Theoretical Tools of Public Finance

131 Undergraduate Public Economics

Emmanuel Saez

UC Berkeley

THEORETICAL AND EMPIRICAL TOOLS

Theoretical tools: The set of tools designed to understand the mechanics behind economic decision making.

Empirical tools: The set of tools designed to analyze data and answer questions raised by theoretical analysis.

CONSTRAINED UTILITY MAXIMIZATION

Economists model individuals' choices using the concepts of utility function maximization subject to budget constraint.

Utility function: A mathematical function representing an individual's set of preferences, which translates her well-being from different consumption bundles into units that can be compared in order to determine choice.

Constrained utility maximization: The process of maximizing the well-being (utility) of an individual, subject to her resources (budget constraint).

UTILITY MAPPING OF PREFERENCES

Indifference function: A utility function is some mathematical representation:

$$U = u(X_1, X_2, X_3, ...)$$

where X_1, X_2, X_3, and so on are the goods consumed by the individual and $u(.,.,...)$ is some mathematical function that describes how the consumption of those goods translates to utility

Example with two goods: $u(X_1, X_2) = \sqrt{X_1 \cdot X_2}$ with X_1 number of movies, X_2 number of music songs

Individual utility increases with the level of consumption of each good

PREFERENCES AND INDIFFERENCE CURVES

Indifference curve: A graphical representation of all bundles of goods that make an individual equally well off. Because these bundles have equal utility, an individual is indifferent as to which bundle he consumes

Mathematically, indifference curve giving utility level $\bar{U}$ is given by the set of bundles (X_1, X_2) such that $u(X_1, X_2) = \bar{U}$

Indifference curves have two essential properties, both of which follow naturally from the more-is-better assumption:

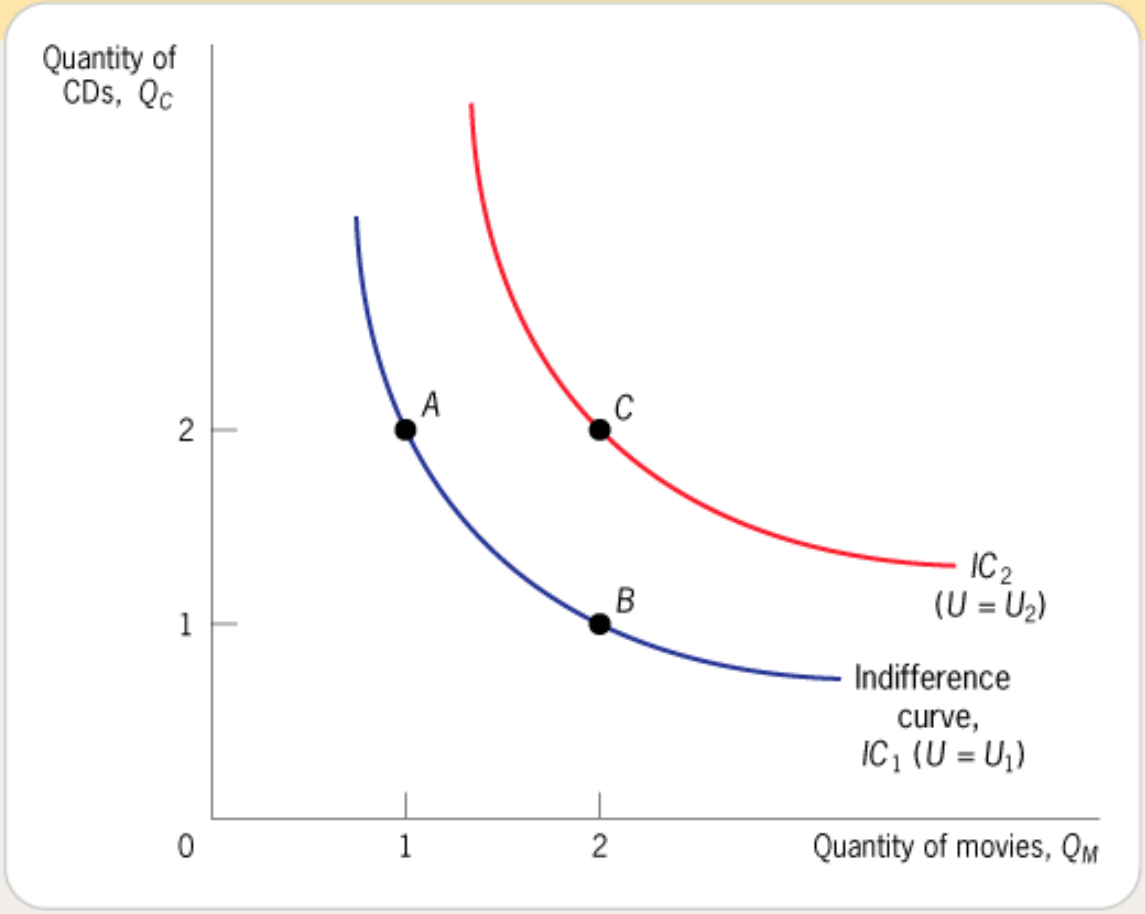
1. Consumers prefer higher indifference curves.

2. Indifference curves are always downward sloping.

Constrained Utility Maximization

Preferences and Indifference Curves

■ FIGURE 2-1



Indifference Curves for Bundles of CDs and Movies • Andrea is indifferent between consuming 2 CDs and 1 movie (point *A*) or 1 CD and 2 movies (point *B*), but she prefers 2 CDs and 2 movies (point *C*) to both. Utility is the same along a given indifference curve; indifference curves farther from the origin represent higher utility levels.

MARGINAL UTILITY

Marginal utility: The additional increment to utility obtained by consuming an additional unit of a good:

Marginal utility of good 1 is defined as:

$$MU_1 = \frac{\partial u}{\partial X_1} \simeq \frac{u(X_1 + dX_1, X_2, X_3, ...) - u(X_1, X_2, X_3, ...)}{dX_1}$$

It is the derivative of utility with respect to X_1 keeping X_2 constant (called the partial derivative)

Example:

$$u(X_1, X_2) = \sqrt{X_1 \cdot X_2} \Rightarrow \frac{\partial u}{\partial X_1} = \frac{\sqrt{X_2}}{2\sqrt{X_1}}$$

This utility function described exhibits the important principle of **diminishing marginal utility**: the consumption of each additional unit of a good gives less extra utility than the consumption of the previous unit

MARGINAL RATE OF SUBSTITUTION

Marginal rate of substitution (MRS): The rate at which a consumer is willing to trade one good for another. The MRS is equal to (minus) the slope of the indifference curve, the rate at which the consumer will trade the good on the vertical axis for the good on the horizontal axis.

Marginal rate of substitution between good 1 and good 2 is:

$$MRS_{1,2} = \frac{MU_1}{MU_2}$$

Individual is indifferent between 1 unit of good 1 and $MRS_{1,2}$ units of good 2.

Example:

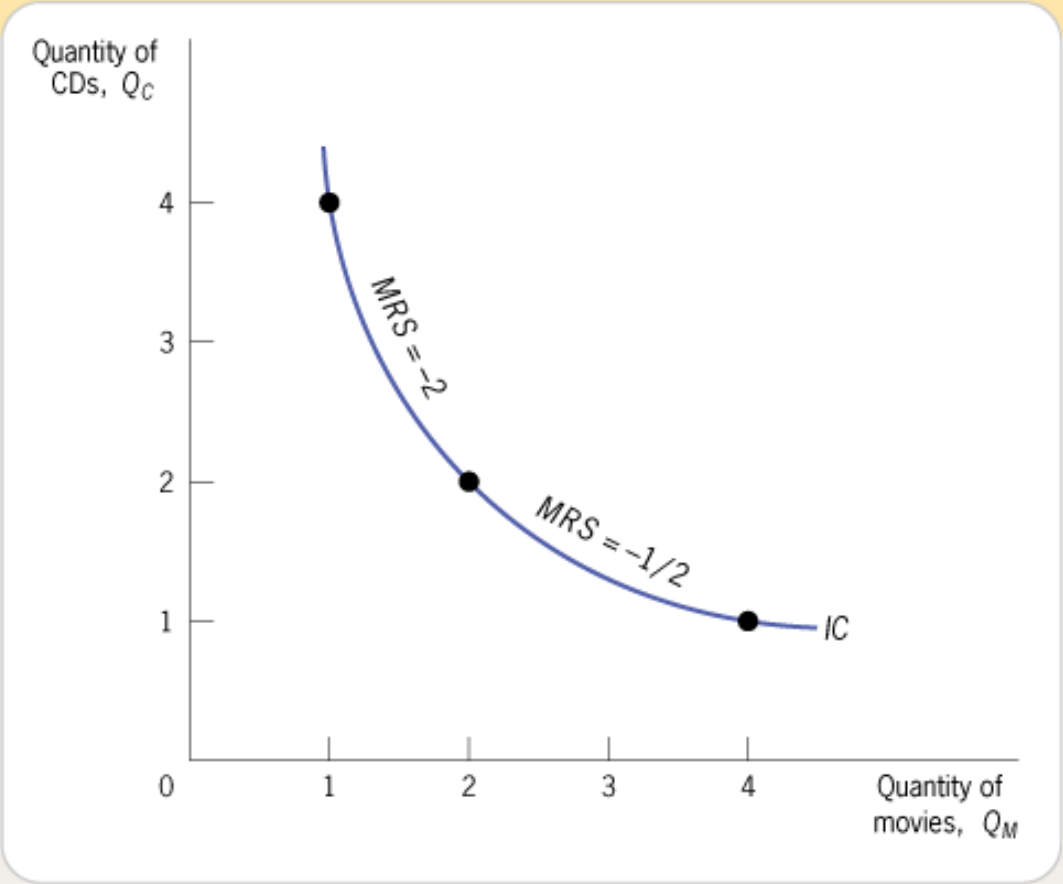
$$u(X_1, X_2) = \sqrt{X_1 \cdot X_2} \Rightarrow MRS_{1,2} = \frac{X_2}{X_1}$$

Constrained Utility Maximization

Utility Mapping of Preferences

Marginal Rate of Substitution

■ FIGURE 2-4



Marginal Rates of Substitution • With a utility function of $U = \sqrt{Q_C \times Q_M}$, *MRS* diminishes as the number of movies consumed increases. At 4 CDs and 1 movie, Andrea is willing to trade 2 CDs to get a movie (*MRS* = –2). At 2 CDs and 2 movies, Andrea is willing to trade 1 CD to get 2 movies (*MRS* = –½).

BUDGET CONSTRAINT

Budget constraint: A mathematical representation of all the combinations of goods an individual can afford to buy if she spends her entire income.

$$p_1 X_1 + p_2 X_2 = Y$$

with p_i price of good i, and Y disposable income.

Budget constraint defines a linear set of bundles the consumer can with its disposable income Y.

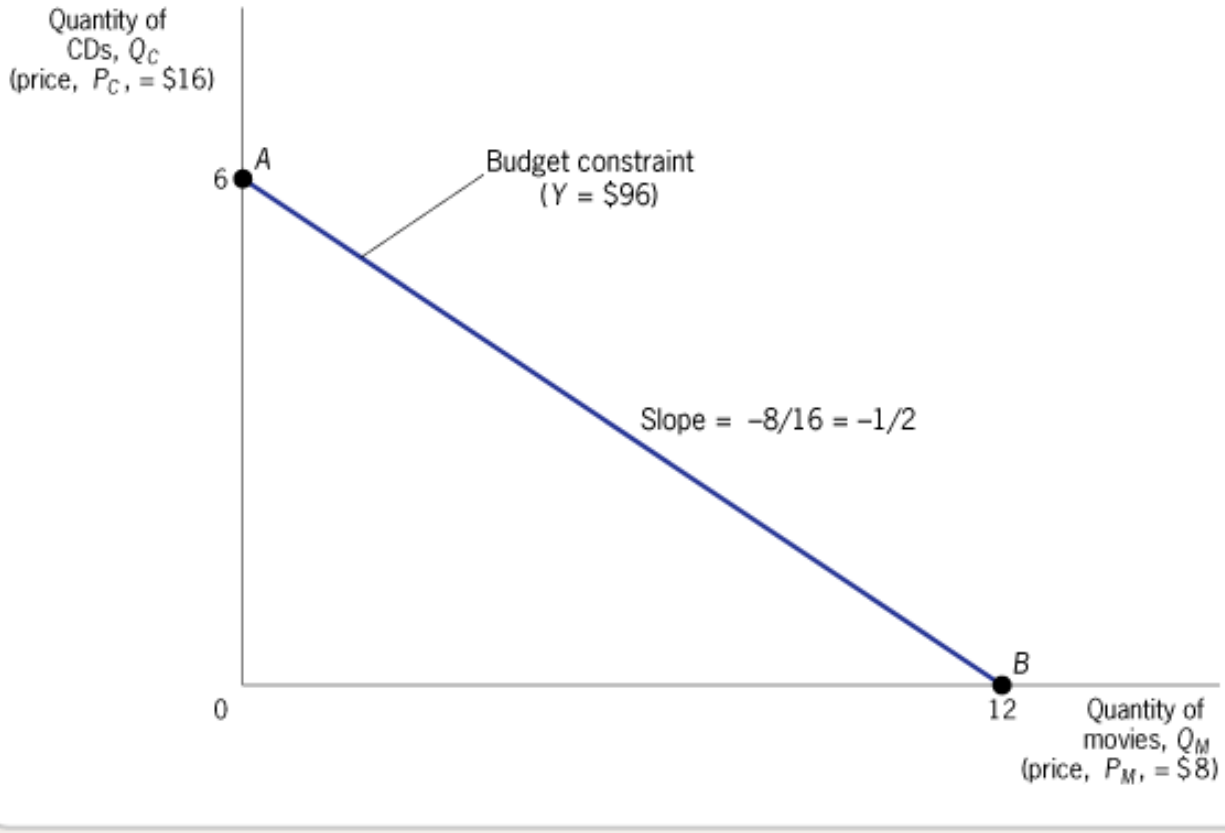
$$X_2 = \frac{Y}{p_2} - \frac{p_1}{p_2} X_1$$

The slope of the budget constraint is $-p_1/p_2$: If the consumer gives up 1 unit of good 1, it has p_1 more to spend, and can buy p_1/p_2 units of good 2.

Constrained Utility Maximization

Budget Constraints

■ FIGURE 2-5



Budget Constraint • With an income, Y, of $96, a price of $16 per CD, and a price of $8 per movie, Andrea can trade off 1 CD for 2 movies, up to a total of either 6 CDs or 12 movies. The slope of the budget constraint is therefore – ½, indicating the ratio of movie-to-CD prices.

UTILITY MAXIMIZATION

Individual maximizes utility subject to budget constraint:

$$\max_{X_1,X_2} u(X_1,X_2) \quad \text{subject to} \quad p_1X_1 + p_2X_2 = Y$$

Solution: $$MRS_{1,2} = \frac{p_1}{p_2}$$

Proof: Budget implies that $X_2 = (Y - p_1X_1)/p_2$

Individual chooses X_1 to maximize $u(X_1, (Y - p_1X_1)/p_2)$.

The first order condition (FOC) is:

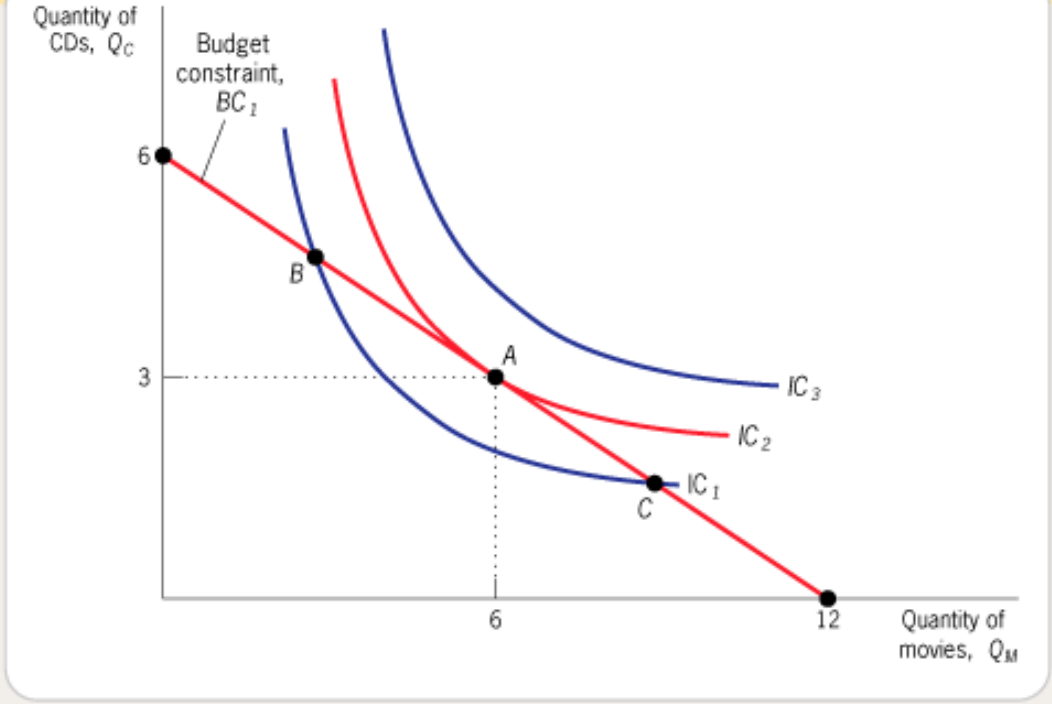
$$\frac{\partial u}{\partial X_1} - \frac{p_1}{p_2} \cdot \frac{\partial u}{\partial X_2} = 0.$$

At the optimal choice, the \$ value for the individual of the last unit of good 1 consumed is exactly equal to p_1: individual is indifferent between 1 more unit of good 1 and using p_1 to purchase p_1/p_2 units of the other good.

Constrained Utility Maximization

Putting It All Together: Constrained Choice

■ FIGURE 2-6



Constrained Optimization • Given a utility function of $U = \sqrt{Q_C \times Q_M}$, an income of $96, and prices of CDs and movies of $16 and $8, respectively, Andrea's optimal choice is 3 CDs and 6 movies (point *A*). This represents the highest indifference curve she can reach, given her resources and market prices. She can also afford points such as *B* and *C*, but they leave her on a lower indifference curve (IC_1 instead of IC_2).

Quick Hint

Marginal analysis, the consideration of the costs and benefits of an additional unit of consumption or production, is a central concept in modeling an individual's choice of goods and a firm's production decision.

INCOME AND SUBSTITUTION EFFECTS

Let us denote by $p = (p_1, p_2)$ the price vector

Individual maximization generates demand functions $X_1(p, Y)$ and $X_2(p, Y)$

How does $X_1(p, Y)$ vary with p and Y?

Those are called price and income effects

INCOME EFFECTS

Income effect is the effect of giving extra income Y on the demand for goods: How does $X_1(p, Y)$ vary with Y?

Normal goods: Goods for which demand increases as income Y rises: $X_1(p, Y)$ increases with Y (most goods are normal)

Inferior goods: Goods for which demand falls as income Y rises: $X_1(p, Y)$ decreases with Y (example: you use public transportation less when you are rich enough to buy a car)

Example: if leisure is a normal good, you work less (i.e. get more leisure) if you are given a stipend

PRICE EFFECTS

How does $X_1(p, Y)$ vary with p_1?

Changing p_1 affects the slope of the budget constraint and can be decomposed into 2 effects:

1) Substitution effect: Holding utility constant, a relative rise in the price of a good will always cause an individual to choose less of that good

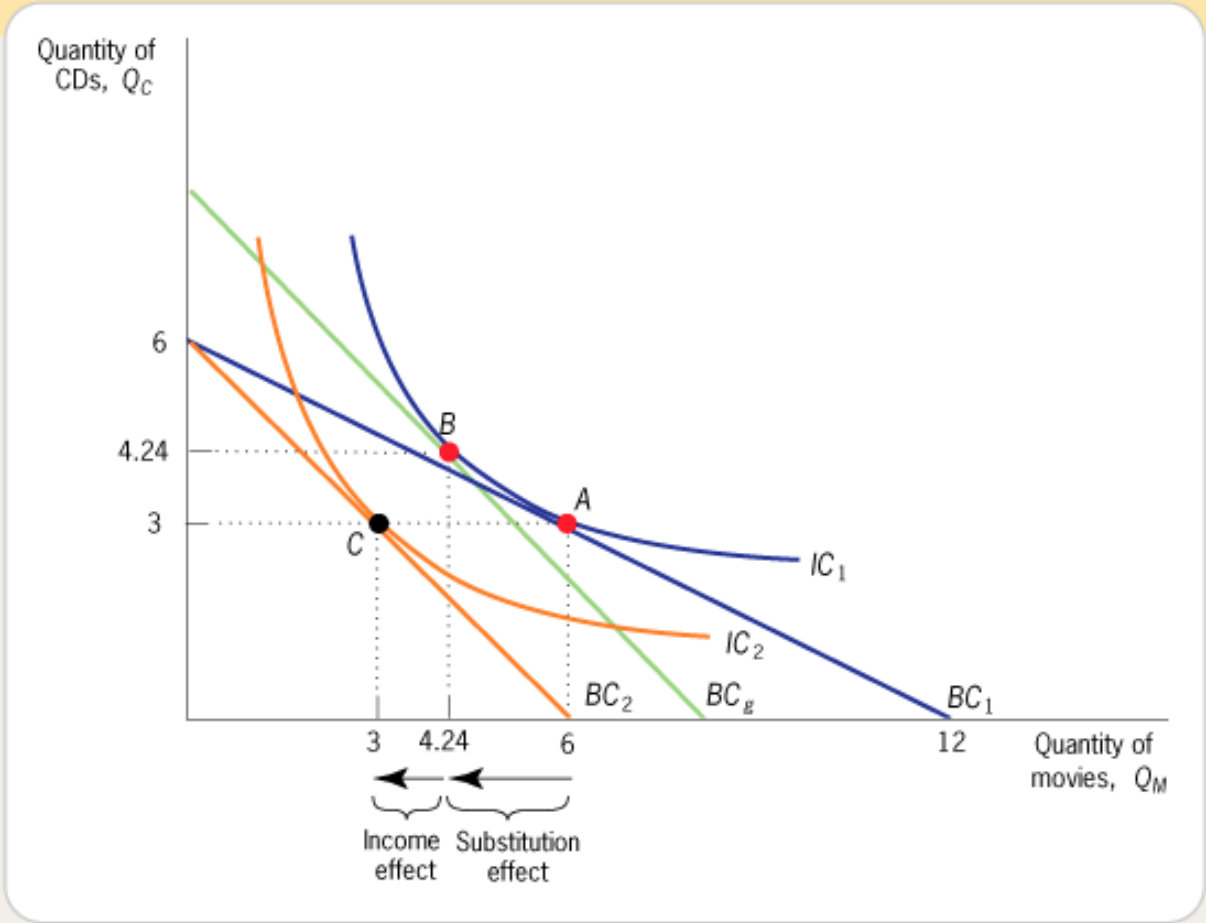
2) Income effect: A rise in the price of a good will typically cause an individual to choose less of all goods because her income can purchase less than before.

For normal goods, an increase in p_1 reduces $X_1(p, Y)$ through both substitution and income effects

Constrained Utility Maximization

The Effects of Price Changes: Substitution and Income Effects

■ FIGURE 2-7

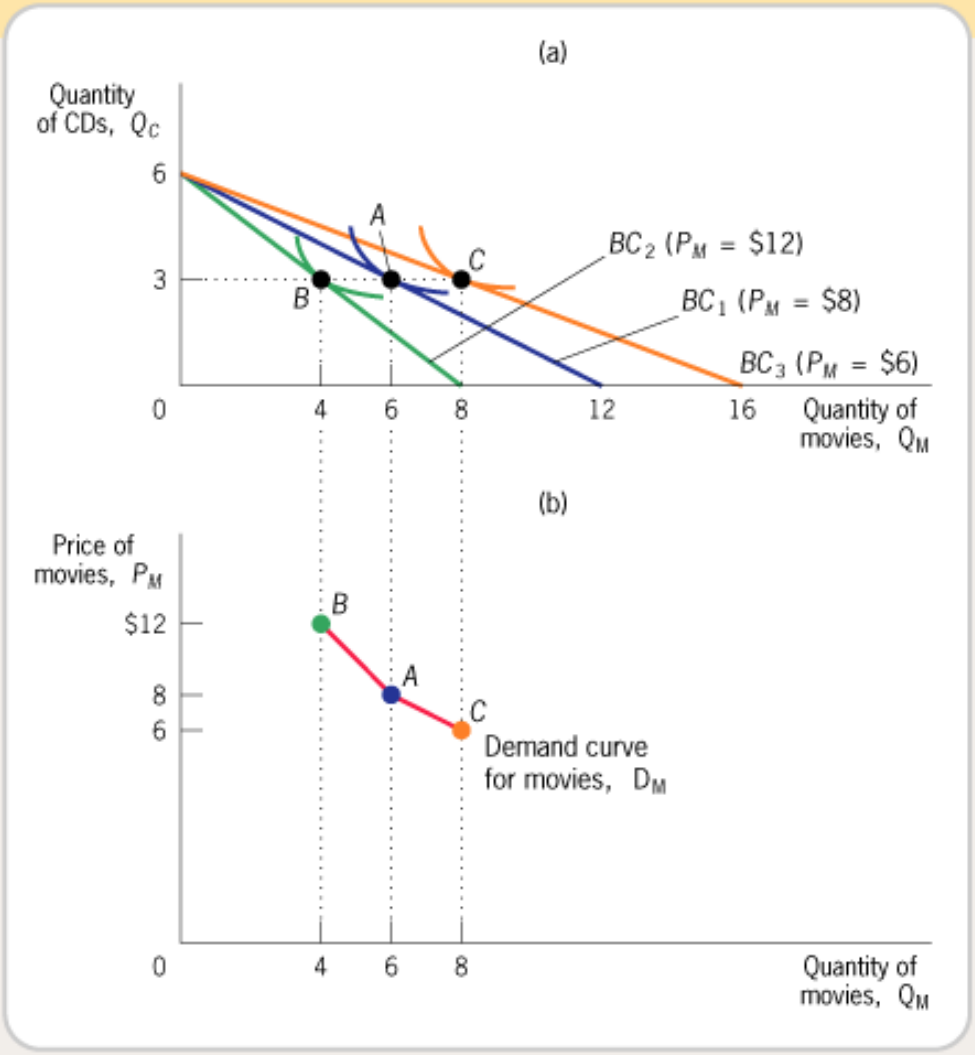


Substitution and Income Effects • When the price of movies increases, it has two effects. First, holding utility constant, there is a substitution effect, which causes Andrea to demand fewer movies since they are relatively more expensive (moving from point *A* to point *B*). Second, holding relative prices constant, there is an income effect, which causes her to demand fewer movies because she is poorer (moving from point *B* to point *C*).

Equilibrium and Social Welfare

Demand Curves

■ FIGURE 2-12



Deriving the Demand Curve • Changes in the price of movies shift the budget constraint, changing the number of movies demanded by individuals. When the price of movies rises to \$12, then the number of movies demanded falls to 4, and when the price of movies demanded falls to \$6, the number of movies demanded rises to 8. We can use this relationship between the price and utility-maximizing choices to trace out the demand curve for movies, D_M, as shown in panel (b).

ELASTICITY OF DEMAND

Each individual has a demand for each good that depends on prices Aggregating across all individuals, we obtain aggregate demand $D(p)$ that depends on the price p of that good

At price p, demand is $D(p)$ and p is \$ value for consumers of the marginal (last) unit consumed

Sensitivity of demand with respect to price is measured using the elasticity of demand with respect to price

Elasticity of demand: The % change in demand caused by each 1% change in the price of that good.

$$\varepsilon = \frac{\%\text{ change in quantity demanded}}{\%\text{ change in price}} = \frac{\Delta D/D}{\Delta p/p} = \frac{p}{D}\frac{dD}{dp}$$

Economists like to use elasticities to measure things because elasticities are unit free

ELASTICITY OF DEMAND

There are several key points to make about elasticities of demand:

1) They are typically negative, since quantity demanded typically falls as price rises.

2) They are typically not constant along a demand curve.

3) A vertical demand curve is one for which the quantity demanded does not change when price rises; in this case, demand is **perfectly inelastic**.

4) A horizontal demand curve is one where quantity demanded changes infinitely for even a very small change in price; in this case, demand is **perfectly elastic**.

5) The effect of one good's prices on the demand for another good is the cross-price elasticity, and with the particular utility function we are using here, that **cross-price elasticity** is zero. Typically, however, a change in the price of one good will affect demand for other goods as well.

PRODUCERS

Producers (typically firms) use technology to transform inputs (labor and capital) into outputs (consumption goods)

Goal of producers is to maximize profits = sales of outputs minus costs of inputs

Production decisions (for given prices) define supply functions

SUPPLY CURVES

Supply curve: A curve showing the quantity of a good that firms in aggregate are willing to supply at each price:

Marginal cost: The incremental cost to a firm of producing one more unit of a good

Profits: The difference between a firm's revenues and costs, maximized when marginal revenues equal marginal costs.

Supply curve $S(p)$ typically upward sloping with price due to decreasing returns to scale

At price p, producers produce $S(p)$, and the \$ cost of producing the marginal (last) unit is p

Elasticity of supply is defined as $\varepsilon = \frac{p}{S}\frac{dS}{dp}$

EQUILIBRIUM

Market: The arena in which demanders and suppliers interact.

Equilibrium: The combination of price and quantity that satisfies both demand and supply, determined by the interaction of the supply and demand curves

Mathematically, the equilibrium is the price p^* such that $D(p^*) = S(p^*)$

In the simple diagram, p^* is unique if $D(p)$ decreases with p and $S(p)$ increases with p

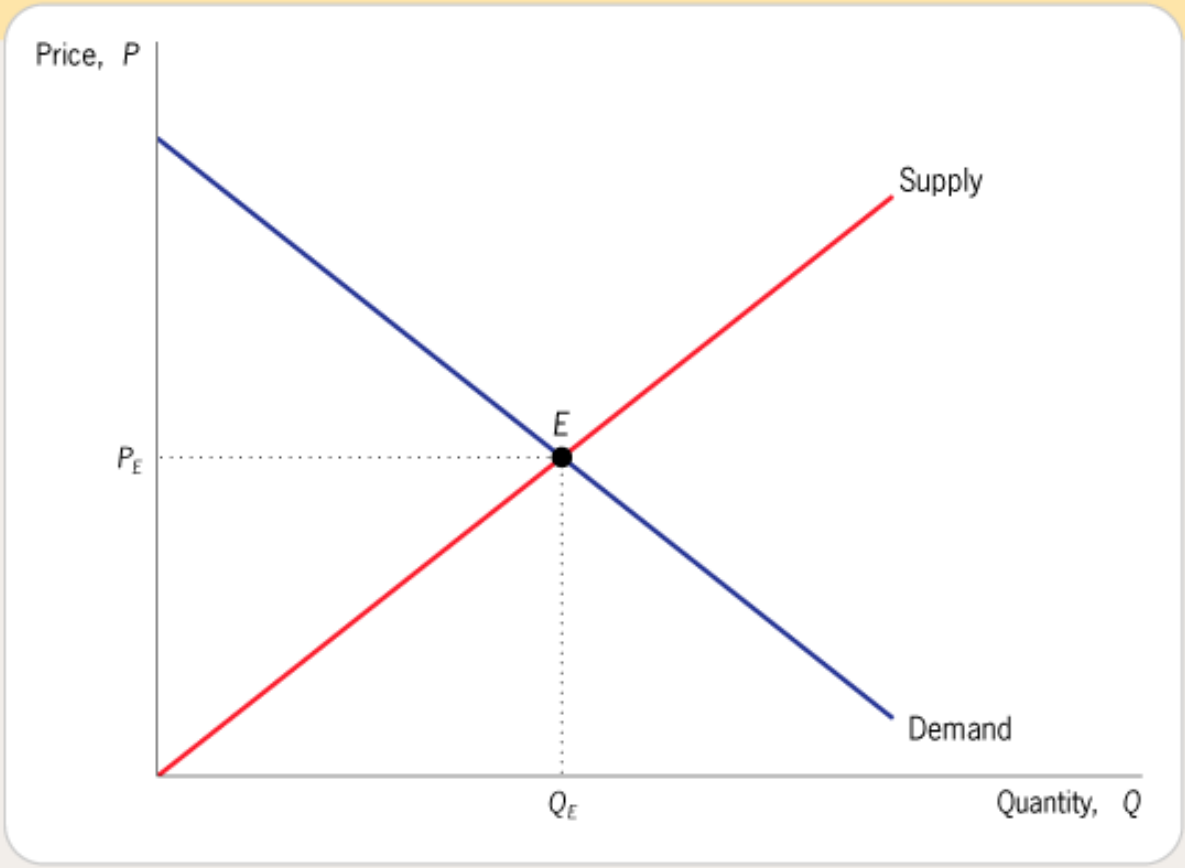
If $p > p^*$, then supply exceeds demand, and price needs to fall to equilibrate supply and demand

If $p < p^*$, then demand exceeds supply, and price needs to increase to equilibrate supply and demand

Equilibrium and Social Welfare

Equilibrium

■ FIGURE 2-13



Market Outcome • The supply and demand curves for movies intersect at the equilibrium point *E*, where both consumers and suppliers are satisfied with price and quantity.

SOCIAL EFFICIENCY

Social efficiency represents the net gains to society from all trades that are made in a particular market, and it consists of two components: consumer and producer surplus.

Consumer surplus: The benefit that consumers derive from consuming a good, above and beyond the price they paid for the good. It is the area below demand curve and above market price.

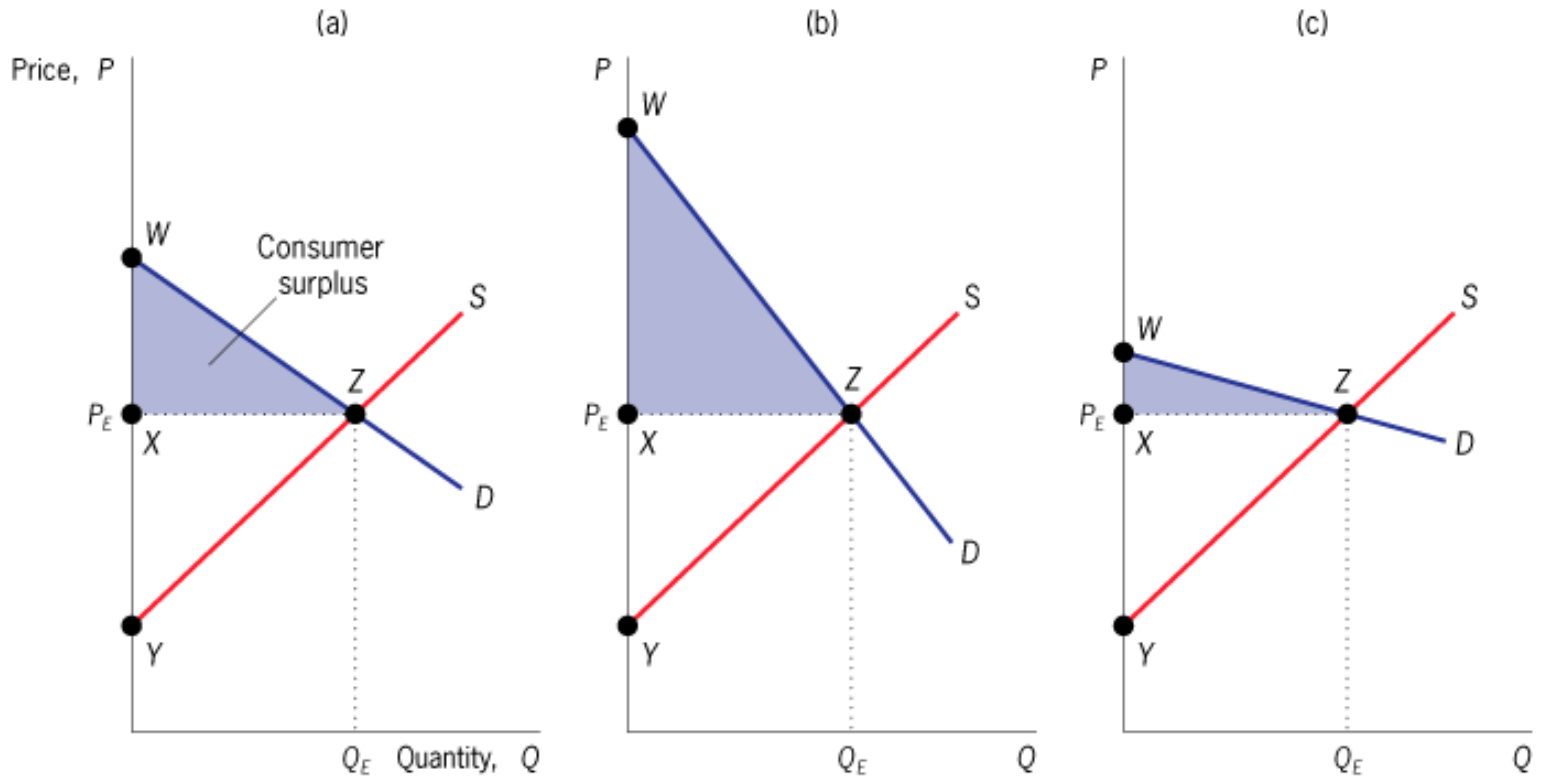
Producer surplus: The benefit producers derive from selling a good, above and beyond the cost of producing that good. It is the area above supply curve and below market price.

Total social surplus (social efficiency): The sum of consumer surplus and producer surplus. It is the area above supply curve and below demand curve.

Equilibrium and Social Welfare

Social Efficiency

■ FIGURE 2-14

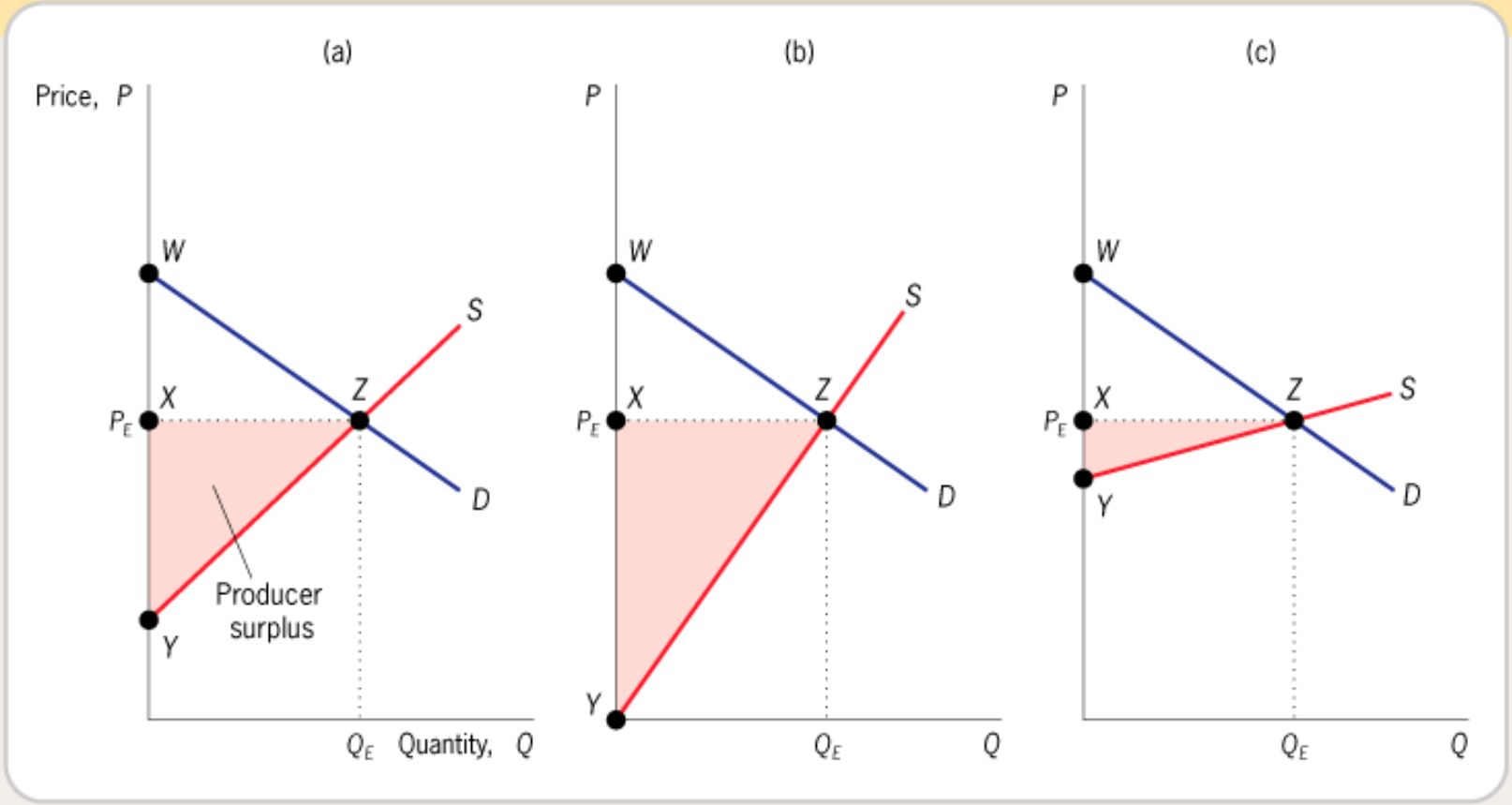


Consumer Surplus • The consumer surplus is the area below the demand curve and above the equilibrium market price, the shaded area *WZX* in all three panels of this graph. This represents the value to consumers of consuming goods above and beyond the price paid for those goods. As demand becomes more inelastic, consumer surplus rises; as demand becomes more elastic, consumer surplus falls.

Equilibrium and Social Welfare

Producer Surplus

■ FIGURE 2-15

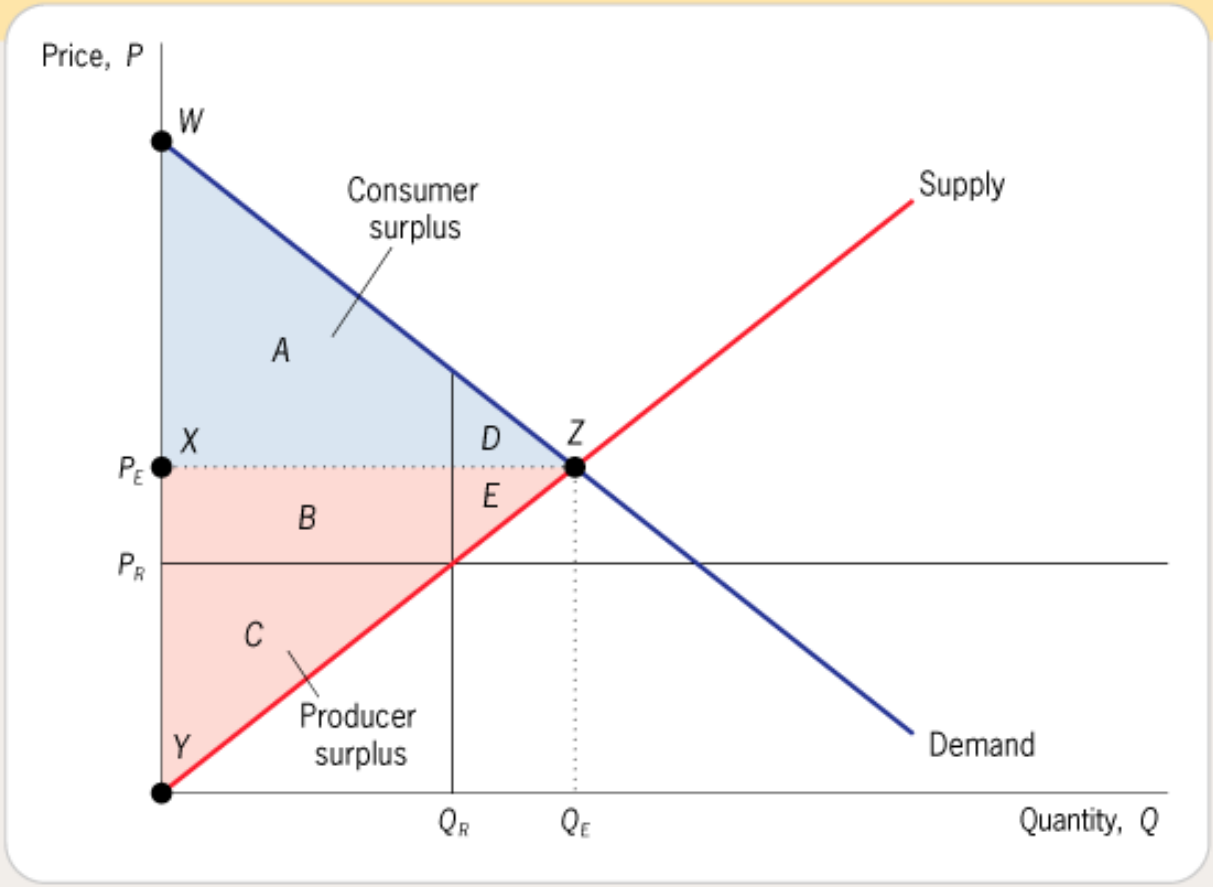


Producer Surplus • The producer surplus is the area below the equilibrium market price and above the supply curve, the shaded area *XZY* in all three panels of this graph. This represents the profit earned by firms on all units sold at the market price. As supply becomes more inelastic, producer surplus rises; as supply becomes more elastic, producer surplus falls.

Equilibrium and Social Welfare

Social Surplus

■ FIGURE 2-16



Competitive Equilibrium Maximizes Social Surplus • The sum of consumer surplus (the area below the demand curve and above the price) and producer surplus (the area above the supply curve and below the price) is maximized at the competitive equilibrium. A restriction on price to P_R lowers quantity supplied to Q_R and creates a deadweight loss of $D + E$.

COMPETITIVE EQUILIBRIUM MAXIMIZES SOCIAL EFFICIENCY

First Fundamental Theorem of Welfare Economics: The competitive equilibrium where supply equals demand, maximizes social efficiency.

Deadweight loss: The reduction in social efficiency from denying trades for which benefits exceed costs when quantity differs from the socially efficient quantity

It is sometimes confusing to know how to draw deadweight loss triangles. The key to doing so is to remember that deadweight loss triangles point to the social optimum, and grow outward from there.

The simple efficiency result from the 1-good diagram can be generalized into the first welfare theorem (Arrow-Debreu, 1940s)

Generalization: 1st Welfare Theorem

Most important result in economics (=markets work)

1st Welfare Theorem: If (1) no externalities, (2) perfect competition, (3) perfect information, (4) agents are rational, then private market equilibrium is **Pareto efficient**

Pareto efficient: Impossible to find a technologically feasible allocation that improves everybody's welfare

Pareto efficiency is desirable but a very weak requirement (a single person owning everything is Pareto efficient)

Government intervention may be particularly desirable if the assumptions of the 1st welfare theorem fail, i.e., when there are market failures $\Rightarrow$ Govt intervention can potentially improve everybody's welfare

First part of class considers such market failure situations

2nd Welfare Theorem

Even with no market failures, free market outcome might generate substantial inequality. Inequality is seen as the biggest issue with capitalism.

2nd Welfare Theorem: Any Pareto Efficient allocation can be reached by

(1) Suitable redistribution of initial endowments [individualized **lump-sum** taxes based on individual characteristics and not behavior]

(2) Then letting markets work freely

$\Rightarrow$ No conflict between efficiency and equity

2nd Welfare Theorem fallacy

In reality, 2nd welfare theorem does not work because redistribution of initial endowments is not feasible (information pb)

$\Rightarrow$ govt needs to use **distortionary** taxes and transfers based on economic outcomes (such as income or working situation)

$\Rightarrow$ Conflict between efficiency and equity: **Equity-Efficiency trade-off**

2nd part of class considers policies that trade-off equity and efficiency

FROM SOCIAL EFFICIENCY TO SOCIAL WELFARE: THE ROLE OF EQUITY

Equity-efficiency trade-off: The choice society must make between the total size of the economic pie and its distribution among individuals.

Social welfare function (SWF): A function that combines the utility functions of all individuals into an overall social utility function.

UTILITARIAN SWF

With a utilitarian social welfare function, society's goal is to maximize the sum of individual utilities:

$$SWF = U_1 + U_2 + ... + U_N$$

The utilities of all individuals are given equal weight, and summed to get total social welfare

If marginal utility of money decreases with income (satiation), utilitarian criterion values redistribution from rich to poor

Taking \$1 for a rich person decreases his utility by a small amount, giving the \$1 to a poor person increases his utility by a large amount $\Rightarrow$ Transfers from rich to poor increase total utility

RAWLSIAN SWF

John Rawls suggested that society's goal should be to maximize the well- being of its worst-off member. The Rawlsian SWF has the form:

$$SWF = \min(U_1, U_2, ..., U_N)$$

Since social welfare is determined by the minimum utility in society, social welfare is maximized by maximizing the well-being of the worst-off person in society

Rawlsian criterion is even more redistributive than utilitarian criterion: society wants to extract as much tax revenue as possible from the middle and rich to make transfers to the poor as large as possible

OTHER EQUITY CRITERIA

Standard welfarist approach is based on individual utilities. This fails to capture important elements of actual debates on redistribution and fairness

Commodity egalitarianism: The principle that society should ensure that individuals meet a set of basic needs (seen as rights), but that beyond that point income distribution is irrelevant.

Equality of opportunity: The principle that society should ensure that all individuals have equal opportunities for success, but not focus on the outcomes of choices made.

Principles of responsibility and compensation: Individuals should be compensated for inequalities they are not responsible for (e.g., family background, inheritance, intrinsic ability) but not for inequalities they are responsible for (being hard working vs. loving leisure)

Conclusion: Two General Rules for Government Intervention

1) Failure of 1st Welfare Theorem: Government intervention can help if there are market failures

2) Fallacy of the 2nd Welfare Theorem: Distortionary Government intervention is required to reduce economic inequality