A Model of Aggregate Demand and Unemployment

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Context and Objective

- Key elements to study fiscal policy over the business cycle: AD & unemployment
- Workhorse unemployment model: matching model
- In matching model: AD is not modeled explicitly
- Objective: develop matching model with AD

Four Key Ingredients

- Matching frictions
  - as in Diamond-Mortensen-Pissarides framework
- Rigid price schedules
  - as in Hall [2005]
  - but any schedule admissible (infinite price bands)
- Concave production function and utility function
  - as in Michaillat [2012]
- Nonproduced good
  - common in Keynesian literature of 1970s and 1980s

Results

1. Equilibrium model of AD and unemployment
2. Predictions to separate AD and AS shocks
3. Design of fiscal policy over the business cycle:
   - government consumption
   - transfer payments
   - payroll tax

Outline

1. **Simple model: market for services**
2. Macro model: product market & labor market
3. Conclusion

Idle Labor as Unemployment

Outline

1. Simple model: market for services
   - **assumptions and equilibrium**
   - aggregate demand & aggregate supply shocks
   - government consumption
   - transfer payments
2. Macro model: product market & labor market
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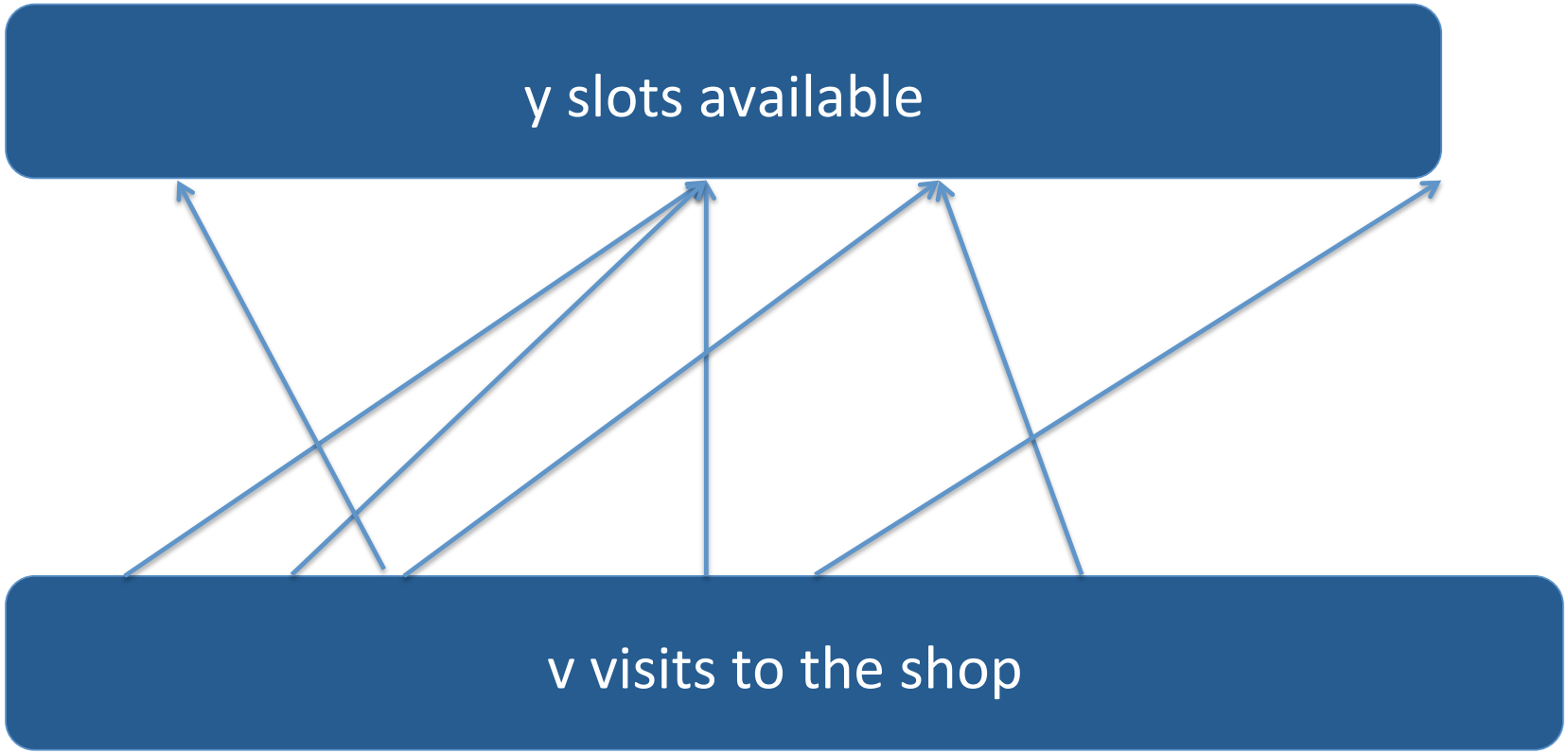
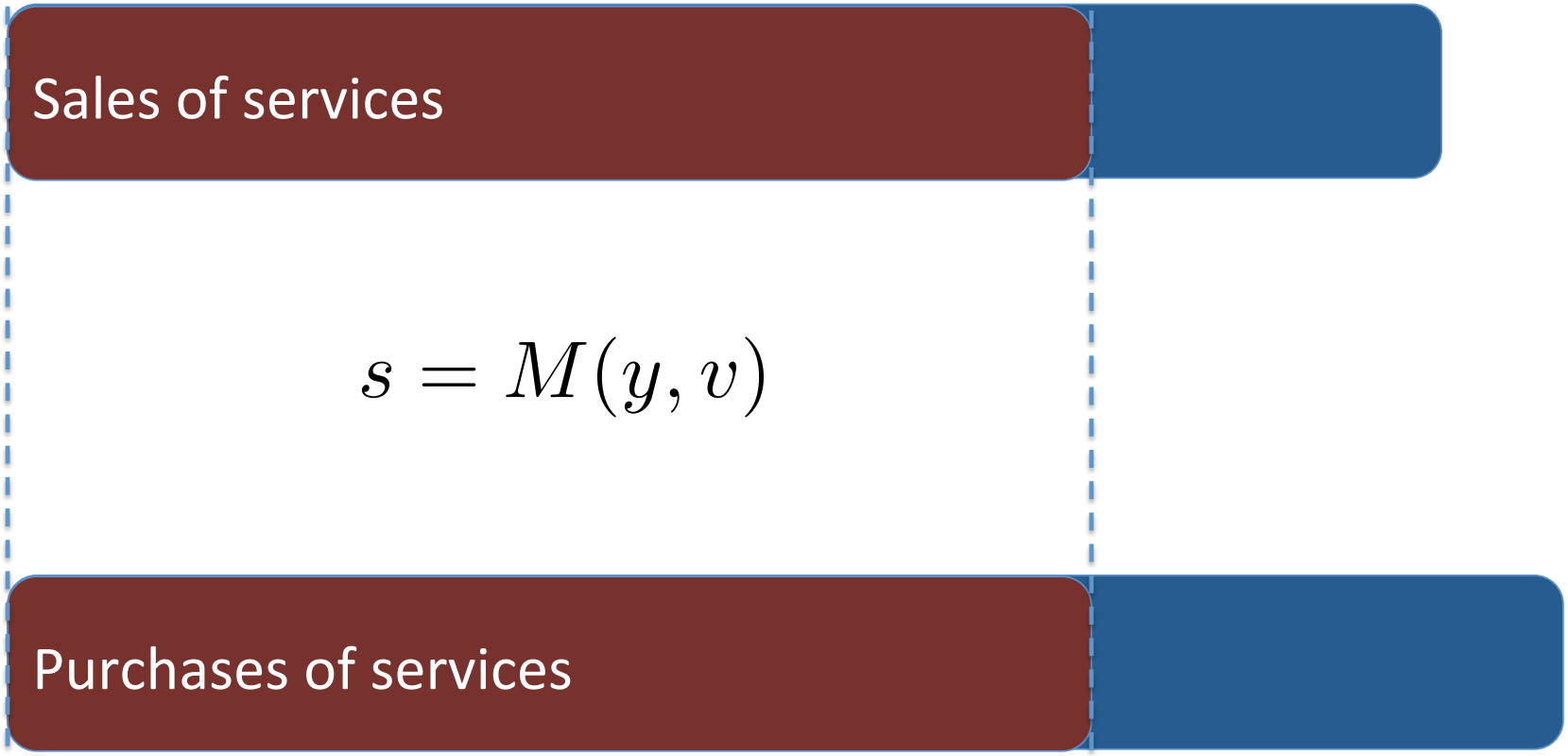
Matching Function



Matching Function

Matching function *M* with **constant returns to scale**:



Matching Function: Market Tightness

Market tightness: $x = \dfrac{v}{y}$

Sales = $y \cdot M(1, x) = y \cdot f(x)$

$$s = M(y, v)$$

Purchases = $v \cdot M\left(\dfrac{1}{x}, 1\right) = v \cdot q(x)$

Matching Function

Market tightness x $\downarrow$

Selling probability $f(x)\downarrow$

Quantity s $\downarrow$

Purchasing probability $q(x)\uparrow$

Matching Function

Market tightness x ↑

Selling probability $f(x)$ ↑

Quantity s ↓

Purchasing probability $q(x)$ ↓

A Visit Requires ρ Services

$$\underbrace{s}_{\text{purchases of services}} = \underbrace{c}_{\text{consumption}} + \underbrace{\rho \cdot v}_{\text{matching expenses}}$$

$$\Rightarrow s = c + \rho \cdot \frac{s}{q(x)}$$

$$\Rightarrow s \cdot \left(1 - \frac{\rho}{q(x)}\right) = c$$

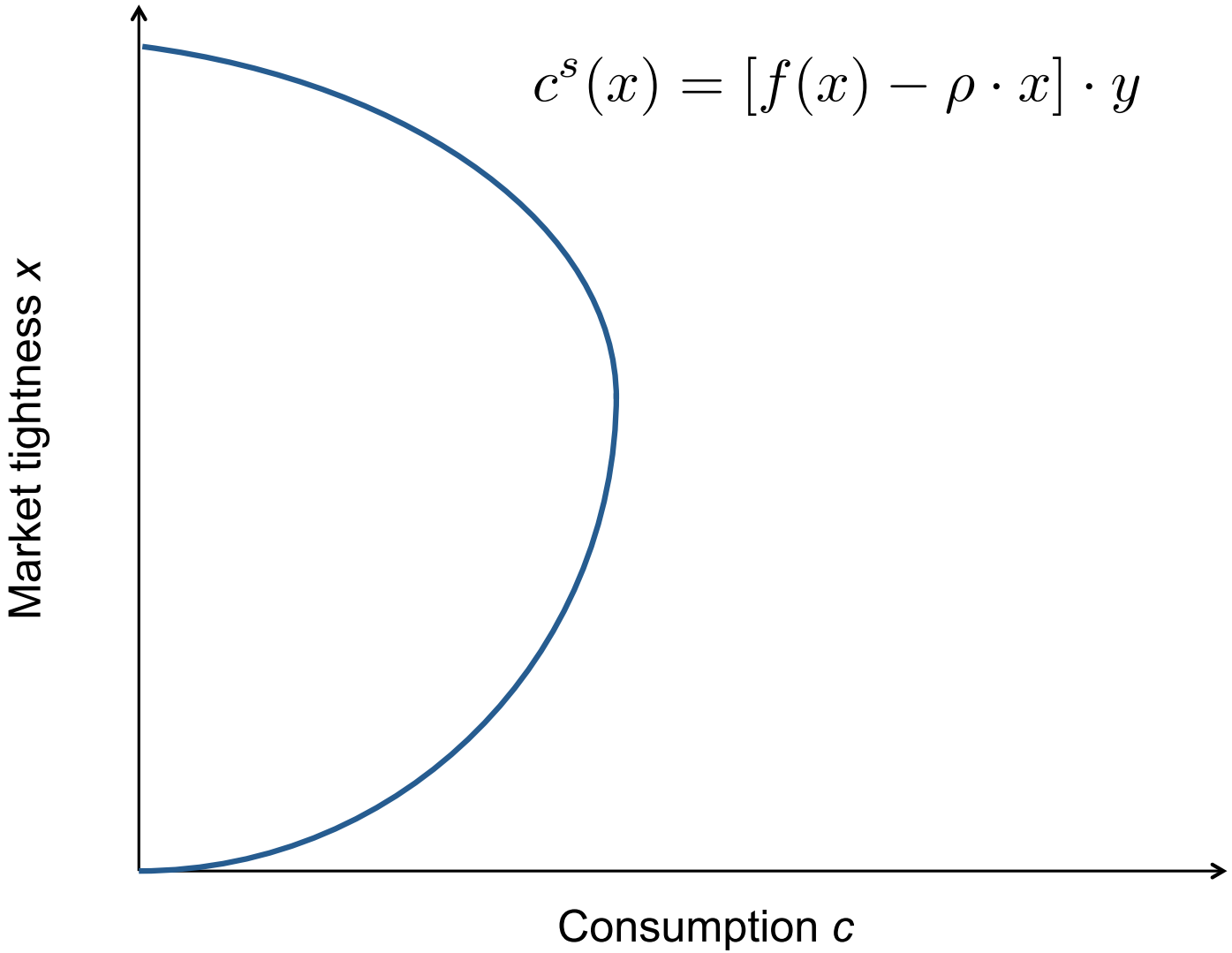
$$\Rightarrow \underbrace{s}_{\text{purchases}} = \underbrace{\left(1 + \frac{\rho}{q(x) - \rho}\right)}_{1+\tau(x)} \cdot \underbrace{c}_{\text{consumption}}$$

Self-Employed Workers

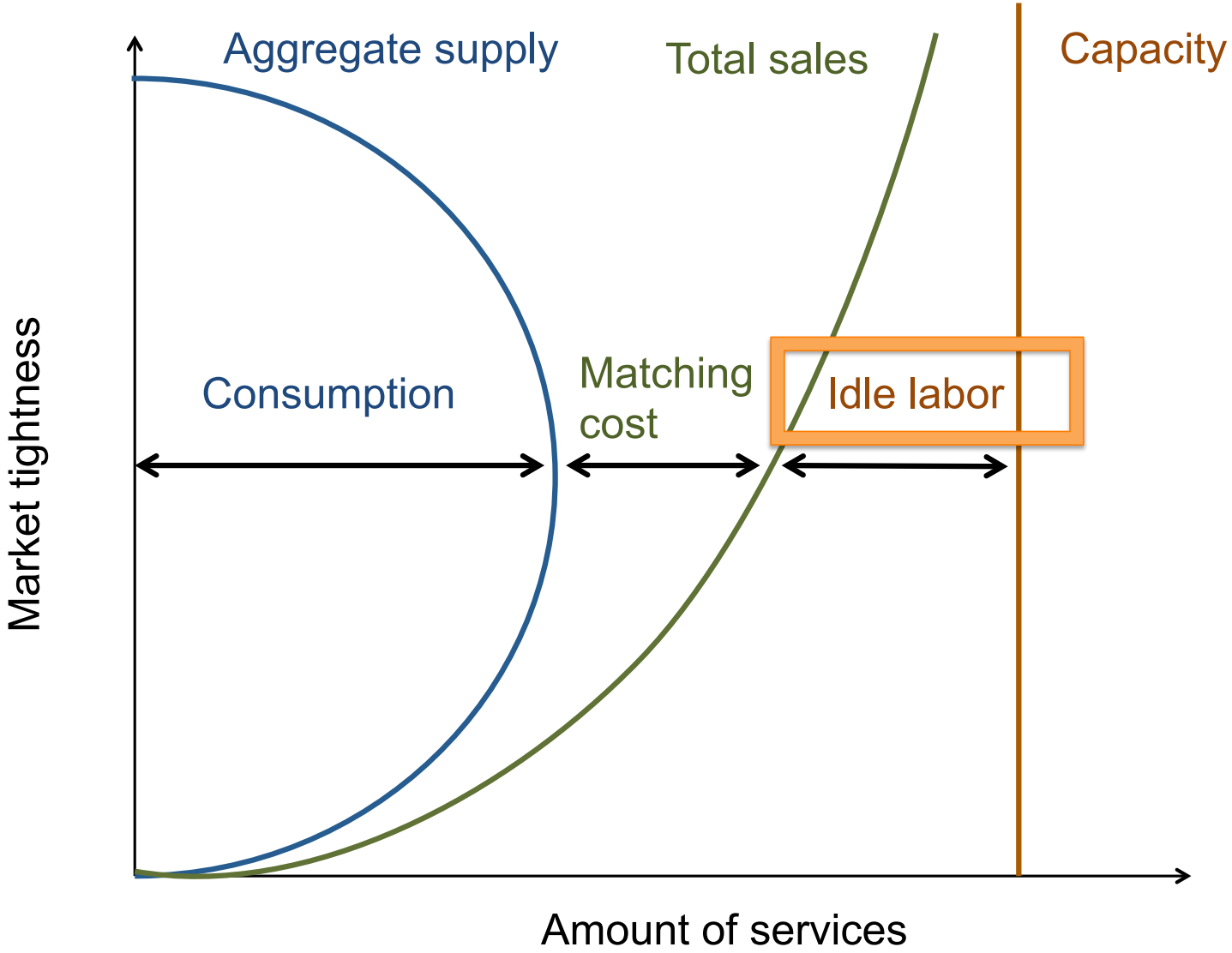
- Amount of services available is y
- Probability to sell is $f(x)$
- Consumption is fraction $1/(1+\tau(x))$ of sales
- Aggregate supply:

$$c^s(x) = \frac{1}{1+\tau(x)} \cdot f(x) \cdot y = (f(x) - \rho \cdot x) \cdot y$$

Aggregate Supply



Idle Labor


Aggregate supply
Total sales
Capacity
Consumption
Matching cost
Idle labor
Market tightness
Amount of services

Consumers

- Take price p and market tightness x as given
- Choose $c,\ m$ to maximize utility

$$\left[\underbrace{\chi \cdot c^{\frac{\epsilon-1}{\epsilon}}}_{\text{services}} + \underbrace{(1-\chi)\cdot m^{\frac{\epsilon-1}{\epsilon}}}_{\text{nonproduced good}}\right]^{\frac{\epsilon}{\epsilon-1}}$$

- Subject to budget constraint

$$\underbrace{m}_{\text{numeraire}} + p \cdot \underbrace{(1+\tau(x))\cdot c}_{\text{purchase of services}} = \underbrace{\mu}_{\text{endowment}} + \underbrace{p \cdot f(x) \cdot y}_{\text{labor income}}$$

Optimal Consumption Decision

- First-order condition

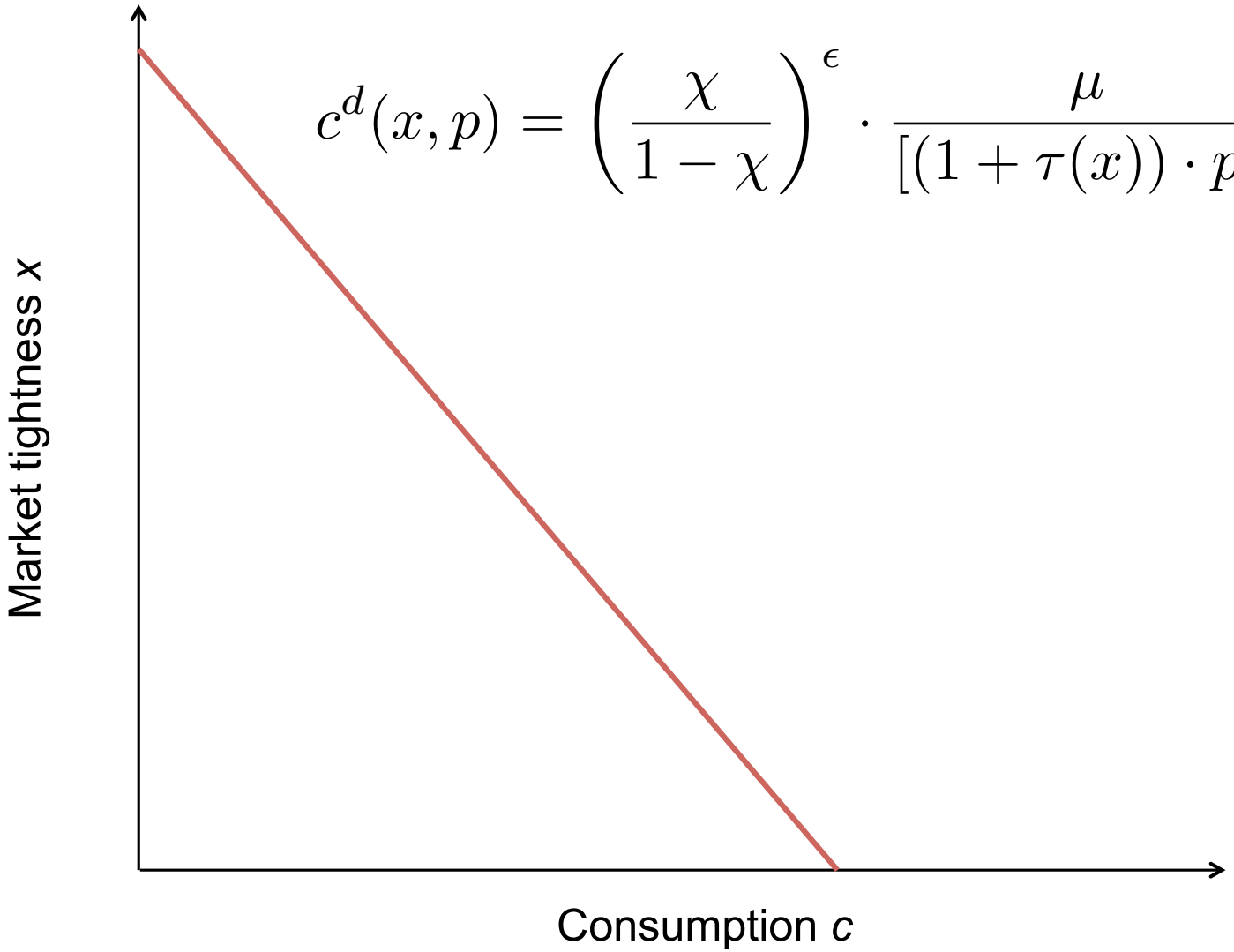
$$\underbrace{(1+\tau(x))\cdot p}_{\text{price of services}}\cdot \underbrace{(1-\chi)\cdot m^{-\frac{1}{\epsilon}}}_{\text{MU of nonproduced good}} = \underbrace{\chi\cdot c^{-\frac{1}{\epsilon}}}_{\text{MU of services}}$$

- Aggregate demand (as $m=\mu$):

$$c^d(x,p) = \left(\frac{\chi}{1-\chi}\right)^{\epsilon}\cdot\frac{\mu}{(1+\tau(x))^{\epsilon}\cdot p^{\epsilon}}$$

Aggregate Demand

$$c^d(x,p) = \left(\frac{\chi}{1-\chi}\right)^{\epsilon} \cdot \frac{\mu}{[(1+\tau(x))\cdot p]^{\epsilon}}$$



Equilibrium with Matching Frictions

- Equilibrium is (c, x, p) such that supply = demand:

$$\begin{cases} c^s(x) &= c^d(x,p) \\ c &= c^s(x) \end{cases}$$

- 3 variables, 2 equations: indeterminacy
- **Short-run equilibrium: p is fixed**
- Any p is acceptable (price band is infinite)

Business Cycle: the Hair Salon Example

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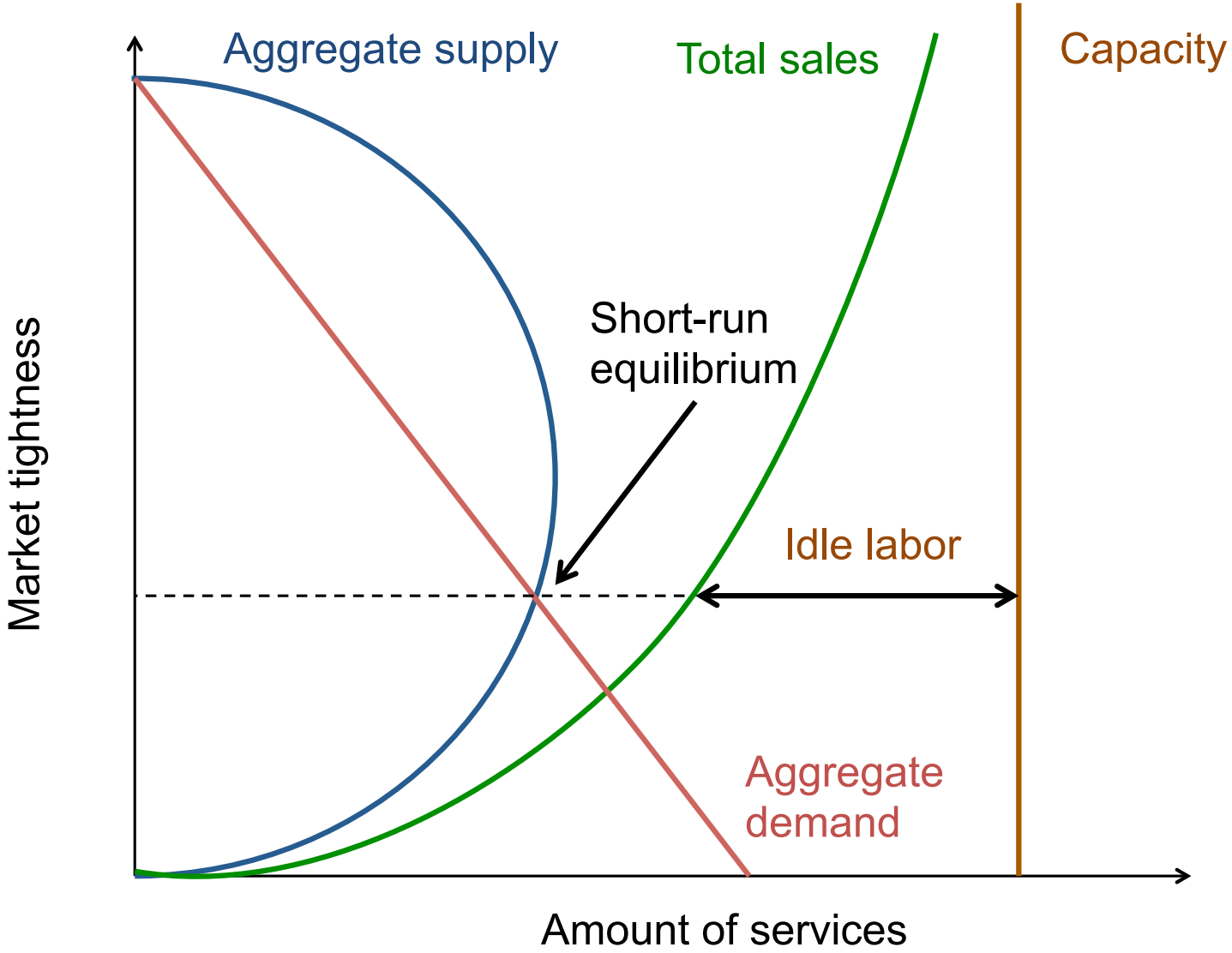
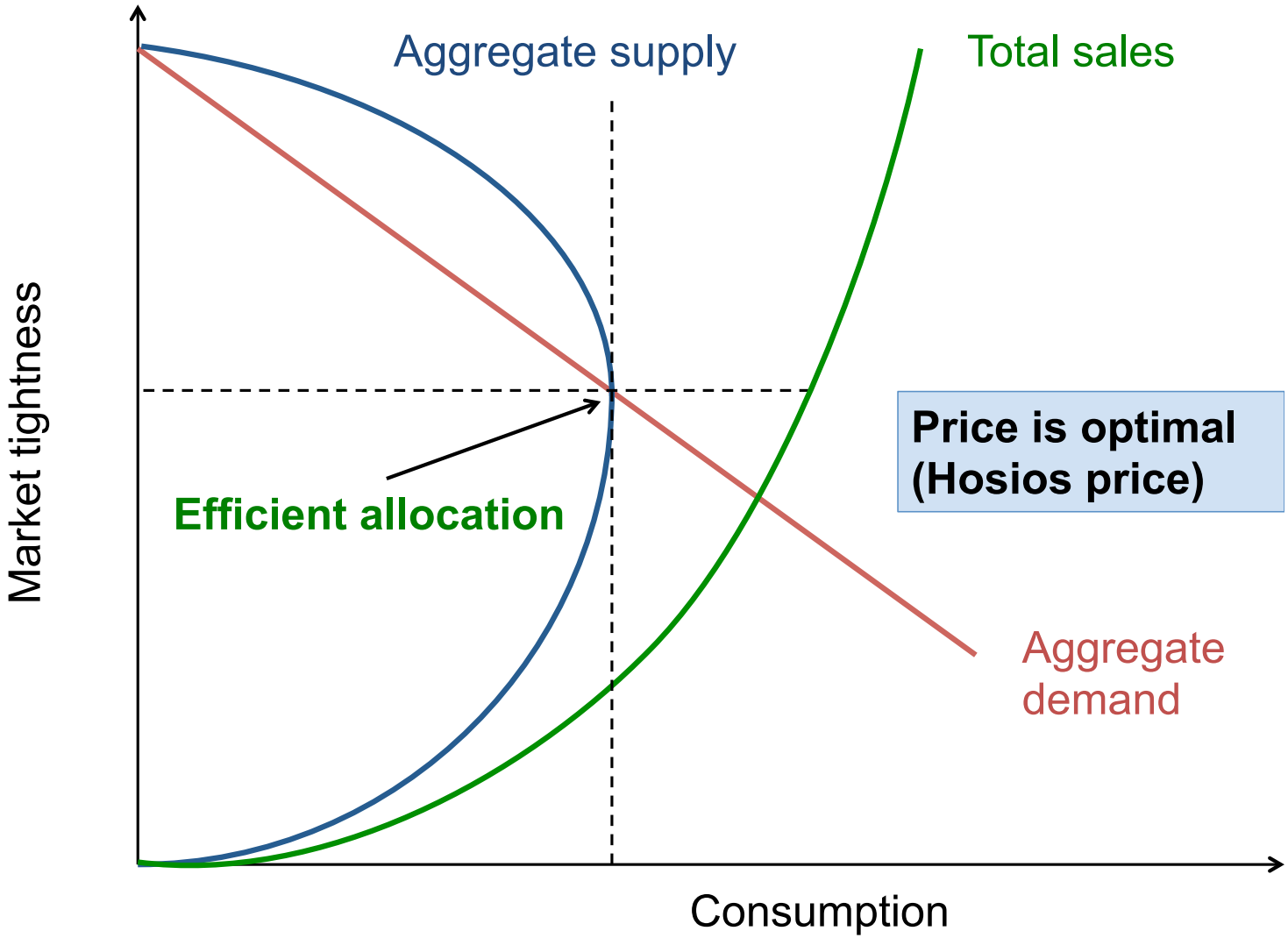
Short-Run Equilibrium



Welfare Properties



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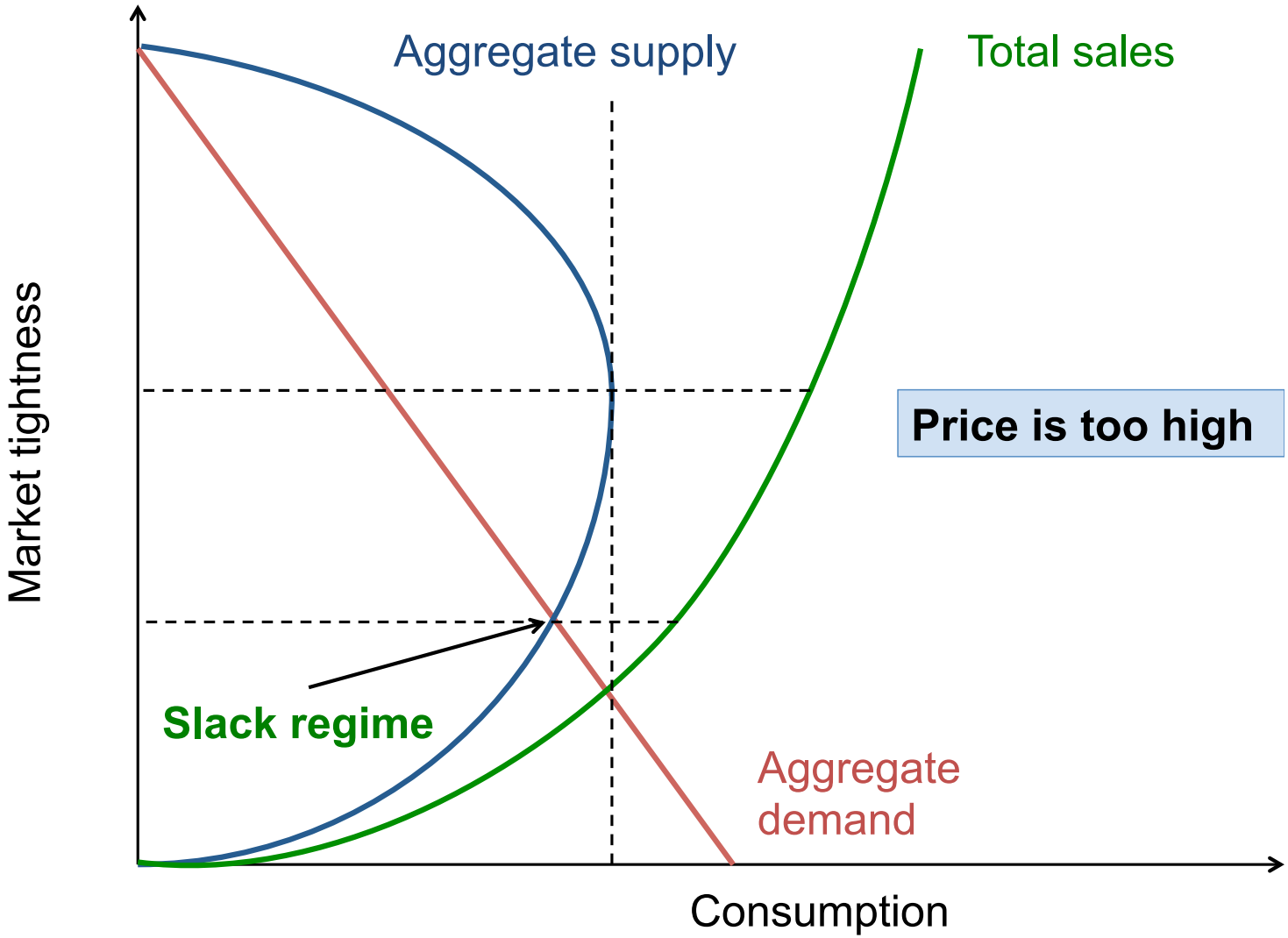
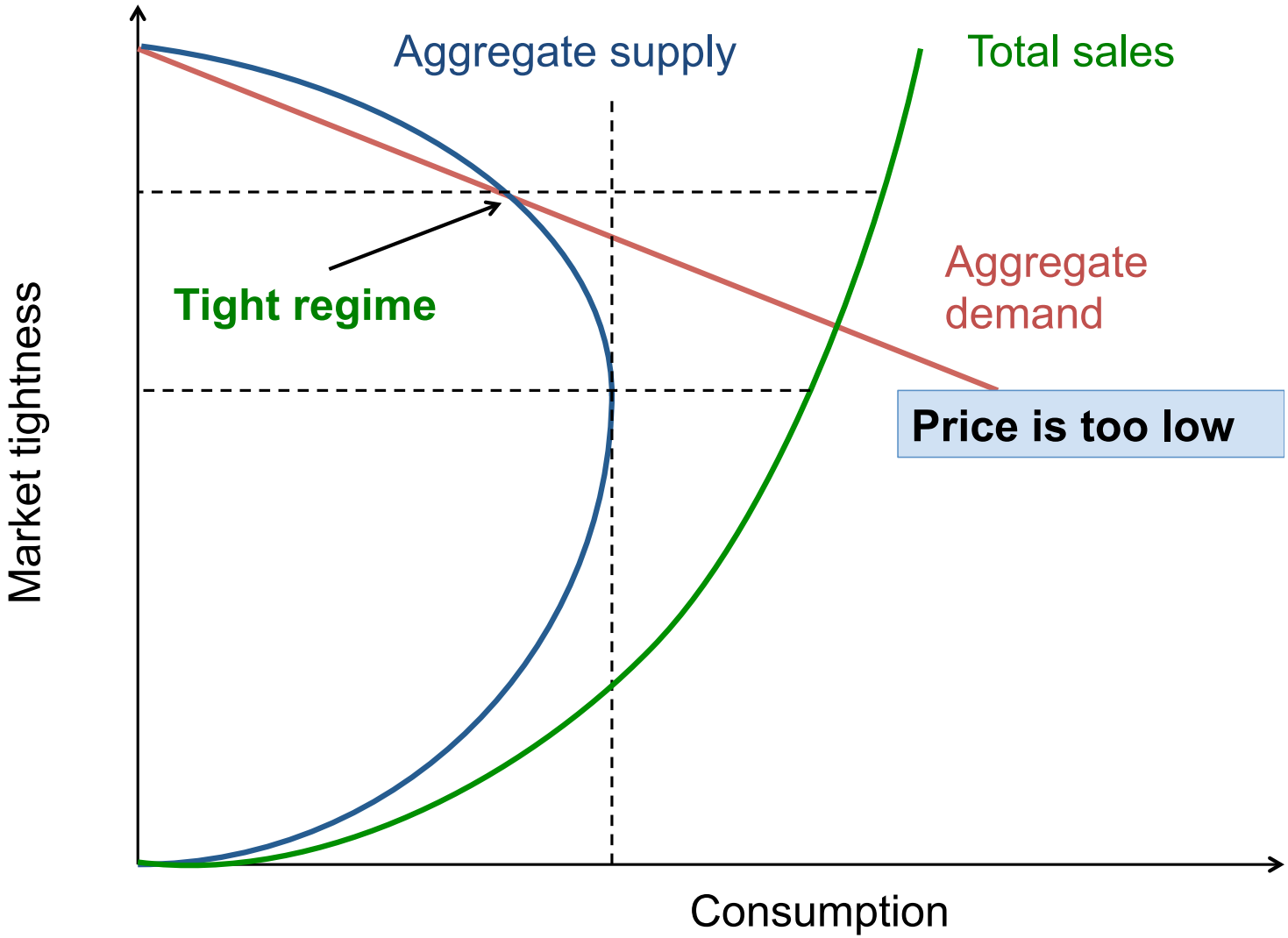


Welfare Properties



Convergence to Optimum in Dynamic Model?

- Short run: fixed price
  - price may not be optimal
- Medium run: new markets are created
  - competitive search mechanism of Moen [1997]
  - price increases if market is tight
  - price decreases if market is slack
- Long run: price at Hosios level
  - maximizes social welfare

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1. Simple model: market for services
   - assumptions and equilibrium
   - **aggregate demand & aggregate supply shocks**
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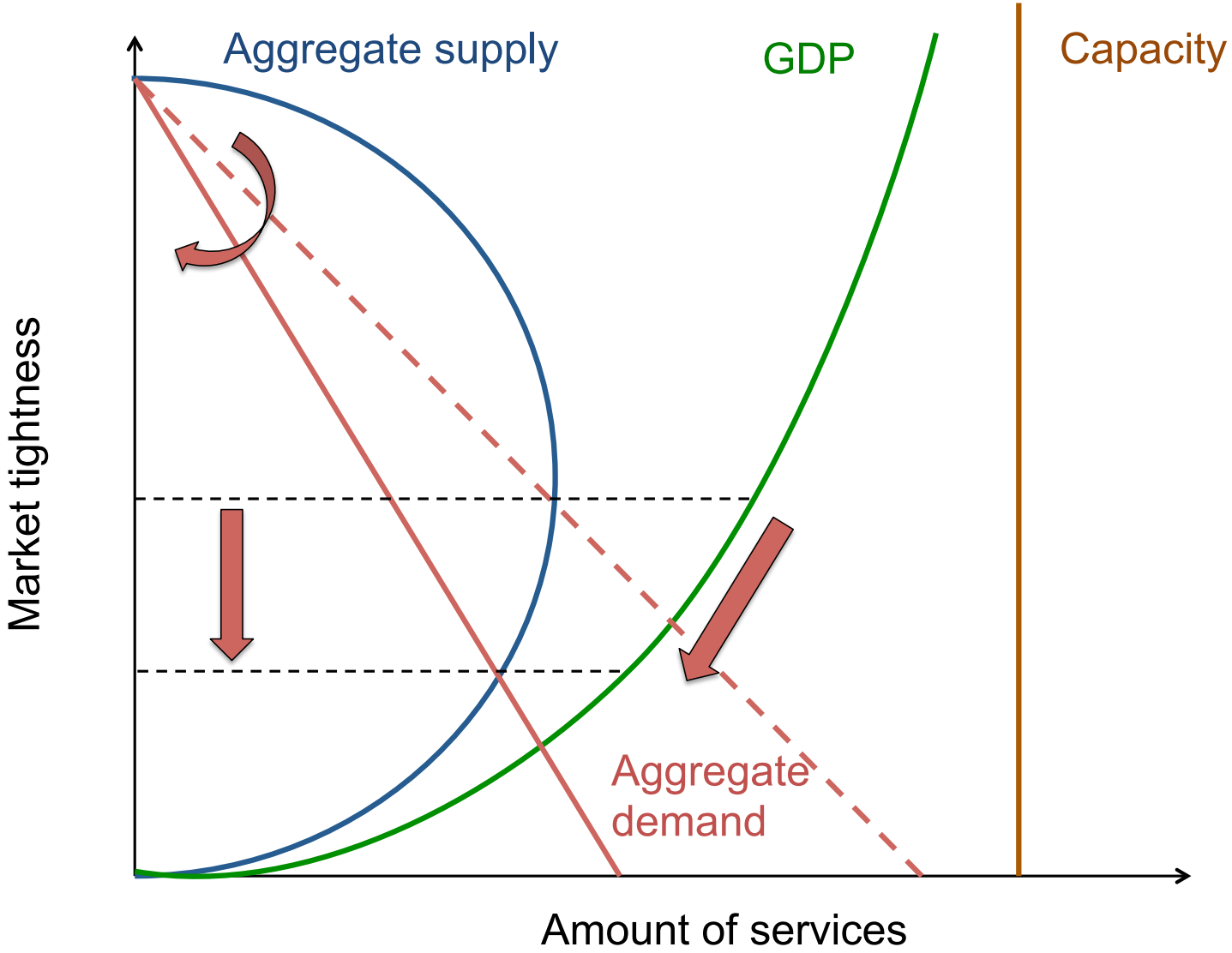
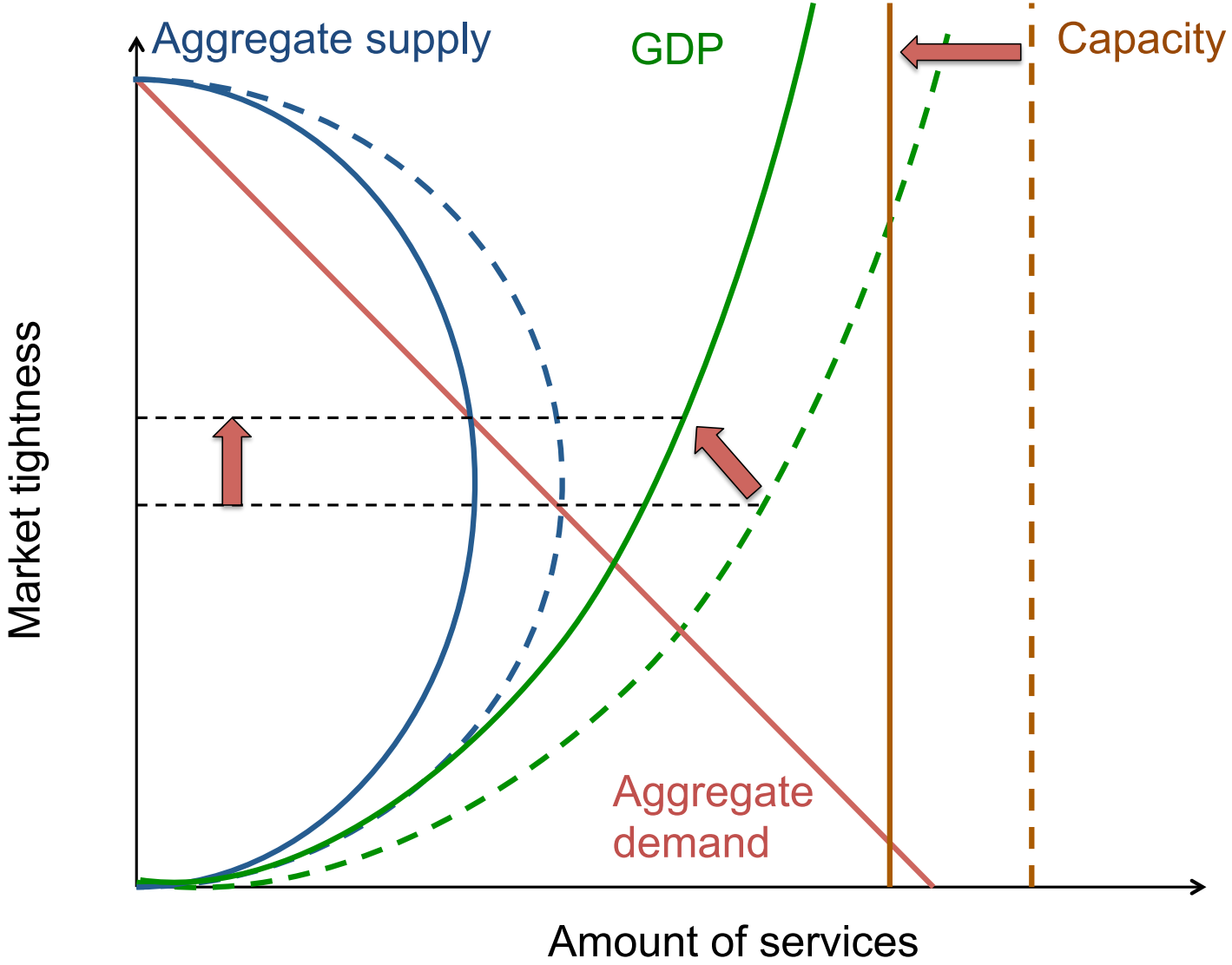
Negative AD Shock: Tightness Falls



Negative AS Shock: Tightness Rises



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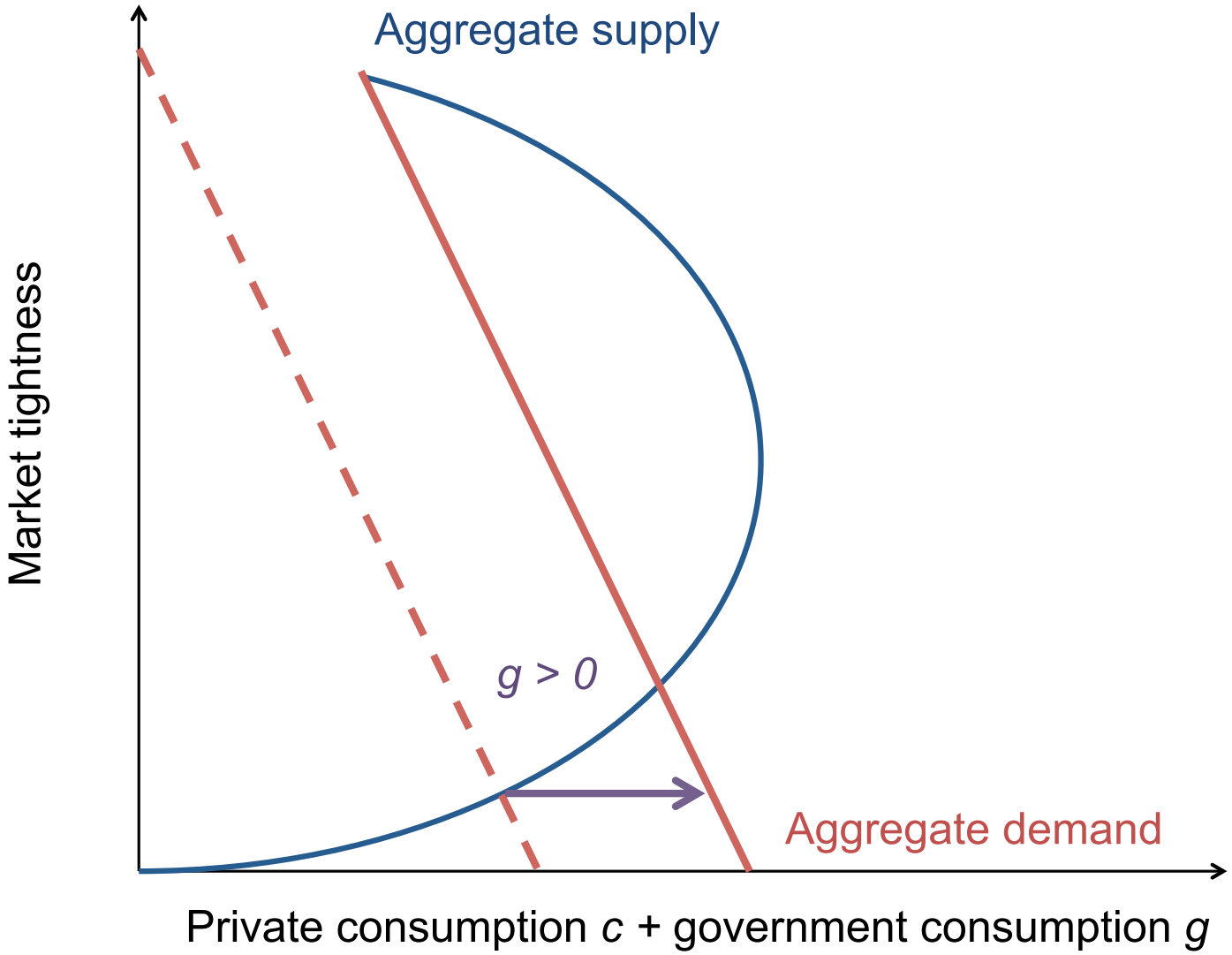
Slack Regime: $0 <$ Multiplier < 1



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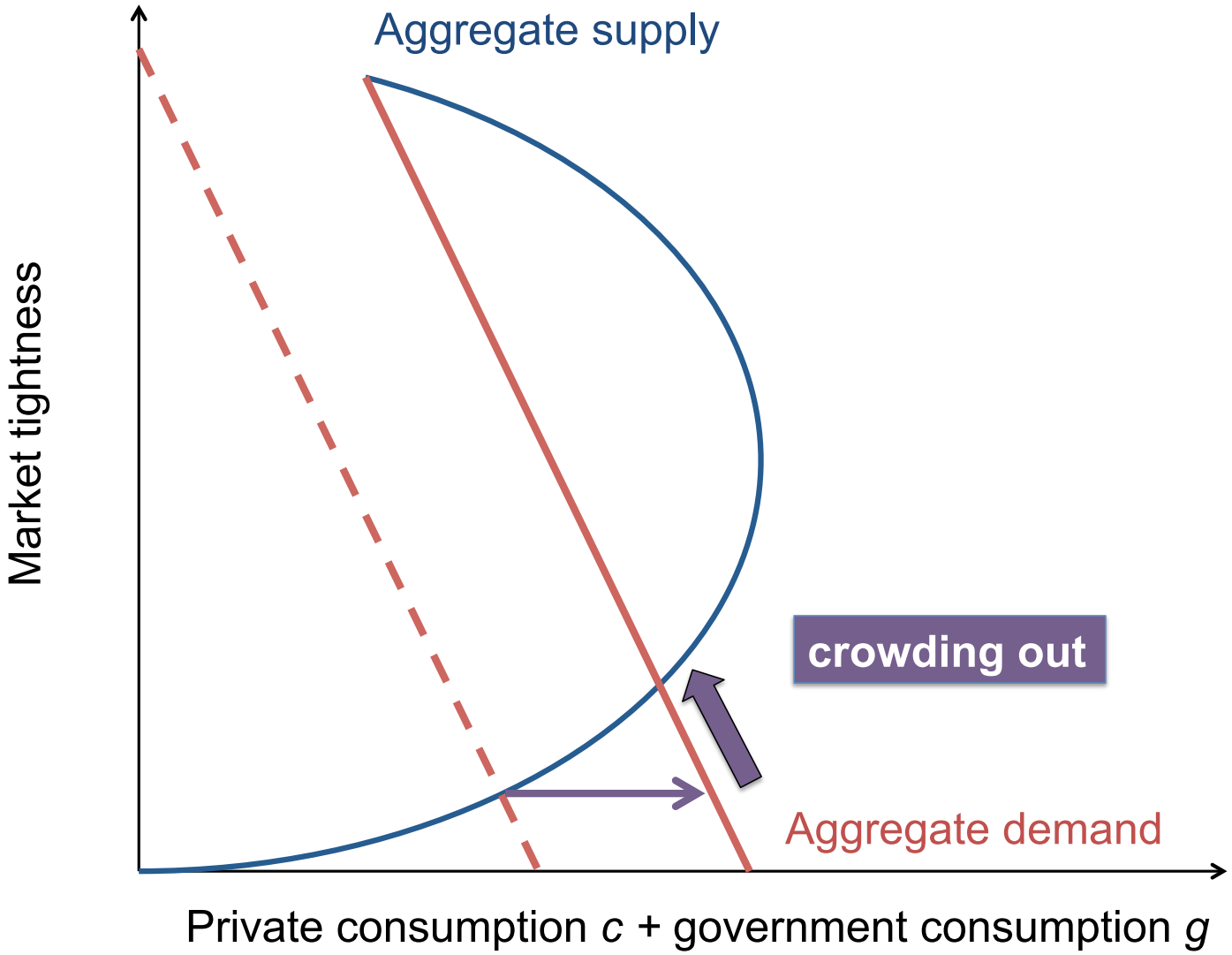

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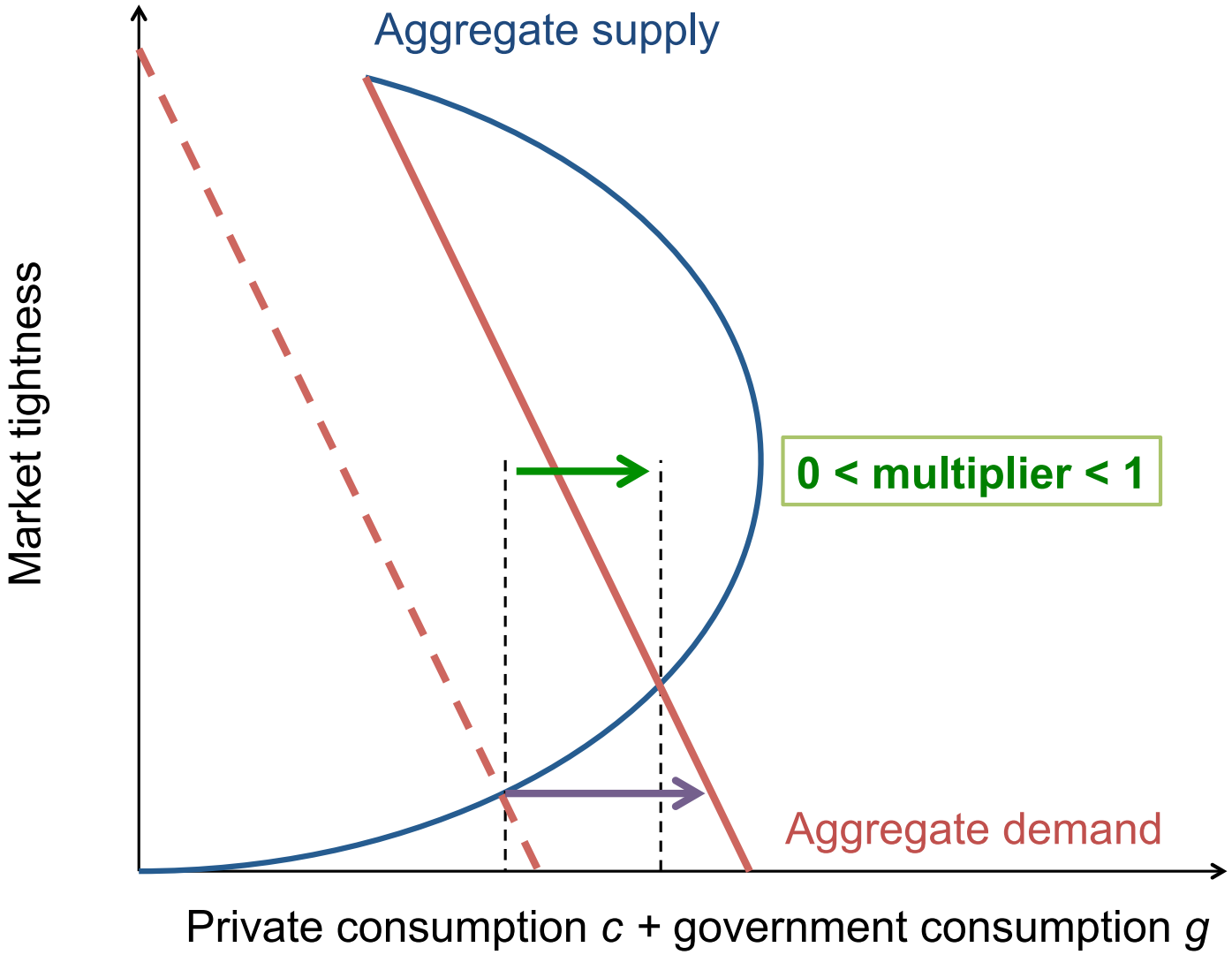
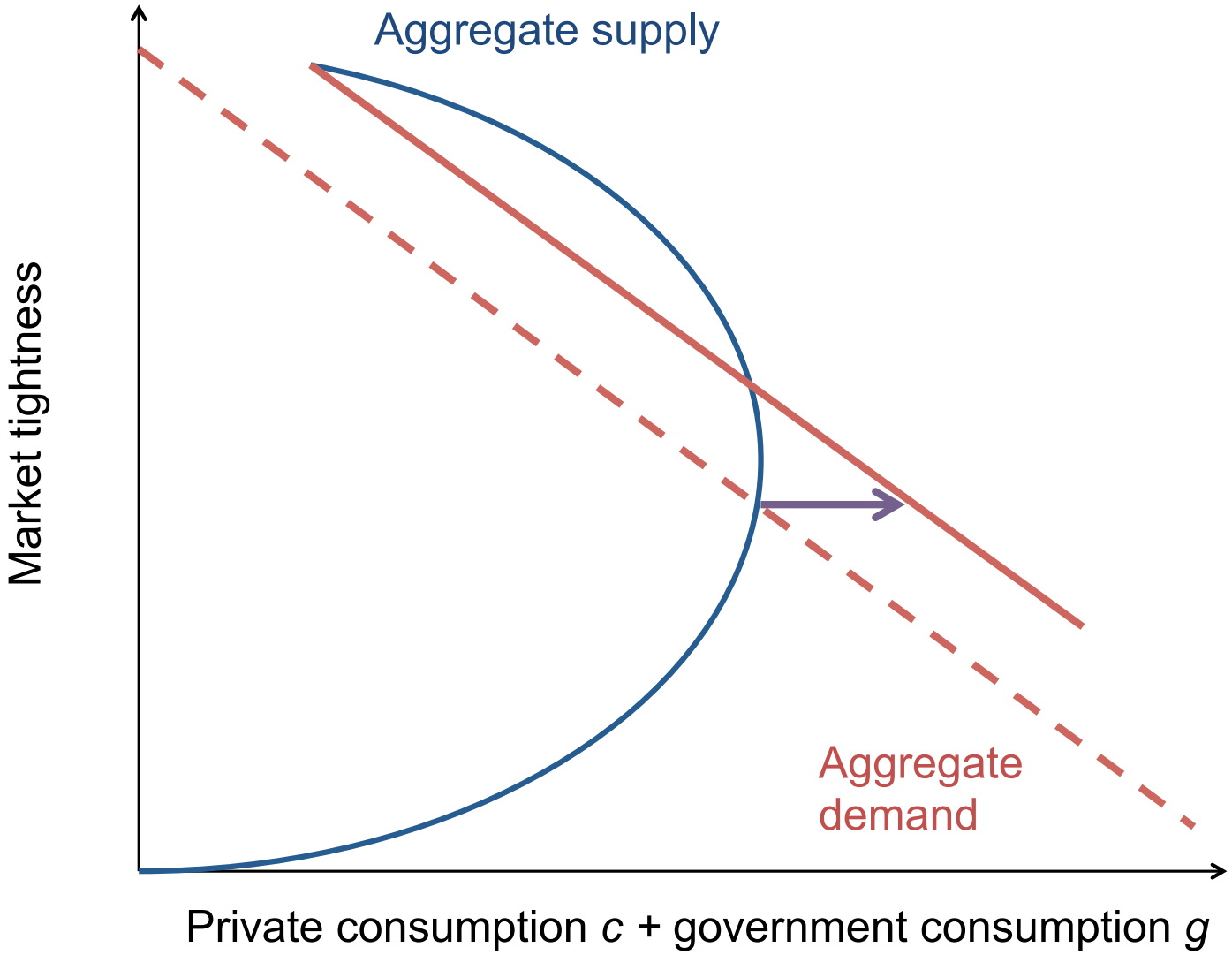


Efficient Regime: Multiplier = 0



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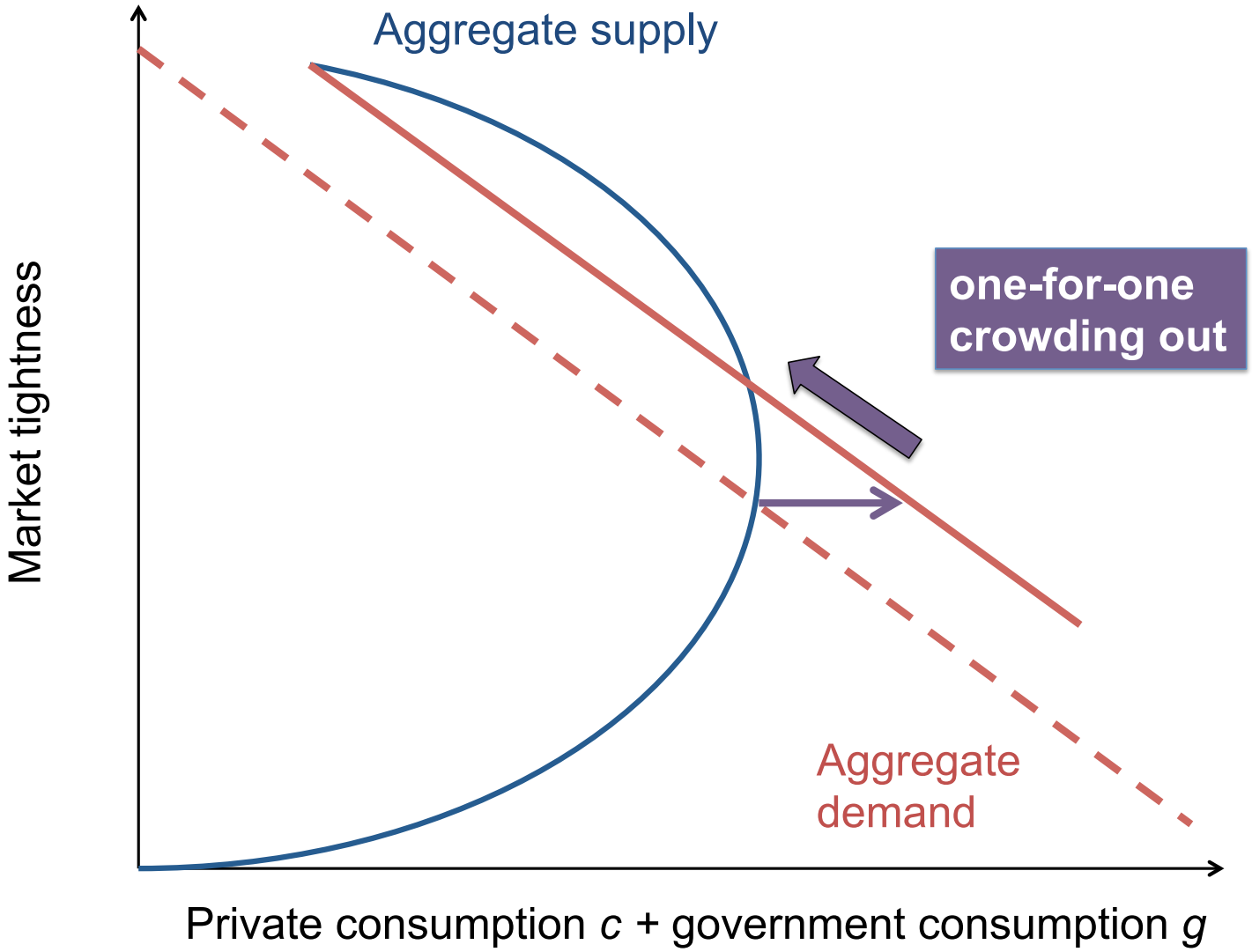

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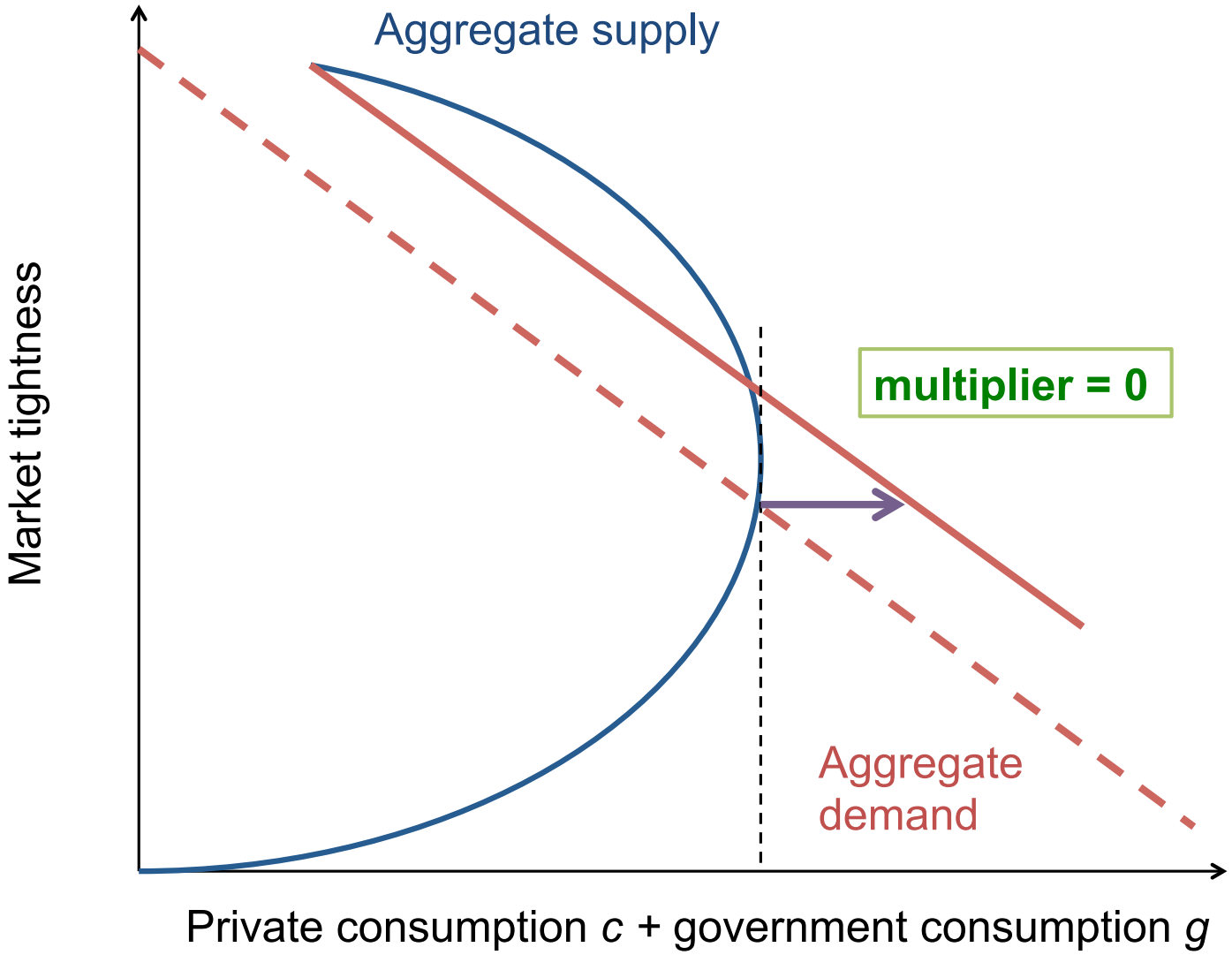
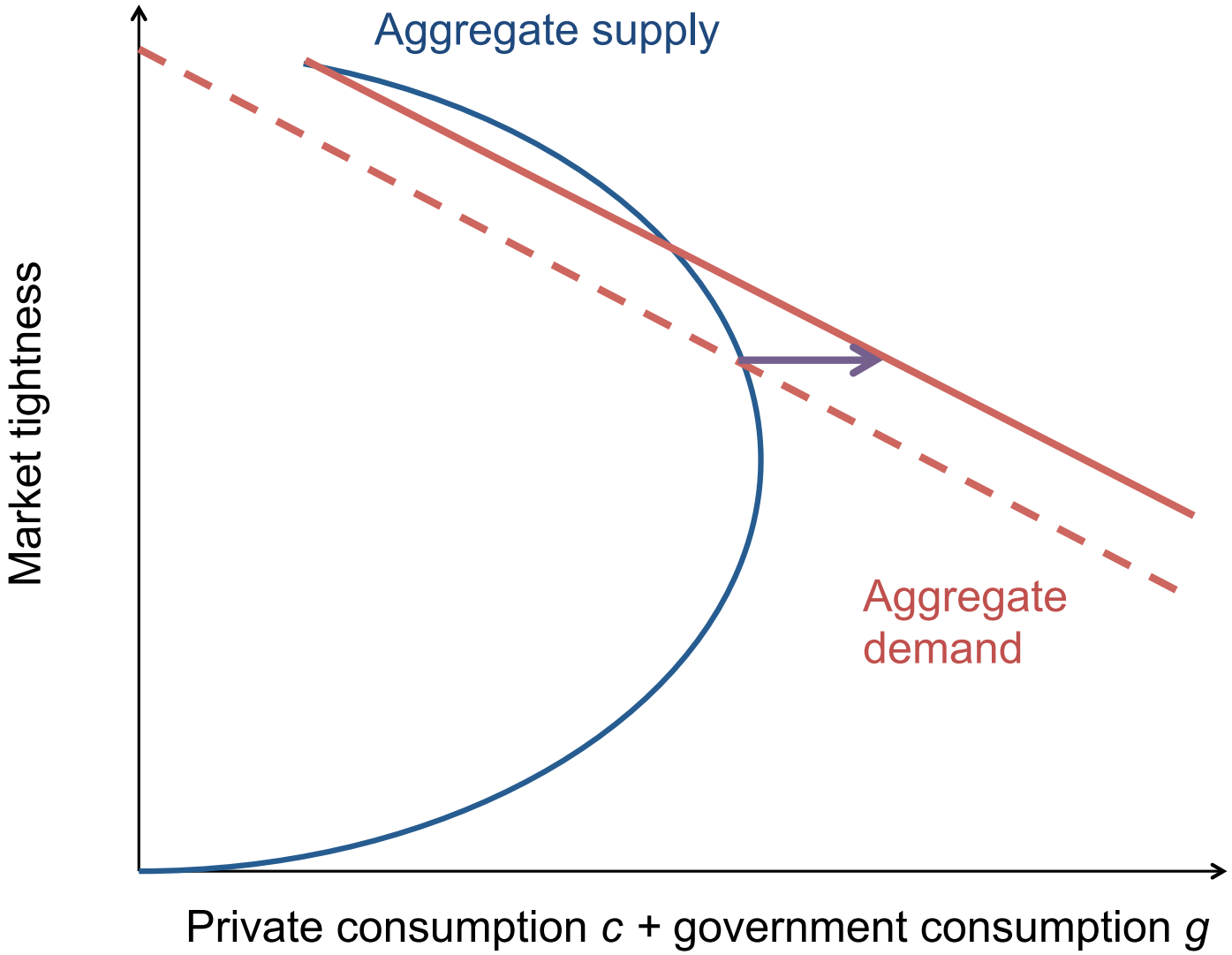
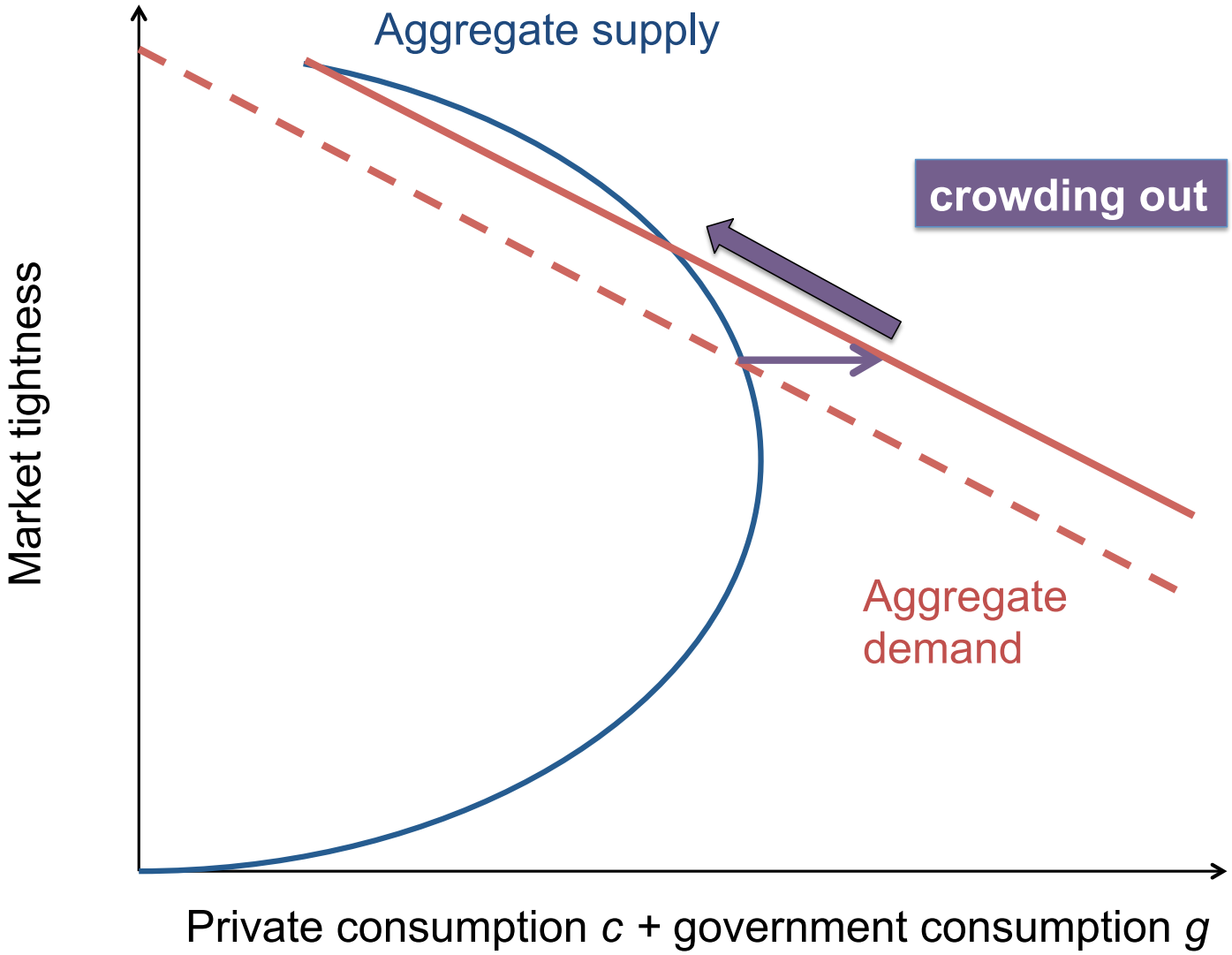


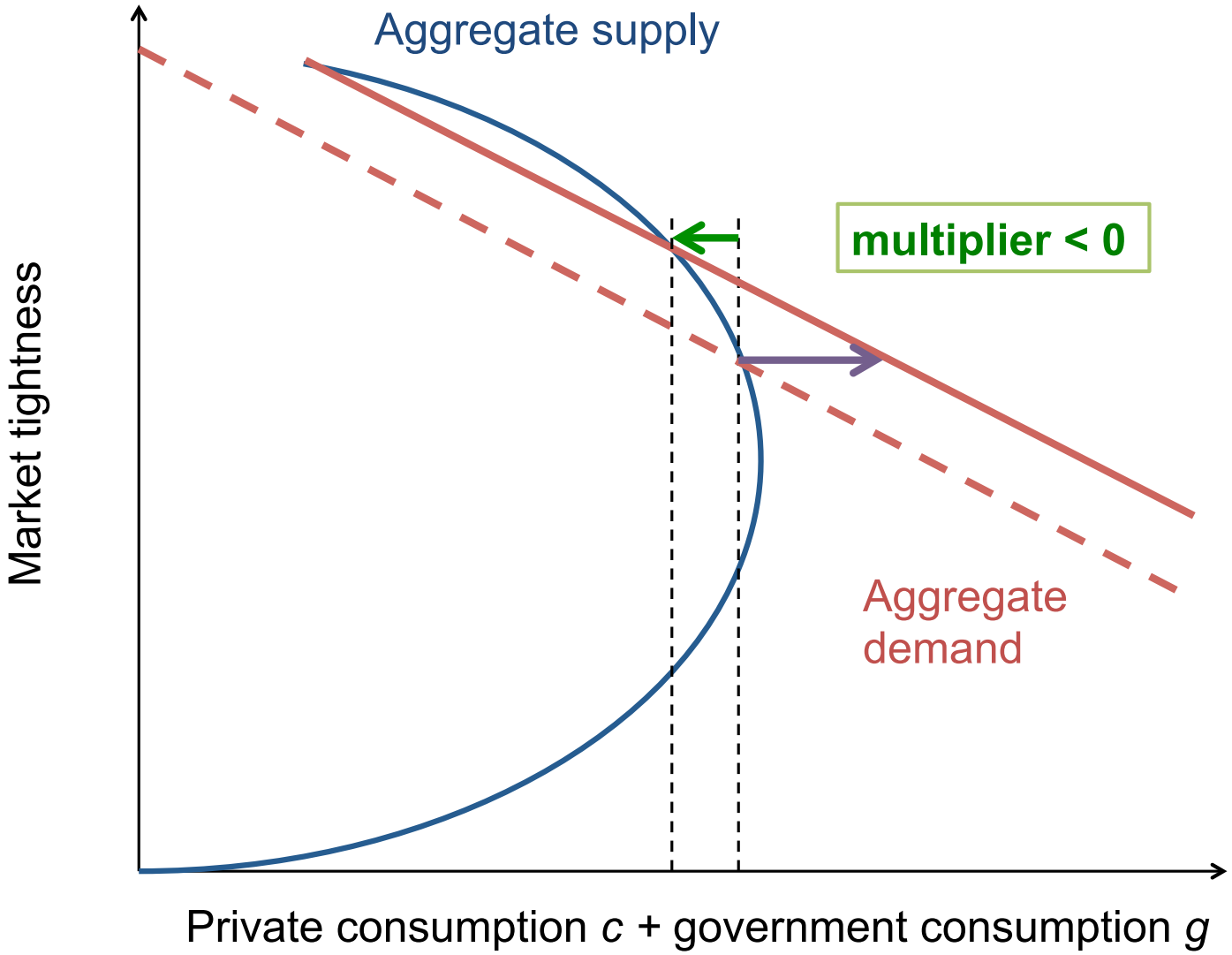
Tight Regime: Multiplier < 0



Tight Regime: Multiplier < 0



Tight Regime: Multiplier < 0



Normative Implications of the Multiplier

- Optimal government consumption g satisfies

$$\underbrace{1 = \frac{\text{MU of } g}{\text{MU of } c}}_{\text{Samuelson's rule}} + \text{multiplier}$$

- If multiplier > 0: optimal $g >$ Samuelson's rule
- If multiplier < 0: optimal $g <$ Samuelson's rule
- Optimal government consumption is countercyclical

General Optimality Principle

- Optimal government consumption g satisfies

$$\underbrace{1 = \frac{\text{MU of } g}{\text{MU of } c}}_{\text{public-finance formula}} + \underbrace{\frac{c}{x} \cdot \frac{\partial x}{\partial g}}_{\text{effect of g on x}} \cdot \underbrace{[(1-\eta) - \eta \cdot \tau(x)]}_{\text{deviation from Hosios condition}}$$

- Economy is slack iff $(1-\eta) - \eta \cdot \tau(x) > 0$
- Optimal UI satisfies formula with same structure

  [Landais, Michaillat, & Saez, 2010]

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Wealth Inequality

- I heterogeneous groups of measure $1/I$
- Group i's per person utility:

$$\chi_i \cdot \ln(c_i) + (1 - \chi_i) \cdot \ln(m_i)$$

- Group i's per person budget:

$$m_i + (1 + \tau(x)) \cdot p \cdot c_i = \mu_i + p \cdot f(x) \cdot y$$

AD and Transfer Payments

AD depends on joint distribution of (μ_i, χ_i):

$$c^d(x,p) = \frac{\sum_i \mu_i \cdot \chi_i}{\sum_i (1-\chi_i)} \cdot \frac{1}{p \cdot (1+\tau(x))}$$

If transfer endowment μ_i from low χ_i to high χ_i:

- AD increases since $\sum_i \mu_i \cdot \chi_i$ increases
- Market tightness increases
- Consumption increases iff economy is slack

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   - **assumptions and equilibrium**
   - more on macro shocks
   - payroll tax
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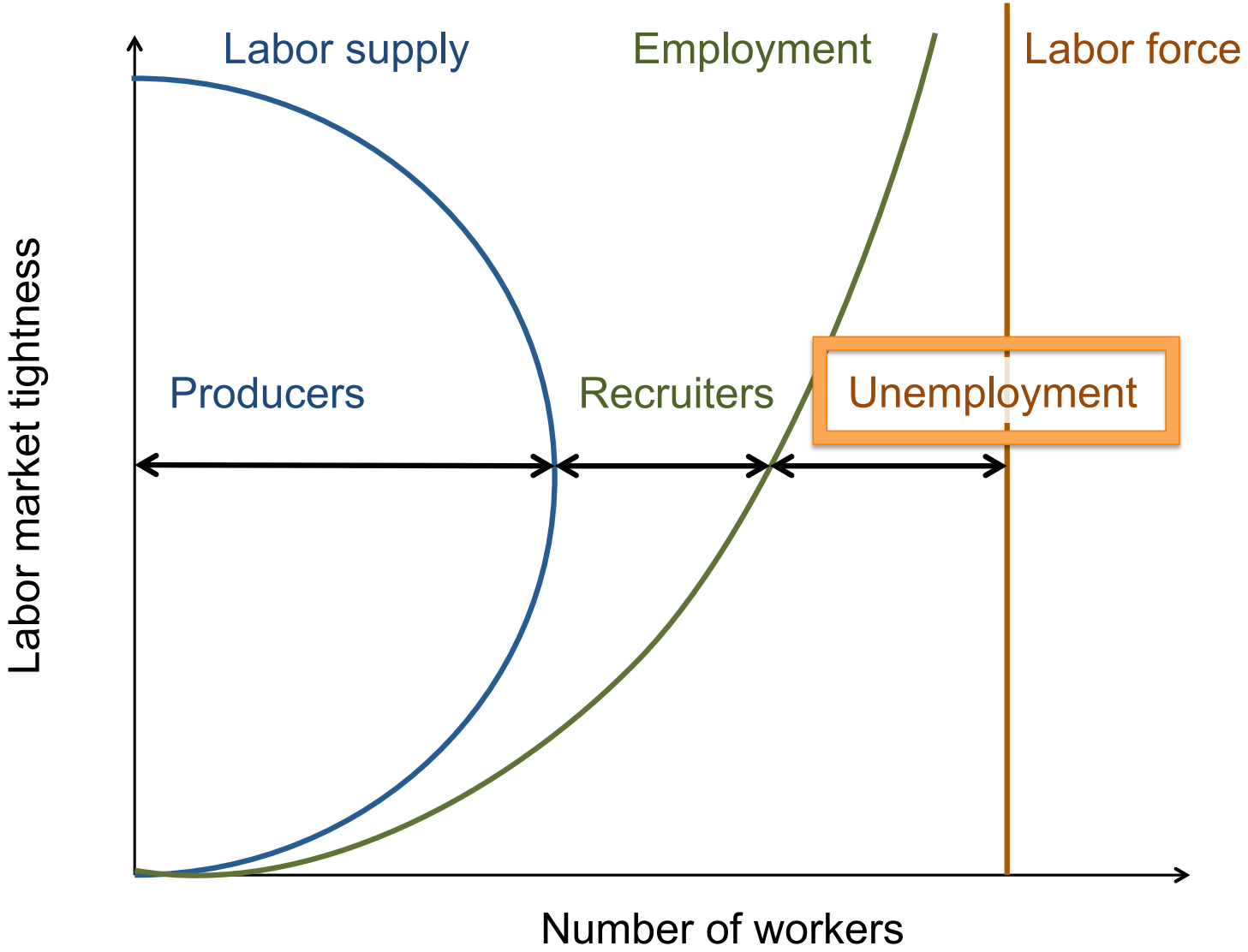
Labor Supply and Unemployment



Firms

- Employ producers and recruiters and sell production
- Take real wage w and tightnesses x and θ as given
- Choose number of producers n to maximize profits

$$\underbrace{f(x)}_{\text{selling probability}} \cdot \underbrace{a \cdot n^{\alpha}}_{\text{production}} - \underbrace{[1 + \hat{\tau}(\theta)] \cdot w \cdot n}_{\text{wage of producers + recruiters}}$$

Optimal Employment Decision

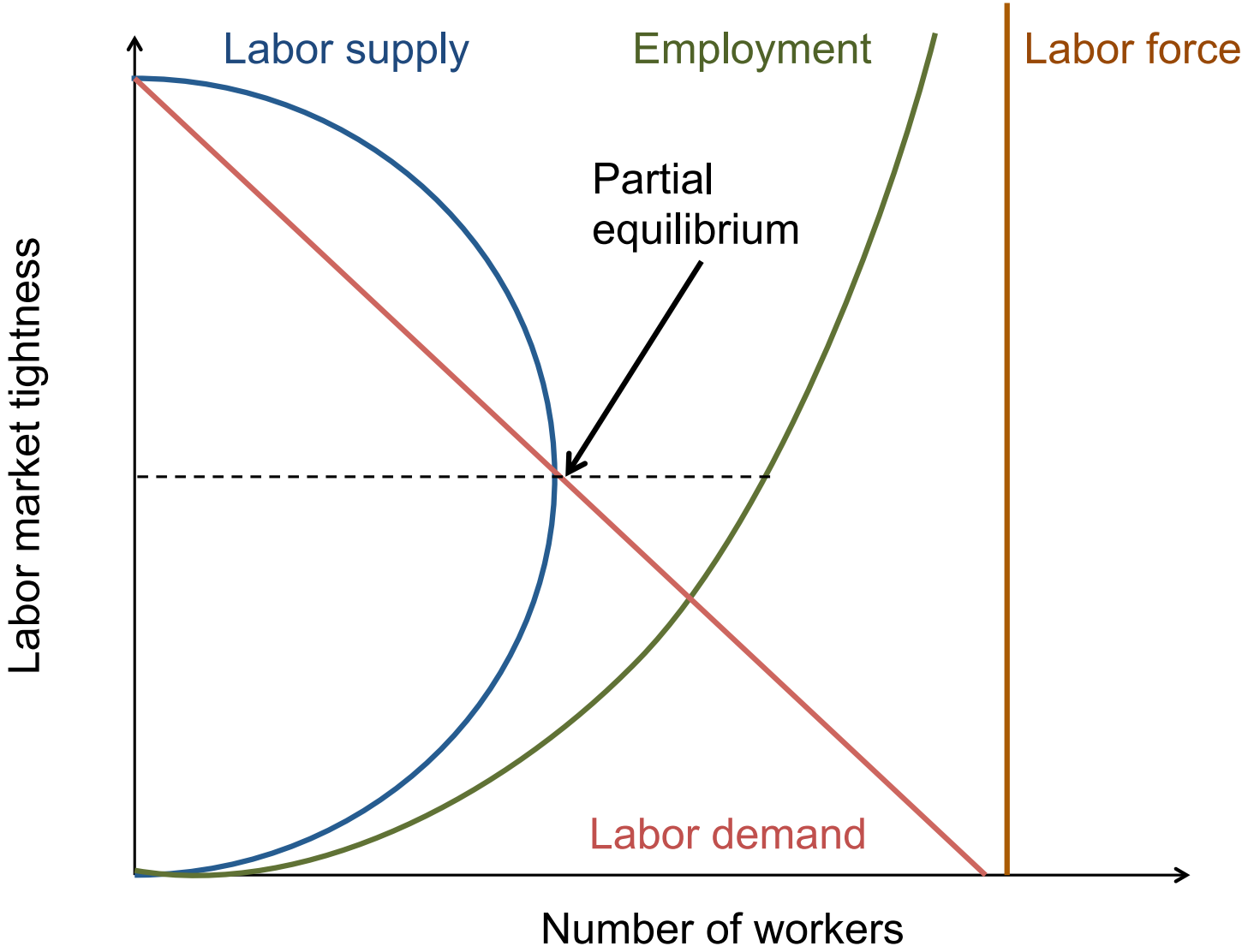
- First-order condition:

$$\underbrace{f(x)}_{\text{selling probability}} \cdot \underbrace{a \cdot \alpha \cdot n^{\alpha-1}}_{\text{MPL}} = \underbrace{[1+\hat{\tau}(\theta)]}_{\text{matching cost}} \cdot \underbrace{w}_{\text{real wage}}$$

- Labor demand:

$$n^d(\theta, x, w) = \left[\frac{f(x) \cdot a \cdot \alpha}{(1+\hat{\tau}(\theta)) \cdot w}\right]^{\frac{1}{1-\alpha}}$$

Partial Equilibrium on Labor Market



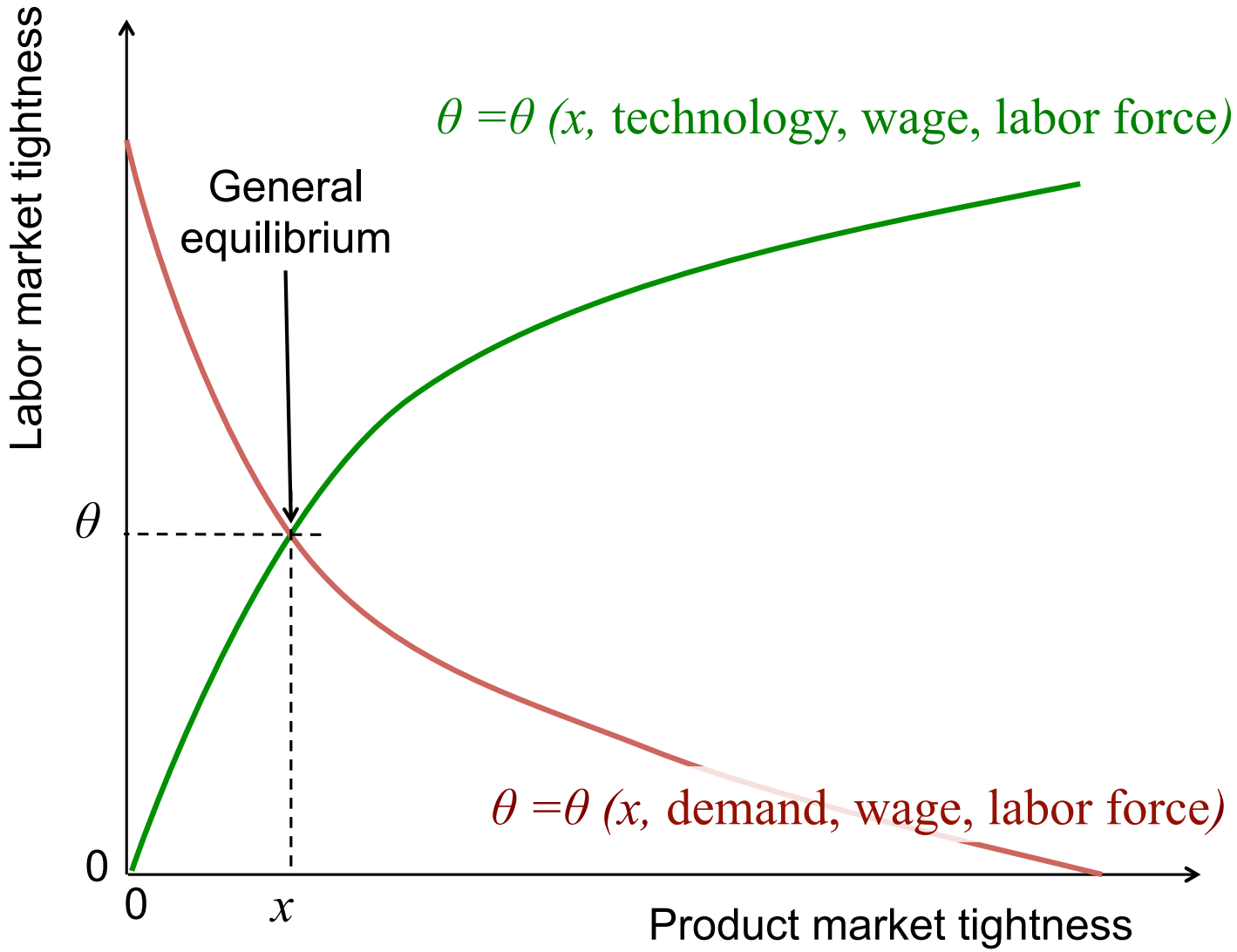
General Equilibrium (c, n, x, θ, p, w)

- Supply = demand on product and labor markets

$$\begin{cases} c^s(x,n) & = & c^d(x,p) \\ c & = & c^s(x) \\ n^s(\theta) & = & n^d(\theta, x, w) \\ n & = & n^s(\theta) \end{cases}$$

- 4 equations, 6 variables: indeterminacy
- **Short-run equilibrium: p & w are parameters**

General Equilibrium Representation



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General-Equilibrium Correlations

		Effect on:			
Increase in:	Regime:	x	θ	GDP	u
Demand		+	+	+	−
Labor force		−	−	+	+
Technology		−	+	+	−
Real wage	labor-slack	+	−	−	+
	labor-tight	−	−	+	+

Possibility to Reject Technology Shocks

		Effect on:			
Increase in:	Regime:	x	θ	GDP	u
Demand		+	+	+	−
Labor force		−	−	+	+
Technology		−	+	+	−
Real wage	labor-slack	+	−	−	+
	labor-tight	−	−	+	+

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Inequality in Profit Income

- Group i's per person budget:

$$m_i + [1 + \tau(x)]\, pc_i = \mu_i + p\sigma_i \pi + p\,[1 + \hat{\tau}(\theta)]\, wn$$

- AD depends on wage:

$$c^d(x, \theta, p) = \left[\frac{\sum_i \mu_i \cdot \chi_i}{\sum_i (1 - \chi_i) \cdot \sigma_i} \cdot \frac{1}{p \cdot (1 + \tau(x))}\right] + \left[\frac{\sum_i \chi_i \cdot (1 - \sigma_i)}{\sum_i (1 - \chi_i) \cdot \sigma_i}\right] \frac{1}{1 + \tau(x)} \cdot w \cdot \hat{f}(\theta)$$

- Increase in w stimulates AD if **savers own firms**

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   - **connection with existing business-cycle models**
   - summary and future research

Perfect Competition

- No matching costs
- Assumptions on price and wage schedules:

$$w = \text{MPL at } (n = h) = a \cdot \alpha \cdot h^{\alpha - 1}$$

$$p = \text{MRS at } (n = h) = \frac{\chi}{1 - \chi} \cdot \left(\frac{\mu}{a \cdot h^{\alpha}} \right)^{\frac{1}{\epsilon}}$$

$\Rightarrow$ Tightnesses $\rightarrow +\infty$

$\Rightarrow$ Unemployment $= 0$, unsold production $= 0$

- AD shocks have no effect

Matching-Bargaining

- Linear production function: $y = a \cdot n$
- Linear utility function: $\chi \cdot c + (1 - \chi) \cdot m$
- Assumptions on price and wage schedules:

$$w = \text{Nash bargained wage} = \hat{\beta} \cdot a \cdot f(x)$$

$$p = \text{Nash bargained price} = \beta \cdot \frac{\chi}{1 - \chi}$$

- AD shocks have no effect

Monopolistic Competition

- Utility function: $\chi \cdot \ln(c) + (1-\chi) \cdot \ln(m) - \nu \cdot h^{\frac{1+\xi}{\xi}}$
- Assumptions on market tightnesses:

$$f(x) = \frac{1}{\text{product markup}}$$

$$\hat{f}(\theta) = \left(\frac{1}{\text{labor markup}}\right)^{\frac{\xi}{\xi+1}}.$$

$\Rightarrow$ Same FOCs, same labor wedge = MPL/MRS

- AD shocks have no effect

Fixprice-Fixwage

- No matching costs
- Special matching functions:

$$s = \min\{v, y\}$$

$$l = \min\{\hat{v}, 1\}$$

$\Rightarrow$ Replicate fixprice-fixwage with proportional rationing

- AD shocks have effects, but 4 equilibrium systems

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A New Kind of Business-Cycle Model

- Microfoundations for AD and AS
- Unemployment & unsold production in equilibrium
- 3 variables per market: price, quantity, tightness
- Comovements of tightnesses and quantities $\Rightarrow$ macro shocks
- Gap between actual tightness and optimal tightness $\Rightarrow$ optimal macro policies

Future Research

1. Optimal macro policies over the business cycle
   - fiscal policy and insurance programs
   - monetary policy
2. Empirical exploration of market tightness
   - measure tightness
   - estimate gap between actual and optimal tightness
3. Quantitative dynamic model
   - adjustment of prices
   - saving and investment