Optimal Minimum Wage Policy in Competitive Labor Markets*

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Abstract

This paper provides a theoretical analysis of optimal minimum wage policy in a perfectly competitive labor market. We show that a binding minimum wage – while leading to unemployment – is nevertheless desirable if the government values redistribution toward low wage workers and minimum wage induced unemployment hits the lowest surplus workers first. This result remains true in the presence of optimal nonlinear taxes and transfers. In that context, a minimum wage effectively rations low skilled labor which is subsidized by the optimal tax/transfer system, and improves upon the second-best tax/transfer optimum. When labor supply responses are along the extensive margin, a minimum wage and low skill work subsidies are complementary policies, and therefore, the co-existence of a minimum wage with a positive tax rate for low skill work is always (second-best) Pareto inefficient. We derive formulas for the optimal minimum wage (with or without optimal taxes) as a function of the elasticities of labor supply and demand and the redistributive tastes of the government and present some illustrative numerical simulations.

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1 Introduction

The minimum wage is a widely used but controversial policy tool. Minimum wages are a potentially useful tool for redistribution because they increase low skilled workers’ wages at the expense of other factors of production such as higher skilled workers or capital. They may, however, also lead to involuntary unemployment and hence worsen the welfare of workers who lose their jobs because of the minimum wage. An enormous empirical literature has studied the extent to which minimum wages affect the wages and employment of low-skilled workers.\footnote{See e.g., Brown et al. (1982), Card and Krueger (1995), Dolado et al. (1996), Brown (1999), or Neumark and Wascher (2006) for extensive surveys.}

The normative literature on the minimum wage, however, is much less extensive.\footnote{Although we believe that efficient rationing is the most natural assumption, we also discuss in detail how our results are modified if unemployment hits low skilled workers independently of surplus, what we call “uniform rationing”.}

This paper provides a normative analysis of the optimal minimum wage in a conventional competitive labor market model, using the standard social welfare framework adopted in the optimal tax theory literature following the seminal contributions of Diamond and Mirrlees (1971) and Mirrlees (1971). In most of our analysis, we adopt the important “efficient rationing” assumption – that unemployment induced by the minimum wage hits workers with the lowest surplus first.\footnote{Although simple, this analysis does not seem to have been formally derived in the previous literature.}

Our goal is to use this framework to make explicit the trade-offs involved when a government sets a minimum wage, and to shed light on the appropriateness of a minimum wage in the context of optimal taxes and transfers.

The first part of the paper considers a competitive labor market with no taxes and transfers. Although unrealistic, this case transparently illustrates the key trade-off at play when choosing a minimum wage rate.\footnote{Although simple, this analysis does not seem to have been formally derived in the previous literature.} We show that a binding minimum wage is desirable as long as the government places a non-zero value on redistribution from high- to low wage workers, the demand elasticity of low-skilled labor is finite, and the supply elasticity of low-skilled labor is positive. Unsurprisingly, the optimal minimum wage is decreasing in the demand elasticity because a minimum wage has larger unemployment effects when the demand elasticity is higher. The optimal minimum wage is increasing in the supply elasticity because a high supply elasticity implies that marginal workers have a low surplus from working (since many would leave the labor force if the wages were slightly reduced). The size of the optimal minimum wage

\[ W^* = \frac{1}{\alpha + \beta} \]
wage follows an inverted U-shape with the degree of the government’s redistributive tastes: there is no role for the minimum wage if the government does not value redistribution nor if the government has extreme Rawlsian preferences (as the costs of involuntary unemployment dominate the value of transfers to low skilled workers).

The second part of the paper considers how the results change when the government also uses taxes and transfers to achieve redistributive goals. As described below in more detail, our key innovation is to abstract from the hours of work decision and focus only in the job choice and work participation decision. In that context, the government observes only occupation choices and the corresponding wage but not the utility work costs incurred by individuals. Therefore, the informational constraints that the government faces when imposing a minimum wage policy and a nonlinear tax/transfer system are well defined and mutually consistent. In such a model, we show that a minimum wage is desirable if rationing is efficient and the government values redistribution toward low skilled workers. This result can be seen as an application of the Guesnerie (1981) and Guesnerie and Roberts (1984) theory of quantity controls in second best economies: When the government values redistribution toward low skilled workers, the optimal tax/transfer system over-encourages the supply of low skilled labor. In that context, a minimum wage effectively rations over-supplied low skilled labor which is socially desirable. Put in another way, with a minimum wage rationing low skilled jobs, the government can increase redistribution toward those low skills workers without inducing any adverse supply response. Theoretically, the minimum wage under efficient rationing sorts individuals into work and unemployment based on their unobservable cost of work. As a result, the minimum wage partially reveals costs of work in a way the tax and transfer system cannot.⁴

When labor supply responses are along the participation margin, we show that a minimum wage should always be associated with work subsidies (such as the Earned Income Tax Credit in the United States). Consequently, imposing positive tax rates on work on minimum wage workers is second-best Pareto inefficient: Cutting taxes on low income workers while reducing the (pre-tax) minimum wage leads to a Pareto improvement. This latter result remains true even if rationing is not efficient and might be of wide application in many OECD countries.

⁴Unsurprisingly, we show that if rationing is uniform (and hence does not reveal anything on costs of work), then the minimum wage cannot improve upon the optimal tax/transfer allocation.
which have significant minimum wages combined with high tax rates on low skilled work.

We derive optimal formulas for the jointly optimal tax/transfer system and minimum wage. The formulas as well as numerical simulations show that – as in the basic case without taxes and transfers – the optimal minimum wage with optimal taxes continues to be decreasing in the demand elasticity for low skilled work, increasing in the supply elasticity for low skilled work, and follows an inverted U-shape pattern with respect to the strength of redistributive tastes.

The remainder of the paper is organized as follows. Section 2 provides an overview of the existing literature most relevant to our analysis. Section 3 presents the basic two skill model with extensive labor supply responses and analyzes optimal minimum wage policy in a situation with no taxes. Section 4 introduces taxes and transfers and analyzes joint optimal minimum wage policy and taxes and transfers. Section 5 presents illustrative numerical simulations. Section 6 offers a brief conclusion. Formal technical proofs of our propositions are presented in appendix A while appendix B contains several extensions such as “uniform rationing” or more general labor supply responses.

2 Existing Literature

The basic point that a large demand elasticity for low skilled workers implies that the negative employment effects of a minimum wage will be large has been recognized for a long time (see e.g. the classic studies by Pigou 1920 and Stigler 1946). A well-known related point is that, if the demand elasticity is larger than one in absolute value, then a minimum wage reduces total pay going to low skilled workers (see e.g. Freeman 1996, Dolado, Felgueroso, and Jimeno 2000, or Danziger, 2006). By contrast, our analysis reveals no special significance to the absolute demand elasticity being one, and additionally highlights the importance of labor supply elasticities. We can divide the recent normative literature on the minimum wage into two strands.

One literature, most closely associated with labor economics, focuses on efficiency effects of the minimum wage in the presence of labor market imperfections. It is well known, at least since Robinson (1933), that if the labor market is monopsonistic, then a minimum wage can actually increase both employment and low skilled wages and hence improve efficiency (see
e.g., Card and Krueger 1995 or Manning 2003 for recent expositions). A number of papers have shown that the monopsony logic for the desirability of the minimum wage extends to other models of the labor market with frictions or informational asymmetries such as efficiency wages (Drazen, 1986, Jones, 1987, Rebiter and Taylor, 1995), bargaining models (Cahuc, Zylberberg, and Saint-Martin, 2001), signalling models (Lang, 1987), search models (Swinnerton, 1996, Acemoglu 2001, Flinn, 2006), Keynesian macro models (Foellmi and Zweimüller, 2007), or endogenous growth models (Cahuc and Michel, 1996). These studies focus on efficiency and generally abstract from the government’s redistributive goals, and do not consider the role of the minimum wage when taxes and transfers are available to achieve these goals.

A second smaller literature in public economics has investigated whether the minimum wage is desirable for redistributive reasons in situations where the government can also use optimal taxes and transfers for redistribution. The general principle, following Allen (1987) and Guesnerie and Roberts (1987), is that a minimum wage is desirable if it expands the redistributive power of the government by relaxing incentive compatibility constraints. In the context of the two-skill Stiglitz (1982) model with endogenous wages, Allen (1987) and Guesnerie and Roberts (1987) show that a minimum wage can sometimes usefully supplement an optimal linear tax but is never useful to supplement an optimal nonlinear tax even in the most favorable case where unemployment is efficiently shared. This result is obtained because a minimum wage does not prevent in any way high skilled workers from imitating low skilled workers in the Stiglitz (1982) model. This is in contrast to our occupational model and we later return to this important difference. By contrast, Boadway and Cuff (2001), using a continuum of skills model as in Mirrlees (1971), show that a minimum wage policy combined with forcing non-working welfare recipients to look for jobs and accept job offers indirectly reveals skills at the bottom of the distribution and this feature can be exploited by the government to target welfare on low skilled individuals and improve upon the standard Mirrlees (1971) allocation.

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5Allen (1987) notes, consistently with our results, that the minimum wage is more likely to be desirable when the labor supply elasticity is high.

6Marceau and Boadway (1994) build upon those papers and show that a minimum wage can be desirable when a participation constraint for low skilled workers is introduced. Although Marceau and Boadway do not explicitly model this participation constraint using fixed costs of work as we do, their paper can be seen as a first step in incorporating the labor force participation decision in the problem.

7Remarkably, this result is obtained in a fixed wage model where the minimum wage destroys all jobs below the minimum wage.
As recognized by Guesnerie and Roberts (1987), these contrasting results stem in part from informational inconsistencies that arise when a minimum wage is introduced: The minimum wage implementation requires observing the wage rates while the income tax is based on earnings because wage rates and hours of work are assumed to be not observable separably for tax purposes. If wage rates are directly observable, then the government can achieve any first best allocation by conditioning taxes and transfers on immutable wage rates (and obviously, no minimum wage would be needed). The negative results on the desirability of the minimum wage of Allen (1987) appear in an environment where the government implicitly observes the wage rates for low-skilled workers – a necessity when implementing a minimum wage – yet ignores this extra information in choosing the income tax. On the other hand, the positive results of Boadway and Cuff (2001) are obtained because the government uses other tools that implicitly exploit the information revealed by the minimum wage.\textsuperscript{8} Our analysis resolves this informational inconsistency by abstracting from the hours of work decision and focusing only on job choice and work participation decisions.\textsuperscript{9}

Finally, some recent studies have brought together those two literature strands and explored the issue of joint optimal minimum wages and optimal taxes and transfers in imperfect labor markets. Blumkin and Sadka (2005) consider a signalling model where employers do not observe productivities perfectly and show that a minimum wage can be desirable to supplement the optimal tax system in that context. Cahuc and Laroque (2007) show that, in a monopsonistic labor market model, with participation labor supply responses only, the minimum wage should not be used when the government can use optimal nonlinear income taxation. Hungerbuhler and Lehmann (2007) analyze a search model and show that a minimum wage can improve welfare even with optimal income taxes if the bargaining power of workers is sufficiently low. There, however, if the government can directly increase the bargaining power of workers, then the desirability of the minimum wage vanishes. These latter two papers are closest to ours because they also abstract from the hours of work choice and consider

\textsuperscript{8} Some papers have actually explicitly modelled limitations on the use of taxes and transfers using political economy arguments. In that context, a minimum wage can be a useful tool for redistribution (see e.g., Drèze and Gollier 1993 and Bacache and Lehmann 2005).

\textsuperscript{9} Although informational consistency is conceptually appealing, governments do use minimum wages based on hours of work and income taxes based on earnings. Hence, it is still useful to consider the constrained optimization problem combining taxes on earnings and minimum wage rates. Therefore, we will try and explain in detail the deeper economic reasons why our results differ from those of Allen (1987).
only the participation margin for labor supply. Our analysis, however, considers the simpler case of perfect competition with no market frictions. Therefore, we see our contribution as complementary to those of Cahuc and Laroque (2007) and Hungerbuhler and Lehmann (2007).

3 Optimal Minimum Wage with no Taxes/Transfers

3.1 Basic Model

• Demand Side

We consider a simple two labor input model where production of a unique consumption good $F(h_1, h_2)$ depends on the number of low skilled workers $h_1$ and the number of high skilled workers $h_2$. We assume perfectly competitive markets so that firms take the wages $(w_1, w_2)$ as given. The production sector chooses labor demand $(h_1, h_2)$ to maximize profits: $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2$, which leads to the standard first order conditions where wages are equal to marginal product:

$$w_i = \frac{\partial F}{\partial h_i}, \quad (1)$$

for $i = 1, 2$. We assume that, in any equilibrium, $w_1 < w_2$. We also assume constant returns to scale so that there are no profits in equilibrium: $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2 = 0$.

• Supply Side

In the basic model, we assume that each individual is either low skilled or high skilled. We normalize the population of workers to one and denote by $h_1^0$ and $h_2^0$ the fraction of low and high skilled with $h_1^0 + h_2^0 = 1$. Each worker faces a cost of working $\theta$ representing disutility of work. In order to generate smooth supply curves, we assume that $\theta$ is distributed according to smooth cumulated distributions $P_1(\theta)$ and $P_2(\theta)$ for low and high skill individuals respectively. There are three groups of individuals: group 0 for individuals (either low or high skilled) out of work and with zero earnings, group 1 for low skilled workers earning $w_1$, and group 2 for high skilled workers earning $w_2$. We denote by $h_i$ the fraction of individuals in each group $i = 0, 1, 2$.

In this section, we assume that there are no taxes and transfers. For simplicity and throughout the paper, we assume no income effects in the labor supply decision.\textsuperscript{10} An individual with

\textsuperscript{10}The presence of income effects would not change our key results as we show in Appendix B.3.
skill $i$ and cost of work $\theta$ makes its binary labor supply decision $l = 0, 1$ in order to maximize utility $u = w_i \cdot l - \theta \cdot l$. Therefore, $l = 1$ if and only if $\theta \leq w_i$. Hence, the aggregate labor supply functions for $i = 1, 2$ are:

$$h_i = h_i^0 \cdot P_i(w_i). \quad (2)$$

We denote by $e_i$ the elasticity of labor supply $h_i$ with respect to the wage $w_i$:

$$e_i = \frac{w_i}{h_i} \frac{\partial h_i}{\partial w_i} = \frac{w_i \cdot p_i(w_i)}{P_i(w_i)},$$

where $p_i = P_i'$ is the density distribution of $\theta$.

**Competitive Equilibrium and Labor Demand**

Combining the demand and supply side equations (1) and (2) defines a single undistorted competitive equilibrium denoted by $(w^*_1, w^*_2, h^*_1, h^*_2)$.

We depict on Figure 1 the competitive equilibrium for low skilled labor using standard supply and demand curve representation. The supply curve is defined as $h_1 = h_1^0 P_1(w_1)$. Because of constant returns to scale in production, only the ratio $h_1/h_2$ is well defined on the demand side. For our purposes, we define the demand for low skilled work $h_1 = D_1(w_1)$ as follows: $D_1(w_1)$ is the level of demand when $w_1$ is set exogenously by the government (such as with a minimum wage policy) and $(h_2, w_2)$ is defined as the market clearing equilibrium on the high skilled labor market. Therefore, Figure 1 captures implicitly general equilibrium effects as well.\(^\text{11}\) The low skilled labor demand elasticity $\eta_1$ is defined as:

$$\eta_1 = -\frac{w_1}{h_1} \cdot D'_1(w_1), \quad (3)$$

where the minus sign normalization is used so that $\eta_1 > 0$.

**Government Social Welfare Objective**

We assume that the government evaluates outcomes using a standard social welfare function of the form: $SW = \int G(u)dv$ where $u \rightarrow G(u)$ is an increasing and concave transformation of the individual money metric individual utilities $u = w_i - \theta \cdot l$. The concavity of $G(.)$ can

\(^{11}\)For example, in the case of a CES production function $F(h_1, h_2) = (a_1 h_1^{(\sigma-1)/\sigma} + a_2 h_2^{(\sigma-1)/\sigma})^{\sigma/\sigma-1}$, the ratio of the demand side equations (1) implies that $h_1 = h_2 \cdot (a_1/a_2)^\sigma \cdot (w_2/w_1)^\sigma$. The no profit condition $F = w_1 h_1 + w_2 h_2$ implies that $a_1^w w_1^{1-\sigma} + a_2^w w_2^{1-\sigma} = 1$, which defines $w_2(w_1)$ as a function of $w_1$. The supply equation $h_2 = h_2^0 P_2(w_2)$ then defines $h_2(w_1)$ as a function $w_1$. Therefore, we have $D_1(w_1) = h_2(w_1) \cdot (a_1/a_2)^\sigma \cdot (w_2(w_1)/w_1)^\sigma$. 

7
represent either the individuals risk aversion and/or the redistributive tastes of the government. Given the structure of our basic model, we can write social welfare as:

\[
SW = (1 - h_1 - h_2)G(0) + h_1^0 \int G(w_1 - \theta)p_1(\theta)d\theta + h_2^0 \int_0^{w_2} G(w_2 - \theta)p_2(\theta)d\theta. \tag{4}
\]

With no minimum wage, integration in the second term of (4) goes from \( \theta = 0 \) to \( w_1 \) but not when a minimum wage is binding as we discuss below. It is useful for our analysis to introduce the concept of social marginal welfare weights at each occupation. Formally, we define \( g_0 = G'(0)/\lambda \) and \( g_i = h_i^0 \int G'(w_i - \theta)p_i d\theta/(\lambda \cdot h_i) \) the average social marginal welfare weight of individuals in occupation \( i = 1, 2 \). The normalization factor \( \lambda > 0 \) is chosen so that those weights average to one: \( h_0g_0 + h_1g_1 + h_2g_2 = 1.12 \)

Intuitively, \( g_i \) measures the social marginal value of redistributing one dollar uniformly across all individuals in occupation \( i \).

In our model, because individuals cannot be forced to work, workers are better off than non-workers, hence concavity of \( G(.) \) implies that \( g_0 > g_1 \) and \( g_0 > g_2 \).

### 3.2 Desirability of the Minimum Wage

Starting from the market equilibrium \( (w_1^*, w_2^*, h_1^*, h_2^*) \) and as illustrated on Figure 1, we introduce a small minimum wage just above the low skill wage \( w_1^* \), which we denote by \( \bar{w} = w_1^* + d\bar{w} \). Formally, the small minimum wage creates changes \( dw_1, dw_2, dh_1, dh_2 \) in our key variables of interest. By definition, \( dw_1 = d\bar{w} \). From \( \Pi = F(h_1, h_2) - w_1h_1 - w_2h_2 \), we have \( d\Pi = \sum_i(\partial F/\partial h_i)dh_i - w_i dh_i - h_i dw_i = -h_1 dw_1 - h_2 dw_2 \) using (1). The no profit condition \( \Pi = 0 \) then implies that \( d\Pi = 0 \) and hence:

\[
h_1 dw_1 + h_2 dw_2 = 0. \tag{5}
\]

Equation (5) is fundamental and shows that the earnings gain of low skilled workers \( h_1 dw_1 > 0 \) (the red dashed rectangle on Figure 1) due to the small minimum wage is exactly compensated by an earnings loss of high skilled workers \( h_2 dw_2 < 0 \). If \( g_2 > g_1 \), i.e., the government values redistribution from high skilled workers to low skilled workers, such a transfer is socially desirable.

However, in addition to this transfer, the minimum wage also creates involuntary unemployment as depicted on Figure 1. To evaluate the welfare cost of the involuntary unemployment,

\[12\text{In Section 4, we will show that } \lambda \text{ is naturally the multiplier of the government budget constraint when the government uses taxes and transfers.}\]
we will make the important assumption of efficient rationing.

**Assumption 1 Efficient Rationing:** Workers who involuntary lose their job because of the minimum wage are those with those with the least surplus from working.

Conceptually, the minimum wage creates involuntary unemployment and hence an allocation problem: which workers become involuntarily unemployed due to the minimum wage? Under costless Coasian bargaining, this allocation problem would be resolved efficiently: a worker with low surplus would be willing to let an unemployed worker with high surplus from working take her job in exchange for a private transfer, leading to efficient rationing overall. In practice, the efficient allocation might be reached because workers with the least surplus would be the most likely to quit through natural attrition and because, if turnover is costly, employers may seek to first lay off workers who are least likely to be stable employees (i.e., those with low surplus from the job).\(^\text{13}\)

In the end, which workers lose their job because of the minimum wage is an empirical question. Unfortunately, empirical work on this question is thin. In the United States, evidence of unemployment effects is stronger among teenagers and secondary earners (Neumark and Wascher 2006) who are likely to be more elastic and hence lower surplus suggesting that rationing might be efficient. More directly, Luttmer (2007) has shown using variations in state minimum wages that (proxies for) reservation wages do not increase following an increase in the minimum wage suggesting that minimum wage induced rationing if efficient.\(^\text{14}\) Obviously, the case with efficient rationing is the most favorable to minimum wage policy. Therefore, we also explore in detail in appendix B.1 how our result change if we assume instead that unemployment losses are distributed independently of surplus.

Under efficient rationing, as can be seen on Figure 1, as long as the supply elasticity is positive (non vertical supply curve) and the demand elasticity is finite (non horizontal demand curve), those who lose their job because of \(d\bar{w}\) have infinitesimal surplus. Therefore, the welfare loss due to involuntary unemployment due to the minimum wage is second order and represented by the dashed green triangle (exactly as in the standard Harberger deadweight burden analysis). Therefore, we can state our first result.

\(^{13}\)It is conceivable, however, that resources (such as search costs or queuing costs, could be dissipated in reaching the efficient allocation.

\(^{14}\)This is in contrast to a situation with low turnover as in the housing market with rent control as in Glaeser and Luttmer (2003).
Proposition 1  With no taxes and transfers and under the efficient rationing assumption 1, introducing a minimum wage is desirable if (1) the government values redistribution from high skilled workers toward low skilled workers \((g_1 > g_2)\), (2) the demand elasticity for low skilled workers is finite, and (3) the supply elasticity of low skilled workers is positive.

The formal proof is presented in Appendix A.1. It is useful to analyze briefly the desirability of the minimum wage when either of those three conditions breaks down. Obviously, condition (1) is necessary. The condition obviously fails if the government does not care about redistribution at all \((g_1 = g_2)\). It also fails in the extreme case where the government has Rawlsian preferences and cares only about those out of work and hence values equally (at zero) marginal income to low and high skilled workers \((g_1 = g_2 = 0)\). Therefore, a minimum wage is desirable only for intermediate redistributive tastes. Even, in that case, condition (1) may fail if minimum wage workers actually belong to well-off families (for example teenagers or secondary earners).15

Condition (2) is also necessary. If the demand elasticity is infinite (which in the context of our simple model is equivalent to assuming that low and high skill workers are perfect substitutes, (so that \(F = a_1h_1 + a_2h_2\) with fixed parameters \(a_1, a_2\)), then any minimum wage set above the competitive wage \(w^*_1 = a_1\) will shut down entirely the low skilled labor market and hence cannot be desirable. A large body of empirical work has shown that the demand elasticity for low skilled labor is not infinite (see e.g. Hamermesh (1996) for a survey). Related, evidence of a spike in the wage density distribution at the minimum wage also implies a finite demand elasticity (Card and Krueger 1995).

When condition (3) breaks down and the supply elasticity is zero, then there are no marginal workers with no surplus from working. Therefore, the unemployment welfare loss is no longer second order. In that context, whether a minimum wage is desirable depends on the parameters of the model (reservation wages of low skilled workers and the size of the demand elasticity).16 Empirically, a large body of work has shown that there are substantial participation supply elasticities for low skilled workers (see e.g., Blundell and MaCurdy, 1999

15 It would be straightforward to capture such an effect in our model by assuming that utility depends also on other household members income. Johnson and Browning (1983) and Burkhauser, Couch, and Glenn (1996) analyze empirically this issue in the United States.

16 The well known result that a minimum wage cannot be desirable if \(\eta_1 > 1\) is based on such a model with fixed labor supply.
for a survey).

3.3 Optimal Minimum Wage

Let us now derive the optimal minimum wage when the conditions of Proposition 1 are met. As displayed in Figure 2, with a non infinitesimal minimum wage $\bar{w} > w^*_1$, we can define $\bar{w}$ as the reservation wage (or equivalently the cost of work) of the marginal low skilled worker (i.e., the worker getting the smallest surplus from working). Formally, $\bar{w}$ is defined so that $h^0_1 P_1(\bar{w}) = D_1(\bar{w})$. The government picks $\bar{w}$ to maximize

$$\text{SW} = (1 - D_1(\bar{w}) - h_2)G(0) + h^0_1 \int_0^\bar{w} G(w_1 - \theta)p_1(\theta)d\theta + h^0_2 \int_0^{\bar{w}_2} G(w_2 - \theta)p_2(\theta)d\theta,$$

subject to the constraints that $w_i = \partial F/\partial h_i$ for $i = 1, 2$, the no profit condition $h_1 w_1 + h_2 w_2 = F(h_1, h_2)$, and $h_2 = h^0_2 P_2(w_2)$. We formally solve this maximization problem in appendix A.1.

In order to obtain an intuitive understanding of the first order condition for the optimal minimum wage $\bar{w}$, we consider a small change $d\bar{w}$ around $\bar{w}$. Figure 2 shows that this change has two effects.

First, it creates a transfer $h_1 d\bar{w}$ toward low skilled workers at the expense of high skilled workers (as $h_2 dw_2 = -h_1 d\bar{w}$ from the no-profit condition (5)). Using the definition of $g_i$ introduced earlier, the net social value of this transfer is $dT = \left[ g_1 - g_2 \right] h_1 d\bar{w}$.

Second, the minimum wage increases involuntary unemployment by $dh_1 = D'_1(\bar{w})d\bar{w} = -\eta_1 h_1 d\bar{w}/\bar{w}$. Using the efficient rationing assumption, those marginal workers have a reservation wage equal to $w$. Therefore, each worker becoming unemployed generates a social welfare cost equal to $G(\bar{w} - w) - G(0)$. We can define $g^*_0 = [G(\bar{w} - w) - G(0)]/[\lambda \cdot (\bar{w} - w)]$ as the marginal welfare weight put on earnings lost due to unemployment. Thus, the welfare cost due to unemployment is $dU = -g^*_0 \cdot (\bar{w} - w) \cdot \eta_1 \cdot h_1 d\bar{w}/\bar{w}$.

Note that the change $dh_2 < 0$ does not generate welfare effects because marginal workers in the high skill sector have no surplus from working and hence the welfare cost is second order. At the optimum, we have $dT + dU = 0$, which implies:

$$\frac{\bar{w} - w}{w} = \frac{g_1 - g_2}{\eta_1 \cdot g^*_0}.$$

Formula (7) shows that the optimum minimum wage wedge (defined as $(\bar{w} - w)/\bar{w}$) is decreasing in the labor demand elasticity $\eta_1$ as a higher elasticity creates larger negative unemployment.
effects. The optimum wedge is increasing with $g_1 - g_2$ which measures the net value of transferring $\$1$ from high skilled workers to low skilled workers, and decreasing in $g_0^e$ which measures the social cost of earnings losses due to involuntary unemployment. Obviously $g_0^e$, $g_1$, and $g_2$ are endogenous parameters and depend on the primitive social welfare function $G(.)$ but also on the level of the minimum wage. At the optimum, however, we have $g_0^e \geq g_1 \geq g_2$. Increasing the redistributive tastes of the government by choosing a more concave $G(.)$ function has an ambiguous effect on the level of the optimum $\bar{w}$ because it is likely to increase both $g_1 - g_2$ and $g_0^e$. As discussed above, the minimum wage should not be used if the government does not value redistribution at all ($g_1 = g_2$) or if the government has has extreme Rawlsian tastes ($g_1 = g_2 = 0$). Therefore, the level of the optimum $\bar{w}$ is expected to follow an inverted U-shape with the level of redistributive tastes.

Formula (7) is not an explicit formula because it depends on $w$ which itself depends on $\bar{w}$ through the supply function (as illustrated on Figure 2). However, if we assume that the elasticities of demand $\eta_1$ and supply $e_1$ are constant, then we can obtain explicit formulas. In that case $D_1(w_1) = D_0 \cdot w_1^{-\eta_1}$ and $S_1(w_1) = S_0 \cdot w_1^{e_1}$ so that $S_0 \cdot w_1^{e_1} = D_0 \cdot w_1^{-\eta_1}$ and $S_0 \cdot w^{e_1} = D_0 \cdot \bar{w}^{-\eta_1}$. This implies that $w = w_1^{e_1} \cdot (w_1^{e_1}/\bar{w})^{\eta_1/e_1}$, and hence:

$$\frac{\bar{w} - w}{\bar{w}} = 1 - \left(\frac{w_1^{e_1}}{\bar{w}}\right)^{1+\eta_1/e_1}. $$

Formula (7) can be rewritten as:

$$\frac{\bar{w}}{w_1^{e_1}} = \left(1 - \frac{g_1 - g_2}{g_0^e \cdot \eta_1}\right)^{-\frac{e_1}{\eta_1}} \approx 1 + \frac{e_1}{\eta_1} \cdot \frac{g_1 - g_2}{g_0^e \cdot \eta_1},$$

(8)

where the approximation holds in the case of a small minimum wage (i.e., when $(g_2 - g_1)/(g_0^e \cdot \eta_1)$ is small). The formula shows that the optimum minimum wage $\bar{w}$ is decreasing in the supply elasticity $e_1$. The intuition can be easily understood from Figure 2. A higher supply elasticity, implies a flatter supply curve, and hence lower costs from involuntary unemployment. If the supply elasticity is high, then a small change in $w_1$ has large effects on supply, implying that workers derive little surplus from working and hence do not lose much from minimum wage induced unemployment. This result is very important because, as is well known, redistribution through taxes and transfers is hampered by a high supply elasticity. Conversely, when the supply elasticity is low, redistribution through the minimum wage is costly while redistribution through taxes and transfers is efficient.
Formula (8) shows that there are two channels through which a higher demand elasticity \( \eta_1 \) reduces the optimal minimum wage. The first channel is the standard *unemployment level effect* mentioned above when discussing (7) that higher demand elasticity creates a larger unemployment response to the minimum wage. The second channel is an *unemployment cost effect* which works through the link between the wedge \((\bar{w} - w)/\bar{w}\) and the minimum wage markup \(\bar{w}/w^*_1\). A higher demand elasticity implies that a given minimum wage markup is associated with a larger wedge, hence higher unemployment costs for the marginal worker. The distinction between those two channels is important because we will see that the first classical unemployment level effect disappears with optimal taxes and transfers but the unemployment cost effect remains.

The logic of the optimal minimum wage formula we have derived easily extends to a more general model with many labor inputs (including a continuum with a smooth wage density), a capital input or pure profits, and many consumption goods. In that context, \(g_2\) is the average social welfare weight across each factor bearing the incidence of the minimum wage increase. Some of the factors can have a negative weight in this average. For example, if there are neo-classical spillovers of a minimum wage increase to slightly higher paid workers (as in Teulings, 2000), it is conceivable that \(g_2\) could be negative. Conversely, if a minimum wage increase leads to higher consumption prices for goods consumed by low income families (such as fast food), then \(g_2\) would be higher (and conceivably even above \(g_1\) if minimum wage workers belong to families better-off than fast food consumers).

4 Optimal Minimum Wage with Taxes and Transfers

4.1 Introducing Taxes and Transfers

We assume that the government can observe job outcomes (not working, work in sector 1 paying \(w_1\), or work in sector 2 paying \(w_2\)) but does not observe costs of work. Therefore, the government can condition tax and transfers only on those observable work outcomes. Let us denote by \(T_i\) the tax (or transfer if \(T_i < 0\)) on occupation \(i\). We denote by \(c_i = w_i - T_i\) the disposable income in occupation \(i = 0, 1, 2\). This is a fully general nonlinear income tax on earnings.

As in our previous model without taxes, an individual with skill \(i = 1, 2\) who decides to
work earns $w_i$ but increases his disposable by $c_i - c_0$. Hence we can naturally define a tax rate $\tau_i$ on skill $i$ workers: $1 - \tau_i = (c_i - c_0)/w_i$. An individual of skill $i = 1, 2$ and with costs of work $\theta$ works if and only if $\theta \leq c_i - c_0 = (1 - \tau_i)w_i$. Hence, the aggregate labor supply functions for $i = 1, 2$ are:

$$h_i = h_i^0 \cdot P_i((1 - \tau_i)w_i) = h_i^0 \cdot P_i(c_i - c_0). \tag{9}$$

As above, we denote by $e_i$ the elasticity of labor supply with respect to the net-of-tax wage rate $w_i(1 - \tau_i) = c_i - c_0$:

$$e_i = \frac{(1 - \tau_i)w_i}{h_i} \frac{\partial h_i}{\partial (1 - \tau_i)w_i} = \frac{(1 - \tau_i)w_i \cdot P_i((1 - \tau_i)w_i)}{P_i((1 - \tau_i)w_i)},$$

The demand side of the economy is unchanged. For given parameters $c_0$, $\tau_1$, $\tau_2$ defining a tax and transfer system, the four equations (1) and (9) for $i = 1, 2$ define the competitive equilibrium $(h_1^*, h_2^*, w_1^*, w_2^*)$.

Assuming no exogenous spending requirement, the government budget constraint can be written as:

$$h_0c_0 + h_1c_1 + h_2c_2 \leq h_1w_1 + h_2w_2. \tag{10}$$

We denote by $\lambda$ the multiplier of the government budget constraint.

### 4.2 Minimum Wage Desirability with Fixed Tax Rates

Let us first analyze how our previous analysis on the desirability of the minimum wage is affected in the presence of taxes and transfers assuming that $\tau_1$, $\tau_2$ are exogenously fixed and that the transfer $c_0$ adjusts automatically to meet the government budget constraint when a small minimum wage $\bar{w} = w_1^* + d\bar{w}$ is introduced. We assume that the minimum wage applies to wages before taxes and transfers.\(^{18}\) This assumption does not affect the desirability of a minimum wage and is the most convenient convention.

**Proposition 2** With fixed tax rates $\tau_1, \tau_2$, under the efficient rationing assumption 1 and assuming $e_1 > 0$ and $\eta_1 < \infty$, introducing a minimum wage is desirable iff

$$g_1 \cdot (1 - \tau_1) - g_2 \cdot (1 - \tau_2) + \tau_1 - \tau_2 - \tau_2 \cdot e_2 - \tau_1 \cdot \eta_1 > 0. \tag{11}$$

\(^{17}\)None of our results would be changed if we assumed a positive exogenous spending requirement for the government.

\(^{18}\)In practice, the legal minimum wage applies to wages net of employer payroll taxes but before employee payroll taxes, income taxes, and transfers. $\bar{w}$ should be interpreted as the minimum wage including employer taxes.
The proof is presented in appendix A.2.

When \( \tau_1 = \tau_2 = 0 \), equation (11) boils down to \( g_1 - g_2 > 0 \) (Proposition 1). Equation (11) shows with taxes and transfers, introducing a minimum wage creates four fiscal effects that need to be taken into account in the welfare analysis: first, transferring $1 pre-tax from high skilled workers to low skilled workers through the minimum wage implies a $ \( (1 - \tau_1) \) post tax transfer to low skilled workers and a $ \( (1 - \tau_2) \) post tax loss to high skilled workers, hence the factors \( (1 - \tau_i) \) multiplying \( g_1 \) and \( g_2 \) in (11). Second and related, such a $1 transfer creates a direct net fiscal effect \( \tau_1 - \tau_2 \). Third, the reduction in \( w_2 \) leads to a supply effect which further reduces taxes paid by the high skilled by \( e_2 \cdot \tau_2 \) per dollar transferred. Finally, involuntary unemployment also creates a tax loss equal to \( -\tau_1 \cdot \eta_1 \) per dollar transferred.\(^{19}\)

It is important to note that a minimum wage cannot be replicated with taxes and transfers. Coming back to Figure 1 when there are no taxes, it is tempting to think that a small tax on low skilled work creates indeed the same wedge between supply and demand as the minimum wage. However, to replicate the minimum wage, this small tax should be rebated lump-sum to low skilled workers only. Obviously, if the tax is rebated to low skilled workers, those who dropped out of work because of the tax would want to come back to work. Without a rationing mechanism preventing this labor supply response, taxes and transfers cannot achieve the minimum allocation.

Cahuc and Laroque (2007) make the point that a minimum wage can be replicated by a knife-edge nonlinear income tax such that \( T(w) = w \) for \( w < \bar{w} \) (as nobody would want to work in a job paying less than \( \bar{w} \), employers would be forced to pay at least \( \bar{w} \) to attract workers) and conclude therefore that a minimum wage is redundant with a fully general nonlinear income tax. This argument is mathematically correct but such a knife-edge income tax would effectively be a minimum wage. Our model rules out such knife edge income taxes because we consider tax rates that are occupation specific (rather than wage level specific). However, a fully general knife-edge income tax could not do better than the combination of our occupation specific tax rates combined with a minimum wage. Therefore, we think that the definition of the tax and minimum wage tools we use is the most illuminating to understand the problem.

\(^{19}\)Note that when low skilled work is subsidized (\( \tau_1 < 0 \)), then the unemployment created by a small minimum wage creates a positive fiscal externality proportional to the demand elasticity \( \eta_1 \). In such a situation, introducing a minimum wage would actually be desirable even without redistributive tastes (\( g_1 = g_2 = 1 \)) if \( -\tau_1 \cdot \eta_1 > \tau_2 \cdot e_2 \).
of joint minimum wage and tax optimization.

4.3 Optimal Tax Formulas with no Minimum Wage

The government chooses $c_0, c_1, c_2$ in order to maximize social welfare

$$SW = (1 - h_1 - h_2)G(c_0) + h_1^0 \int_0^{c_1 - c_0} G(c_1 - \theta)p_1(\theta)d\theta + h_2^0 \int_0^{c_2 - c_0} G(c_2 - \theta)p_2(\theta)d\theta,$$

subject to the budget constraint (10) with multiplier $\lambda$. As shown in appendix A.3, we have the following conditions at the optimum:

$$h_0 \cdot g_0 + h_1 \cdot g_1 + h_2 \cdot g_2 = 1, \quad (12)$$

$$\frac{\tau_i}{1 - \tau_i} = \frac{1 - g_i}{e_i}, \quad (13)$$

for $i = 1, 2$. Equation (12) implies that the average of marginal welfare weights across the three groups $i = 0, 1, 2$ is one. Indeed, the value of distributing one dollar to everybody is exactly the average marginal social weight and the cost of distributing one dollar in terms of revenue lost is also one dollar as we have assumed away income effects.\(^{20}\)

Equation (13) can be understood from Figure 3a. Starting from an allocation $(c_0, c_1, c_2)$, and increasing $c_1$ by $dc_1 > 0$ leads to a positive direct welfare effect $h_1 g_1 dc_1 > 0$, a mechanical loss in tax revenue $-h_1 dc_1 < 0$, and a behavioral response increasing work by $dh_1 = dc_1 \cdot e_1 (1 - \tau_1) > 0$ and creating a fiscal effect equal to $\tau_1 w_1 dh_1 = dc_1 \cdot h_1 \cdot e_1 \cdot \tau_1 / (1 - \tau_1)$. The sum of those three effects is zero implying (13).

If $g_1 > 1$, then the optimal tax rate on low skilled work should be negative because the first two terms net out positive so that the fiscal effect due to the behavioral response has to be negative, requiring $\tau_1 < 0$.\(^{21}\)

Formulas (13) are identical to those derived by Saez (2002) in the same model but with fixed wages. Indeed, it is well known since Diamond and Mirrlees (1971), that optimal tax formulas remain the same when producer prices are endogenous.\(^{22}\) Figure 3b illustrates this key point for our subsequent analysis. When $w_1, w_2$ are endogenous, the small reform $dc_1$ leads

\(^{20}\)See appendix B.3. for an analysis with income effects.

\(^{21}\)This was the key result emphasized by Diamond (1980), Saez (2002), Laroque (2005), Choné and Laroque (2005, 2006): an EITC type transfer for low wage workers is optimal in a situation where individuals respond only along the extensive margin.

\(^{22}\)Piketty (1997) and Saez (2004) have shown that the occupational model we consider inherits this important property of the Diamond and Mirrlees (1971) model.
to changes in $h_1$ and hence to changes $dw_1$ and $dw_2$ through demand side effects. However, assuming that $c_2$ and $c_1 + dc_1$ are kept unchanged, the effect of $dw_1$ and $dw_2$ is fiscally neutral because $h_1 dw_1 + h_2 dw_2 = 0$ through the no-profit condition (5).

Let us denote by $(w_i^T, c_i^T)$ the tax/transfer optimum with no minimum wage.

### 4.4 Desirability of the Minimum Wage

As illustrated on Figure 4, starting from the tax/transfer optimum $(w_i^T, c_i^T)$, let us introduce a minimum wage set at $\bar{w} = w_1^T$. Such a minimum wage is just binding and has no direct impact on the allocation. Let us now increase $c_1$ by $dc_1$ while keeping $c_0$ and $c_2$ constant. As we showed above, such a change provides incentives for some low skilled individuals to start working. However, as we showed in Figure 3b, such a labor supply response would reduce $w_1$ through demand side effects. However, in the presence of a minimum wage set at $w_1^T$, $w_1$ cannot fall, which implies that those individuals willing to start working cannot work and actually shift from voluntary to involuntary unemployment. The assumption of efficient rationing is key here as these are precisely the individuals with the lowest surplus from working. Given that the labor supply channel is effectively shut down by the minimum wage, the $dc_1$ change is like a lumpsum tax reform and its net welfare effect is simply $[g_1 - 1]h_1 dc_1$. This implies that if $g_1 > 1$, introducing at minimum wage improves upon the tax/transfer optimum allocation.\textsuperscript{23}

This result is in line with the theory of optimum quantity controls developed by Guesnerie (1981) and Guesnerie and Roberts (1984) showing that, in an optimum tax model, introducing a quantity control on subsidized goods is desirable. In our model, a minimum wage is an indirect way for the government to introduce rationing on low skilled work.\textsuperscript{24}

We show in appendix B.2 that this result generalizes easily to a more general model with many skills and fully general labor supply responses functions where individuals can respond along the (discrete) intensive margin by shifting to lower paid occupations in response to taxes. The logic of the minimum wage desirability remains exactly the same as the one displayed on Figure 4: Even if higher skilled workers wanted to shift to occupation $w_1$ when $c_1$ increases, a minimum wage set at $w_1^T$ would effectively block such a labor supply response (again under

\textsuperscript{23}The fact that a minimum wage is desirable if $g_1 > 1$ can also be seen from Proposition 2 by plugging the optimal tax rates from equations (13). In that case, equation (11) boils down to $-\tau_1 \cdot (e_1 + \eta_1) > 0$ which is indeed equivalent to $g_1 > 1$.

\textsuperscript{24}Guesnerie and Roberts (1987) proposed an analysis of optimal minimum wage. However, the model they considered was not directly related to their quota theory.
our key assumption of efficient rationing).

This remark can help understand why our results contrast with the negative results of Allen (1987) or Guesnerie and Roberts (1987) obtained in the context of the Stiglitz (1982) model of optimal nonlinear taxation. The key theoretical difference between the Stiglitz model and the occupation model we use is that, in the Stiglitz model, high skilled individuals who imitate low skilled individuals just cut their hours of work but remain in the high skill sector and hence the minimum wage makes it easier for them to imitate low skilled workers. In contrast, in our model, the minimum wage effectively prevents higher skilled workers from occupying minimum wage jobs (by rationing low skilled work). Perhaps more importantly practically, absent the minimum wage, everybody works in the Stiglitz model. Therefore, the Stiglitz cannot capture the participation decision of low skilled workers which strikes us as central to the minimum wage problem in the real world.\textsuperscript{25}

Comparing with the case with no taxes of Section 3, we note that the condition $g_1 > 1$ is stronger than the condition $g_1 > g_2$ we had in the case with no taxes (as the $g_i$’s average to one and hence $g_2 < 1$). However, if the government has redistributive tastes, then $g_1 > 1$ is a weak condition as the low skilled sector can be chosen to represent the very lowest income workers. This also implies that, when the government uses taxes optimally and in the presence of many factors of production or many output goods, the incidence of the minimum wage on other factors (captured by the term $g_2$ in the case with no taxes) becomes irrelevant: the government can effectively undo the incidence effects by adjusting taxes on other factors so as to keep their net-of-tax rewards constant.\textsuperscript{26} In particular, whether the minimum wage creates spill-over effects on slightly higher wages and whether the minimum wage increases prices of goods disproportionately consumed by low income families becomes irrelevant when assessing the desirability of the minimum wage in the presence of optimal taxes. The only relevant factor is whether the government values redistribution to minimum wage workers relative to an across the board lumpsum redistribution (i.e., the condition $g_1 > 1$).

Finally, we show in appendix B.1. that the desirability of the minimum hinges crucially

\textsuperscript{25}Indeed, Marceau and Boadway (1994) show that a minimum wage can be desirable in a Stiglitz type model by implicitly adding fixed costs of work (and hence a participation decision) for low skilled workers. Our model has the advantage of explicitly modelling the participation decision and also avoids the information inconsistency inherent to the Stiglitz model with minimum wage.

\textsuperscript{26}This is directly related to the important fact that incidence on pre-tax prices is irrelevant in optimal Diamond-Mirrlees tax formulas.
on the “efficient rationing” assumption. We show that, under “uniform rationing” (where unemployment strikes independently of surplus), the minimum wage cannot improve upon the optimal tax allocation. Indeed, with efficient rationing, the minimum wage effectively reveals the marginal workers to the government. Because costs of work are not observable, this is valuable as it allows the government to sort workers into a more (socially albeit not privately) efficient set of occupations. Therefore, the minimum wage is desirable. In contrast, with uniform rationing, the minimum wage does not reveal anything about costs of work (as unemployment strikes randomly). As a result, the minimum wage just creates (privately) inefficient sorting across occupations but without revealing anything of value to the government. It is not surprising that the minimum would not be desirable in such a context.

4.5 Optimal Minimum Wage with Taxes and Transfers

Formally, the government chooses $\bar{w}, c_0, c_1, c_2$ to maximize

$$SW = (1 - h_1 - h_2)G(c_0) + h_0 \int_0^{\bar{w}(1-h_1)} G(c_1 - \theta) p_1(\theta) d\theta + h_2 \int_{c_0}^{c_2} G(c_2 - \theta) p_2(\theta) d\theta. \quad (14)$$

subject to its budget constraint (with multiplier $\lambda$). As above, $\bar{w}$ is defined as the reservation wage of the marginal worker: $h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$. The first order condition with respect to $c_0$ implies that $h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$. The first order condition with respect to $c_2$ leads to the standard formula (13): $\tau_2/(1 - \tau_2) = (1 - g_2)/e_2$ as the minimum wage does not impact the trade-off for the choice of $c_2$.

We solve this maximization problem formally in Appendix A.4. The first order condition with respect to $c_0$ implies that $h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$. The first order condition with respect to $c_2$ leads to the standard formula (13): $\tau_2/(1 - \tau_2) = (1 - g_2)/e_2$ as the minimum wage does not impact the trade-off for the choice of $c_2$.

With a binding minimum wage, as we illustrated on Figure 4, increasing $c_1$ is a lumpsum transfer. Therefore, the government will increase $c_1$ up to point where $g_1 = 1$. Therefore, the minimum wage allows the government to redistribute to low skill workers at no efficiency cost and hence achieve “full redistribution to low skilled workers”, making the minimum wage a powerful redistributive tool. We show in appendix B.2 that this result generalizes easily to a model with many labor inputs and more general labor supply responses.

Finally, there is a first order condition for the optimal choice of $\bar{w}$. Increasing $\bar{w}$ by $d\bar{w}$ and
keeping \(c_0, c_1, c_2\) constant leads to an increase in involuntary unemployment: \(dh_1 < 0\). Such involuntary unemployment leads to a (negative) welfare effect on those individuals equal to 
\[dh_1[G(c_0 + (\bar{w} - w)(1 - \tau_1)) - G(c_0)]/\lambda < 0\] 
and a fiscal effect equal to \(dh_1 \cdot \tau_1 \cdot \bar{w}\).\(^{27}\) Therefore, the two effects due to \(dh_1\) need to cancel out at the optimum. Hence the fiscal effect needs to be positive which requires \(\tau_1 < 0\) as \(dh_1 < 0\). We then have the following first order condition:
\[-\tau_1 \cdot \bar{w} = \frac{G(c_0 + (\bar{w} - w)(1 - \tau_1)) - G(c_0)}{\lambda}.
\]
As we did in Section 3, we can introduce the social marginal weight on earnings losses due to (marginal) involuntary unemployment: 
\[g_0^e = \frac{[G(c_0 + (\bar{w} - w)(1 - \tau_1)) - G(c_0)]/[\lambda(\bar{w} - w)(1 - \tau_1)]}{\lambda(\bar{w} - w)(1 - \tau_1)}\]
in order to rewrite (15) as:
\[\frac{\bar{w} - w}{\bar{w}} = -\frac{\tau_1}{1 - \tau_1} \cdot \frac{1}{g_0^e} > 0.
\]
We summarize all those results in the following proposition (that is formally proven in Appendix A.4):

**Proposition 3** Under the efficient rationing assumption 1, assuming \(e_1 > 0\) and \(\eta_1 < \infty\), if \(g_1 > 1\) at the optimum tax allocation (with no minimum wage) then introducing a minimum wage is desirable. Furthermore, at the joint min wage and tax optimum, we have:

- \(h_0g_0 + h_1g_1 + h_2g_2 = 1\) (Social welfare weights average to one)
- \(\tau_2/(1 - \tau_2) = (1 - g_2)/e_2 > 0\) (Formula for \(\tau_2\) unchanged)
- \(g_1 = 1\) (Full redistribution to low skilled workers)
- \((\bar{w} - w)/\bar{w} = -\tau_1/[\lambda(1 - \tau_1) \cdot g_0^e] > 0\) (Negative bottom tax rate \(\tau_1 < 0\))

Quantitatively, \(\tau_1\) is primarily determined to meet the condition \(g_1 = 1\). Then, the optimal minimum wage wedge \((\bar{w} - w)/\bar{w}\) is determined by equation (16) and is increasing in the size of the absolute subsidy \(|\tau_1|\) and decreasing in the social weight on unemployment earnings losses \(g_0^e\). As we discussed in Section 3, we can define the implicit market wage rate \(w_1\) as the wage rate that would prevail under the same tax rates \(\tau_1, \tau_2\) but with no minimum wage. In that case, and assuming constant elasticity of supply and demand, we showed that the minimum wage markup over the market wage rate \(\bar{w}/w_1\), for a given minimum wage wedge

\(^{27}\)As usual, the changes in \(dw_1\) and \(dw_2\) induced by the minimum wage change do not have any fiscal consequence as we have \(h_1dw_1 + h_2dw_2 = 0\) due to the no profit condition (5).
was increasing in $e_1$ and decreasing in $\eta_1$. This implies that our previous result that the optimum minimum wage increases with $e_1$ and decreases with $\eta_1$ carries over to the case with optimal taxes. It is important to note that a high demand elasticity leads to a smaller minimum wage not because this creates more unemployment but rather because a large demand elasticity makes unemployment more costly by increasing the wedge $(\bar{w} - w)/\bar{w}$.

The result that the optimum minimum wage follows an inverted U-shape pattern with the strength of redistributive tastes also carries over to the case with optimal taxes. Extreme redistributive tastes such as Rawlsian tastes imply that $g_1 = 0 < 1$ and hence no minimum wage is desirable. Conversely, no redistributive tastes imply that $g_0 = g_1 = g_2 = 1$, a situation where no minimum wage is desirable either.

### 4.6 A Minimum Wage with $\tau_1 > 0$ is 2nd Best Pareto Inefficient

The last result from Proposition 3 on the negativity of $\tau_1$ at the joint minimum wage and tax optimum has a very important corollary:

**Proposition 4** In our model with extensive labor supply responses, a binding minimum wage associated with a positive tax rate at the bottom $\tau_1 > 0$ is second-best Pareto inefficient. This result remains a-fortiori true when rationing is not efficient.

Proposition 4 is illustrated on Figure 5. Suppose that the minimum wage binds and that $\tau_1 > 0$. Suppose that the government reduces the minimum wage by $d\bar{w} < 0$ while keeping $c_0, c_1, c_2$ constant. Reducing the minimum wage leads to a positive employment effect $dh_1 > 0$ as involuntary unemployment is reduced which improves the welfare of the newly employed workers and also increases tax revenue as $\tau_1 > 0$. The increase $dh_1 > 0$ also leads to a change $dw_2 > 0$. However, because $h_1d\bar{w} + h_2dw_2 = 0$ (through the no-profit condition (5)), the mechanical fiscal effect of $d\bar{w}$ and $dw_2$ while keeping $c_1$ and $c_2$ constant is zero. Because $c_0, c_1, c_2$ remain constant, nobody’s welfare is reduced.\(^{28}\) The increase in welfare due to the reduction in unemployment remains a-fortiori true if rationing is not efficient. Therefore, this reform is a (second-best) Pareto improvement.

The results of Proposition 3 do not necessarily carry over to a model with general labor supply functions. For example, if workers respond along the intensive margin, then the min-

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\(^{28}\)Because, $c_2 - c_0$ remains constant, $h_2$ does not change either.
minimum wage generates not only involuntary unemployment but also involuntary over-work as high skilled workers are also rationed out. In that case, a minimum wage decrease would induce higher skilled workers to become minimum wage workers, and this would reduce government revenue. However, the fact that the minimum wage can create over-work is hardly ever discussed in empirical studies suggesting that the intensive response channel is not very important empirically.

Proposition 4 may have wide applicability because many OECD countries, especially in continental Europe, combine significant minimum wages (OECD 1998, Immervoll 2007) with very high tax rates on low skilled work (Immervoll et al. 2007). The very high tax rates are generated by substantial payroll tax rates financing social security benefits and by the high phasing-out rates of traditional means-tested transfer programs.

In practice, the reform described in Proposition 4 could be achieved by cutting the employer payroll taxes for low income workers which lowers the (gross) minimum wage without affecting the net minimum wage after taxes and transfers. Such a policy should stimulate low skilled employment and increase higher wages. Thus, the direct loss in tax revenue due to the payroll tax cut on low skilled workers could be recouped by adjusting upward taxes on higher earning workers. Such policy reforms have actually already been implemented in a number of OECD countries.

5 Numerical Simulations

• Case with no Taxes or Fixed Taxes

We make the following parametric assumptions. (1) We assume a CES production function with elasticity of substitution $\sigma > 0$. (2) We assume constant labor supply elasticities $e_i > 0$ by choosing $P_i(w) = (w/\bar{\theta}_i)^{e_i}$. We assume $(h_1^0, h_2^0) = (1/4, 3/4)$. We assume a CRRA social welfare function $G(u) = (u + B)^{1-\gamma}/(1 - \gamma)$ with risk aversion parameter $\gamma > 0$ and where $B > 0$ is a constant that is used to avoid infinitely negative utility or infinite social marginal utility for non-workers. We calibrate the production function so that $(w_1^*, w_2^*) = (1, 3)$

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29 Politically, it is extremely difficult to cut directly the legal minimum wage.

30 For example, France has started to reduce employer payroll tax on low income workers in the early 1990s (see Crépon and Desplat, 2002 for an empirical analysis).

31 $B$ could represent for example a uniform lumpsum transfer, whose cost is not affected by behavioral responses.
and the labor supply functions so that \((h_0^*, h_1^*, h_2^*) = (0.2, 0.2, 0.6)\) at the no minimum wage equilibrium. We always assume that \(e_2 = 0.25\) and \(B = 0.5\).

Panel A in Table 1 displays the optimum minimum wage markup over the undistorted market wage \(w_1^*\) as well as the involuntary unemployment rate (among all low skilled individuals) under various scenarios for \(e_1\), \(\sigma\), and \(\gamma\). The table confirms that the optimum minimum wage is increasing in \(e_1\) (comparing columns (1), (2), (3)), decreasing in \(\sigma\) (comparing columns (4), (5), (6)), and has an inverted U-shape pattern with \(\gamma\) (comparing panels A1, A2, and A3). The optimal minimum wage is small for a high \(\gamma = 3\) value.

Panel B in Table 1 illustrates numerically that, starting from a substantial flat rate tax where \(\tau_1 = \tau_2 = 0.35\) (and using the same parametrization as in Panel A), the optimal minimum wage is much lower than in the case with no taxes (and is actually useless when \(\sigma = e_1 = 0.25\)).

- **Case with Optimal Taxes**

Table 2 provides some numerical simulation illustrations using the same parametrization as in the situation with no taxes and transfers of Table 1. Table 2 shows the optimal tax rates with no minimum wage, and then displays the optimal tax rates, optimal minimum wage markup (and associated unemployment level among the unskilled) in the case of joint minimum wage/tax optimization. Table 2 confirms our key findings that the minimum wage should be associated with higher low skilled work subsidies than in the case of optimal tax rates with no minimum wage. Table 2 also shows that the optimal minimum wage is increasing with \(e_1\) and decreasing with \(\sigma\). Finally, the minimum wage is useless in the high redistributive case \(\gamma = 3\) as \(g_1 < 1\) at the pure tax optimum.\(^{32}\) Interestingly, comparing Tables 1 and 2 suggests that the minimum wage with optimal taxes is not necessarily smaller than in the case with no taxes, especially in the case where redistributive tastes are not too large \((\gamma = 0.5)\).

\(^{32}\)The fact that the minimum wage is zero is in large part the consequence of the two skill model assumption. A model with many skills would generate \(g_1 > 1\) at the tax optimum except for extreme Rawlsian redistributive tastes. As discussed below, such a model could cast light on where in the wage distribution should the minimum wage be set.
6 Conclusion

Our paper has proposed a theoretical analysis of optimal minimum wage policy for redistribution purposes in a perfectly competitive labor market, considering first the case with no taxes and transfers and then analyzing the case with optimal taxes and transfers. In light of the previous literature on this topic, we find that the standard competitive labor market model offers a surprisingly strong case for using the minimum wage when we make the efficient rationing assumption. A minimum wage is a useful tool if the government values redistribution toward low wage workers and this result remains true in the presence of optimal nonlinear taxes and transfers. In that context, our model of occupational choice abstracting from hours of work allows us to overcome the informational inconsistency that has plagued previous work analyzing minimum wage policy with optimal income taxation. Our model fits into the general theory of rationing developed by Guesnerie (1981) and Guesnerie and Roberts (1984) as a minimum wage effectively rations low skilled labor. Such rationing is desirable because the optimal tax/transfer over-encourages the supply of low skilled labor.

When low skilled labor supply is along the extensive margin, as empirical studies suggest, a minimum wage should always be associated with in-work subsidies: the co-existence of minimum wages with high participation tax rates for low skilled workers is (second-best) Pareto inefficient. In that situation, which is common in most OECD countries, a cut in employer payroll taxes which decreases the gross minimum wage while keeping the net minimum wage constant, combined by an offsetting tax increase on higher skilled workers is Pareto improving.

There are a number of issues that we have abstracted from in our very stylized model that are worth pointing out as caveats and potential avenues for future research.

First, as mentioned, we abstract from the hours of choice decision which allows us to develop a model with no informational inconsistencies. However, the fact remains that, in practice, taxes and transfers are based on earnings while minimum wages are based on hourly rates. In reality, the government can observe both earnings and hours of work of employees as this information is in general included in the payroll accounting of employers and is actually sometimes required to be reported to the government for administering payroll taxes or maximum hours laws. Therefore, the puzzle remains for why taxes and transfers are based on earnings rather than wage rates. A possible explanation is that hours of work are not very
elastic and that most of the labor supply response takes place along the occupation decision and in particular the participation decision. If hours were very elastic, taxes and transfers should be based (at least in part) on wage rates.\textsuperscript{33} We conjecture that our results on the desirability of the minimum wage, would carry over to that case as well.

Second, a minimum wage rationing mechanism operates very differently from a tax and transfer which alters prices but lets markets clear freely. The rationing induced by the minimum wage creates an allocation problem for with no natural market. It is conceivable that the allocation problem might not lead to the efficient rationing allocation or that the transaction costs (such as search costs or queuing costs) necessary to reach that efficient allocation are not negligible. Evaluating such costs using a model with frictions would be valuable.\textsuperscript{34} It is also conceivable, that the rationing and hence involuntary unemployment creates additional psychological costs (such as feelings of low self-worth) that are not captured in standard models (including those with search frictions), and that would make minimum wage policies less attractive in practice.

Finally, our numerical simulations have been purely illustrative and it would be worth trying to calibrate the simulations using empirically estimated parameters for the wage distribution, labor demand and supply elasticities, and the degree of efficiency of the rationing created by the minimum wage.

\textsuperscript{33}Some transfer programs are based partly on hours information. For example, the British Working Families Credit is given only to families where one earner works at least 16 hours a week. Similarly, the current US welfare program TANF imposes work requirements which is an indirect way of conditioning transfers on hours of work.

\textsuperscript{34}Hungerbuhler and Lehmann (2007) have made an important step in this direction by analyzing optimal minimum wage policy with optimal tax in a search model.
A Appendix: Formal Proofs

A.1 Proof of Proposition 1 and Formula (7)

Social welfare is given by:

\[ SW(\bar{w}) = [1 - D_1(\bar{w}) - h_0^0 \cdot P_2(w_2)]G(0) + h_1^0 \int_0^{\bar{w}} G(\bar{w} - \theta)p_1(\theta)d\theta + h_2^0 \int_0^{\bar{w}} G(w_2 - \theta)p_2(\theta)d\theta, \]

where \( \bar{w} \) is defined as \( h_0^0 \cdot P_1(\bar{w}) = D_1(\bar{w}) \). We have:

\[
\frac{dSW}{d\bar{w}} = -D_1'(\bar{w})G(0) - h_0^0 \cdot \frac{dw_2}{d\bar{w}} p_2(w_2)G(0) + h_1^0 \cdot G(\bar{w} - \bar{w})p_1(\bar{w}) \cdot \frac{dw}{d\bar{w}} + h_1^0 \int_0^{\bar{w}} G'(\bar{w} - \theta)p_1(\theta)d\theta
\]

\[ + h_2^0 \cdot \frac{dw_2}{d\bar{w}} \int_0^{w_2} G'(w_2 - \theta)p_2(\theta)d\theta + h_2^0 \cdot \frac{dw_2}{d\bar{w}} G(0) \cdot p_2(w_2). \]

The second and last term cancel out (as marginal workers are indifferent between working or not). The no-profit condition \( F(h_1, h_2) = w_1 h_1 + w_2 h_2 \) implies that \( h_1 d\bar{w} + h_2 dw_2 = 0 \) so that \( dw_2/d\bar{w} = -h_1/h_2 \). Furthermore, \( h_1^0 \cdot P_1(\bar{w}) = D_1(\bar{w}) \) implies that \( h_1^0 \cdot p_1(\bar{w})d\bar{w} = D_1'(\bar{w})d\bar{w} \). Therefore, we have:

\[
\frac{dSW}{d\bar{w}} = D_1'(\bar{w})[G(\bar{w} - \bar{w}) - G(0)] + h_1^0 \int_0^{\bar{w}} G'(\bar{w} - \theta)p_1(\theta)d\theta - h_1^0 \cdot \frac{P_1}{P_2} \int_0^{w_2} G'(w_2 - \theta)p_2(\theta)d\theta
\]

\[ = -\eta_1 \cdot g_0^e \cdot \frac{\bar{w} - \bar{w}}{\bar{w}} \cdot h_1 \cdot \lambda + [g_1 - g_2] \cdot h_1 \cdot \lambda, \]

where we have used the definitions of \( \eta_1, g_0^e, g_1, g_2 \) in the last equality. Thus, starting from the competitive equilibrium where \( \bar{w} = w = w_1^* \), the first term is zero and hence the minimum wage is desirable if and only if \( g_1 > g_2 \) which proves Proposition 1. At the optimum \( \bar{w} \), \( dSW/d\bar{w} = 0 \) which leads immediately to formula (7). □

A.2 Proof of Proposition 2

Social welfare is given by:

\[ SW(\tilde{w}) = [1 - D_1(\tilde{w}) - h_0^0 \cdot P_2(w_2(1 - \tau_2))]G(c_0) + h_1^0 \int_0^{\tilde{w}(1 - \tau_1)} G(c_0 + \tilde{w}(1 - \tau_1) - \theta)p_1(\theta)d\theta \]

\[ + h_2^0 \int_0^{w_2(1 - \tau_2)} G(c_0 + w_2(1 - \tau_2) - \theta)p_2(\theta)d\theta, \]

where \( \tilde{w} \) is defined as \( h_0^0 \cdot P_1(\tilde{w}(1 - \tau_1)) = D_1(\tilde{w}) \). The government budget constraint is \( c_0 \leq D_1(\tilde{w})\tau_1 \tilde{w} + h_0^0 P_2(w_2(1 - \tau_2))\tau_2 w_2 \). We denote by \( \lambda \) the multiplier of the budget constraint and we introduce the Lagrangian

\[ L = SW(\tilde{w}) + \lambda \cdot [D_1(\tilde{w})\tau_1 \tilde{w} + h_0^0 P_2(w_2(1 - \tau_2))\tau_2 w_2 - c_0]. \]
The first order condition with respect to $c_0$ is:

$$\frac{dL}{dc_0} = h_0 G'(c_0) + h_1^0 \int_0^{w_1(1-\tau_1)} G'(c_1 - \theta)p_1(\theta)d\theta + \int_0^{w_2(1-\tau_2)} G'(c_2 - \theta)p_2(\theta)d\theta - \lambda = 0.$$ 

Using the definitions of $g_0, g_1, g_2$, we obtain immediately $h_1 g_0 + h_1 g_1 + h_2 g_2 = 1$.

Starting from the competitive equilibrium with no minimum wage $\bar{w} = w = w_1$, we have:

$$\frac{dL}{d\bar{w}} |_{\bar{w}=w_1} = -D_1'(w_1) \cdot G(c_0) - h_2^0(1-\tau_2) \cdot \frac{dw_2}{d\bar{w}} \cdot p_2(w_2(1-\tau_2)) \cdot G(c_0) +$$

$$+ h_2^0(1-\tau_2) \int_0^{w_2(1-\tau_2)} G'(c_0 + w_2(1-\tau_2) - \theta)p_2(\theta)d\theta + h_2^0(1-\tau_2) \frac{dw_2}{d\bar{w}} \cdot G(c_0) \cdot p_2(w_2(1-\tau_2))$$

$$+ \lambda \cdot \left[ D_1(w_1) \tau_1 + D_1'(w_1) \tau_1 w_1 + \tau_2 h_2 \frac{dw_2}{d\bar{w}} + w_2(1-\tau_2) h_2^0 p_2(w_2(1-\tau_2)) \frac{dw_2}{d\bar{w}} \right].$$

The second and sixth terms cancel out. From $h_1^0 \cdot P_1(w_1(1-\tau_1)) = D_1'(w)$, we have $h_1^0 \cdot p_1(w_1(1-\tau_1)) \cdot (1-\tau_1) \frac{dw}{d\bar{w}} = D_1'(w_1)$ at $\bar{w} = w = w_1$. Hence, the first and third terms cancel out. The no-profit condition $F(h_1, h_2) = \bar{w} h_1 + w_2 h_2$ implies $h_1 d\bar{w} + h_2 dw_2 = 0$ and hence $dw_2/d\bar{w} = -h_1/h_2$. Hence, using the definitions $e_2 = w_2(1 - \tau_2) \cdot p_2/P_2$ and $\eta_1 = -w_1 D_1'/h_1$, we have:

$$\frac{dL}{d\bar{w}} |_{\bar{w}=w_1} = (1-\tau_1) h_1 g_1 \cdot \lambda + (1-\tau_2) h_2 g_2 \cdot (-h_1/h_2) \cdot \lambda + \lambda \cdot [h_1 \tau_1 - \eta_1 h_1 \tau_1 + h_2 (1+e_2) \tau_2 \cdot (-h_1/h_2)].$$

Hence,

$$\frac{1}{\lambda \cdot h_1} \cdot \frac{dL}{d\bar{w}} |_{\bar{w}=w_1} = (1-\tau_1) \cdot g_1 - (1-\tau_2) \cdot g_2 + \tau_1 - \eta_1 \cdot \tau_1 - \tau_2 \cdot (1+e_2),$$

which is the condition (11) in Proposition 2. □

A.3 Optimal Tax Formulas (13) with no Minimum Wage

Let us introduce $\Delta c_1 = c_1 - c_0$ and $\Delta c_2 = c_2 - c_0$. The government chooses $c_0, \Delta c_1, \Delta c_2$ to maximize social welfare $SW$ subject to its budget constraint $h_0 c_0 + h_1 c_1 + h_2 c_2 \leq w_1 h_1 + w_2 h_2$ which can be rewritten as $c_0 + h_1 \Delta c_1 + h_2 \Delta c_2 \leq h_1 w_1 + h_2 w_2$. Therefore, the Lagrangian of the government maximization problem can be written as:

$$L = (1-h_1^0 P_1(\Delta c_1) - h_2^0 P_2(\Delta c_2)) G(c_0) + h_1^0 \int_0^{\Delta c_1} G(c_0 + \Delta c_1 - \theta)p_1(\theta)d\theta + h_2^0 \int_0^{\Delta c_2} G(c_0 + \Delta c_2 - \theta)p_2(\theta)d\theta$$
\[ +\lambda \cdot [h_1^0 P_1(\Delta c_1)(w_1 - \Delta c_1) + h_2^0 P_2(\Delta c_2)(w_2 - \Delta c_2) - c_0], \]

The first order condition in \( c_0 \) (keeping \( \Delta c_1 \) and \( \Delta c_1 \) constant) is:

\[ \frac{dL}{dc_0} = h_0 G'(c_0) + h_1 \int_0^{\Delta c_1} G'(c_1 - \theta)p_1(\theta)d\theta + \int_0^{\Delta c_2} G'(c_2 - \theta)p_2(\theta)d\theta - \lambda = 0. \]

Using the definitions of \( g_0, g_1, g_2 \), we obtain immediately \( h_1g_0 + h_1g_1 + h_2g_2 = 1 \). The first order condition in \( \Delta c_1 \) is:

\[ 0 = \frac{dL}{\Delta c_1} = -h_1^0 \cdot p_1(\Delta c_1)G(c_0) + h_1^0 \int_0^{\Delta c_1} G'(c_1 - \theta)p_1(\theta)d\theta + h_2^0 G(c_0)p_1(\Delta c_1) + \lambda \left[ h_1^0 p_1(\Delta c_1)(w_1 - \Delta c_1) - h_1^0 P_1(\Delta c_1) + h_1 \cdot \frac{dw_1}{d\Delta c_1} + h_2 \cdot \frac{dw_2}{d\Delta c_1} \right]. \]

The first and third term cancel out (marginal workers are indifferent between working or not working). The no profit condition \( F(h_1, h_2) = w_1h_1 + w_2h_2 \) implies that \( h_1dw_1 + h_2dw_2 = 0 \) and hence \( h_1dw_1/d\Delta c_1 + h_2dw_2/d\Delta c_1 \) so that the last two terms cancel out. Therefore, we have:

\[ 0 = \frac{1}{h_1 \cdot \lambda} \frac{dL}{\Delta c_1} = h_1 \cdot g_1 - h_1 + h_1 \frac{w_1 - \Delta c_1}{\Delta c_1} \cdot \frac{\Delta c_1 \cdot p_1(\Delta c_1)}{P_1(\Delta c_1)}. \]

Recognizing that \( \Delta c_1 = w_1(1-\tau_1) \), we have \( w_1 - \Delta c_1 = w_1\tau_1 \), and by definition \( e_1 = \Delta c_1 \cdot p_1/P_1 \), therefore:

\[ 0 = \frac{1}{h_1 \cdot \lambda} \frac{dL}{\Delta c_1} = g_1 - 1 + \frac{\tau_1}{1 - \tau_1} \cdot e_1, \]

which implies equation (13) for \( i = 1 \). The proof for \( i = 2 \) is exactly the same. \( \square \)

**A.4 Proof of Proposition 3**

The government chooses \( c_0, \Delta c_1, \Delta c_2, \bar{w} \) to maximize social welfare \( SW \) subject to its budget constraint \( c_0 + h_1\Delta c_1 + h_2\Delta c_2 \leq h_1w_1 + h_2w_2 \). Hence, the Lagrangian of the government maximization problem is:

\[ L = (1-D_1(\bar{w})-h_2^0 P_2(\bar{w}))G(c_0) + h_1 \int_0^{\bar{\theta}} G(c_0 + \Delta c_1 - \theta)p_1(\theta)d\theta + h_2 \int_0^{\Delta c_2} G(c_0 + \Delta c_2 - \theta)p_2(\theta)d\theta \]

\[ + \lambda \cdot \left[ D_1(\bar{w})(\bar{w} - \Delta c_1) + h_2^0 P_2(\Delta c_2)(w_2 - \Delta c_2) - c_0 \right]. \]

where \( \bar{\theta} \) in the firm integral term is defined so that the number of low skilled workers meets exactly the demand: \( h_1^0 \cdot P_1(\bar{\theta}) = D_1(\bar{w}) \). The first order condition with respect to \( c_0 \) (keeping \( \Delta c_1 \) and \( \Delta c_1 \) constant) implies \( h_1g_0 + h_1g_1 + h_2g_2 = 1 \) (same proof as in Appendix A.3). The
first order condition with respect to $\Delta c_2$ implies $\tau_2/(1 - \tau_2) = (1 - g_2)/e_2$ (same proof as in Appendix A.3).

The first order condition with respect to $\Delta c_1$ is:

$$0 = \frac{dL}{d\Delta c_1} = h_1^0 \int_0^\theta G'(c_1 - \theta)p_1(\theta)d\theta - \lambda \cdot D_1(\bar{w}),$$

which implies $g_1 = 1$.

Finally, the first order condition with respect to $\bar{w}$ is:

$$0 = \frac{dL}{d\bar{w}} = -D'_1(\bar{w})G(c_0) + h_1^0 \cdot \frac{d\theta}{d\bar{w}}G(c_0 + \Delta c_1 - \bar{w})p_1(\theta) + \lambda \left[ D'_1(\bar{w})(\bar{w} - \Delta c_1) + D_1(\bar{w}) + h_2 \frac{dw_2}{d\bar{w}} \right].$$

By definition of $\tau_1$, we have $\Delta c_1 = \bar{w}(1 - \tau_1)$. Introducing the reservation wage $w$ of the marginal worker defined as $w(1 - \tau_1) = \bar{w}$ as in the text, and noting that $h_1^0 \cdot P_1(\bar{\theta}) = D_1(\bar{w})$, we have $h_1^0 \cdot p_1(\bar{\theta})d\theta/d\bar{w} = D'_1(\bar{w})$. Finally, the no-profit condition $F(h_1, h_2) = \bar{w}h_1 + w_2h_2$ implies $h_1d\bar{w} + h_2dw_2 = 0$ and hence $dw_2/d\bar{w} = -h_1/h_2$. As a result, the last two terms in the square expression cancel out. Hence, we have:

$$0 = \frac{dL}{d\bar{w}} = D'_1(\bar{w})[G(c_0 + (1 - \tau_1)(\bar{w} - w)) - G(c_0)] + \lambda \cdot D'_1(\bar{w})\bar{w}\tau_1,$$

which implies

$$-\frac{\tau_1}{1 - \tau_1} = \frac{\bar{w} - w}{\bar{w}} \cdot \frac{G(c_0 + (\bar{w} - w)(1 - \tau_1)) - G(c_0)}{\lambda(\bar{w} - w)(1 - \tau_1)} = \frac{\bar{w} - w}{\bar{w}} \cdot g_0^e,$$

where we have used the definition $g_0^e$ in the last equality. □
B Appendix: Extensions

B.1 Uniform Rationing

As discussed above, our previous results are derived under the key assumption of efficient rationing, the situation the most favorable to the minimum wage. We explore briefly how results change if we adopt instead the polar opposite “uniform rationing” assumption whereby unemployment is distributed across workers independently of surplus.\(^{35}\)

**Case with no Taxes**

In the case of uniform rationing with no taxes, the government chooses \(\bar{w}\) to maximize:

\[
SW = (1 - D_1(\bar{w}) - h_2^0P_2(w_2))G(0) + D_1(\bar{w})\int_0^{\bar{w}} G(\bar{w} - \theta) \frac{p_1(\theta)}{P_1(\bar{w})} d\theta + h_2^0\int_0^{w_2} G(w_2 - \theta)p_2(\theta)d\theta.
\]

(17)

The second term in equation (17) reflects the facts that all workers with work costs \(\theta \in (0, \bar{w})\) have the same probability of being employed but that the total number of low skilled workers is given by the demand function \(D_1(\bar{w})\).

Suppose that \(\bar{w}\) is increased by \(d\bar{w}\) under the “uniform rationing” scenario. The redistributive value of introducing a small minimum wage \(d\bar{w}\) remains the same: \(T = [g_1 - g_2]h_1d\bar{w}\).

The minimum wage reduces employment through a demand effect by \(dh_1 = -\eta_1h_1d\bar{w}/\bar{w}\). However, the minimum wage will induce workers with cost of work \(\theta \in (\bar{w}, \bar{w} + d\bar{w})\) to look for a job as well. There are \(e_1h_1^Sd\bar{w}/\bar{w}\) such workers where \(h_1^S = h_0^0P_1(\bar{w})\) is the number of low skilled individuals willing to work for wage \(\bar{w}\). Under efficient rationing, those marginal workers would stay out of work. Under uniform rationing, however, a fraction \(h_1/h_1^S\) of those new workers will join the labor force and will displace some other workers as unemployment is distributed uniformly. That excess labor supply creates involuntary unemployment. As involuntary unemployment is distributed uniformly across all low skilled workers, the average welfare cost per displaced worker is \(\int_0^\bar{w}[G(\bar{w} - \theta) - G(0)]p_1(\theta)d\theta/P_1(\bar{w})\). The number of displaced workers is \(h_1(e_1 + \eta_1)d\bar{w}/\bar{w}\). Thus, the welfare loss due to involuntary unemployment is equal to \(U = -h_1(d\bar{w}/\bar{w})(e_1 + \eta_1)\int_0^\bar{w}[G(\bar{w} - \theta) - G(0)]p_1(\theta)d\theta/P_1(\bar{w})\). At the optimum, we

\(^{35}\)“Uniform rationing” amounts to assuming that unemployment strikes randomly and that Coasian re-trading is prohibitively expensive and hence does not happen at all.
have $U + T = 0$ which implies

$$
\int_0^{\bar{w}} \frac{[G(\bar{w} - \theta) - G(0)]p_1(\theta)d\theta}{\bar{w}P_1(\bar{w})} \cdot (e_1 + \eta_1) = g_1 - g_2. \tag{18}
$$

If at $\bar{w} = w^*_1$, the left-hand-side is smaller than the right-hand-side of (18), then a minimum wage is desirable (and conversely). The key point is that a minimum wage is no longer necessarily desirable under “uniform rationing”.

We can introduce a welfare weight on employment losses defined as

$$
g_0^u = \int_0^{\bar{w}} [G(\bar{w} - \theta) - G(0)]p_1(\theta)d\theta / \int_0^{\bar{w}} (\bar{w} - \theta)p_1(\theta)d\theta. \tag{19}
$$

This equation is an implicit formula for the optimum minimum wage. Presumably, the welfare weight ratio $(g_1 - g_2)/g_0^u$ is decreasing with $\bar{w}$. Formula (19) implies that the minimum wage should be increased up to the point where the welfare weight ratio is equal to the elasticity ratio $(e_1 + \eta_1)/(1 + e_1)$. Obviously, if at $\bar{w} = w^*_1$, the welfare weight ratio is already below the elasticity ratio, then no minimum wage is desirable. Note that the elasticity ratio is increasing in $\eta_1$ and, hence, the optimum minimum wage is decreasing in $\eta_1$. If $g_0^u \geq g_1$, equation (19) implies that the right-hand-side is less than one, and hence a minimum wage can be desirable only if $\eta_1 < 1$.

When $\eta_1 < 1$, the elasticity ratio increases with $e_1$. This implies that the optimum minimum wage is decreasing in $e_1$. This stands in contrast to our results with efficient rationing and can be understood as follows. A large supply elasticity makes unemployment less costly as workers have lower surplus from working on average but a large supply elasticity induces more formerly out of work individuals to start looking for jobs and displace workers with higher surplus which is inefficient. When $\eta_1 < 1$, the latter effect is stronger than the former effect explaining why the minimum wage decreases with $e_1$.  

Empirically, we would expect rationing to be in between efficient rationing and uniform rationing. It is very easy to extend the model to a mixed situation where a fraction $\delta$ of

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36For example, this holds for constant supply elasticity $e_1$ and constant risk aversion functions $G(\cdot)$.

37Note also that, with uniform rationing, and if workers can smooth consumption across unemployment spells, then we have $g_0^u = g_1$. The standard result about the pivotal $\eta_1 = 1$ can be seen as a particular case of (19) when $e_1 = 0$ (no supply elasticity), $g_2 = 0$ (no value assigned to high skilled workers), and $g_0^u = g_1$ (unemployment spells are shared and consumption is smoothed).
unemployment strikes uniformly while a fraction $1 - \delta$ of the unemployment is efficiently allocated. In that case, the formula for the optimum minimum wage is a straight average of (7) and (19), namely $g_1 - g_2 = (1 - \delta)g_0\eta_1(\bar{w} - \bar{w})/\bar{w} + \delta g_0^u(\eta_1 + e_1)/(\eta_1 + e_1).$ This shows that our efficient rationing results are robust to the introduction of a little bit of uniform rationing.

- **Case with Optimal Taxes**

In the case with taxes, the government chooses, $c_0, c_1, c_2,$ and $\bar{w}$ to maximize:

$$SW = (1 - D_1(\bar{w}) - h_2^0P_2(c_2 - c_0))G(c_0) + D_1(\bar{w}) \int_0^{c_1 - c_0} \frac{G(c_1 - \theta)p_1(\theta)}{P_1(c_1 - c_0)} d\theta + h_2^0 \int_0^{c_2 - c_0} G(c_2 - \theta)p_2(\theta)d\theta,$$

subject to the standard budget constraint and the fact that demand for labor is competitively set. The second term in equation (20) reflects the facts that all workers with work costs $\theta \in (0, c_1 - c_0)$ have the same probability of being employed but that the total number of low skilled workers is given by the demand function $D_1(\bar{w}).$ The first order condition with respect to $c_0$ (keeping $c_1 - c_0, c_2 - c_0, \bar{w}$ constant) implies the standard result $h_0g_0 + h_1g_1 + h_2g_2 = 1.$ The first order condition with respect to $c_2$ leads to the standard optimal tax formula for $\tau_2,$ namely $\tau_2/(1 - \tau_2) = (1 - g_2)/e_2.$ The first order condition with respect to $c_1$ leads to:

$$\frac{g_1 - 1}{e_1} = g_0^u \cdot \int_0^{c_1 - c_0} \left(1 - \frac{\theta}{c_1 - c_0}\right) \frac{p_1(\theta)}{P_1(c_1 - c_0)} d\theta,$$

where $g_0^u = \int_0^{c_1 - c_0} [G(c_1 - \theta) - G(c_0)]p_1(\theta)d\theta/(\lambda \cdot \int_0^{c_1 - c_0} (c_1 - c_0 - \theta)p_1(\theta)d\theta)$ is the welfare weight on unemployment losses.

The first order condition with respect to $\bar{w}$ leads to:

$$-\frac{\tau_1}{1 - \tau_1} = g_0^u \cdot \int_0^{c_1 - c_0} \left(1 - \frac{\theta}{c_1 - c_0}\right) \frac{p_1(\theta)}{P_1(c_1 - c_0)} d\theta.$$

Therefore and strikingly, combining those two first order conditions, we find that the optimal tax formula for $\tau_1$ in the presence of the optional minimum wage is the same as with no minimum wage, namely $\tau_1/(1 - \tau_1) = (1 - g_1)/e_1.$ Intuitively and following the derivation from Figure 3b, this can be understood as follows. Suppose $c_1$ is increased by $dc_1$ and at the same time, the minimum wage $\bar{w}$ is reduced by $d\bar{w}$ such that $dc_1 \cdot p_1/P_1 = d\bar{w} \cdot D'_1(\bar{w})/D_1.$ In that case, a fraction $D_1/P_1$ of those $p_1 dc_1$ workers willing to join the labor force because of $dc_1$ can do so and hence the fiscal effect of the reform is $(T_1 - T_0)p_1 dc_1 \cdot D_1/P_1 = D_1 dc_1 \cdot e_1 \cdot \tau_1/(1 - \tau_1)$ and hence the standard formula goes through. We can then obtain the following proposition.
Proposition 5  With optimal taxes and transfers and uniform rationing, if the welfare weight on unemployment losses is larger than the welfare weight on low skilled workers \( g_0^u \geq g_1 \) and the supply elasticity \( e_1 \) is constant, then a minimum wage is not desirable.

Proof: Under the assumption of a constant \( e_1 \) and if the minimum wage binds, the integral term in the right-hand-side of (21) is equal to \( 1/(1 + e_1) \) and hence (21) can be rewritten as \( (g_1 - 1)/e_1 = g_0^u/(1 + e_1) \). However, \( (g_1 - 1)/e_1 < g_1/(1 + e_1) \leq g_0^u/(1 + e_1) \), where the first inequality follows from that fact that \( g_1 < 1 + e_1 \) (as \( \tau_1 = (1 - g_1)/(1 - g_1 + e_1) \))\(^{38}\) and the second inequality from our assumption that \( g_1 \leq g_0^u \). This creates a contradiction showing that the minimum wage cannot be binding. \( \square \)

As in the case with no taxes, it is easy to extend the model to a mixed situation where a fraction \( \delta \) of unemployment strikes uniformly while a fraction \( 1 - \delta \) of the unemployment is efficiently allocated. In particular, a minimum is desirable if and only if \( g_1 > 1 + \delta g_0^u e_1/(1 + e_1) \) at the tax optimum with no minimum wage. When a minimum wage is desirable, at the optimum we have, \( g_1 = 1 + \delta g_0^u e_1/(1 + e_1) \) and \( -\tau_1/(1 - \tau_1) = \delta g_0^u/(1 + e_1) + (1 - \delta) g_0^u (\bar{w} - w)/\bar{w} \). This shows that our results under efficient rationing are also robust to the introduction of a little bit of uniform rationing (small \( \delta \)) in the case with optimal taxes.

B.2  General Labor Supply Function

We consider a general model with \( I \) occupations (instead of 2) and a general production function.\(^{39}\) Most importantly, the model will allow for any labor supply responses, instead of only considering the extensive margin as in the previous section.

• Model and Optimal Taxation

The model we use is the general occupation model described in the appendix of Saez (2002) and in Saez (2004). There are \( I + 1 \) occupations, paying wages \( w_0 = 0, w_1, \ldots, w_I \). Occupation 0 denotes unemployment. There is a constant return to scale production function \( F(h_1, \ldots, h_I) \) so that \( w_i = \partial F/\partial h_i \). We assume that in equilibrium, occupations are ordered so that \( 0 < w_1 < \ldots < w_I \). Each individual is characterized by a cost parameter \( \theta = (\theta_0 = 0, \theta_1, \ldots, \theta_I) \)

\(^{38}\)If \( g_1 > 1 + e_1 \), then reducing \( \tau_1 \) is strictly desirable which cannot happen at the optimum.

\(^{39}\)Introducing a capital input would also be possible as long as we assume that returns on capital can be taxed at a specific rate \( \tau_K \). Similarly, pure profits can also be introduced as long as the government can tax them away fully.
which describes the labor supply cost for the individual to work in each occupation \( i = 1, \ldots, I \).

By assumption, being out of work is costless. We assume that \( \theta \) is distributed according to a measure \( \nu(\theta) \) on \( \Theta \), with total population normalized to one.

The government can apply a general income tax and transfer system \( T = (T_0, \ldots, T_I) \). We denote by \( c_i = w_i - T_i \) the disposable income (after taxes and transfers) in occupation \( i \).

An individual with cost \( \theta \) picks the occupation \( i \) which maximizes \( c_j - \theta_j \) for \( j = 0, \ldots, I \).

Hence, the set \( \Theta \) is partitioned into \( I + 1 \) subsets \( \Theta_0, \ldots, \Theta_I \) so that individuals with \( \theta \in \Theta_i \) choose occupation \( i \). We denote by \( h_i = \nu(\Theta_i) \) the fraction of individuals in occupation \( i \). Those supply functions are functions of \( c = (c_0, \ldots, c_I) \) and hence denoted by \( h_i(c_0, \ldots, c_I) \).

We assume that \( \theta \) is distributed smoothly across individuals so that the supply functions \( h_i \) are continuously differentiable. This is a fully general supply model with no income effects.

The participation model from our previous section is a special case. Similarly, the intensive labor supply of Mirrlees (1971) can be represented in this discrete model by assuming that individuals of “type \( i \)” can work in job \( i - 1 \) at no cost or work in job \( i \) at cost \( \theta_i > 0 \) (see Saez 2002 for details).

Abstracting first from the minimum wage, the government chooses \( c = (c_0, \ldots, c_I) \) in order to maximize:

\[
SW = \int_{\theta \in \Theta} G(c_i - \theta_i) d\nu(\theta) \text{ subject to the budget constraint: } \sum_{j=0}^I (w_j - c_j) \cdot h_j(c) \geq 0.
\]

\( G(\cdot) \) is increasing and concave and where index \( i \) inside in integral for \( SW \) denotes the utility maximizing job choice of individual \( \theta \). We denote again by \( \lambda \) the multiplier of the budget constraint.

The first order condition with respect to \( c_i \) is simply:

\[
(1 - g_i)h_i = \sum_{j=0}^I T_j \cdot \frac{\partial h_j}{\partial c_i}, \tag{23}
\]

where \( g_i \) is the average social marginal welfare weight in occupation \( i \), defined as \( g_i = \int_{\theta \in \Theta_i} G'(c_i - \theta_i) d\nu(\theta)/(\lambda \cdot h_i) \).

The derivation is straightforward once one recognizes that (1) the welfare effect of a small increase \( dc_i \) due to switching jobs behavioral responses is zero (because of a standard envelope theorem argument), (2) the wages changes \( dw_1, \ldots, dw_I \) due to \( dc_i \) have no fiscal consequence because the no profit condition \( F = w_1 h_1 + \ldots + w_I h_I \) implies that \( h_1 dw_1 + \ldots + h_I dw_I = 0 \).

The no income effects assumption implies that \( \sum_{j=0}^I g_i \cdot h_i = 1 \). This can be obtained by increasing every \( c_i \) by \( dc \) uniformly. This generates no behavioral responses and hence the
fiscal cost \( dc \) must be equal to the welfare gain \( dc \cdot \sum_j h_j g_j \). This implies that the average of \( g_i \) is one.

**Desirability of Minimum Wage Rationing**

We can generalize proposition 3 as follows: If \( g_1 > 1 \) at the tax optimum, then, if rationing is efficient, introducing a minimum wage is desirable. The proof remains the same: Starting from the tax optimum with no minimum wage, setting \( \bar{w} = w_1 \), and increasing \( c_1 \) improves social welfare when \( g_1 > 1 \) without triggering any behavioral response because those who would like to move to occupation 1 cannot do so because of the minimum wage rationing. Those already in occupation 1 are not displaced because of the efficient rationing assumption.

Theoretically, the occupation model can be seen as a generalized Diamond-Mirrlees optimal tax model, which inherits most of the structure and properties of the Diamond-Mirrlees model. In particular, the analysis of minimum wages parallels the theory of rationing in second-best optimal tax models developed by Guesnerie (1981) and Guesnerie and Roberts (1984). Following Samuelson (1951), using the symmetry result (\( \partial h_i / \partial c_j = \partial h_j / \partial c_i \)), the optimal tax formula (23) can be rewritten as:

\[
\frac{1}{h_i} \sum_{j=0}^{I} -T_j \cdot \frac{\partial h_i}{\partial c_j} = g_i - 1.
\]

The left-hand-side measures the percentage change in \( h_i \) created by the tax system (which changes \( c_j \) from \( w_j \) to \( w_j - T_j \)). Hence, if \( g_i > 1 \ (g_i < 1) \), the optimal tax system encourages (discourages) the supply of labor in occupation \( i \). Hence, the optimal tax system (absent a minimum wage) subsidizes goods going to disadvantaged individuals (here low skilled work). As a result, low skilled is socially over-supplied at the second best tax optimum. It is then socially desirable to ration subsidized low skill labor using a minimum wage.

The “full redistribution to minimum wage workers” result of Proposition (3) also extends to this general model. At the join tax and minimum wage optimum, the optimum minimum wage \( \bar{w} \) covers occupations \( i = 1, .., i^* \) (we assumed that occupations were ordered). Then all those occupations pay the same wage \( \bar{w} \). As a result, the government can no longer distinguish across those occupations and hence the government is forced to tax (or subsidize) them uniformly so that \( c_1 = .. = c_{i^*} = \bar{c} \). We denote by \( \bar{T} = \bar{w} - \bar{c} \) the net tax on minimum wage workers.
Again, increasing $\tilde{c}$ does not produce any behavioral labor supply response (as occupations $1, .., i^*$ are rationed by the minimum wage). Hence, the government should increase $\tilde{c}$ up to the point that $\tilde{g} = 1$ where $\tilde{g} = (h_1g_1 + .. + h_{i^*}g_{i^*})/(h_1 + .. + h_{i^*})$ is the average social marginal welfare weight on minimum wage workers.

- Many Consumption Goods and Production Efficiency

It is also possible to extend the tax model to a situation with many goods. In that context, we can show that the standard theorems of public finance, namely the production efficiency theorem of Diamond and Mirrlees (1971) and the no commodity taxation result of Atkinson and Stiglitz (1976) carry over to the model with optimal minimum wage with taxes and transfers.

The production efficiency theorem implies that, at the joint minimum wage and tax optimum, there should be production efficiency: producers should maximize profits using pre-tax prices for labor inputs and consumption outputs. This result is trivial to verify in the two skill model and remains true with many labor inputs and many consumption goods. As is well known, the production efficiency result implies that there should be no tariffs in the context of an open economy. Therefore, this important result carries over when the government uses a minimum wage.

The Atkinson and Stiglitz (1976) implies that, if utility functions are separable between consumption goods and labor costs and the sub-utility of consumption is homogenous across all consumers, then the optimum tax/minimum wage system should tax labor only and not impose any differentiated taxes on consumption goods. This result also carries over to the joint tax and minimum wage optimum.

B.3 Income Effects

In order to introduce income effects in the 2-skill model used in the text, we can define individual utility as $u(c) - \theta \cdot l$ where $u(.)$ is increasing and concave. Thus, an individual of skill $i$ works iff $\theta \leq u(c_i) - u(c_0)$. Denoting $u_i = u(c_i)$, for $i = 0, 1, 2$, the labor supply function becomes $h_i = h_0^i \cdot P_i(u_i - u_0)$. We can again evaluate social welfare as:

$$SW = (1 - h_1 - h_2)G(u_0) + h_1^1 \int G(u_1 - \theta)p_1(\theta)d\theta + h_2^0 \int_{u_2-u_1} G(u_2 - \theta)p_2(\theta)d\theta, \quad (25)$$

where $G(.)$ is a concave and increasing transformation. Note that (25) is identical to social welfare with no income effects once $c_i$ substituted by $u_i$.  

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Let us denote again by \( \lambda \) the multiplier of the government budget constraint \( h_0 c_0 + h_1 c_1 + h_2 c_2 \leq h_1 w_1 + h_2 w_2 \) which can be rewritten as:
\[
h_0 u^{-1}(u_0) + h_1 u^{-1}(u_1) + h_2 u^{-1}(u_2) \leq h_1 w_1 + h_2 w_2.
\]
We define social marginal welfare weights as: \( g_0 = G'(u_0)u'(c_0)/\lambda \) and \( g_i = \int G'(u_i - \theta)p_i(\theta)d\theta \cdot u'(c_i)/\lambda \) for \( i = 1, 2 \).

With no minimum wage, the government chooses \( c_0, c_1, c_2 \) or equivalently \( u_0, u_1, u_2 \) to maximize \( SW \) subject to budget constraint. Increasing \( u_0, u_1, \) and \( u_2 \) by \( du \) leads to the first order condition:
\[
h_0 G'(u_0) + h_1 \int G'(u_1 - \theta)p_1 d\theta + h_1 \int G'(u_1 - \theta)p_1 d\theta = \lambda \cdot \sum \frac{h_i}{u'(c_i)},
\]
which can be rewritten as:
\[
\tilde{h}_0 g_0 + \tilde{h}_1 g_1 + \tilde{h}_2 g_2 = 1,
\]
where \( \tilde{h}_i = (h_i/u'(c_i))/\left(\sum_j h_j/u'(c_j)\right) > 0 \) can be interpreted as occupation weights renormalized by the marginal utility of consumption. Using those weights, social marginal weights \( g_i \) average again to one.

The first order condition with respect to \( u_i \) leads to the usual optimal tax formula \( \tau_i/(1 - \tau_i) = (1 - g_i)/e_i \) where the supply elasticity is defined as \( e_i = [(c_i - c_0)/h_i]\partial h_i/\partial c_i|_{c_0} = (c_i - c_0)u'(c_i) \cdot p_i(u_i - u_0)/P_i(u_i - u_0). \)

We can show again that a minimum wage is desirable if \( g_1 > 1 \) at the tax optimum. At the joint minimum wage and tax optimum, we have \( \tilde{h}_0 g_0 + \tilde{h}_1 g_1 + \tilde{h}_2 g_2 = 1, \) \( g_1 = 1, \)
\( \tau_2/(1 - \tau_2) = (1 - g_2)/e_2. \) Furthermore, the first order condition in \( \tilde{w} \) takes a similar form
\[
[G(u_1 - \bar{\theta}) - G(u_0)]/\lambda = -w_1 \cdot \tau_1 < 0 \text{ (where } \bar{\theta} \text{ is the cost of work of the marginal worker).}
\]
Hence, Proposition (4) showing that \( \tau_1 > 0 \) along with a binding minimum wage is Pareto dominated extends to the case with income effects as well.
References


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Review 36, 358-365.


Figure 1: Desirability of Small Minimum Wage

Wage $w_1$

Transfer $h_1\Delta w$ from other factors to min wage workers

Efficiency Loss due to unemployment: 2nd order

$h^*_1 + \Delta h_1$ $h^*_1$

Supply $S(w_1)$

Demand $D(w_1)$

$w = w^*_1 + \Delta w_1$
Figure 2: Deriving the Optimal Minimum Wage

Wage $w_1$

Transfer from other factors to min wage workers: $h_1dw$

Weight $g_1-g_2$

$w+w+dw$

min wage $w$

Unemployment loss: $(w-w)dh_1$

Weight $g_0$

$w+dw$

Supply $S(w_1)$

Demand $D(w_1)$

Labor $h_1$

$dh_1<0$
Consumption c
Wage w

Net Welfare effect: $h_1dc_1(g_1-1)>0$

Labor Supply: $dh_1w_1\tau_1<0$

At the optimum:
$dh_1w_1\tau_1 + h_1dc_1(g_1-1)=0$
implies
$\tau_1/(1-\tau_1)=(1-g_1)/e_1<0$

Figure 3a: Optimal Tax/Transfer Derivation (assuming $g_1>1$)
Figure 3b: Optimal Tax/Transfer Derivation (assuming $g_1 > 1$)

- Consumption
- Wage $w$
- Endogenous wages do not affect optimal formula as $h_1 dw_1 + h_2 dw_2 = 0$ (no profits) and tax $= (w_1 - c_1) h_1 + (w_2 - c_2) h_2$
- Labor Supply: $dh_1 w_1 \tau_1 < 0$
- Net Welfare effect: $h_1 dc_1 (g_1 - 1) > 0$
Figure 4: Desirability of Min Wage with Optimal Taxes

Consumption $c$

Wage $w$

Net Welfare effect: $h_1dc_1(g_1-1)>0$

Labor Supply: $dh_1=0$

With min wage set at $w_1$, $dc_1>0$ does not affect labor supply because $w_1$ cannot go down: $dh_1=0$
Fig. 5: Pareto Improving Policy when $\tau_1 > 0$ and min wage binds

- Consumption $c$
- Wage $w$

Unemployment decreases: $dh_1 > 0$ \Rightarrow Welfare effect $> 0$

Fiscal effect: $\tau_1 w_1 dh_1 > 0$

Reduce $\bar{w}$ while keeping $c_1, c_2$ constant:
- Fiscal effect $> 0$ and welfare effect $> 0$
- No direct fiscal effect of $dw, dw_2$ as $h_1 dw + h_2 dw_2 = 0$ (no profits)
- And tax $= (w_1 - c_1) h_1 + (w_2 - c_2) h_2$
### Table 1: Optimal Minimum Wage with No Taxes or Fixed Taxes

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#### A. Optimum Minimum Wage with no taxes and transfers

**A1. Case $\gamma=1$**

| Minimum Wage / Market Wage | 1.12 | 1.21 | 1.34 | 1.44 | 1.21 | 1.07 |
| Unemployment Rate          | 7.6% | 16.8% | 39.8% | 24.8% | 16.8% | 9.2% |

**A2. Case $\gamma=3$**

| Minimum Wage / Market Wage | 1.03 | 1.08 | 1.17 | 1.18 | 1.08 | 1.03 |
| Unemployment Rate          | 2.3% | 6.9% | 20.1% | 10.9% | 6.9% | 3.8% |

**A3. Case $\gamma=0.5$**

| Minimum Wage / Market Wage | 1.11 | 1.19 | 1.29 | 1.41 | 1.19 | 1.06 |
| Unemployment Rate          | 6.9% | 15.0% | 34.9% | 23.1% | 15.0% | 8.1% |

#### B. Optimum Minimum Wage with exogenous taxes (uniform tax rate $\tau=0.35$)

**B1. Case $\gamma=1$**

| Minimum Wage / Market Wage | 1.00 | 1.01 | 1.04 | 1.13 | 1.01 | 1.00 |
| Unemployment Rate          | 0.0% | 0.5% | 5.5% | 8.0% | 0.5% | 0.0% |

Notes: The table reports the minimum wage (relative to market wage rate $w^*_1$) and the induced unemployment rate among the low skilled as a function of the elasticity of substitution $\sigma$ between low and high skilled labor in production, the elasticity of labor supply of low skilled workers $e_1$ (the high skilled labor supply elasticity $e_2=0.25$ in all cases), and the risk aversion $\gamma$ of the social welfare function. The production function is CES with elasticity of substitution $\gamma$, calibrated so that market equilibrium with no minimum wage is $(w^*_1,w^*_2)=(1,3)$. The supply functions are calibrated so that $(h^*_0,h^*_1,h^*_2)=(0.2,0.2,0.6)$. The social welfare function is such that $G(u)=(u+0.5)^{1-\gamma}/(1-\gamma)$. 
### Table 2: Optimal Minimum Wage with Optimal Taxes

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<td><strong>A1. Optimal Tax Rates with no Minimum Wage</strong></td>
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<td>-25.9%</td>
<td>-23.3%</td>
<td>-17.8%</td>
</tr>
<tr>
<td>Tax rate on high skilled workers τ₂</td>
<td>32.4%</td>
<td>33.6%</td>
<td>35.2%</td>
<td>34.4%</td>
<td>33.6%</td>
<td>32.0%</td>
</tr>
<tr>
<td><strong>C2. Optimal Tax Rates and optimal Minimum Wage</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax rate on low skilled workers τ₁</td>
<td>-41.0%</td>
<td>-81.3%</td>
<td>-153.3%</td>
<td>-93.2%</td>
<td>-81.3%</td>
<td>-34.1%</td>
</tr>
<tr>
<td>Tax rate on high skilled workers τ₂</td>
<td>31.3%</td>
<td>30.7%</td>
<td>29.1%</td>
<td>31.5%</td>
<td>30.7%</td>
<td>31.1%</td>
</tr>
<tr>
<td>Minimum Wage / Market Wage</td>
<td>1.08</td>
<td>1.21</td>
<td>1.49</td>
<td>1.35</td>
<td>1.21</td>
<td>1.04</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>5.5%</td>
<td>21.9%</td>
<td>91.5%</td>
<td>23.8%</td>
<td>21.9%</td>
<td>4.7%</td>
</tr>
</tbody>
</table>

Notes: The table reports optimal tax rates (on low and high skilled) with no minimum wage and the joint optimal tax rates and minimum wage (relative to market wage rate $w^*_1$) and the induced unemployment rate among the low skilled as a function of the elasticity of substitution $\sigma$ between low and high skilled labor in production, the elasticity of labor supply of low skilled workers $e_1$ (the high skilled labor supply elasticity $e_2=0.25$ in all cases), and the risk aversion $\gamma$ of the social welfare function.

The production function is CES with elasticity of substitution $\gamma$, calibrated so that market equilibrium with no minimum wage is $(w^*_1,w^*_2)=(1,3)$. The supply functions are calibrated so that $(h^*_0,h^*_1,h^*_2)=(0.2,0.6)$. The social welfare function is such that $G(u)=(u+0.5)^{1-\gamma}/(1-\gamma)$. 

The table shows the optimal tax rates and minimum wage for different cases with varying elasticities of substitution $\sigma$ and labor supply elasticities $e_1$. The optimal tax rates and minimum wage are calculated to maximize social welfare, considering the induced unemployment rate among the low skilled.