regressors:

\[ d_i = 1\{w_i' \delta_0 + u_i > 0\}, \]
\[ x_i = z_i' \pi_0 + v_i, \]
\[ y_i = d_i \cdot \max\{0, x_i' \beta_0 + \epsilon_i\}, \]

where \( d_i, y_i, x_i, w_i, \) and \( z_i \) are observable random variables and vectors. It is assumed that the unobservable errors \( u_i, v_i, \epsilon_i \) are jointly independent of the exogenous variables \( w_i \) and \( z_i \) with a smooth but unknown joint density function with lots of moments; the exogenous variables \( w_i \) and \( z_i \) are also jointly continuously distributed with well-behaved densities and moments.

Discuss identification of the unknown parameters (a) \( \delta_0 \), (b) \( \pi_0 \), and (c) \( \beta_0 \) under the assumption that the joint distribution of \( w_i \) and \( z_i \) and the functions

\[ p(w_i) = E[d_i|w_i] \]
\[ g(z_i) = E[x_i|z_i], \] and
\[ m(x_i, w_i, z_i, d_i) = \text{median}[y_i|x_i, w_i, z_i, d_i] \]

are known for all \( w_i \) and \( z_i \). Be explicit about any normalizations you need to impose. [Hint: for part (c), consider the value of \( m(\cdot) \) when \( d_i = 1 \).]