A Macroeconomic Approach to Optimal Unemployment Insurance: Applications

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Abstract

This paper applies Landais, Michaillat, and Saez’s [2015] theory of optimal unemployment insurance (UI) to determine how optimal UI varies over the business cycle. The optimal UI replacement rate is the Baily-Chetty rate plus a correction term. The statistics entering the correction term are evaluated using empirical evidence for the United States: since 1990 the correction term is usually positive and is very countercyclical; hence, the optimal replacement rate is usually somewhat higher than the Baily-Chetty rate and is much higher in slumps. Simulations of a matching model fitting the empirical evidence confirm that optimal UI is markedly countercyclical.
Unemployment insurance (UI) is a key component of social insurance in modern welfare states, and whether to increase or decrease the generosity of UI during slumps is an important yet unresolved public policy question. For instance, in the United States the generosity of UI automatically increases with the unemployment rate, but these automatic increases remain debated.\(^1\) It is clear that an increase in the generosity of UI leads to a reduction in unemployed workers’ search effort. Proponents of increasing UI in slumps argue that the reduction in search effort does not raise the unemployment rate much in slumps because high unemployment is due to a scarcity of jobs, not insufficient search. Proponents of reducing UI in slumps respond that in slumps, the reduction in search effort remains detrimental to unemployment in slumps because flows on the US labor market remain large so search remains important. Proponents of reducing UI in slumps also worry that higher UI may lead to higher wages through bargaining and thus lower job creation by firms and higher unemployment.\(^2\)

In a companion paper [Landais, Michaillat and Saez, 2015], we developed a theory of optimal UI in matching models and showed that the optimal replacement rate is the conventional Baily-Chetty rate plus a correction term. In this paper, we apply the theory to explore how the generosity of UI should vary over the business cycle. The theory is well adapted to the problem because it captures the different aspects of the debate—effect of UI on search effort, possible scarcity of jobs in slumps, large labor market flows, and effect of UI on wages and job creation. Our approach is to study the fluctuations of the correction term over the business cycle. Our work complements the important contribution of Kroft and Notowidigdo [2015], who study the fluctuations of the Baily-Chetty rate over the business cycle. We find based on empirical evidence for the United States that the correction term is strongly countercyclical—it is much higher in slumps than in booms. This reduced-form evidence suggests that the optimal replacement rate is more countercyclical than the Baily-Chetty rate. Simulations of a structural model fitting the reduced-form evidence further show that the optimal replacement rate is also countercyclical in absolute terms.

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\(^1\)In the United States, UI benefits have a duration of 26 weeks in normal times. The Extended Benefits program automatically extends duration by 13 weeks in states where the unemployment rate is above 6.5\% and by 20 weeks in states where the unemployment rate is above 8\%. Duration is often further extended in severe recessions.

We begin in section I by extending the static model from our companion paper into a rich dynamic model that can be brought to the data and used for various quantitative applications. The model is well adapted to study UI over the business cycle because it accounts for all the aspects of the UI debate. First, the models offer an appealing description of labor market flows and matching between jobseekers and job vacancies. Second, it captures how unemployed workers adjust job search and self-insurance to the generosity of UI. Third, it accurately describes the cost of unemployment by factoring in the UI program, self-insurance through home production, and a nonpecuniary cost of unemployment. Fourth, it describes how firms adjust their hiring decision based on wages and the state of the labor market; in particular, firms’ labor demand can easily generate a scarcity of jobs. Fifth, the model accommodates a broad range of wage mechanisms, including some in which wages respond to UI. And sixth, the model is well adapted to study business cycles because in equilibrium the unemployment rate may be efficient, inefficiently high, or inefficiently low. This implies that the model can accommodate not only efficient unemployment fluctuations but also inefficient ones.

We find that in this richer model the optimal UI formula is virtually identical to the formula obtained in our companion paper. The optimal replacement rate of UI is the sum of the conventional Baily-Chetty replacement rate and a correction term. The correction term measures the effect of UI on welfare through labor market tightness and therefore is the product of two terms: the effect of UI on tightness and the effect of tightness on welfare. Furthermore, these effects can be measured in the data exactly as described in our companion paper; this is what we do in Sections II and III.

An increase in UI raises tightness if and only if the macroelasticity of unemployment with respect to UI is smaller than the microelasticity. In section II, we review the evidence on this issue, including several recent studies whose analysis is guided in part by the theory developed in our companion paper. Although there remains significant uncertainty about the estimates, most studies find that the microelasticity is larger than the macroelasticity. Moreover, there is compelling evidence of rat-race effects in several case studies but no clear evidence that an increase in UI raises wages, as happens with the job-creation mechanism. Based on this evidence, we conclude that an increase in UI raises tightness.

An increase in tightness raises welfare when the value of having a job relative to being un-
employed is high enough compared to the share of labor devoted to recruiting. In section III we
develop a method that exploits this insight to assess whether tightness is efficient, inefficiently
low, or inefficiently high at any point in time. This is a difficult empirical question but it is central
for optimal UI, and for a broad range of stabilization policies. Our method should be seen as a
blueprint that should be amended as better empirical evidence becomes available; we are not able
to provide a definitive answer because the values of several required statistics are not yet solidly
established. The two main steps of our method are to determine the share of labor devoted to
recruiting and the utility loss from unemployment. First, using several sources from the Bureau
of Labor Statistics (BLS) and Census Bureau, we construct a quarterly time series for the share
of labor devoted to recruiting covering 1990–2014. The share of labor devoted to recruiting av-
erages 2.3% and is sharply procyclical. Second, we combine a variety of sources measuring the
consumption drop upon unemployment, risk aversion, and the nonpecuniary cost of unemploy-
ment. In particular, numerous studies using different approaches find that the nonpecuniary cost
of unemployment—stemming for instance from the toll taken by unemployment on mental and
physical health—is very large. This result contrasts sharply with the standard view in economics,
which treats unemployment as leisure. Overall, we find that tightness is somewhat too low on
average, is much too low in slumps (especially in 1992, 2001, and 2009), and is too high in strong

Combining the results from Sections II and III we infer that the optimal UI replacement rate
is usually somewhat higher than the Baily-Chetty rate, is much higher in slumps, and is lower
only in strong booms. Because of the endogeneity of all the statistics entering the optimal UI
formula, however, we cannot quantify the optimal replacement rate simply by plugging exist-
ing estimates of the statistics in the formula. To overcome this difficulty, Section IV calibrates
and simulates a structural model to quantify the business-cycle fluctuations of the optimal re-
placement rate. We use the job-rationing model developed by Michaillat [2014]. We choose this
model because it generates a macroelasticity of unemployment with respect to UI smaller than the
microelasticity, consistent with the evidence from Section II whereas other common matching

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3 Michaillat and Saez [2014, 2015] have applied the same matching framework to study optimal monetary policy,
debt policy, and government purchases. In all these cases, the optimal policy depends on the gap between actual
tightness and efficient tightness.
models do not. Furthermore, because real wages are somewhat rigid in this model, technology shocks lead tightness to be sometimes inefficiently low and sometimes inefficiently high, in line with the evidence from Section III. The simulations indicate that the optimal replacement rate is markedly countercyclical, almost doubling from 40% to 75% when the unemployment rate rises from 4% to 12%.

I. Optimal Unemployment Insurance in a Dynamic Model

In this section we propose a rich dynamic model of UI that builds on the static model from our companion paper [Landais, Michaillat and Saez, 2015]. This model includes partial self-insurance against unemployment through home production and a nonpecuniary cost of unemployment. These two features improves the description of the cost of unemployment; they are introduced as in Section IV of our companion paper. This model also includes long-term employment relationships and labor market flows—from unemployment to employment and from employment to unemployment. The dynamic model therefore provides a better description of the real world than the static model because large flows prevail on the labor market at all times. The optimal UI formula is nearly identical and it has the same interpretation as that in our companion paper, even though the presence of labor market flows slightly modifies its derivation. In the next sections, we use this formula for various applications.

A. The Model

Given that our companion paper details the structure of the model, we keep the presentation short. The model is set in continuous time. There is a measure 1 of identical workers and a measure 1 of identical firms. At time $t$, the number of employed workers is $l(t)$ and the number of unemployed workers is $u(t) = 1 - l(t)$. Each firm posts $v(t)$ vacancies to recruit workers. Each unemployed worker searches for a job with effort $e(t)$. The matching function $m$ determines the number of new worker-firm matches that are formed at time $t$: $m(t) = m(e(t) \cdot u(t), v(t))$, where $m(t)$ is the number of workers who find a job, $e(t) \cdot u(t)$ is aggregate job-search effort, and $v(t)$ is aggregate vacancies. The function $m$ has constant returns to scale and is differentiable and increasing in both arguments. The labor market tightness $\theta(t)$ is defined by the ratio of aggregate vacancies
to aggregate job-search effort: $\theta(t) = v(t)/(e(t) \cdot u(t))$. The job-finding rate per unit of search effort is $f(\theta(t)) = m(t)/(e(t) \cdot u(t)) = m(1, \theta(t))$ and the job-finding rate therefore is $e(t) \cdot f(\theta)$. The vacancy-filling rate is $q(\theta(t)) = m(t)/v(t) = m(1/\theta(t), 1)$. We denote by $1 - \eta$ and $-\eta$ the elasticities of $f$ and $q$ with respect to $\theta$. Jobs are destroyed at an exogenous rate $s > 0$.

Technically, employment is a state variable with law of motion $\dot{l}(t) = e(t) \cdot f(\theta(t)) \cdot (1 - l(t)) - s \cdot l(t)$. If $e$, $f(\theta)$, and $s$ remain constant over time, employment converges to the steady-state level

$$l = \frac{e \cdot f(\theta)}{s + e \cdot f(\theta)}. \tag{1}$$

In US data, employment reaches this steady-state level quickly because labor market flows are large. In fact, Hall [2005b] and Michaillat and Saez [2015] show that the employment rate obtained from equation (1) and the actual employment rate are indistinguishable. Therefore, as Hall [2005a] and Pissarides [2009] do, we simplify the analysis by ignoring the transitional dynamics of employment and assuming that employment is a jump variable that depends on search effort and tightness according to equation (1).

The representative firm employs $l(t)$ workers in total: $n(t)$ workers who produce output and $l(t) - n(t)$ workers who recruit employees by posting vacancies. All workers are paid a real wage $w(t)$. The firm’s production function is $y(n(t))$. The function $y$ is differentiable, increasing, and concave. Posting a vacancy requires $\rho$ recruiters. Labor market flows are balanced so $s \cdot l(t) = v(t) \cdot q(\theta(t))$ and the number of vacancies posted by firms is $v(t) = s \cdot l(t)/q(\theta(t))$. Hence the number of recruiters in a firm with $l(t)$ employees is $l(t) \cdot s \cdot \rho/q(\theta(t))$, and the number of producers in the firm is $n(t) = l(t) \cdot (1 - s \cdot \rho/q(\theta(t)))$. The recruiter-producer ratio in the firm therefore is $\tau(\theta(t)) = s \cdot \rho/(q(\theta(t)) - s \cdot \rho)$, and the numbers of employees and producers are related by $l(t) = (1 + \tau(\theta(t))) \cdot n(t)$.

The firm sells its output on a perfectly competitive market. At time $t$, the firm takes $\theta(t)$ and $w(t)$ as given, and it chooses $l(t)$ to maximize profits $y(l(t)/(1 + \tau(\theta(t)))) - w(t) \cdot l(t)$. Because we ignore the transitional dynamics of employment, firms maximize a static objective at each instant. This type of approximation works well as long as the job-destruction rate, $s$, is much higher than the interest rate; this condition is satisfied in US data where $s = 3.3\%$ per month (see
Table 1). The labor demand \( l^d(\theta(t), w(t)) \) gives the optimal number of employees for the firm.

The government’s UI program provides employed workers with consumption \( c^e(t) \) and unemployed workers with consumption \( c^u(t) < c^e(t) \). The generosity of UI is measured by the replacement rate \( R(t) = 1 - \Delta c(t)/w(t) \), where \( \Delta c(t) = c^e(t) - c^u(t) \). At time \( t \), the government must satisfy the budget constraint \( y(n(t)) = (1 - l(t)) \cdot c^u(t) + l(t) \cdot c^e(t) \).

Employed workers consume \( c^e(t) \), which yields utility \( U(c^e(t)) \). The function \( U \) is differentiable, increasing, and concave. In addition to consuming \( c^u(t) \), unemployed workers consume an amount \( h(t) \) produced at home. The utility from consumption is \( U(c^u(t) + h(t)) \). The utility cost of home production is \( \lambda(h(t)) \). The function \( \lambda \) is differentiable, increasing, convex, and \( \lambda(0) = 0 \). The disutility from job-search effort is \( \psi(e(t)) \). The function \( \psi \) is differentiable, increasing, convex, and \( \psi(0) = 0 \). Finally, unemployed workers suffer a nonpecuniary cost of unemployment \( z \). Accordingly, the utility of an unemployed worker is \( U(c^u(t) + h(t)) - z - \lambda(h(t)) - \psi(e(t)) \). The nonpecuniary cost of work for employed workers has been normalized to zero; with a nonzero cost of work, we could redefine \( z \) as the nonpecuniary cost of unemployment net of the nonpecuniary cost of work.

Unemployed workers choose \( h(t) \) to maximize \( U(c^u(t) + h(t)) - \lambda(h(t)) \). The home-production supply \( h^* \) gives the optimal level of home production. It is implicitly defined by the first-order condition of the maximization:

\[
\lambda'(h^*(c^u(t))) = U'(c^u(t) + h^*(c^u(t))).
\] (2)

With home production the consumption of unemployed workers is \( c^h(t) = c^u(t) + h^*(c^u(t)) \); importantly, the consumption level \( c^h(t) \) only depends on \( c^u(t) \).

At time \( t \), the representative worker takes \( \theta(t), c^e(t), c^u(t), \) and \( c^h(t) \) as given, and it chooses \( e(t) \) to maximize its expected utility

\[
\frac{e(t) \cdot f(\theta(t))}{s + e(t) \cdot f(\theta(t))} \cdot U(c^e(t)) + \frac{s}{s + e(t) \cdot f(\theta(t))} \cdot \left[ U(c^h(t)) - z - \lambda(h^*(c^u(t))) - \psi(e(t)) \right].
\] (3)
Straightforward algebra shows that the optimal search effort satisfies

$$\psi'(e(t)) = \frac{f(\theta(t))}{s + e(t) \cdot f(\theta(t))} \cdot (\Delta U(t) + \psi(e(t))),$$  \hspace{1cm} (4)

where

$$\Delta U(t) = U(c^e(t)) - U(c^h(t)) + z + \lambda(h^u(c^u(t)))$$  \hspace{1cm} (5)

is the instantaneous utility gain from work.

The effort supply $e^s(f(\theta(t)),\Delta U(t))$ gives the optimal search effort at time $t$; it is implicitly defined by equation (4). The labor supply $l^s(\theta(t),\Delta U(t))$ gives the number of workers who have a job when job search is optimal. It is defined by

$$l^s(\theta(t),\Delta U(t)) = e^s(f(\theta(t)),\Delta U(t)) \cdot f(\theta(t)),$$  \hspace{1cm} (6)

Finally, the wage is given by a general wage mechanism: $w(t) = w(\theta(t),\Delta U(t))$.

An equilibrium at time $t$ is parameterized by the generosity of UI, $\Delta U(t)$. In equilibrium, tightness equalizes labor supply and demand: $l^s(\theta(t),\Delta U(t)) = l^d(\theta(t),w(\theta(t),\Delta U(t)))$. This equation defines the equilibrium level of tightness as an implicit function of $\Delta U(t)$, denoted $\theta(\Delta U(t))$. Once $\theta(t) = \theta(\Delta U(t))$ is determined, it is simple to determine the equilibrium level of the other variables: $l(t)$ is determined by $l(t) = l^s(\theta(t),\Delta U(t))$, $e(t)$ by $e(t) = e^s(f(\theta(t),\Delta U(t)))$, $n(t)$ by $n(t) = l(t)/(1 + \tau(\theta(t)))$, $w(t)$ by $w(t) = w(\theta(t),\Delta U(t))$, and $c^e(t)$ and $c^u(t)$ by the government’s budget constraint and (5).

**B. The Optimal Unemployment Insurance Formula**

At any point in time, the government chooses $\Delta U(t)$ to maximize social welfare, given by (3), subject to the equilibrium constraints. The government maximizes a static objective at each instant, although the model is dynamic, because we abstract from the transitional dynamics of employment. This simplification allows us to describe the optimal UI in the dynamic model with a formula that is almost identical to the one obtained in the static model. To simplify notation,
we omit the time index on the variables below.

The optimal UI formula relies on three statistics. The microelasticity of unemployment with respect to UI is

$$
\varepsilon^m = \frac{\Delta U}{1 - l} \cdot \frac{\partial l^s}{\partial \Delta U} \Bigg|_{\theta}.
$$

(7)

The discouraged-worker elasticity is

$$
\varepsilon^f = \frac{f(\theta)}{e} \cdot \frac{\partial e^s}{\partial f} \Bigg|_{\Delta U}.
$$

(8)

Last, the macroelasticity of unemployment with respect to UI is

$$
\varepsilon^M = \frac{\Delta U}{1 - l} \cdot \frac{dl}{d\Delta U}.
$$

(9)

In this dynamic model, the optimal UI replacement rate is given by the following formula:

$$
R = \frac{l}{w} \cdot \frac{\Delta U}{\varepsilon^m} \cdot \left(\frac{1}{U'(c^e)} - \frac{1}{U'(c^h)}\right)
+ \left(1 - \frac{\varepsilon^M}{\varepsilon^m}\right) \cdot \frac{1}{1 + \varepsilon^f} \cdot \left[\Delta U + \psi(e)\frac{w}{\phi} + \left(1 + \varepsilon^f\right) \cdot R - \frac{\eta \cdot \tau(\theta)}{1 - \eta \cdot u}\right],
$$

(10)

where $\phi = 1/\left[l/U'(c^e) + (1-l)/U'(c^h)\right]$ is the harmonic mean of workers’ marginal consumption utilities. The complete derivation of the formula is presented in Appendix A; it closely follows the derivation presented in our companion paper.

The formula is nearly identical to the formula for a static model [Landais, Michaillat and Saez, 2015 formula (23)], and it has the same interpretation.4 The optimal replacement rate is

4There are two small differences between the efficiency term in formula (10) and the efficiency term in the formula for a static model [Landais, Michaillat and Saez, 2015 formula (23)]. These differences can be resolved once the formulas are expressed with the correct statistics. The first difference is that $\Delta U$ is replaced by $\Delta U + \psi(e)$. This is because the utility gap between employed and unemployed workers is $\Delta U + \psi(e)$ in the dynamic model, in which only unemployed workers search for jobs, but $\Delta U$ in the static model, in which all workers initially search for jobs. The second difference is that $\tau(\theta)$ is replaced by $\tau(\theta)/u$. This is because the number of workers who search for a job is $u$ in the dynamic model but 1 in the static model, in which as everybody is initially unemployed.
the sum of the conventional Baily-Chetty replacement rate plus a correction term. The Baily-Chetty replacement rate is the optimal replacement rate when tightness does not respond to UI. The correction term governs the adjustment that must be made to the Baily-Chetty rate to obtain the optimal replacement rate when tightness responds to UI.

The correction term measures the effect of UI on welfare through tightness. When UI has a positive effect on welfare through tightness, the correction term is positive and the optimal replacement rate is more generous than the Baily-Chetty rate; conversely when UI has a negative effect on welfare through tightness, the correction term is negative and the optimal replacement rate is less generous than the Baily-Chetty rate.

More precisely, the correction term equals the product of the effect of UI on tightness times the effect of tightness on welfare, keeping UI constant. The effect of UI on tightness is measured by the elasticity wedge (see equation (A9) in Appendix A), and the effect of tightness on welfare is measured by the efficiency term (see equation (A5) in Appendix A). In Sections II and III, we measure the elasticity wedge and efficiency term to determine the deviation of the optimal replacement rate from the Baily-Chetty rate over the business cycle.

II. The Effect of Unemployment Insurance on Labor Market Tightness

Our macroeconomic theory of optimal UI departs from the microeconomic theory of Baily [1978] and Chetty [2006a] because it takes into account the effect of UI on labor market tightness. Our companion paper [Landais, Michaillat and Saez, 2015] shows that in theory, UI can lower tightness through a job-creation mechanism or raise tightness through a rat-race mechanism. The overall effect of UI on tightness depends on which mechanism dominates. Measuring the effect of UI on tightness is critical because the optimal replacement rate departs from the conventional Baily-Chetty replacement rate when the effect is nonzero.

There is an empirical criterion to evaluate whether an increase in the generosity of UI raises or lowers tightness. An increase in UI raises tightness if the wedge $1 - \epsilon^M / \epsilon^m$ between the macroelasticity of unemployment with respect to UI, $\epsilon^M$, and the microelasticity of unemployment with respect to UI, $\epsilon^m$, is positive (equivalently, if $\epsilon^M < \epsilon^m$); conversely, an increase in UI lowers tightness if the wedge $1 - \epsilon^M / \epsilon^m$ is negative (equivalently, if $\epsilon^M > \epsilon^m$).
The intuition is simple. The microelasticity measures the increase in unemployment caused by an increase in UI, accounting for the reduction in job search but keeping tightness constant. The macroelasticity, on the other hand, measures the increase in unemployment caused by an increase in UI, accounting both for the reduction in job search and the equilibrium response of tightness. When an increase in UI raises tightness, it increases the job-finding rate, which dampens the increase in unemployment caused by the reduction in job search. In this case, the macroelasticity is smaller than the microelasticity. Conversely, when an increase in UI lowers tightness, it reduces the job-finding rate, which exacerbates the increase in unemployment caused by the reduction in job search. In this case, the macroelasticity is larger than the microelasticity.

This section reviews empirical evidence on the elasticity wedge. The evidence suggests that the elasticity wedge is positive, with an estimate for the United States around $1 - \frac{\varepsilon^M}{\varepsilon^m} = 0.3$.

### A. Estimates of the Elasticity Wedge

The ideal experiment to estimate the elasticity wedge is a design with double randomization: (i) some randomly selected labor markets are treated and some are not, and (ii) within treated labor markets, all but a randomly selected subset of jobseekers are treated. The treatment is to offer higher or longer UI benefits. Nontreated jobseekers in nontreated markets are a pure control group. Nontreated jobseekers in treated markets are affected only by the change in tightness in their labor market. Hence, the elasticity wedge can be estimated by comparing the unemployment durations of nontreated jobseekers in nontreated markets to that of nontreated jobseekers in treated markets. Here we discuss the few studies that aim to estimate the elasticity wedge. We start with studies on US data before discussing a study on European data.

In an early study, Levine [1993] finds that the decrease in the search effort of jobseekers eligible for UI induced by an increase in UI has a positive effect on the job-finding probability of jobseekers ineligible for UI in the same labor market. This result is obtained using a variety of microdata—Current Population Survey (CPS) and National Longitudinal Survey of Youth—as well as state-level data on unemployment and UI recipients. This finding implies a positive elasticity wedge. His Table 5 shows that an increase in the UI replacement rate has a positive effect on the unemployment rate of workers eligible for UI (column 1), a negative effect on
the unemployment rate of workers ineligible for UI (column 2), and an insignificant effect on the overall unemployment rate (columns 3 and 4). This suggests that the macroelasticity is 0 and hence the elasticity wedge is 1 (although standard errors are relatively large). The main identification threat is that UI eligibility is not randomly assigned.

Using longitudinally matched CPS data, [Valletta 2014] uncovers spillover effects of UI extensions on unemployed workers ineligible for UI during the Great Recession (2007–2011). He finds those ineligible for UI have higher job-finding rates when UI duration is longer, but the spillover effect is only present in states with high unemployment (Table 6, column 2). This suggests that the elasticity wedge is close to zero when unemployment is low but positive when unemployment is high.

In a recent study, [Marinescu 2014] follows another route to assess the sign and magnitude of the elasticity wedge: she directly estimates the effect of a change in UI on labor market tightness. The changes in UI that she considers are the long UI extensions implemented in the US during the recent Great Recession from 2008 to 2013. Using detailed information on vacancies and job applications from [CareerBuilder.com], the largest American online job board, she computes the effect of a change in UI on aggregate search effort, measured by the number of job applications sent, and on vacancies. At the state level, she finds that an increase in UI has a negative effect on job applications but no effect on vacancies. Since tightness is the ratio of vacancies to aggregate search effort, these results imply that an increase in UI raises tightness and thus that the elasticity wedge is positive. The measured response of tightness to changes in UI implies an elasticity wedge of $1 - \frac{\varepsilon^M}{\varepsilon^m} = 0.29$ (p.33).

[Johnston and Mas 2015] analyze an unexpected reduction in benefit duration in Missouri in 2011. First, using a regression discontinuity design on state-level administrative data, they find that a one-month reduction in potential duration of benefits decreases the average spell of nonemployment by 10 days, which translates into a microelasticity of unemployment with respect to maximum benefit duration, $\varepsilon^m_D = 0.65$. Then, using state-level administrative data
and a difference-in-differences estimator taking other US states as a control, they find a reduction in unemployment rate of 0.78 to 0.94 percentage points (Table 5, columns (1)–(3), row Missouri × post). Since this macro effect is commensurate to the micro effect of UI, they conclude that the elasticity wedge is about zero.

Finally, [Lalive, Landais and Zweimüller [2015] estimate the elasticity wedge in Austrian data. They use a natural experiment that offers the desired design: the Regional Extended Benefit Program implemented in 1988–1993. The treatment was an increase in benefit duration from 52 to 209 weeks for eligible unemployed workers in a subset of regions. They provide convincing nonparametric evidence that noneligible unemployed in treated labor markets experienced significantly lower unemployment duration as a result of the program. This result implies that the elasticity wedge is positive; they report an estimate of \( 1 - \frac{\varepsilon^M}{\varepsilon^m} = 0.21 \) (p.3).

There is a presumption that the elasticity wedge could also be measured by estimating separately the microelasticity and macroelasticity and comparing their magnitudes. It is true that this indirect approach has the advantage of being conceptually simple, but it suffers from the drawback that elasticities are nonstructural constructs that need not be stable across contexts or policy variations. In fact, the fairly broad range of microelasticity estimates obtained in the literature suggest that the microelasticity may not be stable across contexts. Moreover, the simulations in Section IV show that the macroelasticity can fluctuate significantly over the business cycle (see Figure 6). Therefore, as Johnston and Mas [2015] do, one needs to identify and estimate the micro and macro responses to the same UI change, in the same context, to measure the elasticity wedge properly. Measures of the elasticity wedge obtained by comparing estimates of microelasticity and macroelasticity obtained separately are not really compelling.

An implication is that macroelasticity estimate obtained in isolation, without a corresponding microelasticity estimate, is not sufficient to draw normative conclusions. For instance, Hagedorn et al. [2013] use the large extensions in benefit duration implemented in the United States in the 2009–2013 period and compare border counties across states with different benefit durations. They find a macroelasticity of \( \varepsilon^M_D = 0.39.6 \) The estimate \( \varepsilon^M_D = 0.39 \) falls in the range of micro-
elasticity estimates in the literature, and it therefore does not provide conclusive evidence about the sign of the elasticity wedge, which is the crucial statistic for welfare. With a microelasticity \( \varepsilon^m_D = 0.65 \) as in Johnston and Mas [2015], the macroelasticity \( \varepsilon^M_D = 0.39 \) implies a positive and fairly large elasticity wedge \( 1 - \varepsilon^M_D / \varepsilon^m_D = 0.4 \). With a microelasticity \( \varepsilon^m_D = 0.43 \) as in the classic study of Katz and Meyer [1990], the macroelasticity \( \varepsilon^M_D = 0.39 \) implies a small positive elasticity wedge \( 1 - \varepsilon^M_D / \varepsilon^m_D = 0.09 \). With a smaller microelasticity estimate as in recent work using data from the Great Recession, the macroelasticity \( \varepsilon^M_D = 0.39 \) could even imply a small negative elasticity wedge.

B. Evidence on the Rat-Race and Job-Creation Mechanisms

The elasticity wedge summarizes the overall effect of UI on tightness. In matching models, UI affects tightness through two mechanisms: the rat-race mechanism, which raises tightness, and the job-creation mechanism, which lowers tightness. A number of papers study these two effects in isolation, thus providing additional evidence on the effect of UI on tightness. Overall, the absence of evidence of job-creation mechanisms together with the evidence of rat-race mechanisms corroborates direct evidence suggesting that the elasticity wedge is positive.

**Rat-Race Mechanism.** Numerous papers measure the effect of an increase in the search effort of some jobseekers, induced for example by placement programs, on the job-finding probability of other jobseekers (see Card, Kluve and Weber [2010] for a survey). Finding that an increase in the search effort of some jobseekers has a negative effect on the job-finding probability of other jobseekers would suggest that the rat-race mechanism is present. Crepon et al. [2013] offer the most compelling identification by analyzing a large randomized field experiment in France in which some young educated jobseekers are treated by receiving job placement assistance. The experiment has a double-randomization design: (i) some areas are treated and some are not, (ii) within treated areas some jobseekers are treated and some are not. Interpreting the treatment as an increase in search effort from \( e_C^C \) for control jobseekers to \( e_T^T \) for treated jobseekers, their

99 weeks is quite sizable: the effect on unemployment is 110%, meaning that such a permanent increase would increase the long-run average unemployment rate from 5 to 10.5%.” Therefore, the estimated macroelasticity of unemployment with respect to benefit duration is \( (10.5 - 5) / 5 \) \( / \) \( (99 - 26) / 26 \) = 0.39. Katz and Meyer [1990] Table 4 find \( \varepsilon^M_D = 0.43 \) in administrative data covering several US states in the 1980s.
empirical results for long-term employment translate into an elasticity wedge $1 - \varepsilon^M/\varepsilon^m = 0.4$.\footnote{As shown in Table IX, Panel B, column 1, treated jobseekers face a higher job-finding probability than control jobseekers in the same area: $[e^T - e^C] \cdot f^T = 5.7\%$. But control jobseekers in treated areas face a lower job-finding probability than control jobseekers in control areas: $e^C \cdot [f^T - f^C] = -2.1\%$. Therefore the increase in the job-finding probability of treated jobseekers in treated areas compared to control jobseekers in control areas is only $[e^T \cdot f^T] - [e^C \cdot f^C] = 5.7 - 2.1 = 3.6\%$. Since $\varepsilon^m$ is proportional to $[e^T - e^C] \cdot f^T$ and $\varepsilon^M$ is proportional to $[e^T \cdot f^T] - [e^C \cdot f^C]$, the implied elasticity wedge is $1 - \varepsilon^M/\varepsilon^m = 1 - 3.6/5.7 = 0.4$.}

Job-Creation Mechanism. The best way to measure the job-creation mechanism is to look directly at whether a more generous UI increases wages. Using microdata, a number of studies have investigated whether more generous UI benefits affect the re-employment wage. Most studies find no effect on wages or even slightly negative effects [for example, Card, Chetty and Weber 2007; Johnston and Mas 2015]. The only exception is the recent work by Hagedorn et al. [2013]; in US macrodata, they find significant effects of UI extensions on wages.

Note however that more generous benefits, by inducing longer unemployment durations, may have a negative effect on wages if the duration of unemployment affects the productivity of unemployed workers or is interpreted by employers as a negative signal of productivity, as in the compelling experiment by Kroft, Lange and Notowidigdo [2013]. It is difficult to disentangle this negative effect from the positive effect of UI on wages through bargaining, which is the relevant effect for our analysis. Using administrative data for Germany, Schmieder, von Wachter and Bender [2015] attempt such a decomposition by controlling for the duration of the unemployment spell and find a negative effect of UI on wages through longer unemployment durations but zero effect through wage bargaining. Lalive, Landais and Zweimüller [2015] use the same methodology and find a positive but tiny effect of UI through wage bargaining in Austria. In sum, while there is no consensus on the magnitude of the job-creation mechanism in the literature, the absence of strong and significant effect of UI on re-employment wages suggests that the job-creation effect is likely to be small.

C. The Elasticity Wedge: Summary and Discussion

Direct estimates of the elasticity wedge suggest that the elasticity wedge is positive, with an estimate for the United States around $1 - \varepsilon^M/\varepsilon^m = 0.3$. The absence of evidence in favor of the
job-creation mechanism, coupled with convincing evidence in favor of the rat-race mechanism, corroborates the finding that the elasticity wedge is positive.

The finding that the elasticity wedge is positive means that an increase in the generosity of UI raises the labor market tightness. This property of UI implies that the conventional Baily-Chetty formula is invalid as soon as the level of tightness is inefficient—in the sense that variations in tightness have first-order effects on welfare. As shown in Table 2 of our companion paper [Landais, Michaillat and Saez, 2015], with a positive elasticity wedge the optimal UI replacement rate is more generous than the Baily-Chetty replacement rate when the efficiency term is positive (an indication that tightness is inefficiently low) and less generous when the efficiency term is negative (an indication that tightness is inefficiently high). In section III we measure the sign and magnitude of the efficiency term in US data to gauge the departures of the optimal replacement rate from the Baily-Chetty rate.

Finally, the finding of a positive elasticity wedge has important implications for the underlying mechanics of the labor market. As explained in Table 3 of our companion paper, different matching models make different predictions regarding the sign of the elasticity wedge, and evidence on this sign can validate or falsify these models. The standard model of Pissarides [2000], with a linear production function and wage bargaining, predicts a negative elasticity wedge; it is therefore inconsistent with a positive elasticity wedge. The fixed-wage model of Hall [2005a], with a linear production function and a fixed wage, predicts zero elasticity wedge and is also inconsistent with the evidence. In contrast, the job-rationing model of Michaillat [2012], with a concave production function and fixed wage, predicts a positive elasticity wedge; therefore, it is the only model consistent with the empirical evidence. Furthermore, as showed in Figure 4 of our companion paper, the job-rationing model is the only model that can generate the rat-race mechanism observed in several studies. In sum, the empirical investigation of the elasticity wedge strongly suggests that the job-rationing model is better suited to describe the labor market and the effects of UI. As a consequence, it is the job-rationing model that we use in our simulations of optimal UI in Section IV.
III. The Effect of Labor Market Tightness on Welfare

The evidence presented in the previous section suggests that an increase in the generosity of UI raises the labor market tightness. This result implies that the optimal UI replacement rate departs from the Baily-Chetty rate as soon as the level of tightness is inefficient, such that a change in tightness, keeping UI constant, has a first-order effect on welfare.

There is an empirical criterion to evaluate the effect of tightness on welfare, keeping UI constant. An increase in tightness raises welfare if the efficiency term in formula (10), given by

$$\frac{1}{1 + \varepsilon_f} \cdot \left[ \frac{\Delta U + \psi(e)}{w \cdot \phi} + (1 + \varepsilon_f) \cdot R - \frac{\eta}{1 - \eta} \cdot \frac{\tau}{u} \right]$$ (11)

is positive; conversely, an increase in tightness lowers welfare if the efficiency term is negative.

Equation (11) shows that an increase in tightness has a positive effect on welfare when the value of having a job relative to being unemployed ($\Delta U + \psi(e)$) and the fiscal cost of unemployment ($R$) are high enough compared to the share of labor devoted to recruiting ($\tau$). Intuitively, the efficient level of unemployment is positive in matching models because some unemployment allows firms to devote fewer workers to recruiting and more to production, thus increasing output. While some unemployment is desirable, too much unemployment is costly because it makes too many workers idle and unproductive and because unemployed workers are worse off than employed workers. Hence, the efficient levels of unemployment and tightness balance the amount of labor devoted to recruiting with the cost of being unemployed.

This section uses both available and new empirical evidence for the United States to compute the efficiency term over the business cycle. To be transparent, we construct the efficiency term in several stages: (i) elasticities describing search and matching ($\varepsilon_f$ and $\eta$); (ii) statistics describing the allocation of labor to unemployment, recruiting, and producing ($u$ and $\tau$); and (iii) statistics determining the cost of unemployment ($R$, $\Delta U + \psi(e)$, and $w \cdot \phi$). The empirical evidence suggests that since 1990 the efficiency term is somewhat positive in normal times, very positive in bad times, and negative only in very good times.
A. Elasticities Describing Search and Matching

The Discouraged-Worker Elasticity ($\varepsilon^f$). The elasticity $\varepsilon^f$ measures how search effort responds to labor market conditions. We know little about $\varepsilon^f$.

Two studies measure $\varepsilon^f$ using the American Time Use Survey (ATUS), in which search effort is directly measured as the amount of time spent searching for a job. These studies suggest that $\varepsilon^f$ is positive. DeLoach and Kurt [2013] find that workers do reduce their search in response to deteriorating labor market conditions. They also find that reductions in household wealth occurring at the same time as increases in unemployment mitigate the discouraged-worker effect, explaining why job search may appear acyclical. Gomme and Lkhagvasuren [2015] find that individual search effort is mildly procyclical. They also show that the search effort of long-term unemployed workers is slightly countercyclical whereas that of short-term unemployed workers is quite procyclical.

Two other studies measure $\varepsilon^f$ mainly from the CPS, in which search effort is proxied by the number of job-search methods used. These studies suggest that $\varepsilon^f$ is zero or slightly negative. Shimer [2004] analyzes the CPS over the 1994–2004 period and finds that labor market participation and search intensity are broadly acyclical, even after controlling for changing characteristics of unemployed workers over the business cycle. This empirical evidence suggests that $\varepsilon^f$ is close to zero. Next, Mukoyama, Patterson and Sahin [2014] jointly analyze the ATUS and CPS over the 1994–2011 period and find that aggregate search effort is countercyclical. Half of the countercyclical movement in search effort, however, is explained by a cyclical shift in the observable characteristics of unemployed workers, and a large share of the remaining countercyclical movement is explained by the fall in housing and stock-market wealth. This evidence therefore suggest that $\varepsilon^f$ is slightly negative.

Overall, these four empirical studies suggest that the response of search effort to the job-finding rate is quite weak, maybe slightly positive; therefore, we simply set $\varepsilon^f = 0$.

The Elasticity $\eta$. The elasticity $\eta$ is defined by $1 - \eta = d \ln(f(\theta))/d \ln(\theta)$. Empirical evidence suggests that the matching function is approximately Cobb-Douglas [Petrongolo and Pissarides 2001]. A Cobb-Douglas matching function is written $m(e \cdot u, v) = \mu \cdot (e \cdot u)^\eta \cdot v^{1-\eta}$. As $f(\theta) \equiv
\[ \frac{m}{(e \cdot u)} = \mu \cdot \left[ \frac{v}{(u \cdot e)} \right]^{1-\eta}, \]

we have

\[
\ln(e \cdot f) = \ln(\mu) + (1 - \eta) \cdot \ln \left( \frac{v}{u} \right) + \eta \cdot \ln(e). 
\]

(12)

We can therefore estimate \(1 - \eta\) by regressing \(\ln(e(t) \cdot f(t))\) on \(\ln(v(t)/u(t))\). Before running this regression, we measure the vacancy-unemployment ratio, \(v/u\), and job-finding rate, \(e \cdot f\).

We start by measuring the vacancy-unemployment ratio, \(v(t)/u(t)\). The number of unemployed workers \(u(t)\) is constructed by the BLS from the CPS. After December 2000 the number of vacancies is measured by the BLS using data collected in the Job Opening and Labor Turnover Survey (JOLTS); but the JOLTS is not available before December 2000. Since there are no long time series for vacancies, we construct our own. We start from the help-wanted advertising index constructed by Barnichon [2010]. This index combines the online and print help-wanted advertising indices constructed by the Conference Board, which are standard proxies for vacancies. We then rescale the Barnichon index to transform it into a number of vacancies. \(^9\) Figure 1, Panel A, plots the number of vacancies \(v(t)\). Figure 1, Panel B, plots \(v(t)/u(t)\).

We can construct the job-finding rate, \(e(t) \cdot f(t)\), from CPS data following the methodology of Shimer [2012]. We assume that unemployed workers find a job according to a Poisson process with arrival rate \(e(t) \cdot f(t)\). Under this assumption, the monthly job-finding rate satisfies \(e(t) \cdot f(t) = -\ln(1 - F(t))\), where \(F(t)\) is the monthly job-finding probability. We construct \(F(t)\) as

\[
F(t) = 1 - \left( u(t+1) - u^s(t+1) \right)/u(t),
\]

where \(u(t)\) is the number of unemployed persons at time \(t\) and \(u^s(t)\) is the number of short-term unemployed persons at time \(t\). We measure \(u(t)\) and \(u^s(t)\) in the data constructed by the BLS from the CPS. The number of short-term unemployed persons is the number of unemployed persons with zero to four weeks duration, adjusted after 1994 as in Shimer [2012]. Figure 1, Panel C, plots \(e(t) \cdot f(t)\).

Using our measures of \(e(t) \cdot f(t)\) and \(v(t)/u(t)\), we estimate the linear regression

\[
\left[ \ln(e(t+1) \cdot f(t+1)) - \ln(e(t) \cdot f(t)) \right] = \beta_0 + \beta_1 \cdot \left[ \ln \left( \frac{v(t+1)}{u(t+1)} \right) - \ln \left( \frac{v(t)}{u(t)} \right) \right] + \varepsilon(t)
\]

\(^9\)The average value of the Barnichon index between December 2000 and December 2014 is 80.59. The average number of vacancies from JOLTS between December 2000 and December 2014 is 3.707 millions. Hence we multiply the Barnichon index by \(3.707 \times 10^6 / 80.59 = 45,996\) to obtain a proxy for the number of vacancies since 1951.

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Figure 1: Labor Market Data for the United States, 1951–2014

Notes: Panel A: The number of vacancies is the quarterly average of the monthly vacancy index constructed by Barnichon [2010], scaled to match the number of vacancies in JOLTS for 2001–2014. Panel B: The vacancy-unemployment ratio is the number of vacancies displayed in Panel A divided by the quarterly average of the seasonally adjusted monthly number of unemployed persons constructed by the BLS from the CPS. Panel C: The job-finding rate is constructed from CPS data following the methodology of Shimer [2012]. Panel D: The unemployment rate is the quarterly average of the seasonally adjusted monthly unemployment rate constructed by the BLS from the CPS. The shaded areas represent the recessions identified by the NBER.
using ordinary least squares. We use first differences because both $\ln(e(t) \cdot f(t))$ and $\ln(v(t)/u(t))$ have a trend over 1951–2014. We find $\beta_1 = 0.34$ with a robust standard error of 0.02.\footnote{Another way to deal with those trends is to run the regression on detrended data, as Shimer [2005]. After detrending the data using a Hodrick-Prescott filter with a standard smoothing parameter of 1,600, the regression yields $\beta_1 = 0.40$, which is close to the estimate obtained with first differences.} Hence, we estimate that $\eta = 0.66$. Our estimate for $\eta$ is close to the estimate of 0.72 obtained by Shimer [2005] using comparable data for the 1951–2003 period. More generally, it is in line with the estimates found in the vast empirical literature describing the matching function: in their survey, Petrongolo and Pissarides [2001] conclude that estimates of $\eta$ fall between 0.5 and 0.7.

The estimate of $\eta$ that we obtain may be biased if $e(t)$ is correlated with $v(t)/u(t)$. Taking first differences does not address this specific problem. Yet, as search effort does not respond much to labor market conditions (as $\epsilon_f \approx 0$), we expect the bias to be small and use $\eta = 0.66$ in the analysis.

### B. The Allocation of Labor to Unemployment, Recruiting, and Producing

#### The Unemployment Rate ($u$). We measure $u$ with the unemployment rate constructed by the BLS from the CPS. Figure 1, Panel D plots the unemployment rate for 1951–2014. The unemployment rate averages 5.9% between 1951 and 2014, and it is sharply countercyclical.

#### The Recruiter-Producer Ratio ($\tau$). As the times series of this ratio has not been formally estimated before, we construct three measures of $\tau$ and combine them to create a composite measure of the recruiter-producer ratio. The composite measure summarizes the available information. The first measure relies on the assumption that the share of recruiting outsourced by firms is constant over time. An advantage of this measure is that it measures directly the resource cost of recruiting. The two other measures rely on the assumption that the cost of posting a vacancy is constant over time. The three measures are displayed in Figure 2. The first two measures are available for 1990–2014, and the third one for 2001–2014. Although they are based on completely different data sources, the three measures generate similar series over the period.

The first measure is based on the size of the recruiting industry. We measure the size of the recruiting industry, denoted $rec(t)$, by the seasonally adjusted monthly number of workers in the
recruiting industry computed by the BLS from the Current Employment Statistics (CES) survey. The recruiting industry is the industry with North American Industry Classification System (NAICS) code 56131. Its official name is “employment placement agencies and executive search services”. It comprises firms primarily engaged in listing employment vacancies and referring or placing applicants for employment, and firms providing executive search, recruitment, and placement services. The series is available for 1990–2014. This industry comprises 280,700 workers on average.

Of course the employees in the recruiting industry constitute only a small fraction of the workers allocated to recruiting. Hence, we scale up \( \text{rec}(t) \) by a factor 8.4 to measure the total amount of labor devoted to recruiting in the economy. Scaling up allows us to account for the many workers who are not in recruiting industry but who spend a lot of time and effort recruiting for their own firm. The scaling factor of 8.4 is chosen to ensure that the share of labor devoted to recruiting in 1997 is 2.5%, thus matching the evidence from the 1997 National Employer Survey. This survey, conducted by the Census Bureau, gathered data from 4500 establishments on their methods for recruiting applicants. Firms in the survey reported spending 2.5% of their labor costs in recruiting activities [Villena Roldan 2010].

Finally we construct the recruiter-producer ratio as

\[
\tau(t) = \frac{8.4 \cdot \text{rec}(t)}{l(t) - 8.4 \cdot \text{rec}(t)},
\]

where \( l(t) \) is the seasonally adjusted monthly number of workers in all private industries computed by the BLS from the CES survey. This first measure of \( \tau \) is the solid, blue line in Figure 2, Panel A.

The second measure of \( \tau \) is based on the job-finding and job-destruction rates measured in data constructed by the BLS from the CPS. To construct \( \tau \), we use the result from Section that the recruiter-producer ratio satisfies

\[
\tau(t) = \frac{s(t) \cdot \rho}{q(t) - s(t) \cdot \rho},
\]

where \( q(t) \) is the vacancy-filling rate, \( s(t) \) is the job-destruction rate, and \( \rho \) is the flow cost of
posting a vacancy, assumed to be constant over the 1990–2014 period. We compute the vacancy-filling rate using \( q(t) = \frac{f(t)}{\theta(t)} = \frac{e(t) \cdot f(t)}{(v(t) + u(t))} \) and the series for \( v(t) + u(t) \) and \( e(t) \cdot f(t) \) constructed above and displayed in Figure 1, Panels B and C. The vacancy-filling rate is displayed in Figure 3, Panel A. We construct the job-destruction rate \( s(t) \) following the method developed by [Shimer, 2012]. The rate \( s(t) \) is implicitly defined by

\[
    u(t + 1) = \left(1 - e^{-e(t) \cdot f(t) - s(t)}\right) \cdot \frac{s(t)}{e(t) \cdot f(t) + s(t)} \cdot h(t) + e^{-e(t) \cdot f(t) - s(t)} \cdot u(t),
\]

where \( h(t) \) is the number of persons in the labor force at time \( t \), \( u(t) \) is the number of unemployed persons at time \( t \), and \( e(t) \cdot f(t) \) is the monthly job-finding rate. We measure \( u(t) \) and \( h(t) \) in the data constructed by the BLS from the CPS, and we use the series for \( e(t) \cdot f(t) \) constructed above and displayed in Figure 1, Panel C. The job-destruction rate is displayed in Figure 3, Panel B. Finally, we compute \( \rho \) to ensure that the share of labor devoted to recruiting in 1997 is 2.5%. The average job-destruction rate in 1997 is 2.8%, the average vacancy-filling rate is 85%, so we set \( \rho = (0.85/0.028) \times 0.025 = 0.77 \). Using our series for \( q(t) \) and \( s(t) \) and our estimate of \( \rho \), we construct a series for \( \tau(t) \) using (14). This second measure of \( \tau \) is the dotted, green line in
Figure 3: The Vacancy-Filling and Job-destruction Rates in the United States, 1990–2014

Notes: Panel A: The solid, blue line is the vacancy-filling rate constructed as \( q(t) = e(t) \cdot f(t)/(v(t)/u(t)) \), where \( e(t) \cdot f(t) \) is the job-finding rate described in Figure 1 Panel C and \( v(t)/u(t) \) is the vacancy-unemployment ratio described in Figure 1 Panel B. The dashed, red line is the vacancy-filling rate constructed as \( q(t) = h(t)/v(t) \), where \( v(t) \) is the number of vacancies in all nonfarm industries and \( h(t) \) is the number of hires in all nonfarm industries. Both \( v(t) \) and \( h(t) \) are quarterly averages of the seasonally adjusted monthly series constructed by the BLS from the JOLTS. Panel B: The solid, blue line is the job-destruction rate constructed from CPS data following the methodology in Shimer [2012]. The dashed, red line is the quarterly average of the seasonally adjusted monthly separation rate in all nonfarm industries constructed by the BLS from the JOLTS. The shaded areas represent the recessions identified by the NBER.

The third measure of \( \tau \) is based on the job-finding and job-destruction rates measured in the data constructed by the BLS from the JOLTS. This series is available for 2001–2014. To construct \( \tau \), we use again (14). We measure \( q(t) \) by \( q(t) = h(t)/v(t) \), where \( v(t) \) is the number of vacancies in all nonfarm industries and \( h(t) \) is the number of hires in all nonfarm industries. The rate \( q(t) \) is displayed in Figure 3 Panel A. We measure \( s(t) \) by the separation rate in all nonfarm industries. The rate \( s(t) \) is displayed in Figure 3 Panel B. Last, we set \( \rho = 0.81 \) such that the average value of \( \tau(t) \) in 2001 is 2.7%, the same as the average value of the first recruiter-producer ratio. This third measure of \( \tau \) is the dashed, red line in Figure 2 Panel A.

All three series of \( \tau \) displayed in Figure 2 are similar, in spite of being constructed from different and largely independent sources. Consistent with the theory, they are highly procyclical, providing direct evidence that recruiting costs are high when the labor market is tight and low when the labor market is slack. The first measure based on CES data for the recruiting industry offers perhaps the most compelling evidence.
Finally, we combine the three measures of recruiter-producer ratio to create a composite measure. The composite measure is the average of the measures from the CES and from the CPS over the 1990–2000 period, and the average of the measures from the CES, from the CPS, and from the JOLTS over the 2001–2014 period. Figure 2, Panel B, displays this composite measure. The correlation of the composite measure with each of the individual measures is above 0.9. We use this composite measure to construct the efficiency term. This measure is sharply procyclical, with an average value of 2.3%.

C. The Cost of Unemployment

The Replacement Rate (R). The UI program in the United States is much more complex than in our model. In the model, UI indefinitely provides unemployment benefits at a replacement rate $R$; in the United States, weekly unemployment benefits replace between 50% and 70% of the last weekly pretax wage of a worker, up to a maximum level of benefits and usually only for a duration of 26 weeks \cite{PavoniViolante2007}. As a simple model cannot possibly capture all the intricacies of the UI program in the United States, we follow \cite{Chetty2008} and summarize the generosity of the UI program by setting $R = 50\%$.

The Utility Gain from Work ($\Delta U + \psi(e)$). With a cost of home production $\lambda(h)$, a cost of job search $\psi(e)$, and a pure nonpecuniary cost of unemployment $z$, the utility gain from work is

$$\Delta U + \psi(e) = U(c^e) - U(c^h) + Z,$$

where $Z = z + \lambda(h) + \psi(e)$ is the total nonpecuniary cost of unemployment.

In a setting closely related to ours, \cite{Chetty2006b} finds that the impact of wage changes on labor supply implies an upper bound on the coefficient of relative risk aversion close to one. We therefore take a coefficient of relative risk aversion equal to one so that the consumption utility is $U(c) = \ln(c)$. Compared to estimates of risk aversion obtained in other settings in the literature, our calibration is on the low side, implying relatively low optimal replacement rates. With log utility, $U(c^e) - U(c^h) = \ln(c^e/c^h)$.

Several studies have documented the drop in consumption upon unemployment in the United
States using measures of food consumption. Using the Panel Study of Income Dynamics (PSID) for 1968–1987, Gruber [1997] estimates that the drop for food consumption upon unemployment is 7%. Estimates are roughly consistent across studies: Stephens [2001] finds that food consumption falls by 9% following a job displacement, and Aguiar and Hurst [2005] find that in unemployment food consumption is lower by 5%. As emphasized by Browning and Crossley [2001], total consumption is more elastic than food consumption to an income change so the consumption drop for overall expenditures should be much larger than 7%. Using the Consumer Expenditure Survey for 1972–1973, Hamermesh [1982] estimates that for unemployed workers the income elasticity of food consumption is 0.36. As a consequence, we expect the consumption drop upon unemployment to be 7%/0.36 = 19% so that \( c^h/c^e = 0.81 \). This estimate is in line with the recent estimates obtained by Kolsrud et al. [2015] using administrative data for Sweden: they find a drop in total expenditure upon unemployment of 19%. In recent work, Kroft and Notowidigdo [2015] determine how the consumption drop varies over the business cycle. Using data from the PSID for 1968–1997, they obtain the important result that the consumption drop does not vary significantly over the business cycle. Therefore, we assume that the ratio \( c^h/c^e \) is acyclical. This means that \( \ln(c^e/c^h) = \ln(1/0.81) = 0.21 \) at any point in time.

Next we measure the total nonpecuniary cost of unemployment (Z). This nonpecuniary cost measures the difference between the well-being of an unemployed and an employed worker, keeping consumption constant. This nonpecuniary cost is high if unemployment has high mental health cost [Theodossiou 1998], if unemployment has high physical health cost [Sullivan and von Wachter 2009], and if home production or job search are costly. It is low if employed workers find working costly or if unemployed workers enjoy leisure.

Using the US General Social Survey for 1972–1994, Di Tella, MacCulloch and Oswald [2003 Table 5] find that being unemployed is extremely costly: controlling for income and other per-

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1. Aguiar and Hurst [2005] find that food expenditures is lower by 19% in unemployment. They argue that food consumption responds much less than food expenditures because households are able to smooth consumption by substituting time for expenditures.

2. The income elasticity of food consumption at home is 0.24 while that of food consumption away from home is 0.82. Furthermore, in the consumption basket of an unemployed worker the share of food consumption at home is 0.164 while that of food consumption away from home is 0.041. Hence the income elasticity of food consumption is \( 0.24 \times [0.164/(0.164+0.41)] + 0.82 \times [0.041/(0.164+0.41)] = 0.36 \.

3. Kolsrud et al. [2015, p.3] write “We find that expenditures drop on average by 19% in the first 20 weeks of unemployment, compared to their pre-unemployment level.”
personal characteristics, it is equivalent to dropping from the top to the bottom income quartile, or to moving from married to divorced. Then, using the Euro-barometer survey that covers twelve European countries for 1975–1992, Di Tella, MacCulloch and Oswald are able to quantify the cost of unemployment. They find that falling unemployed is as bad as losing approximately $3,500 of income a year, which represents 45% of the average GDP per capita across the nations and years in the sample ($7,809). Hence, they find $Z = 0.45$. Other studies using well-being surveys find even larger estimates of $Z$.\footnote{Let $inc$ be the average income per capita. Since $Z$ is the utility cost of falling unemployed, and the marginal value of income is assumed to be the same in the empirical work of Di Tella, MacCulloch and Oswald\cite{Di Tella, MacCulloch and Oswald2003}, we obtain $Z = 0.45 \times inc \times U'(inc)$. With log utility, $U'(inc) = 1/inc$ and $Z = 0.45$.}

One potential issue with the estimate from Di Tella, MacCulloch and Oswald\cite{Di Tella, MacCulloch and Oswald2003} is that it relies on reported well-being and not observed choices. The study by Borgschulte and Martorell\cite{Borgschulte and Martorell2015} addresses this limitation. They use military personnel records linked to post-service civilian earnings to measure how enlisted service members choosing to reenlist or exit the military value an increase in the local unemployment rate $u$ by one percentage point. They find that servicemen are willing to accept a reduction in earnings $b$ of more than 1.5% to avoid the increase in unemployment. With log utility, this implies that $\Delta u \cdot [\ln(c^e) - \ln(c^h) + Z] = \ln(w) - \ln(w \cdot (1 - b))$, where $\Delta u$ is the perceived higher likelihood of being unemployed when the local unemployment rate is higher by one point, $c^e$ consumption if employed, $c^h$ consumption if unemployed, $Z$ the total nonpecuniary cost of being unemployed, $w$ is military re-enlistment earnings, $b$ the percent earnings reduction needed to make servicemen indifferent between entering the low-unemployment and high-unemployment labor markets. Using the above estimates, we find $Z = -\ln(1 - b)/\Delta u - \ln(c^e/c^h) = -\ln(1 - 0.015)/\Delta u - 0.21 = 0.015/\Delta u - 0.21$. For $\Delta u = 0.01$, we obtain $Z = 1.3$. Conceivably, servicemen are young and less educated than average, hence potentially facing a higher unemployment risk than average. With an unemployment rate twice as large as the posted one, $\Delta u = 0.02$ and $Z = 0.54$. Therefore, even conservative estimates of $Z$ drawn from the work of Borgschulte and Martorell are large.

Using the US General Social Survey for 1972–1998, Blanchflower and Oswald\cite{Blanchflower and Oswald2004} find that falling unemployed is as bad as losing approximately $60,000 of income a year, when the average yearly income in their sample ranges from $4,261 in 1973 to $20,457 in 1998. Hence their results suggest a huge nonpecuniary cost of unemployment: $Z \geq 3$. This large number seems to come from a low marginal value of income. Last, using the German Socio-Economic Panel for 1984–1989, Winkelmann and Winkelmann\cite{Winkelmann and Winkelmann1998} also find that the nonpecuniary cost of unemployment is much larger than the pecuniary cost.
Absent a precise estimate for $Z$, we assume that $Z$ remains constant at 0.45, which is a lower bound on the nonpecuniary cost of unemployment.\footnote{Chodorow-Reich and Karabarbounis [2015] offer a first and important attempt to measure cyclicalitiy of $Z$. Their findings suggest that $-Z$ may be procyclical, so $Z$ may be countercyclical, which would amplify the countercyclical movements of the efficiency term displayed in Figure 4.} Using our estimates of $U(c^e) - U(c^h)$ and $Z$, we set $\Delta U = 0.21 + 0.45 = 0.66$ and keep $\Delta U$ constant over the business cycle.

The Term $1/(\phi \cdot w)$. With log utility, we have $1/(\phi \cdot w) = (l \cdot c^e + (1 - l) \cdot c^u)/w = y(n)/w$. If we assume that firms make no profits, then $w \cdot l = y(n)$ and $1/(\phi \cdot w) = l$. With an unemployment rate of 6%, $1/(\phi \cdot w) = 0.94$.\footnote{Firms make no profits if the production function is linear. If the production function is concave with elasticity $\alpha < 1$, firms make some profits and $w \cdot l = \alpha \cdot y(n)$ so that $1/(\phi \cdot w) = l/\alpha$.} The cyclical fluctuations of the employment rate are small, so we assume that $1/(\phi \cdot w)$ is constant over the business cycle.

D. The Efficiency Term: Summary and Discussion

Using the empirical evidence presented in this section, we compute the efficiency term given by (11) for 1990–2014. Figure 4 displays the efficiency term. In normal times, the efficiency term is somewhat positive: its average value over 1990–2014 is 0.33. In bad times, the efficiency term is very positive: it reaches 0.64 in 1992, right after the 1990–1991 recession, 0.50 in 2003, in the wake of the 2001 recession, and 0.8 in 2009, at the end of the Great Recession. The efficiency term is only negative in very good times: it is negative between early 1998 and early 2001 with a trough at -0.31 in 2000. Overall, the efficiency term is subject to sharp countercyclical fluctuations over the business cycle—it is particularly high in slumps and low in booms.

The nonpecuniary cost of unemployment, $Z$, plays an important role in the analysis. In Figure 4 we have calibrated $Z = 0.45$, based on numerous sources indicating that being unemployed is extremely costly. This calibration starkly contrasts with the typical calibration in matching models. In these models, unemployed workers usually enjoy some positive utility that stems from the consumption of UI benefits and leisure. Calibrating the model to include a benefit from leisure when unemployed implies that $Z \leq 0$. The calibration $Z < 0$ has never been justified on empirical grounds, however; it is usually only introduced to increase the rigidity of Nash bargained wages and thus generate larger business cycle fluctuations.\footnote{Indeed, Nash bargained wages are too flexible to generate realistic labor market fluctuations unless unemployed} Calibrating $Z \leq 0$ would
lower the efficiency term in Figure 4. Imagine for instance that $Z = 0$. Since $Z$ is multiplied by $1/\left(1 + e^f\right) \cdot \phi \cdot w = 0.94$ in the efficiency term, given by (11), this alternative calibration would reduce the efficiency term by $0.45 \times 0.94 = 0.42$ at any point in time. Thus, the efficiency term would be broadly zero on average, positive in bad times, and negative in good times. At the end of 2014 the labor market tightness is still too low under our calibration of $Z = 0.45$, but it would be slightly too high under the hypothetical calibration $Z = 0$. The calibration of $Z$ therefore has a direct and important impact on our assessment of labor market efficiency. The level of $Z$, however, does not affect our conclusions about the cyclicality of the efficiency term, and ultimately the cyclicality of optimal UI.

As established [Landais, Michaillat and Saez 2015, Proposition 1], the sign of the efficiency term indicates the effect of an increase in labor market tightness on welfare. Our empirical results therefore mean that an increase in tightness yields a small first-order welfare gain in normal times, a large first-order welfare gains in bad times, and a first-order welfare loss in very good times. Combined with the evidence from Section II that the elasticity wedge is positive and thus that an increase in UI raises tightness, these empirical results dictate how the optimal UI replacement rate departs from the conventional Baily-Chetty rate: in normal times, the optimal replacement rate is somewhat higher than the Baily-Chetty rate; in bad times, the optimal replacement rate is

workers enjoy a very high and fixed utility from leisure [Hagedorn and Manovskii 2008, Shimer 2005].
much higher than the Baily-Chetty rate; and in very good times, the optimal replacement rate is lower than the Baily-Chetty rate.

The empirical evidence also provides a sense of the magnitude of the departures of the optimal replacement rate from the Baily-Chetty rate. The efficiency term depicted in Figure 4 increased from -0.3 in 2000 to 0.5 in 2003 and from 0 in 2007 to 0.8 in 2009; with an elasticity wedge $1 - \frac{\epsilon^M}{\epsilon^m} = 0.3$, this means that the correction term increased by $0.3 \times 0.8 = 0.24$ during the last two recessions. Since in typical calibrations the Baily-Chetty rate is around 0.4, the fluctuations of the correction term are significant; hence, the optimal replacement rate is likely to depart markedly from the Baily-Chetty rate during recessions.

The empirical evidence in this section and the previous one indicates that the correction term in formula (10) is markedly countercyclical. The optimal replacement rate will therefore be countercyclical if the cyclical fluctuations of the Baily-Chetty rate do not undo this cyclicity. Empirical analysis of the statistics entering the Baily-Chetty replacement rate (the microelasticity of unemployment and the consumption drop upon unemployment) suggests that the Baily-Chetty replacement rate is unlikely to be procyclical—if anything, Kroft and Notowidigdo [2015] find that it is countercyclical. Hence, the fluctuations of the Baily-Chetty replacement rate is unlikely to undo the countercyclical fluctuations of the correction term. Nevertheless, because the sufficient statistics entering the formula are endogenous to the replacement rate, we cannot precisely quantify the cyclical fluctuations of the optimal replacement rate at this stage. To quantity these fluctuations more precisely, putting more structure on the underlying model is helpful. We carry out this task in Section [IV] where we calibrate and simulate a specific model.

Last, the finding of a markedly countercyclical efficiency term has implications for the efficiency of labor market fluctuations. Of course whether labor market fluctuations are efficient or not has critical implications for the conduct of numerous policies. In theory, fluctuations can be efficient or inefficient in matching models. The measure of the efficiency term developed in this section directly addresses this question. As far as we know, this is the first empirical methodology that can be used to assess the efficiency of labor market fluctuations. Our findings on the behavior

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19 As discussed in our companion paper [Landais, Michaillat and Saez, 2015], the equilibrium of a matching model is efficient when tightness maximizes welfare for a given UI program. The equilibrium is inefficient when a change in tightness, for a given UI program, generates a first-order change in welfare. The equilibrium is inefficient when the real wage is inefficiently high or low.
of the efficiency term suggest that labor market fluctuations are inefficient and that tightness is always inefficiently low in slumps. The implication is that more should be done to stabilize the labor market in slumps.

IV. Business-Cycle Simulations with a Job-Rationing Model

In the United States, the efficiency term fluctuates sharply over the business cycle (Figure 4) and the elasticity wedge is positive. These observations suggest that the generosity of optimal UI should vary over the business cycle. In this section we use simulations to quantify exactly how this generosity varies over the business cycles.

The model used in the simulations is the matching model with job rationing developed by Michaillat [2012]. We choose this model because it generates a positive elasticity wedge consistent with the empirical evidence presented in Section II, whereas other common matching models do not. Furthermore, because real wages are somewhat rigid in this model, technology shocks lead tightness to be sometimes inefficiently low and sometimes inefficiently high, in line with the evidence presented in Section III.

We calibrate the model to match the empirical evidence presented above. The simulations show that the optimal UI replacement rate is markedly countercyclical, almost doubling from 40% to 75% when the unemployment rate rises from 4% to 12%.

A. The Production Function and Wage Mechanism

We specialize the generic model proposed in Section I to obtain the job-rationing model from Michaillat [2012]. First, we specify a concave production function: \( y(n(t)) = a(t) \cdot n(t)^\alpha \). The exogenous variable \( a(t) > 0 \) measures the technology of the firm at time \( t \) and the parameter \( \alpha \in (0, 1) \) captures decreasing marginal returns to labor. Second, we specify a wage mechanism that is independent of UI and partially rigid with respect to technology: \( w(t) = \omega \cdot a(t)^{1-\gamma} \). The parameter \( \gamma \in (0, 1] \) measures the rigidity of wages with respect to technology. If \( \gamma = 0 \), wages are flexible: they are proportional to technology. If \( \gamma = 1 \), wages are fully rigid: they do not respond to technology.
Taking \( a(t) \) and \( \theta(t) \) as given, the firm chooses \( l(t) \) to maximize profits

\[
a(t) \cdot \left[ \frac{l(t)}{1 + \tau(\theta(t))} \right]^\alpha - \omega \cdot a(t)^{1-\gamma} \cdot l(t).
\]

The labor demand \( l^d(\theta(t), a(t)) \) gives the optimal number of workers employed by the firm:

\[
l^d(\theta(t), a(t)) = \left( \frac{\alpha}{\omega} \right)^{\frac{1}{1-a}} \cdot a(t)^{\frac{\gamma}{1-a}} \cdot (1 + \tau(\theta(t)))^{-\frac{\alpha}{1-a}}.
\]  (15)

The labor demand is unaffected by UI because the wage does not respond to UI. Since \( \tau(\theta) \) is increasing in \( \theta \), the labor demand is decreasing in \( \theta \). When the labor market is tighter, hiring workers is less profitable as it requires a higher share of recruiters; hence, firms employ fewer workers. The labor demand is also increasing in \( a \). When technology is lower, the wage-technology ratio is higher as wages are partially rigid, and hiring workers is less profitable; hence, firms employ fewer workers. In the \((l, \theta)\) plane of Figure 5, the labor demand curve is downward sloping, and it shifts inward when technology falls.

**B. Preliminary Theoretical Results**

We derive a few theoretical results about the job-rationing model. These results will be useful to calibrate the model and understand the simulation outcomes.

We start by computing closed-form expressions for the main sufficient statistics:

**PROPOSITION 1.** The microelasticity of unemployment with respect to UI satisfies

\[
\varepsilon^m = \frac{1 - u}{\kappa} \cdot \frac{\Delta U}{\Delta U + \psi(e)},
\]  (16)

where \( \kappa \) is the elasticity of \( \psi'(e) \) with respect to \( e \). The discouraged-worker elasticity satisfies

\[
\varepsilon^f = \frac{u}{\kappa}.
\]  (17)
The elasticity wedge satisfies
\[
1 - \frac{\varepsilon^M}{\varepsilon^m} = \left( 1 + \frac{\eta}{1 - \eta} \cdot \frac{\alpha}{1 - \alpha} \cdot \frac{1}{1 + \varepsilon^f} \cdot \tau(\theta) \right)^{-1} > 0.
\] (18)

**Proof.** See Appendix B. □

The proposition indicates that \( \varepsilon^m \) is much larger than \( \varepsilon^f \) in normal circumstances because \( \varepsilon^m \) is proportional to \( 1 - u \), which is close to 1, whereas \( \varepsilon^f \) is proportional to \( u \), which is close to 0. As \( \varepsilon^m \) measures the response of search effort to UI and \( \varepsilon^f \) the response of search effort to labor market conditions, it is possible in our model that search responds strongly to UI but weakly to labor market conditions. Our model therefore reconciles the literature estimating a significant response of job search to UI (see Krueger and Meyer [2002] for a review) and the literature estimating a weak response of job search to labor market conditions (see Section II).

Next, we establish that equilibria with low technology are slumps and equilibria with high technology are booms:

**Proposition 2.** For a given utility gain from work, an equilibrium with higher technology has higher tightness and higher employment: \( \partial \theta / \partial a \big|_{\Delta U} > 0 \) and \( \partial l / \partial a \big|_{\Delta U} > 0 \).

**Proof.** The equilibrium condition is \( l^d(\theta, a) = l^s(\theta, \Delta U) \), where \( l^d(\theta, a) \) is given by (15) and \( l^s(\theta, \Delta U) \) by (6). The implicit differentiation of the equilibrium condition yields \( \partial \theta / \partial a = (\partial l^d / \partial a) \cdot (\partial l^s / \partial \theta - \partial l^d / \partial \theta)^{-1} \). We have seen that \( \partial l^d / \partial a > 0, \partial l^s / \partial \theta > 0, \) and \( \partial l^d / \partial \theta < 0. \) Thus \( \partial \theta / \partial a > 0. \) The other result follows since \( l = l^s(\theta, \Delta U) \) and \( \partial l^s / \partial \theta > 0. \) □

The proposition says that when technology is high, tightness and employment are high, as in a boom. Conversely, tightness and employment are low when technology is low, as in a slump. The mechanism is simple. When technology is low, the wage-technology ratio is high by wage rigidity, which depresses labor demand and therefore tightness and employment. In Figure 5, Panel A plots the labor demand curve for a low technology, which represents a slump, and Panel B plots it for a high technology, which represents a boom.

To conclude, we establish that the elasticity wedge is higher in slumps than in booms:
Figure 5: Countercyclicality of the Elasticity Wedge, $1 - \varepsilon^M/\varepsilon^m$, in the Job-Rationing Model

Notes: A slump is an equilibrium with low technology, and a boom is an equilibrium with high technology.

**Assumption 1.** The matching function and marginal disutility of search effort are isoelastic: $m(e \cdot u, v) = \mu \cdot (e \cdot u)^\eta \cdot v^{1-\eta}$ and $\psi'(e) = \delta \cdot e^\kappa$ with $\eta \in (0,1)$, $\kappa > 0$, $\mu > 0$, and $\delta > 0$.

**Proposition 3.** Under Assumption 1, an equilibrium with lower technology has a higher elasticity wedge: $\partial \left[ 1 - \varepsilon^M/\varepsilon^m \right]/\partial a|_{\Delta U} < 0$.

Proof. Proposition 2 implies that $\partial \theta/\partial a|_{\Delta U} > 0$ and $\partial u/\partial a|_{\Delta U} < 0$. Since $\tau(\theta)$ is increasing in $\theta$, we infer that $\partial [\tau(\theta)/u]/\partial a|_{\Delta U} > 0$. Under Assumption 1, $\kappa$ is a parameter, so equation (8) implies that $\partial \varepsilon^l/\partial a|_{\Delta U} < 0$. Last, under Assumption 1, $\eta$ is a parameter so it is independent of $a$. Combining these results with (18) implies that $\partial \left[ 1 - \varepsilon^M/\varepsilon^m \right]/\partial a|_{\Delta U} < 0$. \hfill $\Box$

The proposition shows that the elasticity wedge is higher in slumps than in booms, implying that the rat-race mechanism is stronger in slumps than in booms. This result appears in Figure 5.

The wedge between $\varepsilon^M$ and $\varepsilon^m$ is driven by the slope of the labor supply relative to that of the labor demand. In a slump (Panel A), the labor supply is flat at the equilibrium point because the matching process is congested by search efforts; hence, $\varepsilon^M$ is much lower than $\varepsilon^m$. Conversely, in a boom (Panel B), the labor supply is steep at the equilibrium point because the matching process is congested by the large number of vacancies; hence, $\varepsilon^M$ is close to $\varepsilon^m$.

Formally, let $\varepsilon^{ls} = (\theta/l) \cdot (\partial l^s/\partial \theta)$ and $\varepsilon^{ld} = -(\theta/l) \cdot (\partial l^d/\partial \theta)$ be the elasticities of labor supply and labor demand with respect to tightness ($\varepsilon^{ld}$ is normalized to be positive). We could rewrite the elasticity wedge as $1 -$
The prediction of Proposition 3 is corroborated by empirical work. Lalive, Landais and Zweimüller [2015, Table 9 in online appendix] finds that the elasticity wedge is significantly larger in labor markets in which tightness is initially low. Crepon et al. [2013, Table VI, Table X] also find that the rat-race mechanism is stronger in areas and periods with higher unemployment.

C. Calibration

We calibrate the job-rationing model to US data for 1951–2014. The calibration, summarized in Table I ensures that the sufficient statistics at the heart of our formula match the empirical evidence.

First, the calibration ensures that the level of the aggregate variables matches the evidence for the US labor market. We calibrate the parameters such that the average unemployment rate is \(u = 5.9\%\), the average tightness is \(\theta = 0.49\), and the average matching wedge is \(\tau = 2.3\%\). These values are the means over 1951–2014 of the series for \(u\), \(\theta\), and \(\tau\) constructed in Section III.

Second, the calibration matches the empirical evidence from Section III on the generosity of UI and the cost of unemployment. We set the replacement rate to \(R = 50\%\). We target an average consumption drop upon unemployment of \(c^h/c^e = 0.81\) and an average total nonpecuniary cost from unemployment of \(Z = 0.45\). We also target an average consumption smoothing by UI of \(d\ln(c^h)/d\ln(c^u) = 0.44\). This target comes from Gruber [1997], who estimates that when the replacement rate increases by 1 percentage point from an average value of 58%, the food consumption of an unemployed worker increases by 0.27 percent. The implied elasticity of food consumption to UI benefits therefore is \(0.27/(1/0.58) = 0.16\). Using again the estimate from Hamermesh [1982] to convert the response of food consumption into the response of total consumption, we infer that \(d\ln(c^h)/d\ln(c^u) = 0.16/0.36 = 0.44\). That is, increasing unemployment benefits by 1% increases total consumption when unemployed by 0.44%.

Third, the calibration matches the empirical evidence from Section II on the response of unemployment to UI. First, we target an elasticity wedge \(1 - \varepsilon^M/\varepsilon^m = 0.3\). The value of 0.3\(^{21}\) is relatively small because our times series for \(\tau\) is only available for the 1990–2014 period, we take the average over this period.
corresponds to the estimate found by Marinescu [2014] in US data and is in the range of estimates found by modern studies in other countries [Crepon et al., 2013; Lalive, Landais and Zweimüller, 2015]. Second, we target a microelasticity of unemployment duration with respect to the replacement rate of $\varepsilon^m_R = 0.4$. This microelasticity is defined by

$$
\varepsilon^m_R \equiv -\frac{\partial \ln(e \cdot f(\theta))}{\partial \ln(R)} \bigg|_{\theta, e}
$$

The value of 0.4 comes from Landais [2015]. Using a compelling regression kink design on an administrative dataset covering several US states in the 1980s, Landais finds that $\varepsilon^m_R$ is between 0.05 and 0.75, with an average estimate of $\varepsilon^m_R = 0.4$. Many empirical studies estimate $\varepsilon^m_R$, and 0.4 is in the range, albeit toward the low end, of the estimates in the literature. For instance, using the same data as Landais [2015] but a different identification strategy, the classic study of Meyer [1990, Table VI, columns (6)–(9)] finds an elasticity $\varepsilon^m_R = 0.6$. More recently, using a regression kink design on administrative data from Missouri for 2003–2013, Card et al. [2015, Table 1, column (2)] find that $\varepsilon^m_R$ is between 0.37 and 0.88.

Before starting the calibration, we impose some normalizations. First, we set the average technology to $a = 1$. Second, we target an average search effort of $e = 1$ and a disutility from search effort of $\psi(1) = 0$.

We begin the calibration with the parameters related to matching. We set the job-destruction rate to its average value in the times series constructed in Section III from CPS data for 1951–2014: $s = 3.3\%$. We use a Cobb-Douglas matching function: $m(e \cdot u, v) = \mu \cdot (e \cdot u)^\eta \cdot v^{1-\eta}$. We set $\eta = 0.66$, using the estimate from Section III. To calibrate the matching efficacy, we exploit the relationship $u \cdot e \cdot f(\theta) = s \cdot (1-u)$, which implies $\mu = s \cdot \theta^{\eta-1} \cdot (1-u)/(u \cdot e)$. With $s = 3.3\%$, $\theta = 0.49$, $\eta = 0.66$, $u = 5.9\%$, and $e = 1$, we get $\mu = 0.68$. To calibrate the matching cost, we exploit the relationship $\tau = \rho \cdot s/[\mu \cdot \theta^{\eta} - \rho \cdot s]$, which implies $\rho = \mu \cdot \theta^{\eta} \cdot \tau/[s \cdot (1 + \tau)]$. With $\mu = 0.68$, $s = 3.3\%$, $\theta = 0.49$, and $\tau = 2.3\%$, we obtain $\rho = 0.73$.

Next we calibrate the production-function parameter $\alpha$. Equation (18) shows that the elas-
Table 1: Calibration of the Model Used for Simulations

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u = 5.9% )</td>
<td>Unemployment rate</td>
<td>CPS, 1951–2014</td>
</tr>
<tr>
<td>( \theta = 0.49 )</td>
<td>Labor market tightness</td>
<td>Section III</td>
</tr>
<tr>
<td>( \tau = 2.3% )</td>
<td>Recruiter-producer ratio</td>
<td>Section III</td>
</tr>
<tr>
<td>( R = 50% )</td>
<td>Replacement rate</td>
<td>Pavoni and Violante [2007]</td>
</tr>
<tr>
<td>( \varepsilon_R^m = 0.4 )</td>
<td>Microelasticity of unemployment duration wrt replacement rate</td>
<td>Landais [2015]</td>
</tr>
<tr>
<td>( 1 - \varepsilon_M^R/\varepsilon_m^R = 0.3 )</td>
<td>Elasticity wedge</td>
<td>Marinescu [2014]</td>
</tr>
<tr>
<td>( Z = 0.45 )</td>
<td>Total nonpecuniary cost from unemployment</td>
<td>Di Tella, MacCulloch and Oswald [2003]</td>
</tr>
<tr>
<td>( c_h/c_u = 0.81 )</td>
<td>Consumption drop upon unemployment</td>
<td>Gruber [1997], Hamermesh [1982]</td>
</tr>
<tr>
<td>( d \ln(c_h)/d \ln(c_u) = 0.44 )</td>
<td>Consumption smoothing by UI</td>
<td>Gruber [1997], Hamermesh [1982]</td>
</tr>
<tr>
<td>( a = 1 )</td>
<td>Technology</td>
<td>Normalization</td>
</tr>
<tr>
<td>( e = 1 )</td>
<td>Search effort</td>
<td>Normalization</td>
</tr>
<tr>
<td>( s = 3.3% )</td>
<td>Monthly job-destruction rate</td>
<td>CPS, 1951–2014</td>
</tr>
<tr>
<td>( \eta = 0.66 )</td>
<td>Unemployment elasticity of matching function</td>
<td>Section III</td>
</tr>
<tr>
<td>( \mu = 0.68 )</td>
<td>Matching efficacy</td>
<td>Matches targets</td>
</tr>
<tr>
<td>( \rho = 0.73 )</td>
<td>Matching cost</td>
<td>Matches ( \tau = 2.3% )</td>
</tr>
<tr>
<td>( \kappa = 1.8 )</td>
<td>Disutility from search: convexity</td>
<td>Matches ( \varepsilon_R^m = 0.4 )</td>
</tr>
<tr>
<td>( \delta = 0.62 )</td>
<td>Disutility from search: level</td>
<td>Matches targets</td>
</tr>
<tr>
<td>( \alpha = 0.76 )</td>
<td>Decreasing marginal returns to labor</td>
<td>Matches ( 1 - \varepsilon_M^R/\varepsilon_m^R = 0.3 )</td>
</tr>
<tr>
<td>( \omega = 0.75 )</td>
<td>Real wage: level</td>
<td>Matches targets</td>
</tr>
<tr>
<td>( \gamma = 0.5 )</td>
<td>Real wage: rigidity</td>
<td>Pissarides [2009], Haeferke, Sonntag and van Rens [2008]</td>
</tr>
<tr>
<td>( \sigma = 0.35 )</td>
<td>Disutility from home production: convexity</td>
<td>Matches ( d \ln(c_h)/d \ln(c_u) = 0.44 )</td>
</tr>
<tr>
<td>( \zeta = 2.3 )</td>
<td>Disutility from home production: level</td>
<td>Matches targets</td>
</tr>
<tr>
<td>( z = 0.26 )</td>
<td>Pure nonpecuniary cost from unemployment</td>
<td>Matches ( Z = 0.45 )</td>
</tr>
</tbody>
</table>
ticity wedge is positive in the job-rationing model, consistent with the empirical evidence in Section II and that the magnitude of the elasticity wedge is determined by $\alpha$. Hence, we calibrate $\alpha$ to match $1 - e^M/e^m = 0.3$. In equation (18) we set $1 - e^M/e^m = 0.3$, $\eta = 0.66$, $\tau = 2.3\%$, $u = 5.9\%$, and $\epsilon^{f} = 0$.\textsuperscript{23} We obtain $\alpha = 0.76$.

Then we calibrate the parameters of the wage schedule. To calibrate the wage level, we exploit the relationship $\alpha \cdot n^{\alpha - 1} = \omega \cdot a^{-\gamma} \cdot (1 + \tau)$. With $a = 1$, $\tau = 2.3\%$, $\alpha = 0.76$, and $n = (1 - u)/(1 + \tau) = 0.920$, we obtain $\omega = 0.75$. We calibrate the wage rigidity from microeconometric estimates of the elasticity for wages in newly created jobs—the elasticity that matters for job creation.\textsuperscript{24} In panel data following production and supervisory workers from 1984 to 2006, Haefke, Sonntag and van Rens [2008, Table 6, Panel A, column 4] find that the elasticity of new hires’ earnings with respect to productivity is 0.7. If the composition of the jobs accepted by workers improves in expansion, 0.7 is an upper bound on the elasticity of wages in newly created jobs. A lower bound is the elasticity of wages in existing jobs, estimated between 0.1 and 0.45 [Pissarides 2009]. Hence we set $\gamma = 0.5$, in the range of plausible values.

We now compute the consumption levels implied by the calibration. The definition of the replacement rate implies $c^e - c^a = w \cdot (1 - R)$. The resource constraint imposes $(1 - u) \cdot c^e + u \cdot c^a = a \cdot n^{\alpha}$. Solving this linear system of two equations with $w = \omega = 0.75$, $R = 0.5$, $u = 5.9\%$, $a = 1$, $n = 0.920$, and $\alpha = 0.76$, we obtain $c^e = 0.96$ and $c^a = 0.58$. In Section III we document that $c^h/c^e = 0.81$; therefore, $c^h = 0.78$ and $h = c^h - c^a = 0.19$.

The next steps are to calibrate the parameters of the utility function. Relying on the evidence presented in Section III we set the consumption utility to $U(c) = \ln(c)$. With log utility, a total nonpecuniary cost of unemployment $Z = 0.45$, and a consumption drop upon unemployment $c^h/c^e = 0.81$, the utility gain from work is $\Delta U = \ln(c^e/c^h) + Z = \ln(1/0.81) + 0.45 = 0.66$.

We assume that the disutility from search effort is isoelastic: $\psi(e) = \delta \cdot e^{\kappa + 1}/(\kappa + 1) - \delta/(\kappa + 1)$. We calibrate $\kappa$ to obtain a search behavior consistent with the empirical evidence on the microelasticity of unemployment with respect to UI. We first use the estimate $e^m_R = 0.4$ to

\textsuperscript{23}The value of $\alpha$ does not depend much on the value of $\epsilon^f$ since $\epsilon^f$ is small. As we need the value of $\alpha$ to compute $\epsilon^f$, we first calibrate $\alpha$ before computing $\epsilon^f$. We verify that $\epsilon^f \approx 0$ in the calibrated model: we find $\epsilon^f = 0.03$.

\textsuperscript{24}As discussed in Michaillat [2012], because technology is roughly a random walk, the expected present discounted value of wages paid by a firm to a new worker is approximately determined by the initial wage. The rigidity of wages with respect to technology once the worker-firm relationship has started is therefore irrelevant to firms; only the rigidity of wages with respect to technology in newly created jobs matters.
infer the value of our microelasticity $\varepsilon^m$. Given that $1 - l^s = s/(s + e^s \cdot f(\theta))$, we have

$$
\varepsilon^m_R = \frac{1}{l} \left. \frac{\partial \ln(1 - l^s)}{\partial \ln(R)} \right|_{\theta, c^e} = -\frac{R}{l \cdot (1 - l)} \left. \frac{\partial l^s}{\partial R} \right|_{\theta, c^e}. 
$$

We now consider a change $dR$ keeping $c^e$ and $\theta$ constant. As $\Delta c = (1 - R) \cdot w$, we have $c^u = c^e - (1 - R) \cdot w$) and the change $dR$ implies a consumption change $dc^u = w \cdot dR$. The utility gain from work is $\Delta U = U(c^e) - U(c^u + h^s) + z + \lambda (h^s)$. The change $dc^u$ implies a change $dh^s$ in home production, but this change has not first-order effect on $\Delta U$ because $h^s$ is optimally chosen to minimize $\Delta U$. Accordingly, the effect of $dc^u$ on $\Delta U$ is $d\Delta U = -U'(c^h) \cdot dc^u = -U'(c^h) \cdot w \cdot dR$. Using (19), we infer that

$$
\varepsilon^m = \frac{R}{l \cdot (1 - l)} \cdot U'(c^h) \cdot w \cdot l^s \left|_{\theta} = \frac{R \cdot w \cdot U'(c^h)}{(1 - u) \cdot \Delta U} \varepsilon^m. 
$$

Using this equation and setting $\varepsilon^m_R = 0.4$, $u = 5.9\%$, $R = 0.5$, $w = 0.75$, $U'(c^h) = 1/c^h = 1/0.78 = 1.28$, and $\Delta U = 0.66$, we infer that $\varepsilon^m = 0.51$. Using (16) with $u = 5.9\%$, $\varepsilon^m = 0.51$, and $\psi(e) = 0$, we obtain $\kappa = 1.8$. To calibrate $\delta$, we use equation (4). With $e = 1$ and $\psi(e) = 0$, this equation implies $\delta = (1 - u) \cdot \Delta U$. Setting $u = 5.9\%$ and $\Delta U = 0.66$, we obtain $\delta = 0.62$.

The calibration of $\psi(e)$ has two implications. First, since $\psi(e)$ is isoelastic, the microelasticity is nearly constant over the business cycle. This property is consistent with the evidence from Schmieder, von Wachter and Bender [2012] for Germany.\(^{25}\) Second, equation (17) and $\kappa = 1.8$ imply that $\varepsilon^f = 0.03$. The low value of $\varepsilon^f$ is consistent with the empirical evidence in Section III which suggests that $\varepsilon^f \approx 0$.

We assume that the disutility from home production is $\lambda (h) = \xi \cdot h^{\lambda + \sigma} / (1 + \sigma)$. Equation (2) implies that $\xi \cdot h^\sigma = 1/(c^u + h)$. We implicitly differentiate this equation with respect to $c^u$ and

\[^{25}\text{Schmieder, von Wachter and Bender [2012] compellingly identify the microelasticity of unemployment duration with respect to the potential duration of UI benefits by running a regression discontinuity design on German administrative data. The design exploits sharp variations in the potential duration of benefits by age. Their estimates are broadly constant over the business cycle (Table 4, column 7). In the US context, Kroft and Notowidigdo [2015] analyze data from the Survey of Income and Program Participation for 1985–2000 and find that the response of unemployment duration to variations in UI benefits is smaller when unemployment is high. As they compare some individuals across, not within, labor markets, they estimate a mixture of microelasticity and macroelasticity. In our simulations (see Figure 6), the microelasticity is acyclical but the macroelasticity is sharply countercyclical so a mixture of the two elasticities would be somewhat countercyclical, consistent with the results by Kroft and Notowidigdo.}\]
obtain \((\sigma + h/c^h) \cdot (d \ln(h)/d \ln(c^u)) = -c^u/c^h\). This implies

\[
\frac{dc^h}{dc^u} = 1 + \frac{dh}{dc^u} = 1 - \frac{1}{1 + \sigma \cdot c^h/h} = \frac{\sigma}{\sigma + h/c^h}.
\] (20)

Since \(c^h/c^u = 1.33\) and \(d \ln(c^h)/d \ln(c^u) = 0.44\), we infer that \(dc^h/dc^u = (c^h/c^u) \cdot (d \ln(c^h)/d \ln(c^u)) = 0.58\). Combining this fact with (20) and \(h/c^h = 0.25\), we find \(\sigma = 0.35\). Equation (2) implies that 
\[\xi \cdot h^\sigma = 1/c^h.\]
Using \(c^h = 0.78, h = 0.19\), and \(\sigma = 0.35\), we find \(\xi = 2.3\).

Last, we calibrate the pure nonpecuniary cost from unemployment, \(z\). We target \(Z = z + \psi(e) + \lambda(h) = 0.45\); as \(\lambda(h) = 0.19\) and \(\psi(e) = 0\) in the average state, we set \(z = 0.26\).

### D. Simulations

We represent the business cycle as a succession of equilibria in which technology takes different values: a slump is an equilibrium with low technology and high unemployment; a boom is an equilibrium with high technology and low unemployment.\(^{26}\) We compute a collection of equilibria spanning all the stages of the business cycle. We compare equilibria in which the UI replacement rate remains constant at 50%, the average US value, to equilibria in which the UI replacement rate is optimal.

Figure 6 displays the results of the simulations. Because of wage rigidity, the equilibria with low technology have a high wage-technology ratio and therefore high unemployment: they represent slumps. Conversely, the equilibria with high technology have a low wage-technology ratio and low unemployment: they represent booms. In the simulations, when technology increases from 0.94 to 1.06 and UI remains constant, the unemployment rate falls from 12.4% to 3.8%. This result suggests that the modest amount of wage rigidity observed in microdata, which we used to calibrate the wage schedule, is able to generate large fluctuations in unemployment: the elasticity of unemployment with respect to technology in the simulations is 10.6, larger than the elasticity of 4.2 measured in US data.\(^{27}\) At the same time as the unemployment rate falls, the

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\(^{26}\) Technology shocks are a conventional and convenient way to generate business cycles in matching models. As showed by [Michaillat and Saez 2015], these models can also accommodate other types of shocks, particularly aggregate demand shocks. For our analysis, what matters is that shocks to the labor demand drive business cycles—whether these are aggregate demand or technology shocks is unimportant.

\(^{27}\) The elasticity in the simulations is obtained by measuring the change in unemployment generated by a small change in technology. At \(a = 1\) we have \(u = 5.87\%\) and at \(a = 0.995\) we have \(u = 6.18\%\); therefore, the elasticity is
Figure 6: Simulations of Optimal Unemployment Insurance over the Business Cycle
labor market tightness increases from 0.05 to 1.81.

The microelasticity of unemployment with respect to UI \( (e^m) \) is broadly constant around 0.5 in any state of the labor market. Furthermore, the need for insurance remains broadly constant. As a consequence, the Baily-Chetty replacement rate (defined in formula (10)) does not vary much with the state of the labor market.

Unlike the microelasticity, the macroelasticity of unemployment with respect to UI \( (e^M) \) is sharply procyclical: it increases from 0.09 to 0.46 when technology increases from 0.94 to 1.06. Hence, because of job rationing, UI has a weak influence on unemployment in slumps but a strong influence in booms. Moreover, the macroelasticity is always lower than the microelasticity.

The behavior of the microelasticity and macroelasticity imply that the elasticity wedge is markedly countercyclical, even though it always remains positive: the wedge is around 0.8 for high unemployment rates, 0.3 on average, and around 0.1 for low unemployment rates. The simulation result that the wedge is countercyclical was predicted by Proposition 3 and is corroborated by the empirical work of Lalive, Landais and Zweimüller [2015] and Crepon et al. [2013].

Next, the efficiency term varies drastically with the state of the labor market. The efficiency term is strongly positive (above 1) for high unemployment rates, zero for an unemployment rate of 4.8%, and strongly negative (below −1) for low unemployment rates. These sharp fluctuations reflect the fact that in the model business cycle fluctuations are inefficient: high unemployment rates are inefficiently high, low unemployment rates are inefficiently low, and the efficient unemployment rate is 4.8%.

As a consequence of the combined fluctuations of the efficiency term and elasticity wedge, the correction term (defined in formula (10)) fluctuates sharply over the business cycle. Since the correction term is very positive with high unemployment rates and mildly negative for low unemployment rates, and since the Baily-Chetty replacement rate is stable over the business cycle, the optimal replacement is strongly countercyclical: it falls from 76% to 38% when technology increases from 0.94 to 1.06. In fact, the optimal replacement rate is so generous in bad times that the consumption drop upon unemployment falls below 5%. Even though unemployed workers consume nearly the same amount as employed workers (the consumption of unemployed workers includes home production), workers are not indifferent between employment and unemployment:

\[
\frac{(1/5.87) \cdot (6.18 - 5.87)}{(1 - 0.995)} = 10.6.
\]

The elasticity in US data comes from Michaillat [2012, Table 2].
unemployed workers are worse off because they incur a significant nonpecuniary cost from unemployment, coming from the pure nonpecuniary cost from unemployment, the cost of home production, and the cost of job search. In good times, the optimal replacement rate is much less generous and the consumption drop is above 25%.

Because the equilibrium value of the correction term is so large in bad times under optimal UI, the departure from Baily-Chetty formula is large. Somebody assessing the optimal UI program coming from our model and simulations in bad times using only the conventional Baily-Chetty formula would therefore misperceive our proposed optimal replacement rate as far too generous.

Our macroeconomic theory of UI combined with current empirical estimates of the sufficient statistics imply that the optimal replacement rate is strongly countercyclical and that the optimal UI program is very generous in bad times. This result is especially striking because, as showed in Figure 7, UI does have significant disincentives effects at the micro level. These disincentive effects, caused by moral hazard, operate on job search and home production. The figure shows that in bad times, the increase of the replacement rate from 50% to its optimal level of 76% reduces job search and home production by more than 25%. Unlike in the Baily-Chetty theory, where a high replacement rate is only warranted if the disincentive effect of UI on job search at the micro level is weak, our theory indicates that it may be optimal to have a very generous UI program even when the disincentive effect of UI on job search at the micro level is substantial—this occurs when the labor market is depressed and the elasticity wedge positive.

In addition to its micro effects, UI also has macro effects. The unemployment rate responds
to the adjustment of the replacement rate from its original level of 50% to its optimal level. In slumps, the optimal replacement rate is well above 50% so the unemployment rate increases above its original level; in booms, the optimal replacement rate is below 50% so the unemployment rate decreases below its original level. These effects are not strong, however: in slumps, the large adjustment of UI has little effect on unemployment because the macroelasticity is low; in booms, the adjustment of UI is small so it has a small effect on unemployment.

V. Conclusion

This paper applies the theory of optimal UI developed in our companion paper [Landais, Michailat and Saez, 2015] to explore how the generosity of UI should vary over the business cycle. In our companion paper, we show that the optimal UI replacement rate is the Baily-Chetty rate plus a correction term. In this paper, we empirically measure the correction term in the United States for 1990–2014. We find that the correction term is sharply countercyclical, which suggests that the optimal replacement rate is much more countercyclical than the Baily-Chetty rate.

The paper also simulates the job-rationing model of Michaillat [2012] to quantify the fluctuations of the optimal replacement rate. We choose this model because, unlike other matching models, it is consistent with empirical evidence that the macroelasticity of unemployment with respect to UI is smaller than the microelasticity. The simulations show that the optimal replacement rate almost doubles from 40% to 75% when the unemployment rate rises from 4% to 12%. Of course more empirical work would be valuable to cement our findings.

Beyond its implication for optimal UI, the empirical evidence presented in this paper, while not definitive, has important implications for our understanding of the labor market. First, the empirical evidence in Section II suggests that the macroelasticity of unemployment with respect to UI is smaller than the microelasticity. This evidence indicates that the job-rationing model of Michaillat [2012] is better suited than other common matching models to describe the labor market. This empirical finding has policy ramifications. In common matching models, policies that stimulate labor supply (such as job-search monitoring or job placement support) are effective to reduce unemployment in slumps, whereas policies that stimulate labor demand (such as public employment) are ineffective. In the job-rationing model, all these results are reversed: policies
that stimulate labor supply are ineffective to reduce unemployment in slumps, whereas policies that stimulate labor demand are effective [Michaillat 2012, 2014]. The finding of a positive wedge between macroelasticity and microelasticity therefore implies that policies stimulating labor demand should be favored to reduce unemployment in slumps.

Second, the empirical evidence in Section III suggests that the efficiency term is markedly countercyclical: it is positive in slumps but negative in booms. This finding has critical macroeconomic implications. It supports the view that unemployment is inefficiently high in slumps and inefficiently low in booms and therefore implies that more effort should be done to stabilize the macroeconomy in the United States. In this context, the measure of the efficiency term obtained here could be applied to a broad range of stabilization problems. For instance, the work of Michaillat and Saez [2014, 2015] on optimal monetary policy, optimal debt policy, and optimal government purchases shows that an estimate of the efficiency term is required to determine the optimal response of these stabilization policies to macroeconomic shocks. The reason is that the efficiency term gives a direct measure of the unemployment gap—the gap between the actual and efficient unemployment rates—which determines the need for stabilization.

Moreover, the empirical methodology that we propose to measure the efficiency term could be applied to measure the “natural” rate of unemployment. The natural rate of unemployment corresponds to the efficient unemployment rate in our model: it is reached when the efficiency term is zero. Because existing methods for the measurement of the natural rate are not based on economic considerations—they are statistical methods measuring some trend of the unemployment series—our methodology could help provide better estimates of the natural rate.

References


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Appendix A: Derivation of the Optimal Unemployment Insurance Formula

This appendix derives formula (10). The formula characterizes optimal UI in the dynamic model of Section II. The derivation of the formula closely follows that in our companion paper [Landais, Michaillat and Saez, 2015], so we only present the main steps of the derivations and the calculations that are different. The differences arise because the model is dynamic and not static, and because we introduce home production as well as a nonpecuniary cost of unemployment. All the calculations that are the same as in our companion paper are not repeated.

Social welfare is a function of $\Delta U$ and $\theta$:

$$SW(\theta, \Delta U) = \frac{e^s(\theta, \Delta U) \cdot f(\theta(t))}{s + e^s(\theta, \Delta U) \cdot f(\theta(t))} \cdot (\Delta U + \psi(e(\theta, \Delta U))) + U(c^u(\theta, \Delta U) + h^s(c^u(\theta, \Delta U))) - z - \lambda(h^s(c^u(\theta, \Delta U))) - \psi(e(\theta, \Delta U)).$$

The consumption level $c^u(\theta, \Delta U)$ is implicitly defined by

$$y\left(\frac{l^s(\theta, \Delta U)}{1 + \tau(\theta)}\right) = (1 - l^s(\theta, \Delta U)) \cdot c^u(\theta, \Delta U) + l^s(\theta, \Delta U) \cdot U^{-1}(U(c^u(\theta, \Delta U) + h^s(c^u(\theta, \Delta U))) - z - \lambda(h^s(c^u(\theta, \Delta U))) + \Delta U).$$

We first compute the elasticity of the labor supply with respect to tightness. The labor supply, given by (4), can be written as $l^s(\theta, \Delta U) = L(e^s(\theta, \Delta U) \cdot f(\theta))$, where the function $L$ is defined by $L(x) = x/(s + f(x))$. Given that the elasticity of $L(x)$ is $1 - L(x)$, the elasticity of the labor supply with respect to tightness is

$$\frac{\theta}{l^s} \cdot \partial l^s/\partial \theta \bigg|_{\Delta U} = u \cdot (1 + \varepsilon^f) \cdot (1 - \eta).$$

The only difference with formula (14) in our companion paper is the extra factor $u = 1 - l$, which arises because the labor supply is $L(e^s(\theta, \Delta U) \cdot f(\theta))$ in the dynamic model but $e^s(\theta, \Delta U) \cdot f(\theta)$ in the static model, and the elasticity of $L$ in equilibrium is $u$, the unemployment rate.

Next, we compute the partial derivatives of the social welfare function. We start with the partial derivative with respect to $\theta$. First, we recompute the equation (13) from our companion paper. Since workers choose home production to maximize expected utility, a standard application of the envelope theorem says that changes in $h^s(c^u(\theta, \Delta U))$ resulting from changes in $\theta$ have no impact on social welfare. Hence, the introduction of home production does not add new terms to the partial derivative. The presence of home production only changes $U'(c^u)$ into $U'(c^h)$. Accordingly, equation (13) becomes

$$\frac{\partial SW}{\partial \theta} = u \cdot l^s(\theta) \cdot (1 - \eta) \cdot (\Delta U + \psi(e)) + U'(c^h) \cdot \frac{\partial c^u}{\partial \theta}.$$ (A2)

The factor $u$ in the first term appears because the environment is dynamic, as in (A1). The fact that the environment is dynamic also changes $\Delta U$ into $\Delta U + \psi(e)$ in the first term.

Next we recompute equation (15) from our companion paper. First, in the dynamic environment, equation (A1) implies that $\partial l^s/\partial \theta = u \cdot (l/\theta) \cdot (1 - \eta) \cdot (1 + \varepsilon^f)$. Second, with home
production, the derivative of
\[
c^e(c^u, \Delta U) = U^{-1}(U(c^u + h^u(c^u)) - z - \lambda(h^u(c^u)) + \Delta U)
\]  \hspace{1cm} (A3)

with respect to \(c^u\) is \(\frac{\partial c^e}{\partial c^u} = \frac{U'(c^h)}{U'(c^e)}\). Because unemployed workers choose home production to maximize \(U(c^u + h) - \lambda(h)\), changes in \(h^u\) resulting from changes in \(c^u\) have no impact on \(c^e\). Hence, equation (15) becomes
\[
\frac{\partial c^e}{\partial c^u} = \frac{U'(c^h)}{U'(c^e)}
\]

where \(\Delta c = c^e - c^u\). This equation is the same as equation (15) except for the factor \(u\) in the left-hand side and the change of \(U'(c^u)\) into \(U'(c^h)\).

Combining equations (A2) and (A4) we can recompute equation (10) from our companion paper:
\[
\frac{\partial SW}{\partial \theta} \bigg|_{\Delta U} = u \cdot \frac{1}{\theta} \cdot (1 - \eta) \cdot \phi \cdot w \cdot \left[ \frac{\Delta U + \psi(e)}{\phi \cdot w} + R \cdot \left(1 + \varepsilon^f\right) - \frac{\eta}{1 - \eta} \cdot \frac{\tau(\theta)}{u} \right],
\]  \hspace{1cm} (A5)

where \(\phi\) is the harmonic mean of workers’ marginal consumption utilities:
\[
\frac{1}{\phi} = \frac{l}{U'(c^e)} + \frac{1 - l}{U'(c^u)}.
\]

We continue by computing the partial derivative of social welfare with respect to \(\Delta U\). First, we recompute the equation (16) from our companion paper. Applying again the envelope theorem for the changes in \(h^u\) and \(e^u\) resulting from changes in \(\Delta U\), we find that this equation becomes
\[
\frac{\partial SW}{\partial \Delta U} = l + U'(c^h) \cdot \frac{\partial c^u}{\partial \Delta U}.
\]  \hspace{1cm} (A6)

Next we recompute equation (17) from our companion paper. Using the work that we have done to obtain (A4), we find that equation (17) becomes
\[
\frac{\Delta U}{\Delta U} \cdot w - \frac{l}{U'(c^e)} = \left( \frac{l}{U'(c^e)} + \frac{1 - l}{U'(c^u)} \right) \cdot U'(c^h) \cdot \frac{\partial c^u}{\partial \Delta U}.
\]  \hspace{1cm} (A7)

Combining (A6) and (A7) as in the companion paper, it is straightforward to recompute equation (11):
\[
\frac{\partial SW}{\partial \Delta U} \bigg|_{\theta} = u \cdot \frac{\phi \cdot w}{\Delta U} \cdot \varepsilon^m \cdot \left[ R - \frac{l}{w} \cdot \Delta U \cdot \left( \frac{1}{U'(c^e)} - \frac{1}{U'(c^h)} \right) \right],
\]  \hspace{1cm} (A8)

The last step before obtaining the optimal UI formula is to link the elasticity wedge to the equilibrium response of tightness to UI. Following the approach in the companion paper and
using (A1), we find that
\[ \varepsilon^M = \varepsilon^m + l \cdot (1 - \eta) \cdot \left(1 + \varepsilon^f \right) \cdot \frac{\Delta U}{\theta} \cdot \frac{d \theta}{d \Delta U}. \tag{A9} \]

This equation replaces equation (22) in the companion paper. As expected, the only difference with the original equation is that a factor \( l \) replaces the factor \( 1/(1 - l) \).

The first-order condition of the government’s problem is
\[ 0 = \frac{\partial SW}{\partial \Delta U} \bigg|_{\theta} + \frac{\partial SW}{\partial \theta} \bigg|_{\Delta U} \cdot \frac{d \theta}{d \Delta U}. \]

Using the partial derivatives of \( SW(\theta, \Delta U) \) given by (A5) and (A8) and the derivative \( d \theta/d \Delta U \) implied by (A9), we obtain formula (10).

**Appendix B: Proof of Proposition 1**

This appendix proves Proposition 1.

First, we compute the microelasticity of unemployment with respect to UI (\( \varepsilon^m \)). Let \( \kappa \equiv \left(e/\psi'(e)\right) \cdot \psi''(e) \) be the elasticity of \( \psi' \) with respect to \( e \). Let \( \varepsilon^e_A \equiv (\Delta U/e) \cdot (\partial e^f/\partial \Delta U) \) be the elasticity of effort supply with respect to the utility gain from work. As in Appendix A, let \( L(x) \equiv x/(s + x) \). The elasticity of \( L \) with respect to \( x \) is \( 1 - L(x) \). The effort supply \( e^f(f, \Delta U) \) satisfies equation (4), which can be written as
\[ e^f \cdot \psi'(e) = L(e^f) \cdot (\Delta U + \psi(e)). \tag{A10} \]

Differentiating this condition with respect to \( \Delta U \) yields
\[ \varepsilon^e_A + \kappa \cdot \varepsilon^e_A = (1 - l) \cdot \varepsilon^e_A + \frac{\Delta U}{\Delta U + \psi(e)} + \frac{e \cdot \psi'(e)}{\Delta U + \psi(e)} \cdot \varepsilon^e_A. \]

In equilibrium, \( e \cdot \psi'(e)/(\Delta U + \psi(e)) = l \). Therefore,
\[ \varepsilon^e_A = \frac{1}{\kappa} \cdot \frac{\Delta U}{\Delta U + \psi(e)}. \]

Since the labor supply satisfies \( l^s(\theta, \Delta U) = L(e^s(f(\theta) \cdot \Delta U) \cdot f(\theta) \), the elasticity of \( l^s(\theta, \Delta U) \) with respect to \( \Delta U \) is \( (1 - l) \cdot \varepsilon^e_A \). By definition, \( \varepsilon^m \) is \( l/(1 - l) \) times the elasticity of \( l^s(\theta, \Delta U) \) with respect to \( \Delta U \). Thus, \( \varepsilon^m = l \cdot \varepsilon^e_A \) and we obtain equation (16).

Second, we compute the discouraged-worker elasticity (\( \varepsilon^f \)). We differentiate (A10) with respect to \( f \) and obtain
\[ \varepsilon^f + \kappa \cdot \varepsilon^f = (1 - l) \cdot (\varepsilon^f + 1) + \frac{e \cdot \psi'(e)}{\Delta U + \psi(e)} \cdot \varepsilon^f. \]

In equilibrium, \( e \cdot \psi'(e)/(\Delta U + \psi(e)) = l \). Hence, by rearranging the terms in this equation, we obtain (17).
Third, we compute the elasticity wedge \((1 - \varepsilon^M / \varepsilon^m)\). The elasticity of \(1 + \tau(\theta)\) with respect to \(\theta\) is \(\eta \cdot \tau(\theta)\). From (15), we infer that the elasticity of \(l^d(\theta, a)\) with respect to \(\theta\) is \(-\eta \cdot \tau(\theta) \cdot \alpha / (1 - \alpha)\). By definition, \(\varepsilon^M\) is \(l / (1 - l)\) times the elasticity of \(l\) with respect to \(\Delta U\). Since \(l = l^d(\theta, a)\) in equilibrium, we infer that

\[
\varepsilon^M = -\frac{l}{1-l} \cdot \eta \cdot \frac{\alpha}{1-\alpha} \cdot \tau(\theta) \cdot \frac{\Delta U}{\theta} \cdot \frac{d\theta}{d\Delta U}
\]

We substitute the expression for \((\Delta U / \theta) \cdot (d\theta / d\Delta U)\) from (A9) into this equation and obtain

\[
\varepsilon^M = \frac{\eta}{1 - \eta} \cdot \frac{\alpha}{1 - \alpha} \cdot \frac{1}{1 + \varepsilon^f} \cdot \frac{\tau(\theta)}{\varepsilon^m} \cdot (\varepsilon^m - \varepsilon^M).
\]

Dividing this equation by \(\varepsilon^m\) and rearranging yields (18).