

Optimal Public Expenditure with Inefficient Unemployment

Pascal Michailat, Emmanuel Saez *

March 29, 2017

Abstract

This paper proposes a theory of optimal public expenditure when unemployment is inefficient. The theory is based on a matching model. Optimal public expenditure deviates from the Samuelson rule to reduce the unemployment gap (the gap between the current and efficient rates of unemployment). Such optimal “stimulus spending” is described by a formula expressed with estimable sufficient statistics: the unemployment gap, the unemployment multiplier (the effect of public expenditures on unemployment), and the elasticity of substitution between public and private consumption. Using the formula, we obtain four results. (1) When the unemployment multiplier is positive and unemployment is inefficiently high, optimal stimulus spending is positive and increasing in the unemployment gap. (2) Optimal stimulus spending is zero for a zero multiplier, increasing in the multiplier for small multipliers, largest for a moderate multiplier, and decreasing in the multiplier beyond that. (3) Optimal stimulus spending is increasing in the elasticity of substitution between public and private consumption. In particular, it is zero when extra public goods are useless, and it completely fills the unemployment gap when extra public goods are as valuable as extra private goods. (4) The formula for optimal stimulus spending remains the same whether taxes are distortionary or not.

*Pascal Michailat: Brown University, <http://www.pascalmichailat.org>. Emmanuel Saez: University of California–Berkeley, <http://eml.berkeley.edu/~saez>. We thank George Akerlof, Steven Coate, Emmanuel Farhi, Xavier Gabaix, Roger Gordon, Yuriy Gorodnichenko, Nathaniel Hendren, Henrik Kleven, Michael Peters, David Romer, Stefanie Stantcheva, Matthew Weinzierl, and participants at many seminars and conferences for helpful discussions and comments. This work was supported by the Center for Equitable Growth at the University of California–Berkeley, the British Academy, the Economic and Social Research Council [grant number ES/K008641/1], the Institute for New Economic Thinking, and the Sandler Foundation.

1. Introduction

The theory of optimal public expenditure developed by Samuelson (1954, 1955) is a cornerstone of public economics. This theory shows that public goods should be provided to the point where the marginal rate of substitution between public and private consumption equals the marginal rate of transformation. While the theory has been expanded in numerous directions since its inception (see Kreiner and Verdelin (2012) for a recent survey), one question has not been answered: how is the theory modified in the presence of unemployment, especially inefficient unemployment? This question is relevant because public expenditure is one of the key tools used by governments to tackle high unemployment.¹

In this paper, we expand Samuelson’s theory to situations with inefficient unemployment. To introduce inefficient unemployment, we embed Samuelson’s framework into a matching model of the economy.² We find that when unemployment is efficient, the Samuelson rule remains valid; but when unemployment is inefficient, optimal public expenditure deviates from the Samuelson rule to bring unemployment closer to its efficient level. We denote the deviation of public expenditure from the Samuelson rule as “stimulus spending.” We describe optimal stimulus spending with a formula expressed in terms of estimable sufficient statistics:³ (1) the unemployment gap, which is the gap between the current and efficient rates of unemployment; (2) the unemployment multiplier, which measures the reduction in unemployment rate caused by an increase in public expenditure; and (3) the elasticity of substitution between public and private consumption, which describes the social value of additional public consumption.

Being expressed with sufficient statistics, the formula applies broadly, irrespective of the specification of the utility function, aggregate demand, and price mechanism. Furthermore, our formula addresses a common problem with optimal policy formulas expressed with sufficient statistics: in these formulas, the statistics are implicit functions of the policy, so the formulas cannot explicitly characterize the policy. We resolve this issue by expressing the relevant statistics as explicit functions of the policy and backing out optimal stimulus spending as a function of statistics that do not

¹Other key tools are monetary policy, government debt, and government transfers.

²For other papers using matching models to study optimal policies in the presence of unemployment, see Boone and Bovenberg (2002), Hungerbühler and Lehmann (2009), or Lehmann, Parmentier and Van Der Linden (2011).

³For an overview of the sufficient-statistic approach, see Chetty (2009). For recent papers following the approach, see Chetty (2006, 2008), Hendren (2014, 2016), Farhi and Gabaix (2015), Stantcheva (2015), or Werquin (2016).

depend on the policy. Then, we use this explicit formula to obtain four theoretical results about optimal stimulus spending. For concreteness, here we only discuss the case with positive unemployment multiplier and unemployment gap; but we also have results for negative unemployment multiplier or unemployment gap.

Our first result is that when the unemployment multiplier and unemployment gap are positive, optimal stimulus spending is positive. To understand the result, imagine that public expenditure is at the Samuelson level. Keeping unemployment and thus total consumption fixed, increasing public consumption by 1 unit reduces private consumption by 1 unit. At the Samuelson level, the marginal utilities of public and private consumption are equalized, so the increase in public consumption has no first-order effect on welfare so far. When the unemployment multiplier is positive, however, increasing public consumption lowers unemployment. And when the unemployment gap is positive, unemployment is inefficiently high, which means that reducing unemployment raises welfare. Now the increase in public expenditure has a positive effect on welfare: it is therefore optimal to raise public expenditure above the Samuelson level. Further, optimal stimulus spending is increasing in the unemployment gap.

Our second result is that optimal stimulus spending is zero for a zero multiplier, increasing in the multiplier for small multipliers, maximized for a moderate multiplier, and decreasing in the multiplier for larger multipliers. When the unemployment multiplier is small, optimal stimulus spending is solely determined by how much public expenditure reduces the unemployment gap. A larger multiplier means a larger reduction, so it warrants more stimulus spending. When the multiplier is large, however, this logic breaks down: it becomes optimal to nearly entirely fill the unemployment gap. As less spending is required to fill the gap when the multiplier is larger, optimal stimulus spending is decreasing in the multiplier.

Our third result is that optimal stimulus spending is increasing in the elasticity of substitution between public and private consumption. This result is natural: a higher elasticity of substitution means that extra public goods are more valuable, which makes stimulus spending more desirable. There are two interesting limit cases. With a zero elasticity, extra public goods are useless; since public consumption always crowds out private consumption, it is never optimal to provide public goods beyond the Samuelson rule. With an infinite elasticity, public and private goods are interchangeable, so it is optimal to maximize total consumption, irrespective of its composition; this

requires to provide enough public goods to completely fill the unemployment gap.

Our fourth result is that the formula for optimal stimulus spending remains the same whether the taxes used to finance public expenditure are distortionary or not. However, with distortionary taxation, the output multiplier (the effect of an increase in public expenditure on output) is no longer the same as the unemployment multiplier. This point is important because the debate on stimulus spending generally centers around the output multiplier. When taxes are nondistortionary, unemployment and output multipliers are equal, so all our results can be reformulated with the output multiplier. But when taxes are distortionary, higher taxes reduce labor supply, which reduces output but not unemployment. Hence, the output multiplier becomes smaller than the unemployment multiplier. With strong labor-supply responses, it is even possible to have a negative output multiplier and a positive unemployment multiplier. Accordingly, we cannot use the output multiplier's sign to determine whether stimulus spending should be positive or negative.

Finally, since the statistics in the formula are estimable, we can calibrate the formula to help provide policy recommendations. As an illustration, we calibrate the formula to the Great Recession in the United States. There is some uncertainty about the value of the unemployment multiplier, so we compute the optimal stimulus for a range of multipliers. Even with a small multiplier of 0.2, optimal stimulus spending is quite large: 2.8 percentage points of GDP. Optimal stimulus spending is the largest for a modest multiplier of 0.4: 3.7 points of GDP. Optimal stimulus spending then decreases for larger multipliers; when the multiplier reaches 1.5, optimal stimulus spending has fallen to the level warranted by a small multiplier of 0.1: 1.9 points of GDP. Of course, the stimulus does not have the same impact in the two cases: it has no effect on unemployment for the multiplier of 0.1 but almost completely fills the unemployment gap for the multiplier of 1.5.

Public expenditure plays an important role not only in public economics but also in macroeconomics. The macroeconomic literature has focused on estimating or simulating how public expenditure affects macroeconomic variables such as output, employment, or investment; this literature is surveyed in [Ramey \(2011\)](#). Perhaps surprisingly, only a small number of macroeconomic papers study optimal public expenditure (for example, [Woodford 2011](#)). These papers focus on optimal public expenditure in models with rigid prices, when monetary policy is ineffective. These papers are similar to ours in that they study optimal public expenditure when productive efficiency fails; they are different in that productive inefficiency does not arise from unemployment. One additional

specificity of our approach is to express optimal stimulus spending in terms of estimable statistics; this allows us to derive sharp theoretical properties for optimal stimulus spending. In Section 3 we discuss these macroeconomic papers in more detail.

The paper is organized as follows. Section 2 presents the model. Section 3 derives the sufficient-statistic formula and explores its implications. Section 4 considers distortionary taxation. Section 5 presents some numerical applications. Section 6 concludes.

2. A Matching Model of Inefficient Unemployment

We present the model used for the analysis. The model combines the public-expenditure framework of Samuelson (1954, 1955) with the general-equilibrium matching framework of Michailat and Saez (2015). Because of the matching structure, the equilibrium features unemployment, and the rate of unemployment is generally inefficient.

2.1. Informal Description

The model is not standard in public economics. To help readers understand the economic mechanisms, we therefore begin by describing it informally.

In our model of the economy, there are people and a government. All people work as butlers. Butlers can do everything (think Jeeves in P.G. Wodehouse's work): they garden, cook, clean, educate the children, provide haircuts, do administrative work, and so on. Thus all the output in the economy are butler services.

Nobody can be their own butlers (one needs someone else to bring them tea in bed). Thus, people are butlers for others and use their income to hire their own butlers. This captures the fact that a modern economy is based on market exchanges rather than home production.

Beside spending their income to hire butlers, people can also buy land. People derive utility from their land and also use land to save some of their income. As land is in fixed supply overall, the choice between spending on butler services and saving will determine the aggregate demand for butlers. The relevant price in the economy is the price of butler services in terms of land.

Butlers are hired by people or the government. Butlers hired by people produce private butler services (cleaning or cooking) while butlers hired by the government produce public butler services

(tending public spaces or policing the streets). People value both public and private butler services. To finance its expenditure and thus balance its budget at all time, the government taxes households.

Butler services are hired on a matching market. For butlers, this means that while they are available to buttle for forty hours a week, they cannot be hired the whole time. In fact, abstracting from randomness at the butler level, each butler sells the same number of hours and has the same number of idle hours. Since unemployment is equally spread over the population, all butlers have the same consumption, and we can fully abstract from insurance issues.

For people and the government, the matching market means that they need to post help-wanted ads to find butlers. Posting ads involve a labor cost: some butlers have to spend time creating the ad, reading applications and interviewing prospective butlers. The time spent on recruiting by these human-resource butlers depends on the number of butlers to be hired, the time spent on each ad, and the probability to fill an ad (not all ads are filled). Human-resource butlers are paid like other butlers but their services do not directly provide utility; however, they are necessary to hire butlers whose services provide utility. We also abstract from randomness on the buyer side: a buyer of services determines the number of ads to post based on the number of butlers that they want to hire and the probability to fill an ad. For instance, if they want to hire 2 butlers and the probability to fill an ad is 50%, they will post 4 ads and hire exactly 2 butlers.

Once hired, all butlers are paid the same price for their services. A butler stays with an employer for some time, until the relationship stops for exogenous reasons (for instance, the fit is not good anymore). As butler services are sold by the hour, a butler usually works for several people at the same time: the butler may work afternoons in one household, Monday and Wednesday mornings in another household, and be unemployed on the other mornings.

The state of the butler market is described by a tightness variable, defined as the ratio of help-wanted ads to unemployment. When the tightness is high, it is easy for butlers to find work but hard to recruit new butlers; in this situation the unemployment rate is low and employers devote a large share of their workforce to recruiting. When the tightness is low, it is hard for butlers to find work but easy to recruit new butlers; in this situation the unemployment rate is high and employers devote a small share of their workforce to recruiting. There is an efficient rate of unemployment, which maximizes the amount of services that provide utility. When unemployment is inefficiently high, butlers are idle for too many hours, so the amount of services consumed by people is too low.

When unemployment is inefficiently low, too many butler-hours are devoted to human-resource task, and the amount of butler-hours actually enjoyed by people is too low.

In this economy, two variables—the tightness and price of services—equilibrate the demand for butler services and the supply of butler services. Suppose that the price is high. Then demand for butler services will be low (as land is relatively more attractive), few ads are posted, and many butlers are unemployed, so that tightness is low. Suppose instead that the price is low. Then demand for butler services is high, many ads are posted, and few butlers are unemployed, so that tightness is high. Effectively, for any price, tightness will adjust to equilibrate demand and supply. The price can be determined in many ways—bargained between employer and butler, fixed by a social norm, or set by government regulation—but once the price mechanism is specified, the equilibrium is unique. Moreover, the price may generate inefficiently high or low unemployment.

Finally, what happens when the government purchases more butler services? First, providing more public butler services generates utility (there are more flowers in public parks). But public hiring usually increases tightness, making it more difficult for people to recruit their own butlers, and crowding out private consumption. If unemployment is initially too high and public expenditure reduces unemployment, more public expenditure increases total consumption so that crowding-out is less than one-for-one. In contrast, in the Samuelson model, crowding out is exactly one-for-one. Hence, when unemployment is too high and public expenditure reduces unemployment, public expenditure should be above the Samuelson rule .

2.2. *Supply Side*

We now describe the model formally. We start by describing the supply side of the model.

The model is dynamic and set in continuous time. The economy consists of a government and a measure 1 of identical households. Households are self-employed: they produce services and sell them on a matching market.⁴ There are two types of services: private services, purchased by households, and public services, purchased by the government.⁵ All the services are bought on the same matching market at the same price p . Households value both the private services that

⁴We abstract from firms for simplicity. [Michaillat and Saez \(2015\)](#) show how the model can be extended to include firms hiring workers on a labor market and selling their production on a product market.

⁵We assume that only services are produced in the economy, but the model would also work if perishable goods were produced. The only type of goods that the model cannot accommodate is storable goods.

they purchase and the public services provided by the government.

Each household has a fixed productive capacity $k > 0$; the capacity indicates the maximum amount of services that a household could sell at any point in time. (Here k is exogenous, but in Section 4, we will show that the results remain the same when k is chosen by households to maximize utility.) Since there is a measure 1 of households, the aggregate capacity in the economy is k . Because of the matching process, not all available services are sold at any point in time, so there is always some unemployment. At time t , households sell $C(t)$ services to other households and $G(t)$ services to the government; output $Y(t)$ is the sum of all sales:

$$Y(t) = C(t) + G(t).$$

As households are unable to sell their entire capacity, $Y(t) < k$. The unemployment rate is the share of aggregate capacity that is idle: $u(t) = (k - Y(t))/k$.

Services are sold through long-term relationships. Once a seller and a buyer have matched, the seller serves the buyer at each instant until the relationship exogenously ends. The rate at which relationships separate is $s > 0$. Since $Y(t)$ services are committed to existing relationships at time t , the amount of services available for purchase is $k - Y(t)$.

To generate new relationships, households buying services and the government advertise a total of $v(t)$ vacancies. (In Section 2.3, we will explain how households and the government decide the number of vacancies to post.) A matching function taking as arguments the aggregate number of available services and the aggregate number of vacancies determines the rate $h(t)$ at which new long-term relationships are formed. For convenience, we use a standard Cobb-Douglas specification:

$$h(t) = \omega \cdot (k - Y(t))^\eta \cdot v(t)^{1-\eta},$$

where $\eta \in (0, 1)$ is the matching elasticity, and $\omega > 0$ is the matching efficacy.

With constant returns to scale in matching, the rates at which sellers and buyers form new long-term relationships is determined by the market tightness. The market tightness $x(t)$ is the ratio of the two arguments in the matching function: $x(t) \equiv v(t)/(k - Y(t))$. Each of the $k - Y(t)$ available services is sold at rate $f(x(t)) = h(t)/(k - Y(t)) = \omega \cdot x(t)^{1-\eta}$ and each of the $v(t)$ vacancies is filled at rate $q(x(t)) = h(t)/v(t) = \omega \cdot x(t)^{-\eta}$. The selling rate $f(x)$ is increasing in x and the buying

rate $q(x)$ is decreasing in x . Hence, when tightness is higher, it is easier to sell services but harder to buy them.

In such a model, output is a state variable with law of motion $\dot{Y}(t) = f(x(t)) \cdot (k - Y(t)) - s \cdot Y(t)$. The term $f(x(t)) \cdot (k - Y(t))$ is the number of new relationships forming at time t ; the term $s \cdot Y(t)$ is the number of existing relationships separating at time t . If $f(x)$ and s are constant over time, output converges to the steady-state level

$$(1) \quad Y(x, k) = \frac{f(x)}{f(x) + s} \cdot k.$$

The function $Y(x, k)$ is positive and increasing in x and k , and its elasticity with respect to x is $(1 - \eta) \cdot u(x)$. The unemployment rate is directly related to output: $u = 1 - Y/k$; hence, the steady-state unemployment rate is given by

$$(2) \quad u(x) = \frac{s}{s + f(x)}.$$

The function $u(x)$ is positive and decreasing in x , and its elasticity with respect to x is $-(1 - \eta) \cdot (1 - u(x))$. When tightness is higher, output is higher and unemployment is lower because it is easier to sell services.

In US data, unemployment reaches this steady-state level quickly because labor market flows are large. In fact, [Hall \(2005, Figure 1\)](#) shows that the unemployment rate obtained from (2) and the actual employment rate are indistinguishable. Thus, as [Hall \(2005\)](#) does, we simplify the analysis by ignoring the transitional dynamics of output and unemployment and assuming that both variables are jump variables that depend on tightness according to (1) and (2). To simplify the analysis further, we abstract from transitional dynamics and randomness at the household level, and we assume that when tightness is x , all households exactly sell a share $1 - u(x)$ of their capacity k , and exactly have a share $u(x)$ of the capacity idle.

Posting a vacancy is costly: it costs $\rho > 0$ services per unit time. These services represent the resources devoted by households and the government to matching with appropriate providers of services. These matching services do not provide utility to households, so we need to distinguish between services that are purchases and services that provide utility. Households purchases $C(t)$ services and the government purchases $G(t)$ services. We refer to $C(t)$ as *private expenditure* and

$G(t)$ as *public expenditure*. But households only derive utility from $c(t) < C(t)$ private services and $g(t) < G(t)$ public services; $c(t)$ and $g(t)$ are computed by subtracting matching services used by households and the government from $C(t)$ and $G(t)$. We refer to $c(t)$ as *private consumption*, $g(t)$ as *public consumption*, and $y(t) = c(t) + g(t)$ as *total consumption*.

The wedge between expenditure and consumption is directly determined by tightness. As we have done when with sellers of services, we abstract from transitional dynamics and randomness with buyers of services (households and the government). This means that by posting v_0 vacancies, a buyer can establish exactly $v_0/q(x)$ new matches at any point in time. It also means that a buyer is always in a situation where the same number of relationships form and separate at any time. So if a buyer wants to continuously purchase Y_0 services, $s \cdot Y_0$ new matches must be created at any point in time, to replace the matches that have separated. This requires $v_0 = s \cdot Y_0/q(x)$ vacancies and thus $\rho \cdot v_0 = \rho \cdot s \cdot Y_0/q(x)$ services spent on filling vacancies. As a consequence, only $y_0 = Y_0 - Y_0 \cdot \rho \cdot s/q(x)$ services actually provide utility. We can rewrite this relationship as $Y_0 = y_0 \cdot [1 + \tau(x)]$, where

$$(3) \quad \tau(x) \equiv \frac{\rho \cdot s}{q(x) - \rho \cdot s}.$$

is the wedge between consumption and expenditure. This logic holds for any consumption level y_0 . Thus, if a household or the government desire to consume one service, they need to purchase $1 + \tau(x)$ services—one service for consumption plus $\tau(x)$ services for matching. This logic implies that private consumption is related to private expenditure by $c(t) = C(t)/[1 + \tau(x(t))]$ and public consumption to public expenditure by $g(t) = G(t)/[1 + \tau(x(t))]$. The matching wedge $\tau(x)$ is positive and increasing for $x \in [0, x^m)$, where $x^m \in (0, +\infty)$ is defined by $q(x^m) = \rho \cdot s$ and $\lim_{x \rightarrow x^m} \tau(x) = +\infty$, and the elasticity of $\tau(x)$ with respect to x is $\eta \cdot (1 + \tau(x))$. When tightness is higher, the matching wedge is higher because it is more difficult for a buyer to fill a vacancy.

We write total consumption as a function of tightness and capacity:

$$(4) \quad y(x, k) = \frac{1 - u(x)}{1 + \tau(x)} \cdot k.$$

This function $y(x, k)$ plays a central role in the analysis because it gives the amount of services that can be consumed for a given tightness. We refer to $y(x, k)$ as the *aggregate supply*. Equation (4)

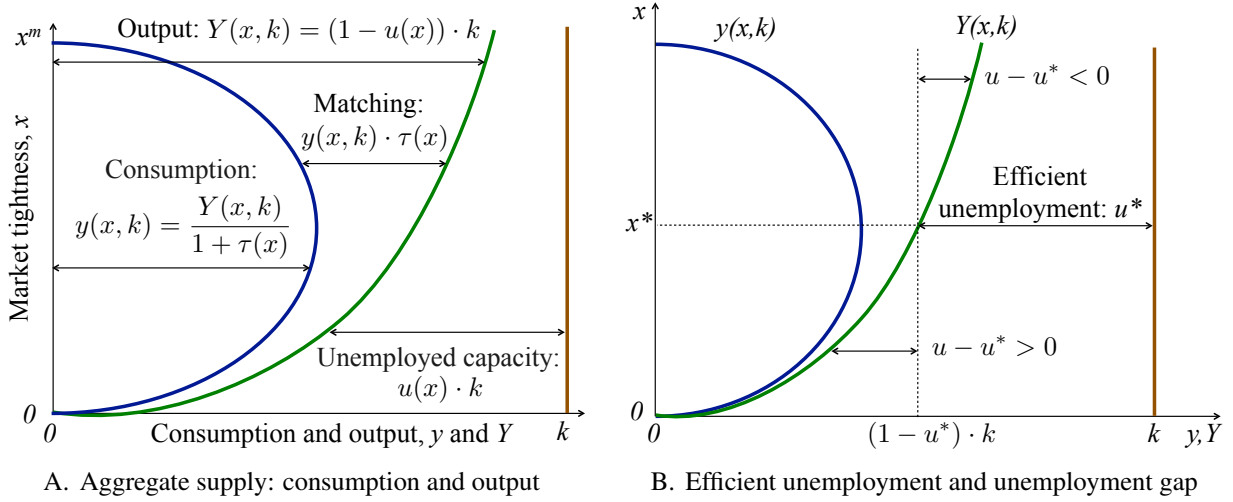


Figure 1: Aggregate Supply and Unemployment

shows that consumption is below the capacity k because some services are not sold ($u(x) > 0$) and some services are used for matching instead of consumption ($\tau(x) > 0$). The function $y(x, k)$ is positive for $x \in [0, x^m)$ and $k > 0$, increasing in k , and with an elasticity with respect to x of $(1 - \eta) \cdot u(x) - \eta \cdot \tau(x)$.

We now define the efficient rate of unemployment:

Definition 1. *Tightness and unemployment rate are efficient if they maximize total consumption for a given productive capacity. The efficient tightness is denoted by x^* and the efficient unemployment rate by u^* . The unemployment gap is $u - u^*$.*

We have seen that the elasticity of $y(x, k)$ with respect to x is $(1 - \eta) \cdot u(x) - \eta \cdot \tau(x)$. This elasticity is $1 - \eta > 0$ for $x = 0$, strictly decreasing in x , and $-\infty$ at $x = x^m$. Thus, we can characterize the efficient rate of unemployment as follows:

Lemma 1. *The efficient tightness is defined by*

$$(5) \quad (1 - \eta) \cdot u(x^*) - \eta \cdot \tau(x^*) = 0.$$

The efficient unemployment rate is $u^ = u(x^*)$.*

The efficiency condition (5) is intuitive. It says that consumption is maximized whenever the marginal reduction in unemployed capacity achieved by higher tightness (measured by $(1 - \eta) \cdot$

u) is equal to the marginal increase in services devoted to matching caused by higher tightness (measured by $\eta \cdot \tau$). Hence, determining whether tightness and unemployment are inefficiently high or low requires to compare the unemployment rate u to the matching wedge τ . For instance, when the unemployment rate is high relative to the matching wedge, such that $u/\tau > \eta/(1 - \eta)$, then tightness is inefficiently low and unemployment inefficiently high.

Panel A of Figure 1 summarizes the supply side of the model. It depicts how total consumption and output depend on tightness. Panel B of Figure 1 depicts the efficient tightness and unemployment rate, situations in which tightness is inefficiently high and unemployment is inefficiently low, and situations in which tightness is inefficiently low and unemployment is inefficiently high. To determine whether unemployment is inefficiently high or low in equilibrium, we need to introduce an aggregate demand. We do this now.

2.3. Demand Side and Equilibrium: General Case

We turn to the demand side and equilibrium of the model. Unlike the supply side, which is specific, the demand side and equilibrium of the model are generic. Specifying the supply side is necessary to compute social welfare and study optimal policy, but the sufficient-statistic approach makes it unnecessary to specify the demand side and equilibrium. We will find sufficient statistics that summarize the relevant features of the demand side and equilibrium, and we will express the formula for optimal public expenditure with these sufficient statistics. Here we discuss the class of models covered by our analysis. Below we provide a specific example.

As we have abstracted from randomness in buying and selling services, all households have the same private consumption c and public consumption g . The representative household derives instantaneous utility $\mathcal{U}(c, g)$, where the utility function \mathcal{U} is strictly increasing in c and g and concave. The marginal rate of substitution between public and private consumption is

$$MRS_{gc} \equiv \frac{\partial \mathcal{U} / \partial g}{\partial \mathcal{U} / \partial c} > 0.$$

We assume that \mathcal{U} is such that MRS_{gc} is a decreasing function of g/c .⁶ For convenience, we also

⁶For instance, \mathcal{U} could be a constant-elasticity-of-substitution utility function. More generally, \mathcal{U} could be a homothetic utility function of the form $\mathcal{U}(c, g) = \mathcal{N}(n(c, g))$ where the function \mathcal{N} is increasing and the function n is increasing in c and g , concave, and homogeneous of degree 1. In that case, as n is homogeneous of degree 1, its

assume that $MRS_{gc}(0) > 1$.

The elasticity of substitution between public and private consumption measures how the marginal rate of substitution varies with g/c :

Definition 2. The elasticity of substitution between public and private consumption, denoted $\varepsilon > 0$, is defined by

$$(6) \quad \frac{1}{\varepsilon} = -\frac{d \ln(MRS_{gc})}{d \ln(g/c)}.$$

The elasticity of substitution is positive because MRS_{gc} is decreasing in g/c . A lower elasticity of substitution implies that the marginal value of public consumption, relative to the marginal value of private consumption, decreases faster with g/c . The elasticity of substitution has two interesting limits. When $\varepsilon \rightarrow 0$, public and private consumption are perfect complements.⁷ This means that a certain number of public services are needed for a given economy, but beyond that, additional public services have zero value and the marginal rate of substitution falls to zero. At this point, public workers dig and fill holes. When $\varepsilon \rightarrow +\infty$, the public and private consumption are perfect substitutes.⁸ This means that households are equally happy to consume a unit of private or public services, such that the marginal rate of substitution is constant at 1.

The demand for services has two components: the demand from the government, and the demand from households. The government chooses an amount $g(t)$ of public consumption. (In Section 3, we will describe the level of public consumption maximizing welfare.) As we have seen, providing $g(t)$ public services requires the government to actually purchase $G(t) = (1 + \tau(x(t))) \cdot g(t)$ services. The extra $\tau(x(t)) \cdot g(t)$ services are used by the government to fill vacancies and thus replace the $s \cdot G(t)$ relationships separating at time t . Finally, to create $s \cdot G(t)$ new relationships at time t , the government needs to post $s \cdot G(t)/q(x(t))$ vacancies at time t . It is equivalent to choose

partial derivatives $\partial n/\partial c$ and $\partial n/\partial g$ are homogeneous of degree 0. This implies

$$MRS_{gc} = \frac{\frac{\partial n}{\partial g}(c, g) \cdot \mathcal{N}'(n(c, g))}{\frac{\partial n}{\partial c}(c, g) \cdot \mathcal{N}'(n(c, g))} = \frac{\frac{\partial n}{\partial g}\left(1, \frac{g}{c}\right)}{\frac{\partial n}{\partial c}\left(\frac{c}{g}, 1\right)}.$$

We see that MRS_{gc} is a function of g/c . Moreover, as n is increasing in c and g , $\partial n/\partial g > 0$ and $\partial n/\partial c > 0$; and as n is concave, $\partial n/\partial g$ is decreasing in its second argument while $\partial n/\partial c$ is decreasing in its first argument. We conclude that MRS_{gc} is decreasing in g/c .

⁷The Leontief utility function $\mathcal{U}(c, g) = \min\{(1 - \gamma) \cdot c, \gamma \cdot g\}$ has $\varepsilon = 0$.

⁸The linear utility function $\mathcal{U}(c, g) = c + g$ has $\varepsilon \rightarrow +\infty$.

a level of consumption (g), a level of expenditure (G), or a number of vacancies. We could equivalently describe the government policy in terms of consumption or expenditure or vacancies, but because consumption matters for welfare, we opt to describe the policy in terms of consumption. Finally, the government balances its budget at all time using a lump-sum tax $T(t) = G(t)$.

To generate a demand for services, households need to have the choice between services and something else. Hence, we assume that households spend part of their labor income on services and save part of it. They choose how much to spend and to save to maximize their intertemporal utility. We assume that the asset used for saving is in fixed supply; thus there are no state variables in the model, and the equilibrium of the model immediately converges to steady state.

In steady state, the representative household demands a quantity $c(x, p, g)$ of private consumption. This demand depends negatively on tightness x because a higher tightness means that households have to devote a larger fraction of their services to matching (the matching wedge $\tau(x)$ is higher), which makes purchasing services less attractive. For obvious reasons, the demand also depends negatively on the price p of services. Finally, the demand depends on public consumption g because g may affect the marginal utility from private consumption. Consumption immediately determines expenditure and vacancies: if the households consumes $c(x, p, g)$, they actually purchase $C(x, p, g) = (1 + \tau(x)) \cdot c(x, p, g)$ services and they post $v(x, p, g) = s \cdot C(x, p, g)/q(x)$ vacancies.

The total demand for consumption is $g + c(x, p, g)$. We refer to $c(x, p, g)$ as *private demand* and $g + c(x, p, g)$ as *aggregate demand*. As we have seen, we could equivalently describe the aggregate demand in terms of consumption or expenditure or vacancies. Since consumption matters for welfare, we have described aggregate supply in terms of consumption (see equation (4)). Thus, we also describe aggregate demand in terms of consumption.

Finally, as in any matching model, we need to specify a price mechanism. Here we assume that the price mechanism is a generic function of tightness and public consumption:

$$p = p(x, g).$$

With this mechanism, the price of services depends on economic conditions and public expenditure. Additionally, since the variables (x, g) determine all other variables in a feasible allocation, this mechanism could be any function of any variable: it is the most generic mechanism possible.

Given the price mechanism and government policy, tightness adjusts such that aggregate supply equals aggregate demand:

$$(7) \quad y(x, k) = c(x, p(x, g), g) + g.$$

This equation implicitly defines equilibrium tightness as a function $x(g)$ of public consumption. Panel A in Figure 2 illustrates how equilibrium tightness $x(g)$ is determined at the intersection of the aggregate-demand and aggregate-supply curves.

On a matching market the price mechanism generally fails to maintain unemployment at its efficient level (Pissarides 2000, Chapter 8).⁹ Hence, policies affecting the price of services could be useful to stabilize the economy—that is, bring unemployment closer to its efficient level. In some contexts, monetary policy could be such a policy, since it determines interest rates and thus affects the choice between consuming and saving. In other contexts, taxes or subsidies on the price of services could be helpful. If these price policies could fully stabilize the economy, unemployment would always be efficient, and our analysis would trivially apply. Here, we also consider the more interesting case in which the price policies are unable to keep unemployment at its efficient level. We take all these other policies as given—they are embedded in the price schedule $p(x, g)$ —and explore how public expenditure can improve stabilization.

For the welfare analysis, all the relevant information about the tightness function $x(g)$ is summarized by the following sufficient statistic:

Definition 3. *The unemployment multiplier is defined by*

$$(8) \quad m = -y \cdot \frac{du}{dg}.$$

The unemployment multiplier measures the decrease of the unemployment rate, measured in percentage points, when public consumption increases by 1 percent of total consumption.

Since unemployment is directly determined by tightness (through (2)), the unemployment multiplier describes how equilibrium tightness $x(g)$ responds to public consumption g . As illustrated in Panel B of Figure 2, public consumption affects equilibrium tightness by shifting the aggregate de-

⁹Since search is random, prices are determined in a situation of bilateral monopoly and are therefore isolated from market forces that could ensure that the price of services maintains unemployment at its efficient level.

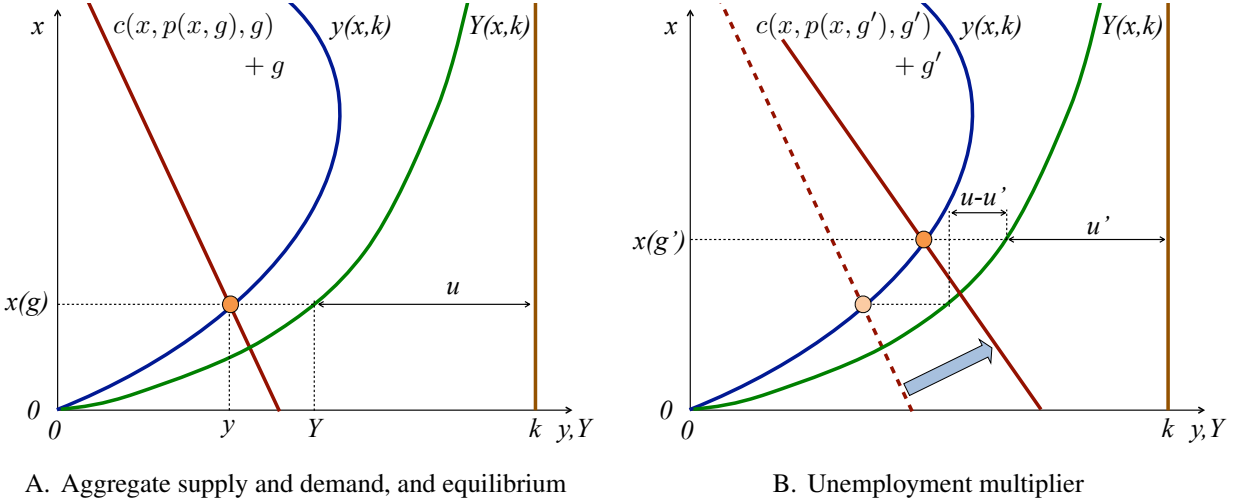


Figure 2: Aggregate Demand and Unemployment Multiplier

mand curve. The shift of aggregate demand occurs through three channels: a mechanical channel, as aggregate demand is the sum of public consumption plus private demand; a price channel, as public consumption may affect the price of services and thus private demand; and a utility channel, as public expenditure may affect the marginal utility from private consumption and thus private demand.¹⁰ Because these channels can take many forms, the unemployment multiplier can take a broad range of values: it can be positive or negative and it can be above 1 or below 1.

2.4. Demand Side and Equilibrium: An Example with Land

We now describe a model in which households save using land. (In [Appendix C](#) we provide other examples.) This example illustrates the type of model covered by the analysis. Furthermore, it shows how demand-side parameters map into sufficient statistics. This subsection is a bit more technical, and with the general description of the demand side and equilibrium given above, it is not required for the welfare analysis. It can therefore be skipped on a first reading.

The representative household purchases a quantity $l(t)$ of land. Land is traded on a perfectly competitive market and is in fixed supply l_0 . In equilibrium, the land market clears so $l(t) = l_0$.¹¹

¹⁰Since private demand is determined by an optimal consumption-saving decision (described by an Euler equation), our framework does not allow for the typical, Keynesian multiplier mechanism. According to this mechanism, more public expenditure means higher autonomous spending for households and through the Keynesian cross, a multiplier over 1. The size of the Keynesian multiplier is determined by households' marginal propensity to consume.

¹¹Iacoviello (2005) and Liu, Wang and Zha (2013) build business-cycle models with the same assumptions: land enters the utility function, is traded on a perfectly competitive market, and is in fixed supply.

The household derives utility from holding $l(t)$ units of land; this utility captures for instance the housing services provided by land. The instantaneous utility function is separable: $\mathcal{U}(c(t), g(t)) + \mathcal{V}(l(t))$. We use a constant-elasticity-of-substitution specification for \mathcal{U} :

$$(9) \quad \mathcal{U}(c, g) = \left[(1 - \gamma)^{\frac{1}{\varepsilon}} \cdot c^{\frac{\varepsilon-1}{\varepsilon}} + \gamma^{\frac{1}{\varepsilon}} \cdot g^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The parameter $\gamma \in (0, 1)$ indicates the value of public services relative to private services, and the parameter $\varepsilon > 0$ gives the elasticity of substitution between public and private consumption. The function \mathcal{V} is strictly increasing and concave. The household's utility at time 0 is

$$(10) \quad \int_0^{+\infty} e^{-\delta t} \cdot [\mathcal{U}(c(t), g(t)) + \mathcal{V}(l(t))] dt,$$

where $\delta > 0$ is the subjective discount rate. The law of motion of the household's land holding is

$$(11) \quad \dot{l}(t) = p(t) \cdot (1 - u(x(t))) \cdot k - p(t) \cdot (1 + \tau(x(t))) \cdot c(t) - T(t).$$

In the law of motion, $p(t) \cdot (1 - u(x(t))) \cdot k$ is the household's labor income, $p(t) \cdot (1 + \tau(x(t))) \cdot c(t)$ is its spending on services, and $T(t)$ is the lump-sum tax used to finance public expenditure.

The household takes $l(0)$ and the paths of $x(t)$, $g(t)$, $p(t)$, and $T(t)$ as given. It chooses the paths of $c(t)$ and $l(t)$ to maximize (10) subject to (11). To solve the household's problem, we set up the current-value Hamiltonian:

$$\mathcal{H}(t, c(t), l(t)) = \mathcal{U}(c(t), g(t)) + \mathcal{V}(l(t)) + \lambda(t) [p(t)(1 - u(x(t)))k - p(t)(1 + \tau(x(t)))c(t) - T(t)]$$

with control variable $c(t)$, state variable $l(t)$, and current-value costate variable $\lambda(t)$. The first-order conditions for an interior solution to the maximization problem are $\partial \mathcal{H} / \partial c = 0$, $\partial \mathcal{H} / \partial l = \delta \cdot \lambda(t) - \dot{\lambda}(t)$, and the appropriate transversality condition. These conditions imply

$$(12) \quad \frac{\partial \mathcal{U}}{\partial c}(c(t), g(t)) = \lambda(t) \cdot p(t) \cdot (1 + \tau(x(t))),$$

$$(13) \quad \mathcal{V}'(l(t)) = \delta \cdot \lambda(t) - \dot{\lambda}(t).$$

Since \mathcal{U} and \mathcal{V} are concave, these conditions are necessary and sufficient.

For a given public consumption, g , an equilibrium consists of paths for $[x(t), c(t), l(t), p(t), \lambda(t)]_{t=0}^{\infty}$ that satisfy $c(t) + g = y(x(t))$, $l(t) = l_0$, $p(t) = p(x(t), g)$, and equations (12) and (13). The first condition is the equality of supply and demand on the market for services, the second is the equality of supply and demand on the market for land, the third is the price mechanism on the market for services, and the fourth and fifth are the first-order conditions of the household's utility-maximization problem.

Since all the variables can be recovered from the costate variable $\lambda(t)$, the equilibrium can be represented as a dynamical system of dimension 1 with variable $\lambda(t)$. The variable $\lambda(t)$ satisfies the differential equation $\dot{\lambda}(t) = \delta \cdot \lambda(t) - \mathcal{V}'(l_0)$. The steady-state value of $\lambda(t)$ is $\lambda = \mathcal{V}'(l_0)/\delta > 0$. Since $\delta > 0$, we infer that the dynamical system is a source. As there is no state variable, the system immediately jumps to the steady state from any initial condition. Thus, the model immediately jumps to its steady-state equilibrium from any initial condition. Since the model is always in steady state, the welfare associated with any equilibrium is the instantaneous welfare, $\mathcal{U}(c, g) + \mathcal{V}(l_0)$.

The main step to describing the steady state is to compute the private demand $c(x, p, g)$. First, we combine the first-order conditions (12) and (13):

$$\frac{\partial \mathcal{U}}{\partial c}(c, g) = (1 + \tau(x)) \cdot p \cdot \frac{\mathcal{V}'(l_0)}{\delta}.$$

Holding one unit of land forever yields utility $\mathcal{V}'(l_0)/\delta$; with one unit of land the household can purchase $1/[p \cdot (1 + \tau(x))]$ services yielding utility $(\partial \mathcal{U} / \partial c) / [p \cdot (1 + \tau(x))]$. The equation therefore implies that at the margin the household must be indifferent between purchasing one unit of land and spending the same amount on private services. This equation also implies that once a seller and buyer have matched, they are always happy to trade at the market price p , because they both derive a positive surplus from the transaction. This is obvious in the seller's case: a service that is not sold is wasted. The equation shows that it is also true in the buyer's case: buying a service at p provides utility $\partial \mathcal{U} / \partial c$ while reneging and purchasing land instead provides utility $p \cdot \mathcal{V}'(l_0)/\delta < \partial \mathcal{U} / \partial c$. The crux is that the matching cost $\tau(x) \cdot p$ is sunk at the time of the match.

Then, using the expression for $\partial \mathcal{U} / \partial c$ in [Appendix B](#), we find that the private demand $c(x, p, g)$

is implicitly defined by

$$(14) \quad \left[(1 - \gamma) + \gamma^{\frac{1}{\varepsilon}} \cdot \left((1 - \gamma) \cdot \frac{g}{c} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} = (1 + \tau(x)) \cdot p \cdot \frac{\mathcal{V}'(l_0)}{\delta}.$$

Equation (14) determines the aggregate demand, $c(x, p, g) + g$. Aggregate-demand shocks are shocks to the marginal utility of land, $\mathcal{V}'(l_0)$, or to the time discount rate, δ . A negative aggregate-demand shock is a higher marginal utility of land or lower time discount rate: after the shock, households desire to save more and consume less, which depresses aggregate demand. If the price is somewhat rigid, such negative aggregate-demand shocks lead to lower tightness and higher unemployment.

The last step to describing the steady state is to specify a mechanism for the price of services. To illustrate the workings of the model, we choose a mechanism that makes it easy to compute the unemployment multiplier:

$$(15) \quad p(g) = p_0 \cdot \left[(1 - \gamma) + \gamma^{\frac{1}{\varepsilon}} \cdot \left((1 - \gamma) \cdot \frac{g}{y^* - g} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1-r}{\varepsilon-1}}.$$

The parameter $p_0 > 0$ governs the price level. The parameter r determines the effect of public consumption on prices: if $r < 1$, the price is increasing in g ; if $r = 1$, the price is fixed ($p(g) = p_0$ for all g); and if $r > 1$, the price is decreasing in g . The parameter r is a key determinant of the unemployment multiplier. Indeed, [Appendix B](#) shows that the multiplier evaluated at the efficient unemployment and optimal public expenditure is

$$(16) \quad m = \frac{r \cdot (1 - u^*)}{\varepsilon \cdot (1 - \gamma)}.$$

The sign of r gives the sign of the multiplier. Beside r , the value of the multiplier also depends on the parameters ε and γ from the utility function, because these parameters influence the shape of the aggregate-demand curve. The multiplier admits a more complicated expression away from the efficient equilibrium.

2.5. *Comparison with the Textbook Matching Model*

Our matching model, which builds on the framework developed by [Michaillat and Saez \(2015\)](#), has many similarities with the textbook matching model developed by [Pissarides \(2000\)](#): the matching function, the trading probabilities f and q , the tightness x , the vacancies v , the vacancy-posting cost ρ , the random search (without searching for the best prices), the long-term relationships and exogenous separations at rate s , and the fixed capacity k . But our model also has several differences with the textbook model, which makes it closer to the public-economic tradition and more adapted to analyze public expenditure. Here we present and justify these differences. This subsection is not required for the welfare analysis, so it could be skipped on a first reading.

To begin with, we have made six cosmetic changes that extend the scope of the matching model and make it closer to the Walrasian model. As the Walrasian model is the workhorse model in public economics, these changes make it easier to use public-economic tools with matching models and compare our results to standard public-economic results. The generalization of the matching model makes it more adapted to analyze study public expenditure.

First, we apply the matching model to a service economy, whereas it is usually applied to the labor market. This means that the traded goods are services instead of labor, the cost of the traded good is the price of services instead of a real wage, the buyers are households (and the government) instead of firms, and the sellers are self-employed households instead of unemployed workers.

Second, we have recast the Beveridge curve as an aggregate-supply curve and the job-creation condition as an aggregate-demand curve.¹² The aggregate supply curve is mathematically equivalent to the Beveridge curve, and the aggregate demand curve to the job-creation curve, but our curves allow us to think with Walrasian supply and demand concepts.

Third, we have recast the equilibrium condition of the model as a supply = demand condition. In fact, it is useful to think of tightness as the second price on the market, and both actual price and tightness ensure that supply and demand are equalized. In this sense, the matching framework generalizes the Walrasian framework by allowing for a second price (tightness) and thus for inefficient equilibria. In a Walrasian equilibrium, when supply equals demand, productive efficiency is respected; that is not the case in the matching model, which is why the model is useful to think

¹²In [Pissarides \(2000\)](#), the Beveridge curve is equation (1.5) and the job-creation condition is equation (1.9). In this paper, the aggregate supply curve is (4) and the aggregate demand curve is (14).

about inefficient unemployment. Since we use the supply-demand formalism, we obtain a different graphical representation of the equilibrium: in the textbook model, the equilibrium is represented as the intersection of the Beveridge and job-creation curves in an (unemployment,vacancy) plan; here, the equilibrium is the intersection of the aggregate-supply and aggregate-demand curves in a (consumption,tightness) plan.¹³

Fourth, we have altered the type of cost incurred for posting a vacancy. In the textbook model, the vacancy-posting cost is measured in terms of a final good, so that there are effectively two goods: labor and a final good. This is hard to handle in a welfare analysis. Here the cost is measured in terms of services, so there is a single good. This is easier to handle when we think about welfare and efficiency: we simply define consumption as output of services net of matching services, and we use consumption in the welfare function. The analysis of the matching market is self-contained since the matching costs are measured in terms of traded goods (here, services).¹⁴

Fifth, while the analysis in the textbook model focuses on atomistic workers and jobs, here we look at households selling and buying a large number of services. This makes the analysis closer to the Walrasian framework, in which agents buy and sell a large number of goods.¹⁵ Furthermore, since our households buy and sell many services, we can abstract from randomness at the household level and thus avoid heterogeneity (for instance, having employed and unemployed workers). Since the households are homogeneous, the welfare analysis is not contaminated by insurance problems.

Sixth, we have reformulated the condition giving the efficient rate of unemployment. In the textbook model, the Hosios (1990) condition gives the efficient rate of unemployment. More precisely, it gives workers' bargaining power such that the wage obtained by Nash bargaining between workers and firms yields the efficient rate of unemployment in equilibrium. Our efficiency condition (5) is mathematically equivalent to the Hosios condition. But it is more general because it is not tied to Nash bargaining: it applies to any price mechanism. Instead of describing the bargaining power such that unemployment is efficient, our condition describes how observable variables (unemployment and matching wedge) are related when unemployment is efficient.

¹³In Pissarides (2000), the equilibrium is depicted in Figure 1.2. In this paper, the equilibrium is depicted in Figure 2.

¹⁴Farmer (2008) makes a similar simplifying assumption in matching models of the labor market.

¹⁵When the labor-market matching model is used in a macroeconomic context, introducing large firms and large households is common (for example, Blanchard and Galí 2010).

In addition to these cosmetic changes, we generalize the textbook model on several dimensions. The generalized model describes a broader range of empirical possibilities, which is especially important when using sufficient statistics. The generalized model yields a broader range of sufficient statistics and maps the data better. First, we allow for a general price mechanism instead of restricting ourselves to an efficient price or Nash bargained price. This allows for a broader range of multipliers and unemployment gaps to better describe what we see in the data. In addition, because of the special functional forms used in the textbook model, the typical labor demand is perfectly elastic in tightness (horizontal in our (y, x) plan) and not affected by public goods. As showed by (Michaillat 2014), an increase in public expenditure has no effect on tightness, so public expenditure crowds out private expenditure one-for-one, and the multiplier is necessarily zero. Following Michaillat (2012), we introduce more general functional forms to obtain an aggregate demand that is less-than-perfectly elastic in tightness (downward-sloping in our (y, x) plan). Thus crowding-out is less than one-for-one and public expenditure can affect unemployment.

3. A Sufficient-Statistic Formula

We derive a sufficient-statistic formula for optimal public expenditure and explore its theoretical implications. The formula shows that when unemployment is inefficient, optimal public expenditure deviates from the Samuelson rule. The deviation is governed by three sufficient statistics: the unemployment gap, the unemployment multiplier, and the elasticity of substitution between public and private consumption.

3.1. Derivation

To obtain the formula for optimal public expenditure, we solve the government's welfare maximization problem. The government chooses public consumption g to maximize welfare $\mathcal{U}(c, g)$. In equilibrium, $c = y(x, k) - g$ and $x = x(g)$. Thus, the optimal g maximizes $\mathcal{U}(y(x(g), k) - g, g)$. We assume that the maximization problem is well-behaved: $g \mapsto \mathcal{U}(y(x(g), k) - g, g)$ admits a unique extremum and the extremum is interior and a maximum. Under this assumption, a first-order condition is necessary and sufficient to describe the solution to the maximization problem.

The first-order condition of the government's problem is

$$(17) \quad 0 = \frac{\partial \mathcal{U}}{\partial g} - \frac{\partial \mathcal{U}}{\partial c} + \frac{\partial \mathcal{U}}{\partial c} \cdot \frac{\partial y}{\partial x} \cdot \frac{dx}{dg}.$$

This equation shows that an increase in public expenditure affects welfare through three channels: it raises public consumption (first term in the right-hand side); for a given level of total consumption, it reduces private consumption one-for-one (second term in the right-hand side); and it affects the level of total consumption and thus private consumption (third term in the right-hand side).

Since $c = y - g$, equation (17) can be rewritten as

$$0 = \frac{\partial \mathcal{U}}{\partial g} + \frac{\partial \mathcal{U}}{\partial c} \cdot \frac{dc}{dg}.$$

This formulation provides a basic insight: at the optimum, public consumption must be out crowding private consumption ($dc/dg < 0$). (Since $\partial \mathcal{U} / \partial c > 0$ and $\partial \mathcal{U} / \partial g > 0$, the condition cannot hold if $dc/dg > 0$.) Hence, while our theory allows for crowding in of private consumption by public consumption ($dc/dg > 0$ or $dy/dg > 1$), crowding in never happens at the optimum. If there is crowding in, then the government should increase g until it starts crowding out private consumption. Crowding out necessarily happens at some point because once unemployment is efficient, total consumption is maximized, so $dy/dg = 0$ and crowding out is one-for-one ($dc/dg = -1$).

Coming back to equation (17) and dividing it by $\partial \mathcal{U} / \partial c$, we obtain the following proposition:

Lemma 2. *Optimal public expenditure satisfies*

$$(18) \quad \underbrace{1 = MRS_{gc}}_{\text{Samuelson rule}} + \underbrace{\frac{\partial y}{\partial x} \cdot \frac{dx}{dg}}_{\text{correction}}.$$

The lemma shows that in a matching model the Samuelson rule needs to be corrected. The correction term is the product of the effect of public expenditure on tightness, dx/dg , times the effect of tightness on total consumption, $\partial y / \partial x$, so it measures the effect of public expenditure on total consumption. For instance, the correction term is positive whenever an increase in public expenditure leads to an increase in total consumption.¹⁶

¹⁶Formula (18) is closely related to the optimal unemployment insurance formula (23) in Landais, Michailat and

When the rate of unemployment is efficient, total consumption is maximized ($\partial y/\partial x = 0$), and the correction term is necessarily zero. Thus, (18) implies that the Samuelson rule remains valid in the presence of unemployment as long as unemployment is efficient. This result shows that the Samuelson rule, which was originally derived in a neoclassical model, is robust: it applies to a broad range of models as long as productive efficiency holds.

However, when the rate of unemployment is inefficient, total consumption is below its maximum ($\partial y/\partial x \neq 0$). In that case, the correction term is nonzero and the Samuelson rule breaks down whenever public consumption affects tightness ($dx/dg \neq 0$). To describe the deviation from the Samuelson rule more concretely, we decompose public expenditure into two components:

Definition 4. *Public expenditure is Samuelson spending plus stimulus spending. Samuelson spending, denoted $(g/c)^* > 0$, satisfies the Samuelson rule:*

$$(19) \quad MRS_{gc}((g/c)^*) = 1.$$

Stimulus spending is $g/c - (g/c)^$.*

Since $MRS_{gc}(0) > 1$ and MRS_{gc} is decreasing in g/c , Samuelson spending is always well-defined. Next, we express the elements of (18) with our three key sufficient statistics: the elasticity of substitution between public and private consumption ε , the unemployment gap $u - u^*$, and the unemployment multiplier m .

Lemma 3. *The term $1 - MRS_{gc}$ can be approximated as follows:*

$$(20) \quad 1 - MRS_{gc} \approx \frac{1}{\varepsilon} \cdot \frac{g/c - (g/c)^*}{(g/c)^*},$$

where ε is evaluated at g/c . The approximation is valid up to a remainder that is $O([g/c - (g/c)^]^2)$.*

The term $\partial y/\partial x$ can be approximated as follows:

$$(21) \quad \frac{x}{y} \cdot \frac{\partial y}{\partial x} \approx \frac{u - u^*}{1 - u^*}.$$

Saez (2016): the two formulas show that in matching models standard public-economics formulas needs to be corrected with a term that is positive whenever the policy improves welfare through tightness.

The approximation is valid up to a remainder that is $O([u - u^*]^2)$. Last, the term dx/dg satisfies

$$(22) \quad \frac{y}{x} \cdot \frac{dx}{dg} = \frac{1}{(1 - \eta) \cdot u \cdot (1 - u)} \cdot m.$$

The proof of the lemma is relegated to [Appendix A](#) but the results are intuitive. Equation (20) directly follows from the definition of the elasticity of substitution between public and private consumption. Equation (22) directly follows from the definition of the unemployment multiplier, given by (8), and the relationship between unemployment and tightness, given by (2). The derivation of equation (21) is more complex, but there is a simple intuition. The goal is to approximate the elasticity of $y(x)$ with respect to x . This elasticity is

$$\frac{x}{y} \cdot \frac{\partial y}{\partial x} = (1 - \eta) \cdot u(x) - \eta \cdot \tau(x) = (1 - \eta) \cdot u(x) \cdot \left[1 - \frac{\eta}{1 - \eta} \cdot \frac{\tau(x)}{u(x)} \right] = (1 - \eta) \cdot u(x) \cdot \left[1 - \frac{u(x^*)}{\tau(x^*)} \cdot \frac{\tau(x)}{u(x)} \right].$$

Now assume that $s \ll f(x)$ and $s \cdot \rho \ll q(x)$. This is what we see in the US labor market, for instance. In that case, we can approximate $s + f(x)$ by $f(x)$ and $q(x) - s \cdot \rho$ by $q(x)$, and since $f(x)/q(x) = x$, we can approximate $\tau(x)/u(x)$ by $\rho \cdot x$ for all x . Accordingly, we get

$$\frac{x}{y} \cdot \frac{\partial y}{\partial x} \approx -(1 - \eta) \cdot u(x) \cdot \frac{x - x^*}{x^*} = -\frac{d \ln(1 - u(x))}{d \ln(x)} \cdot \frac{x - x^*}{x^*} \approx -d \ln(1 - u(x)) \approx \frac{u - u^*}{1 - u^*}.$$

The fact that we can derive equation (21) with some numerical approximations suggests that the first-order approximation in (21) could be accurate even for u far from u^* , a point we investigate numerically in [Section 5.3](#).

Using [Lemma 3](#), [Appendix A](#) proves that (18) can be rewritten as follows:

Lemma 4. *Optimal stimulus spending satisfies*

$$(23) \quad \frac{g/c - (g/c)^*}{(g/c)^*} \approx \frac{1}{z_0} \cdot \varepsilon \cdot m \cdot \frac{u - u^*}{u^*},$$

where ε and m are evaluated at $[g/c, u]$ and $z_0 = (1 - \eta) \cdot (1 - u^*)^2$. The approximation is valid up to a remainder that is $O([u - u^*]^2 + [g/c - (g/c)^*]^2)$.

Formula (23) shows that optimal stimulus spending depends on a public-expenditure multiplier (here, the unemployment multiplier m). This result confirms an intuition that macroeconomists

have had for a long time. However, the multiplier is not sufficient to measure the effect of public expenditure on welfare because an increase in public expenditure also modifies the composition of households' consumption. This is why the elasticity of substitution between public and private consumption ε also enters the formula.

Formula (23) is similar to formula (45) in Woodford (2011): both formulas show that when productive efficiency is not respected, it is optimal to depart from the Samuelson rule, and the departure depends on a public-expenditure multiplier. These two formulas complement each other because they apply to very different models, in which productive inefficiency takes very different forms: we use a matching model with unemployment while Woodford uses a New Keynesian model with monopolistic competition.¹⁷

Formula (23) characterizes optimal public expenditure only implicitly, because the statistics in the right-hand side of the formula are endogenous to the policy (especially u). This means that we cannot use the formula to compute optimal stimulus spending based on the current value of the statistics (especially the current unemployment gap). It also means that we cannot use the formula to perform various comparative statics, such as how optimal stimulus spending depends on the multiplier or elasticity of substitution. This is a typical limitation of the sufficient-statistic approach (Chetty 2009). Here we develop a sufficient-statistic formula that addresses this limitation.

Let us assume that public expenditure is at the Samuelson level $(g/c)^*$ and unemployment is at an inefficient rate $u_0 \neq u^*$. We have in mind the following scenario. Initially everything is going well: unemployment is efficient and accordingly public expenditure satisfies the Samuelson rule. Then a shock occurs and unemployment departs from its efficient level. The shock could be of any type: aggregate-demand shock, aggregate-supply shock, shock to the price of services, shock to the matching function, or shock to the separation rate. Given this initial unemployment gap $u_0 - u^*$, we determine optimal stimulus spending $g/c - (g/c)^*$. As g/c deviates from $(g/c)^*$, unemployment endogenously responds, so we cannot plug $u_0 - u^*$ into (23) to determine optimal stimulus spending. But, taking the response of unemployment into account, we transform (23) into the following explicit sufficient-statistic formula expressed in terms of variables independent of the optimal policy response.

¹⁷Woodford's formula and ours are obtained from static considerations. Werning (2012, pp. 31–33) shows how these formulas are altered by dynamic considerations in a liquidity trap.

Proposition 1. *Suppose that the economy is initially at an equilibrium $[(g/c)^*, u_0]$. Then optimal stimulus spending satisfies*

$$(24) \quad \frac{g/c - (g/c)^*}{(g/c)^*} \approx \frac{1}{z_0} \cdot \frac{\varepsilon \cdot m}{1 + (z_1/z_0) \cdot \varepsilon \cdot m^2} \cdot \frac{u_0 - u^*}{u^*},$$

where ε and m are evaluated at $[(g/c)^*, u_0]$, $z_0 = (1 - \eta) \cdot (1 - u^*)^2$, and $z_1 = (g/y)^* \cdot [1 - (g/y)^*] / u^*$. Under the optimal policy, the unemployment rate is

$$(25) \quad u \approx u^* + \frac{1}{1 + (z_1/z_0) \cdot \varepsilon \cdot m^2} \cdot (u_0 - u^*).$$

The approximations (24) and (25) are valid up to a remainder that is $O([u_0 - u^*]^2 + [g/c - (g/c)^*]^2)$.

Formula (24) links optimal stimulus spending $g/c - (g/c)^*$ to three main sufficient statistics: the elasticity of substitution between public and private consumption ε , defined by (6), the unemployment multiplier m , defined by (8), and the unemployment gap $u_0 - u^*$. Furthermore, the formula involves the Samuelson spending $(g/c)^*$, defined by (19), the efficient unemployment rate u^* , defined by (5), and constants $z_0 > 0$ and $z_1 > 0$, which depend on $(g/c)^*$, u^* , and the matching elasticity η . Formula (25) links the unemployment rate under optimal public expenditure to the same statistics. The advantage of formula (24) over (23) is that the statistics in the right-hand side are not endogenous to the policy. Thus, we can use the formula to compute optimal stimulus spending based on the current value of the statistics.

The proof of the proposition is relegated to [Appendix A](#), but the results can be explained informally. Since the unemployment multiplier m is proportional to du/dg a first-order Taylor expansion of u at $u((g/c)^*) = u_0$ yields

$$u \approx u_0 - \text{constant} \cdot m \cdot \frac{g/c - (g/c)^*}{(g/c)^*}$$

Plugging this expression into (23) then yields

$$\frac{g/c - (g/c)^*}{(g/c)^*} \approx \frac{1}{z_0} \cdot \varepsilon \cdot m \cdot \frac{u_0 - u^*}{u^*} - \frac{\text{constant}}{z_0 \cdot u^*} \cdot \varepsilon \cdot m^2 \cdot \frac{g/c - (g/c)^*}{(g/c)^*}.$$

Reworking this expression then leads to formula (24).

In Section 5 we will complement Proposition 1 in two ways. First, we will show how the sufficient statistics in formula (24) can be calibrated. As an application, we will compute optimal stimulus spending at the onset of the Great Recession in the United States. Second, we will simulate a structural model to verify that formula (24) is accurate. Since the unemployment rate displays large fluctuations, a concern is that the second-order remainder in the formula (24) could be large and the approximation therefore inaccurate. In the simulation, however, this does not happen.

It is true that formula (24) is not based on the output multiplier, which usually features in academic and policy discussions. However, the unemployment and output multipliers are closely related, so we can recast the results in terms of the output multiplier. First, we introduce the empirical unemployment multiplier. This multiplier will be a bridge between the unemployment multiplier m in our formula and the output multiplier. We define the empirical unemployment multiplier by

$$(26) \quad M = -\frac{Y}{1-u} \cdot \frac{du}{dG}.$$

The empirical unemployment multiplier measures the percent increase of the employment rate, $1-u$, when public expenditure increases by 1 percent of GDP. In practice, $1-u \approx 1$ so $M \approx -du/(dG/Y)$: this multiplier approximately measures the decrease of the unemployment rate when public expenditure increases by 1 percent of GDP. Appendix A shows that the theoretical and empirical multipliers are close cousins:

$$(27) \quad m = \frac{(1-u) \cdot M}{1 - \frac{G}{Y} \cdot \frac{\eta}{1-\eta} \cdot \frac{\tau}{u} \cdot M}.$$

Thus, the theoretical unemployment multiplier, m , and the empirical unemployment multiplier, M , always have the same sign. Moreover, m is increasing in M . In practice, the two unemployment multipliers take similar values. In turn, the empirical unemployment multiplier and output multiplier are closely related. Consider a change in public expenditure dG . This change leads to a

change du in unemployment and, since $Y = (1 - u) \cdot k$, to a change $dY = -du \cdot k$ in output. Hence,

$$\frac{dY}{dG} = -k \cdot \frac{du}{dG} = -\frac{Y}{1-u} \cdot \frac{du}{dG} = M.$$

Since the empirical unemployment multiplier and output multiplier are equal, our formula can be expressed with the output multiplier by replacing m by the function of M given by (27) and then substituting dY/dG for M . After this manipulation, it is clear that the output multiplier affects optimal stimulus spending in the same way as the unemployment multiplier m .

An important caveat, however, is that the output multiplier is only useful when the tax changes associated with stimulus spending are nondistortionary. We will show in Section 4 that when taxation is distortionary, the link between unemployment and output multipliers breaks down, so it is necessary to use the unemployment multiplier and not the output multiplier to design stimulus spending.

3.2. Implications

We now explore three implications of sufficient-statistic formula (24): how optimal stimulus spending depends on the unemployment gap, how it depends on the unemployment multiplier, and how it depends on the elasticity of substitution between public and private consumption. We also use (25) to describe the properties of the unemployment gap under optimal stimulus spending.

Formula (24) immediately gives the sign of optimal stimulus spending in various situations:

Proposition 2. *Consider first a positive unemployment multiplier ($m > 0$). If the unemployment gap is positive ($u_0 > u^*$), optimal stimulus spending is positive ($g/c > (g/c)^*$) but does not completely fill the unemployment gap ($u > u^*$). If the unemployment gap is negative ($u_0 < u^*$), optimal stimulus spending is negative ($g/c < (g/c)^*$) but does not completely eliminate the unemployment gap ($u < u^*$). Next, if the unemployment multiplier is negative ($m < 0$), the sign of optimal stimulus spending is the opposite. Last, if the unemployment multiplier is zero ($m = 0$), optimal stimulus spending is zero ($g/c = (g/c)^*$).*

It is only if the unemployment multiplier is always zero or the unemployment gap is always zero that public expenditure should always be at the Samuelson level. In all other situations, optimal

public expenditure deviates from the Samuelson rule and optimal stimulus spending is nonzero. All the possibilities are illustrated in Table 1. The general pattern is that when unemployment is inefficiently high or low, public expenditure should deviate from the Samuelson level to partially fill the initial unemployment gap.

These results have implications for the cyclical behavior of optimal public expenditure. Under the presumption that the unemployment gap is positive in slumps but negative in booms and that the unemployment multiplier is nonzero with a constant sign, then optimal stimulus spending changes sign over the business cycle. Thus optimal public expenditure fluctuates over the business cycle around the Samuelson level. Furthermore, the optimal policy does not eliminate the underlying unemployment gap: it only reduces it. This means that public expenditure cannot restore the efficient equilibrium (where unemployment is efficient and public expenditure satisfies the Samuelson rule); only a policy affecting the price of services could achieve this. Thus, public expenditure is a second-best, not a first-best, policy.

To understand these results, imagine that public expenditure is at the Samuelson level, the unemployment multiplier is positive, and unemployment is inefficiently high. Keeping total consumption constant, increasing public consumption by 1 service reduces private consumption by 1 service. At the Samuelson level, the marginal utilities of public and private consumption are equalized, so the increase in public expenditure has no first-order effect on welfare so far. Yet, since the unemployment multiplier is positive, increasing public consumption lowers unemployment; and since unemployment is inefficiently high, reducing unemployment raises total consumption. Once the effect on public expenditure on unemployment is accounted for, the increase in public expenditure has a positive effect on welfare. Thus, it is optimal to raise public expenditure above the Samuelson level—that is, to have positive stimulus spending.

These results challenge common ideas in macroeconomics. The first is that stimulus spending is desirable in slumps only if public consumption crowds in private consumption (i.e., public spending *increases* private spending). The logic comes from traditional Keynesian models: in these models, public consumption crowds in private consumption through the Keynesian cross, so additional public consumption improves welfare. During the Great Recession, this idea led to a spurt of empirical research to determine whether public consumption was indeed crowding in private consumption (for example, [Cwik and Wieland 2011](#); [Ramey 2013](#)). Our theory shows in

Table 1: Sign of Optimal Stimulus Spending

Unemployment gap	Unemployment multiplier		
	$m < 0$	$m = 0$	$m > 0$
$u_0 - u^* < 0$	$g/c - (g/c)^* > 0$	$g/c - (g/c)^* = 0$	$g/c - (g/c)^* < 0$
$u_0 - u^* = 0$	$g/c - (g/c)^* = 0$	$g/c - (g/c)^* = 0$	$g/c - (g/c)^* = 0$
$u_0 - u^* > 0$	$g/c - (g/c)^* < 0$	$g/c - (g/c)^* = 0$	$g/c - (g/c)^* > 0$

Notes: This table illustrates Proposition 2. It gives the sign of optimal stimulus spending ($g/c - (g/c)^*$) as a function of the initial unemployment gap ($u_0 - u^*$) and the initial unemployment multiplier (m). The Samuelson spending $(g/c)^*$ is defined by (19), the efficient unemployment rate u^* by (5), and the unemployment multiplier m by (8).

contrast that while crowding in is sufficient to justify stimulus spending, it is not at all necessary. In our theory, public consumption crowds out private consumption; nevertheless, stimulus spending is desirable in slumps whenever the unemployment multiplier is positive.

The second is the Keynesian result that public expenditure should be used to fill the output gap. [Mankiw and Weinzierl \(2011\)](#) formally derive this result with a disequilibrium model. In contrast, in our theory, optimal stimulus spending fills the unemployment gap partially, not completely.¹⁸ The difference comes from the crowding out of private consumption by public consumption. In the disequilibrium model, there is no crowding out and public consumption is valuable, so public expenditure should be increased until the output gap is entirely filled. In our model, there is crowding out; in fact crowding out is one-for-one once the unemployment gap is completely filled (since total consumption is maximum at that point). Accordingly, it cannot be optimal to deviate from the Samuelson rule and completely fill the unemployment gap. If the government did that, the situation would be such that one unit of public consumption costs one unit of private consumption (since crowding out is one-for-one when the unemployment gap is zero) but the marginal value of public consumption is higher than the marginal value of private consumption (since public spending is above Samuelson spending). This situation is suboptimal: welfare can be increased by reducing public expenditure.

Next, we determine how optimal stimulus spending depends on the unemployment multiplier. By writing the right-hand side of (24) as a function of m and characterizing this function, it is straightforward to obtain the following proposition:

¹⁸[Woodford \(2011, Figure 4\)](#) and [Werning \(2012, p. 29\)](#) obtain a similar result using New Keynesian models in a liquidity trap.

Proposition 3. Assume that the initial unemployment gap is positive ($u_0 - u^* > 0$). Then optimal stimulus spending is a hump-shaped function of the unemployment multiplier: it is 0 when $m = 0$, increasing in m for $m \in [0, m^\dagger]$, maximized at $m = m^\dagger$, decreasing in m for $m \in [m^\dagger, +\infty]$, and 0 for $m \rightarrow +\infty$. The maximizing multiplier is

$$(28) \quad m^\dagger = \sqrt{\frac{z_0}{z_1 \cdot \varepsilon}}$$

and the maximum optimal stimulus spending is

$$(29) \quad \frac{(g/c)^\dagger - (g/c)^*}{(g/c)^*} = \frac{1}{2} \cdot \sqrt{\frac{\varepsilon}{z_0 \cdot z_1}} \cdot \frac{u_0 - u^*}{u^*}.$$

The unemployment gap under the optimal policy is a decreasing function of the multiplier: it falls from $u_0 - u^*$ when $m = 0$ to 0 when $m \rightarrow +\infty$.

The proposition shows that for a given unemployment gap, optimal stimulus spending is a hump-shaped function of the unemployment multiplier: it is increasing in the multiplier until a threshold, and decreasing after that. The proposition also gives the threshold and the maximum optimal stimulus spending. For concreteness this proposition and the next only consider positive unemployment multipliers and unemployment gaps, but we could of course derive similar results with negative unemployment multipliers and unemployment gaps.

What is the intuition behind the hump-shape result? When public expenditure is optimal, the marginal social cost from consuming too many public services and too few private services equals the marginal social value from reducing unemployment. This marginal social value is determined by two factors: the current unemployment multiplier, which measures how much unemployment can be reduced by additional expenditure, and the current unemployment gap, which measures the social value from lower unemployment. For a given amount of stimulus spending and a given initial unemployment gap, a larger initial multiplier has conflicting effects on the two factors: it means a larger current multiplier (a higher marginal social value) but a smaller current unemployment gap (a lower marginal social value). The first effect advocates for more spending but the second for less spending. It turns out that the first effect dominates for small multipliers, so optimal stimulus spending is increasing in the multiplier; but the second effect dominates for large multipliers, so

optimal stimulus spending is decreasing in the multiplier. In fact, for large multipliers, it becomes optimal to nearly entirely fill the unemployment gap; since less spending is required to fill the gap when the multiplier is larger, optimal stimulus spending is decreasing in the multiplier.

Our results challenge the view that larger multipliers entail larger stimulus spending—the bang-for-the-buck logic. For instance, stimulus skeptics usually believe in small multipliers and infer that stimulus spending should be small or zero in slumps. Following the same logic, stimulus advocates usually believe in large multipliers and infer that stimulus spending should be large in slumps. Indeed, at first glance, formula (23) seems to justify this bang-for-the-buck logic: for a given $u - u^*$, a larger m seems to indicate that the optimal $g/c - (g/c)^*$ is larger. However, our theory shows that a larger unemployment multiplier does not necessarily justify a stronger response of public expenditure to fluctuations in unemployment. Instead, the relationship between optimal stimulus spending and the size of the unemployment multiplier is hump-shaped. The relationship peaks for an unemployment multiplier of m^\dagger , and higher and lower multipliers warrant smaller levels, such that optimal stimulus spending is similar for some small and large multipliers. The reason why the bang-for-the-buck logic fails is that it omits the response of unemployment to public expenditure in formula (23).

Last, we determine how optimal stimulus spending depends on the elasticity of substitution between public and private consumption. By writing the right-hand side of (24) as a function of ε and characterizing this function, we immediately obtain the following proposition:

Proposition 4. *Assume that the unemployment multiplier and initial unemployment gap are positive ($m > 0$ and $u_0 - u^* > 0$). Then optimal stimulus spending is an increasing function of the elasticity of substitution between public and private consumption: it rises from 0 when $\varepsilon = 0$ to*

$$\frac{g/c - (g/c)^*}{(g/c)^*} = \frac{1}{z_1 \cdot m} \cdot \frac{u_0 - u^*}{u^*}$$

when $\varepsilon \rightarrow +\infty$. The unemployment gap under the optimal policy is a decreasing function of the elasticity of substitution: it falls from $u_0 - u^$ when $\varepsilon \rightarrow 0$ to 0 when $\varepsilon \rightarrow +\infty$.*

The proposition shows that both optimal stimulus spending and the share of the unemployment gap filled under the optimal policy are increasing in the elasticity of substitution between public and private consumption.

There are two interesting special cases. The first special case is $\varepsilon \rightarrow 0$. In this situation, additional public services have zero value, so additional public workers dig and fill holes. In that case, optimal stimulus spending is zero, irrespective of the unemployment rate and multiplier. Intuitively, public consumption beyond the Samuelson level is useless. Since public consumption always crowds out private consumption, it is never optimal to provide more public consumption than in the Samuelson rule.¹⁹

The second special case is $\varepsilon \rightarrow +\infty$. In this situation, the public services provided by the government perfectly substitute for the private services purchased by households. In that case, optimal stimulus spending completely fills the unemployment gap such that $u = u^*$. This result holds even if the multiplier is very small and public expenditure severely crowds out private consumption. Intuitively, public and private consumption are interchangeable, so it is optimal to maximize total consumption, irrespective of its composition. Accordingly, it is optimal to completely fill the unemployment gap.

The result that public expenditure should completely fill the unemployment gap when $\varepsilon \rightarrow \infty$ is reminiscent of the Keynesian result that public expenditure should completely fill the output gap, but the logic behind the two results is completely different. The Keynesian result arises because there is no crowding out of private consumption by public consumption in Keynesian models. In our model there is crowding out, but the composition of total consumption does not matter when $\varepsilon \rightarrow \infty$, so crowding out has no welfare cost.

Furthermore, our theory clarifies the link between the usefulness of public expenditure and optimal stimulus spending. A concern of stimulus skeptics is that additional public expenditure could be wasteful. Our theory develops this argument. It is true that when the elasticity of substitution

¹⁹When $\varepsilon \rightarrow 0$, the first-order approximation of MRS_{gc} does not work well. As a result, there is one additional case that is not covered by formula (24): when $\varepsilon \rightarrow 0$, it would be optimal to raise public expenditure above the Samuelson level if public consumption crowds in private consumption ($dc/dg > 0$ or equivalently $dy/dg > 1$). It remains true that public consumption in itself is useless above the Samuelson level, but since more public consumption implies more private consumption, it is optimal to have positive stimulus spending. This case is not visible with formula (24), but it can be seen with (18), which can be written $1 = MRS_{gc}(g/c) + dy/dg$. When $\varepsilon \rightarrow 0$, then $MRS_{gc}(g/c) = 0$ as soon as $g/c > g/c^*$. Nevertheless, it is optimal to increase spending above $(g/c)^*$ if $dy/dg > 1$ at $g/c > (g/c)^*$. In this case it is optimal to spend until dy/dg falls to 1.

However, this situation with $dy/dg > 1$ is unlikely to happen in reality. First, it requires extremely large deviations from u^* . Indeed, at u^* , total consumption is maximized so $dy/dg = 0$. Hence, u_0 must be far enough from u^* that it is possible to reach $dy/dg = 1$. Second, it requires strange effects of public expenditure on the price of services. We can show that if the price of services is unaffected or raised by an increase in public expenditure, then necessarily $dc/dg < 0$ and $dy/dg < 1$. It is only in the strange situation where higher public expenditure means lower prices that we could see $dy/dg > 1$.

between public and private consumption is zero, so that additional public workers dig and fill holes in the ground, public expenditure should remain at the Samuelson level. But in the more realistic case where the elasticity of substitution is positive, some stimulus spending is desirable in slumps.

We also qualify the view of stimulus advocates in the Keynesian tradition who argue that public expenditure should entirely fill the unemployment gap, irrespective of the usefulness of additional public expenditure. It is true that when the elasticity of substitution is infinite, so that public and private consumption are perfect substitute, public expenditure should completely fill the unemployment gap. But in the more realistic case where the elasticity is finite, optimal public expenditure fills the unemployment gap partially, not completely.

4. Distortionary Taxation

We now study how distortionary taxation affects optimal public expenditure. To that end, we introduce an endogenous labor supply and a distortionary income tax. We compare two approaches to taxation: the traditional approach in public economics and macroeconomics, which consists in using a linear income tax; and the modern approach in public economics, which consists in using a nonlinear income tax implemented following the benefit principle.

4.1. Traditional Approach

In the traditional approach to taxation, the government uses a linear income tax τ^L to finance public expenditure. The representative household supplies a productive capacity k at a utility cost $\mathcal{W}(k)$, where the function \mathcal{W} is strictly increasing in k and convex. Choosing k is akin to a labor-supply decision. With the linear income tax, the household's labor income becomes $(1 - \tau^L) \cdot Y(x, k) = (1 - \tau^L) \cdot (1 - u(x)) \cdot k$. To finance public expenditure G , the tax rate must be $\tau^L = G/Y = g/y$.

The household chooses k to maximize utility. Let $MRS_{kc} \equiv \mathcal{W}'(k)/(\partial \mathcal{U} / \partial c)$ be the marginal rate of substitution between labor and private consumption. As usual, the households' optimal labor supply decision is such that the marginal rate of substitution between labor and consumption equals the post-tax real wage. Here, this optimality condition is

$$(30) \quad MRS_{kc} = (1 - \tau^L) \cdot \frac{1 - u(x)}{1 + \tau(x)}.$$

Indeed, one unit of labor is only sold with probability $1 - u$. When it is sold, it only yields $1/(1 + \tau(x))$ units of consumption. Hence, the effective real wage is $(1 - u)/(1 + \tau(x))$ and the post-tax real wage is $(1 - \tau^L) \cdot (1 - u)/(1 + \tau(x))$.²⁰

The supply decision is distorted by the income tax: a higher τ^L implies a lower k . In fact, equation (30) implicitly defines a function $k(g)$ that describes how the aggregate productive capacity responds to a change in public expenditure and the associated change in the income tax rate. Since the income tax is distortionary, the function $k(g)$ is decreasing in g .

The welfare of an equilibrium is $\mathcal{U}(c, g) - \mathcal{W}(k)$. Given a tightness function $x(g)$ and a capacity function $k(g)$, the government chooses g to maximize $\mathcal{U}(y(x(g), k(g)) - g, g) - \mathcal{W}(k(g))$. The first-order condition of the government's problem is

$$0 = \frac{\partial \mathcal{U}}{\partial g} - \frac{\partial \mathcal{U}}{\partial c} - \mathcal{W}'(k) \cdot \frac{dk}{dg} + \frac{\partial \mathcal{U}}{\partial c} \cdot \frac{\partial y}{\partial k} \cdot \frac{dk}{dg} + \frac{\partial \mathcal{U}}{\partial c} \cdot \frac{\partial y}{\partial x} \cdot \frac{dx}{dg}.$$

Dividing the condition by $\partial \mathcal{U} / \partial c$, we obtain

$$1 = MRS_{gc} - \left(MRS_{kc} - \frac{\partial y}{\partial k} \right) \cdot \frac{dk}{dg} + \frac{\partial y}{\partial x} \cdot \frac{dx}{dg}.$$

Households' optimal labor supply, given by (30), implies that $MRS_{kc} = (1 - \tau^L) \cdot (\partial y / \partial k)$. The government's budget constraint implies that $\tau^L = g/y$. Last, from equation (4), $\partial y / \partial k = y/k$. Hence, $-(MRS_{kc} - \partial y / \partial k) = \tau_L \cdot y/k = g/k$ and we have proved the following:

Lemma 5. *In the model of Section 4.1, optimal public expenditure satisfies*

$$\underbrace{1 - \frac{d \ln(k)}{d \ln(g)}}_{\text{modified Samuelson rule}} = MRS_{gc} + \underbrace{\frac{\partial y}{\partial x} \cdot \frac{dx}{dg}}_{\text{correction}}.$$

This optimality condition differs from (18), but the two have the same structure once the Samuelson rule is modified to account for distortionary taxation.²¹ Indeed, this condition can

²⁰Formally, for all the models in Section 2 and Appendix C, the first-order condition with respect to k is $\mathcal{W}'(k) = \lambda \cdot (1 - \tau^L) \cdot (1 - u(x))$, where λ is the costate variable associated with real wealth in the household's Hamiltonian. We combine this equation and the first-order condition with respect to c , given by (12), and obtain (30).

²¹In the public economics literature, the modified Samuelson rule was developed by Stiglitz and Dasgupta (1971), Diamond and Mirrlees (1971), and Atkinson and Stern (1974) to describe optimal public expenditure with a linear income tax. A large literature has built on these two papers. See Ballard and Fullerton (1992) and Kreiner and

be written as the modified Samuelson rule plus a correction equal to $(\partial y/\partial x) \cdot (dx/dg)$. The statistic $1 - d\ln(k)/d\ln(g) > 1$ in the modified Samuelson rule is the marginal cost of funds. It is more than one because the linear income tax distorts the labor supply so it costs more than one private service to produce one public service. Because the marginal cost of funds is greater than one, the modified Samuelson rule recommends a lower level of public expenditure than the regular rule. It is useful generalize the definition of Samuelson spending with distortionary taxation:

Definition 5. Samuelson spending, denoted $(g/c)^*$, is defined by

$$(31) \quad MRS_{gc}((g/c)^*) = 1 - \frac{d\ln(k)}{d\ln(g)},$$

where the elasticity $d\ln(k)/d\ln(g) < 0$ is evaluated at optimal public expenditure.

Although Samuelson spending is lower with a linear income tax, the correction to the Samuelson rule is the same, so as we prove in [Appendix A](#), our sufficient-statistic formula for optimal stimulus spending remains the same:

Proposition 5. Consider the model of [Section 4.1](#), and define Samuelson spending by [\(31\)](#). Suppose that the economy is initially at an equilibrium $[(g/c)^*, u_0]$. Then optimal stimulus spending satisfies [\(24\)](#) and the unemployment rate under the optimal policy satisfies [\(25\)](#), where the statistic z_1 is generalized to allow for supply-side responses:

$$z_1 = \frac{(g/y)^* \cdot [1 - (g/y)^*]}{u^*} \cdot \frac{1}{1 - d\ln(k)/d\ln(g)}.$$

The elasticity $d\ln(k)/d\ln(g)$ is evaluated at $[(g/c)^*, u^*]$.

The unemployment multiplier in formulas [\(24\)](#) and [\(25\)](#) is a policy elasticity, in the sense of [Hendren \(2016\)](#). It measures the change in unemployment for a change in public expenditure accompanied by the change in taxes maintaining a balanced government budget. In [Section 3](#) taxes are not distortionary, so the unemployment multiplier should be measured using a policy reform in which taxes are nondistortionary. Here taxes are distortionary, so the unemployment multiplier should be measured using a policy reform in which the tax change distorts the labor supply.

[Verdelin \(2012\)](#) for surveys and [Kleven and Kreiner \(2006\)](#) for a recent contribution.

When taxation is nondistortionary, equation (27) shows that the unemployment multiplier m in our sufficient-statistic formula is closely related to the empirical unemployment multiplier M . Furthermore, the output multiplier is equal to M , so all our results remain the same if we reformulate them with the output multiplier instead of m . But when taxation is distortionary, things are different, and the output multiplier cannot be used to design optimal public expenditure. With distortionary taxation, equation (27) remains valid, but the link between the output multiplier and M break down. Indeed, output is $Y = (1 - u) \cdot k$ so

$$\frac{dY}{dG} = -k \cdot \frac{du}{dG} + (1 - u) \cdot \frac{dk}{dG} = -\frac{Y}{1 - u} \cdot \frac{du}{dG} + \frac{Y}{k} \cdot \frac{dk}{dG} = M + \frac{Y}{k} \cdot \frac{dk}{dG}.$$

Since taxes are distortionary, $dk/dG < 0$ and

$$M = \frac{dY}{dG} - \frac{Y}{k} \cdot \frac{dk}{dG} > \frac{dY}{dG}.$$

Thus, when a change in taxes distort the capacity supplied by households, the unemployment multiplier M is the output multiplier net of the supply-side response $(Y/k) \cdot (dk/dG)$. The supply-side response measures the percentage change in labor supply when public expenditure increases by 1 percent of GDP. As taxation is distortionary, the supply-side response is negative and the unemployment multiplier is larger than the output multiplier.

The unemployment multiplier is the correct sufficient statistic whether taxation is distortionary or not. With distortionary taxation, there is a wedge between unemployment and output multipliers equal to the supply-side responses, so the output multiplier is not useful to calibrate optimal stimulus spending. To understand why it is the unemployment multiplier but not the output multiplier that is the correct sufficient statistic in our formula, it is useful to realize that a change in public expenditure has two complementary effects: an effect on unemployment that is usually positive and a negative effect on labor supply. The negative effect on labor supply determines the marginal cost of fund and Samuelson spending, but it has nothing to do with the correction to the Samuelson rule. The effect on unemployment, on the other hand, determines the correction to the Samuelson rule but has nothing to do with Samuelson spending. Stimulus spending is determined by the correction to the Samuelson rule, so it depends on the effect on unemployment (measured by the unemployment multiplier) but not the effect on labor supply. As the output multiplier also conveys

information on labor supply effects, it is not directly relevant to optimal stimulus spending.

One concern of stimulus skeptics is that output is already too low in recession and that increasing taxes to fund stimulus spending would further reduce output through supply-side responses.²² How can such a policy be optimal? Starting from Samuelson spending, a small increase in public expenditure reduces unemployment, reduces labor supply, and increases public consumption, which are all good for welfare; but it reduces output and thus private consumption, which is bad for welfare. At the modified Samuelson rule, the cost of lower private consumption offsets the benefit of higher public consumption and lower labor supply; the only remaining effect on welfare is the positive effect from lower unemployment. Overall, the small increase in public expenditure leads to higher welfare, and it is indeed optimal to raise public expenditure. It is true that raising public expenditure reduces output even further. But, because the reduction in output due to lower labor supply is internalized in the modified Samuelson rule, it is only the increase in output due to lower unemployment that determines the deviation from the modified Samuelson rule.

4.2. *Modern Approach*

An important result in modern public-economic theory is that the optimal provision of public expenditure can be disconnected from distortionary taxation. With a nonlinear income tax, a marginal public expenditure can be financed by a tax change that leaves all individual utilities unchanged and does not further alter labor-supply decisions. This is called the benefit principle: taxes are changed to correspond to the extra benefit received from the public expenditure, individual by individual. If public expenditure is below the Samuelson level, such a reform will raise revenue, and conversely, if public expenditure is above the Samuelson level, such a reform will create a deficit. Hence, the Samuelson formula has to hold; otherwise a Pareto improvement is possible.²³

The benefit principle can also be applied in our setting. In this case, even with distortionary taxation, we obtain the same results as in Section 3. More precisely, we assume that households

²²For instance, [Barro and Redlick \(2011\)](#) find in US data that the deficit-financed output multiplier is positive (around 0.5) but because tax increases significantly depress output, the balanced-budget output multiplier is negative (around -0.6).

²³The benefit principle was introduced by [Hylland and Zeckhauser \(1979\)](#) and [Christiansen \(1981\)](#), and was finally fully understood and generalized by [Kaplow \(1996\)](#). For discussions of the benefit principle, see [Kaplow \(2004\)](#) and [Kreiner and Verdellin \(2012\)](#). The benefit principle is closely related to the approach of [Coate \(2000\)](#), [Jacobs \(2016\)](#), and others, who study optimal public expenditure when taxes satisfy redistributive objectives.

choose capacity k to maximize utility, and that public expenditure is funded by a distortionary, nonlinear income tax $T(k)$ (instead of the linear income tax τ_L of Section 4.1). We start from an equilibrium $[c, g, x, k]$. To ease notation, we introduce $\phi(x) \equiv (1 - u(x))/(1 + \tau(x))$. With the income tax, the household's disposable income becomes $(1 - u(x)) \cdot (k - T(k))$. In equilibrium, households' disposable income equals their expenses: $(1 - u(x)) \cdot (k - T(k)) = (1 + \tau(x)) \cdot c$ so $c = \phi(x) \cdot (k - T(k))$.

We implement a small change in public expenditure dg funded by a small tax change $dT(k)$ that satisfies the benefit principle. This change triggers a small change dx in tightness. By the benefit principle, the tax change $dT(k)$ is designed to keep the household's utility constant for any choice of k . For all k , $dT(k)$ satisfies

$$(32) \quad \mathcal{U}(\phi(x) \cdot [k - T(k)], g) = \mathcal{U}(\phi(x + dx) \cdot [k - T(k) - dT(k)], g + dg).$$

The left-hand and right-hand sides of this equation define two identical functions of k . This implies that the household does not change his choice of k after the reform: the labor supply is unaffected by a change dg funded by the benefit principle.

Taking a first-order expansion of the right-hand side of (32) and subtracting the left-hand side from the right-hand side, we obtain

$$\frac{\partial \mathcal{U}}{\partial c} \cdot [\phi'(x) \cdot (k - T(k)) \cdot dx - \phi(x) \cdot dT(k)] + \frac{\partial \mathcal{U}}{\partial g} \cdot dg = 0.$$

Dividing by $\partial \mathcal{U} / \partial c$ and re-arranging yields

$$T(k) \cdot \phi'(x) \cdot dx + \phi(x) \cdot dT(k) = MRS_{gc} \cdot dg + \phi'(x) \cdot k \cdot dx.$$

Accordingly, the effect of the reform on the government budget balance $R = \phi(x) \cdot T(k) - g$ is

$$dR = T(k) \cdot \phi'(x) \cdot dx + \phi(x) \cdot dT(k) - dg = [MRS_{gc} - 1] \cdot dg + \phi'(x) \cdot k \cdot dx = [MRS_{gc} - 1] \cdot dg + \frac{\partial y}{\partial x} \cdot dx.$$

(We used $dk = 0$ and $\phi'(x) \cdot k = \partial y / \partial x$.) At the optimum, $dR = 0$, so we have proved the following:

Lemma 6. *Under the benefit-principle, optimal public expenditure satisfies (18) even with distor-*

tionary taxation.

Under the benefit principle, (18) remains valid and capacity k is not affected by changes in public expenditure. Thus, our sufficient-statistic formula also remains valid and we have:

Proposition 6. *Suppose that the economy is initially at an equilibrium $[(g/c)^*, u_0]$. Then, under the benefit principle, optimal stimulus spending satisfies (24) and the unemployment rate under the optimal policy satisfies (25).*

Under the benefit principle, there are no labor-supply distortions for a marginal increase in public expenditure; therefore, output and unemployment multipliers are equal, and the output multiplier can be used to design optimal stimulus spending.

5. Numerical Applications

We now complement our theoretical results with several numerical applications. First, we calibrate our sufficient-statistic formula and compute optimal stimulus spending at the onset of the Great Recession in the United States. This calibration illustrates quantitatively how much optimal public expenditure may deviate from the Samuelson rule. Second, we calibrate and simulate the matching model with land to show that it generates realistic business-cycle fluctuations in the unemployment rate and the unemployment multiplier. This realism suggests that the matching framework is useful to study optimal public expenditure over the business cycle. Third, we use the same simulations to verify the accuracy of the sufficient-statistic formula. Since the formula is obtained with first-order approximations, it is important to verify that in a fully specified, structural model, it yields a similar policy as the exact optimality condition.

5.1. *The Great Recession in the United States*

Here we use our sufficient-statistic formula, given by (24), to compute the optimal response of public expenditure to the increase in unemployment observed at the onset of the Great Recession in the United States. We examine how such optimal stimulus spending varies with the unemployment multiplier and the elasticity of substitution between public and private consumption. Since the

formula is valid whether taxes are distortionary or not, the numerical results are valid irrespective of the form that taxation takes.

Our starting point is 2008:Q3 in the United States. Then, the unemployment rate was $u = 6\%$ and public expenditure was $G/C = 19.7\%$.²⁴ We consider the unemployment rate in 2008:Q3 as efficient: $u^* = 6\%$. This choice seems a reasonable starting place as the unemployment rate in 2008:Q3 is close to its 25-year average, and there is a presumption, going back at least to [Okun \(1963\)](#), that the economy is efficient on average. If unemployment is efficient in 2008:Q3, it is optimal for the government to follow the Samuelson rule. Hence for simplicity, we set Samuelson spending to the level of public expenditure in 2008:Q3: $(G/C)^* = 19.7\%$. This choice also seems reasonable as G/C in 2008:Q3 is equal its 25-year average.

In 2008, a shock hits the US economy and unemployment starts rising toward an inefficient level $u_0 > u^*$. In theory our sufficient-statistic formula accommodates any type of shock. But since we have calibrated u^* and $(G/C)^*$ based on observations before the shock, our analysis requires that the shock does not affect u^* and $(G/C)^*$. So it could be a shock to the aggregate demand, a shock to the price of services relative to the saving instrument, or a shock to the aggregate supply (either labor-force participation rate or productivity of each worker). But it could not be a shock to the matching function (mismatch shock), for instance.

In our model unemployment immediately reaches the higher level u_0 , but in reality unemployment slowly rises to u_0 . The challenge for policymakers is to forecast u_0 before that level is reached. In the winter 2008–2009, when the US government designed their stimulus package, they forecast $u_0 = 9\%$ ([Romer and Bernstein 2009](#), Figure 1). Hence, we use $u_0 = 9\%$.

To apply our sufficient-statistic formula, we need estimates of three statistics: the elasticity of substitution between public and private consumption (ε), the matching elasticity (η), and the unemployment multiplier (m). The elasticity of substitution reflects the value that society places

²⁴We set u to the seasonally adjusted unemployment rate constructed by the Bureau of Labor Statistics from the Current Population Survey. To construct G/C , we set G to the seasonally adjusted employment level in the government industry and C to the seasonally adjusted employment level in the private industry. Both series constructed are by the Bureau of Labor Statistics from the Current Employment Statistics survey. Over the 1990–2014 period, the average unemployment rate is $u = 6.1\%$ and the average public expenditure is $G/C = 19.7\%$. We could also have set G and C to the government consumption expenditures and personal consumption expenditures constructed by the Bureau of Economic Analysis as part of the National Income and Product Accounts. Over 1990–2014, this measure of public expenditure averages $G/C = 23.0\%$, and in 2008:Q3, this measure of public expenditure is $G/C = 23.8\%$. With this alternative calibration, Samuelson spending would be higher but optimal stimulus spending would not be affected.

on public services, so it is an input into the design of the optimal policy. Accordingly, we consider several values: $\varepsilon = 0.5$, $\varepsilon = 1$, and $\varepsilon = 2$. The case $\varepsilon = 1$ is a useful point of reference because it corresponds to a Cobb-Douglas utility function: $\mathcal{U}(c, g) = c^{1-\gamma} \cdot g^\gamma$. To calibrate the matching elasticity, we rely on a vast literature that estimates matching functions on labor market data. Following Landais, Michaillat and Saez (2017, p. 48), who review this literature, we set $\eta = 0.6$.

The last step is to determine plausible values for the unemployment multiplier m . Since m is difficult to observe, we report estimates for the closely related multiplier M and then translate M into m using (27).²⁵ In fact, as above, we can set $(G/Y)^* = 0.197/(1 + 0.197) = 16.5\%$, $\eta = 0.6$, and $u_0 = 9\%$. The series on resources devoted to matching constructed by Landais, Michaillat and Saez (2017) indicates that when the unemployment rate is around 9%, as in 2009:Q2, then $\tau = 1.7\%$. So we set $\tau_0 = 1.7\%$. Under this calibration, m is almost identical to M :

$$(33) \quad m = \frac{0.91 \times M}{1 - 0.165 \times 1.5 \times (1.7/9) \times M} = \frac{0.91 \times M}{1 - 0.046 \times M}.$$

We use this formula to transform any estimate of M into the estimate of m that enters formula (24).

The unemployment multiplier M is estimated by measuring the response of the unemployment rate (in percentage points) when public expenditure increases by 1% of GDP. Monacelli, Perotti and Trigari (2010, pp. 533–536) estimate a structural vector autoregression (SVAR) on US data and find unemployment multipliers between 0.2 and 0.6. Ramey (2013, pp. 40–42) estimates SVARs on US data with various identification schemes and sample periods. She finds unemployment multipliers between 0.2 and 0.5, except in one specification where the multiplier is 1.²⁶

Overall, the average unemployment multiplier is likely to fall in the 0.2–1 range. If multipliers are larger when unemployment is higher, as suggested by recent research on state-dependent multipliers, the multiplier entering our formula could even be larger. For instance, using regime-

²⁵The multiplier m measures the response to a change in government consumption g , which is public expenditure G minus the resources devoted by the government for matching and therefore difficult to observe. The multiplier M is much easier to estimate because it measures the response to a change in public expenditure G , which are reported in national accounts.

²⁶One potential issue is that these multipliers are estimated using deficit-financed increases in public expenditure instead of balanced-budget increases (Barro and Redlick 2011). This issue is not important here: unemployment multipliers are larger once distortionary taxation is taken into account. This is because a reduction in labor supply following an increase in distortionary taxes leads to higher tightness and thus a lower unemployment rate in the matching model (Michaillat and Saez 2015, pp. 530–532). Hence, these estimates are lower bounds for the balanced-budget unemployment multiplier.

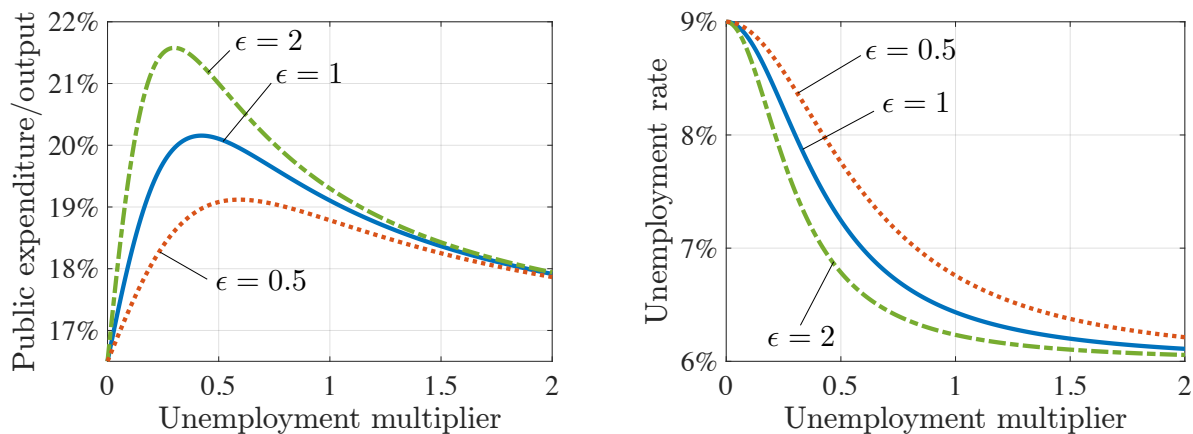


Figure 3: Optimal Public Expenditure, and Unemployment Rate under the Optimal Policy

Notes: In the United States, in 2008:Q3, unemployment was $u = 6\%$ and public expenditure was $G/Y = 16.5\%$. We assume that at that point, unemployment was efficient and public expenditure was optimal. The Great Recession shock hit the US economy around that time, raising the unemployment rate to a projected level of 9% . The left-hand graph displays the optimal response of public expenditure, expressed as a share of output, after the shock. This response is computed using (24), $u^* = 6\%$, $(G/Y)^* = 16.5\%$, $u_0 = 9\%$, $\eta = 0.6$, $\varepsilon = 0.5, 1, \text{ and } 2$, and $M \in [0, 2]$. The right-hand graph displays the unemployment rate under optimal public expenditure. This unemployment rate is computed using (25) with the same calibration.

switching SVARs on US data, [Auerbach and Gorodnichenko \(2012, Table 1, rows 1–3\)](#) find that while the output multiplier is 0.6 in expansions and 1 on average, it is as high as 2.5 in recessions. To account for the uncertainty about the exact estimate of the multiplier, we compute optimal stimulus spending for M between 0 and 2 .

The results are displayed in Figure 3. The left-hand graph displays optimal public expenditure as a share of output, G/Y , as a function of the unemployment multiplier, M . This graph is constructed using (24) and the identity $G/Y = (g/c)/(1 + g/c)$. Initially the unemployment rate is $u_0 = 9\%$, but with the optimal policy the unemployment rate falls below its initial level. The right-hand graph displays the unemployment rate under the optimal policy. This graph is constructed using (25). The numerical application illustrates all the theoretical results obtained in the previous sections. In addition, it provides orders of magnitude for the optimal increase in public expenditure after an increase in unemployment from 6% to 9% . Several observations stand out.

First, even with a small multiplier of 0.2 , optimal stimulus spending is significant. With $\varepsilon = 0.5$, optimal stimulus spending is 1.6 percentage points of GDP. With $\varepsilon = 1$, it is 2.8 points of GDP. And with $\varepsilon = 2$, it is 4.7 points of GDP.

Second, the multiplier that warrants the largest stimulus is fairly modest. With $\varepsilon = 0.5$ the

largest stimulus (2.6 points of GDP) occurs with a multiplier of 0.6. With $\varepsilon = 1$ the largest stimulus (3.7 points of GDP) occurs with a multiplier of 0.4. And with $\varepsilon = 2$ the largest increase (5.1 points of GDP) occurs with a multiplier of 0.3.

Third, optimal stimulus spending is the same for small and large multipliers. For instance, fix $\varepsilon = 1$. Optimal stimulus spending is the same for multipliers of 0.12 and 1.5—1.9 points of GDP. Optimal stimulus spending is also the same for multipliers of 0.08 and 2—1.3 points of GDP. Of course the resulting unemployment rates are very different. The optimal stimulus fills a much larger portion of the unemployment gap with a large multiplier.

Fourth, even though optimal stimulus spending is significant for small multipliers, the unemployment rate barely falls below its initial level of 9%. With the multiplier of 0.2, the unemployment rate only falls to 8.7% with $\varepsilon = 0.5$, to 8.5% with $\varepsilon = 1$, and to 8.1% with $\varepsilon = 2$. Unemployment does not fall much because public expenditure has little effect on unemployment when the multiplier is small.

Fifth, with a multiplier above 1, the stabilization of the unemployment rate is almost perfect. With a multiplier of 1, the unemployment rate achieved with the optimal policy is below 6.8%, so the remaining unemployment gap is less than 0.8 percentage points. With a multiplier of 2, the remaining unemployment gap is less than 0.2 percentage points.

Sixth, the elasticity of substitution between public and private consumption plays a significant role for small to medium multipliers but not for large multipliers. Consider first a multiplier of 0.4. With $\varepsilon = 0.5$, optimal stimulus spending is 2.4 percentage points of GDP; with $\varepsilon = 1$, public expenditure should increase by an additional 1.2 points of GDP; and with $\varepsilon = 2$, public expenditure should increase by another 1.2 points of GDP. Hence, ε significantly influences optimal stimulus spending. In contrast, for a multiplier above 1, optimal stimulus spendings with $\varepsilon = 0.5$, $\varepsilon = 1$, and $\varepsilon = 2$ are nearly indistinguishable. This is because for large multipliers, the optimal policy is to fill the unemployment gap, so it is not influenced by the elasticity of substitution.

Finally, if we pick values for the unemployment multiplier and elasticity of substitution, we can calculate optimal stimulus spending at the onset of the Great Recession. For instance, let's set $M = 0.5$, which is a median estimate of the unemployment multiplier when unemployment is high, and $\varepsilon = 1$, which corresponds to a Cobb-Douglas utility function. With this calibration, optimal stimulus spending is 3.6 percentage points of GDP. Since the US GDP in 2008 is 14,718

billion dollars, our formula suggests that optimal stimulus spending at the beginning of the Great Recession is 530 billion dollars per year. Critically, this estimate applies to a balanced-budget stimulus, and it is valid even if the increase in taxes required to finance the stimulus is distortionary.

5.2. *Quantitative Properties of the Matching Model*

We calibrate and simulate the matching model with land described in Section 2.4. We find that the model provides a good description of the business cycle: in response to aggregate-demand shocks, the model generates realistic, countercyclical fluctuations in the unemployment rate and unemployment multiplier. Hence the matching framework seems adapted to study optimal public expenditure over the business cycle.

The model is calibrated to US data, using the same empirical evidence as in Section 5.1. (The calibration is relegated to Appendix B.) We represent the business cycle as a succession of unexpected permanent aggregate-demand shocks. We focus on these shocks because unlike other shocks, they generate inefficient fluctuations in unemployment as well as negative comovements between tightness and unemployment.²⁷ We parameterize aggregate demand with $\alpha = \delta/\mathcal{Y}'(l_0)$. Since the economy jumps to its new steady-state equilibrium in response to an unexpected permanent shock, we only need to compute a collection of steady states parameterized by $\alpha \in [0.97, 1.03]$. We run two simulations: one in which G/Y is constant at 16.5%, its average value in the United States for 1990–2014, and one in which G/Y is at its optimal level, given by (18).

Figure 4 displays the results of the simulations. We find that the unemployment rate is countercyclical: when $G/Y = 16.5\%$, it rises from 4.4% when aggregate demand is highest ($\alpha = 1.3$), to 6.1% (the average unemployment rate in the United States for 1990–2014) when aggregate demand is average ($\alpha = 1$), and to 11.0% when aggregate demand is lowest ($\alpha = 0.97$). The unemployment rate fluctuates in response to aggregate-demand shocks because the price of services is rigid: when α goes up, the price does not adjust, which stimulates the aggregate demand curve (14) and reduces unemployment.

The unemployment multiplier is also countercyclical: it increases from 0.2 when unemployment is 4.4%, to 0.5 (the midrange of US estimates) when unemployment is 6.1%, to 1.4 when

²⁷For instance, aggregate-supply shocks generate positive comovements between tightness and unemployment, which are counterfactual (Michaillat and Saez 2015).

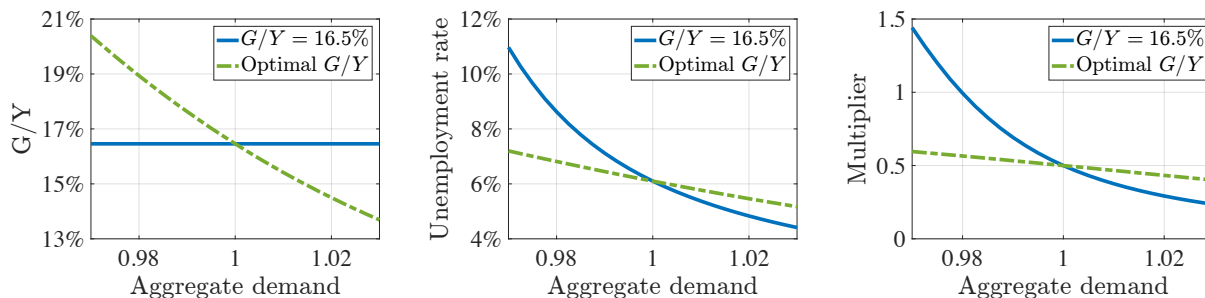


Figure 4: Business Cycle in the Matching Model with Land

Notes: The figure displays business cycles when public expenditure is $G/Y = 16.5\%$ (solid, blue line) and when public expenditure is set using the exact optimality condition (18) (dashed, green line). The business cycles are generated by changes in aggregate demand. The multiplier in the right-hand panel is the unemployment multiplier defined by (26) or the output multiplier (they are equal here). The results are obtained by simulating the matching model with land of Section 2.4 under the calibration in Appendix B.

unemployment is 11.0%. This countercyclicity is consistent with evidence suggesting that in the United States, multipliers are higher when unemployment is higher or output is lower (Auerbach and Gorodnichenko 2012; Candelon and Lieb 2013; Fazzari, Morley and Panovska 2015). The mechanism behind this countercyclicity is described in Michailat (2014). When unemployment is high, there is a lot of idle capacity so the matching process is congested by sellers of services. Hence, an increase in public expenditure has very little effect on other buyers of services. Crowding out of private expenditure by public expenditure is therefore weak, and the multiplier is large. When unemployment is low, the opposite occurs: matching is congested by buyers of services, crowding out of private expenditure by public expenditure is sharp, and the multiplier is small.

We can also compute optimal public expenditure over the business cycle. The unemployment rate is efficient when $\alpha = 1$ (by calibration), inefficiently high when $\alpha < 1$, and inefficiently low when $\alpha > 1$. Furthermore, the unemployment multiplier is positive. Hence, optimal public spending is above Samuelson spending when $\alpha < 1$ and below it when $\alpha > 1$. Indeed, optimal public spending is markedly countercyclical, decreasing from $G/Y = 20.4\%$ to $G/Y = 13.7\%$ when α increases from 0.97 to 1.03.

Finally, unemployment responds when public expenditure is adjusted from $G/Y = 16.5\%$ to its optimal level. When aggregate demand is low, optimal public expenditure is higher than $G/Y = 16.5\%$ so unemployment falls below its original level. For instance, at $\alpha = 0.97$ the unemployment rate falls from 11.0% to 7.2%. When aggregate demand is high, optimal public expenditure is

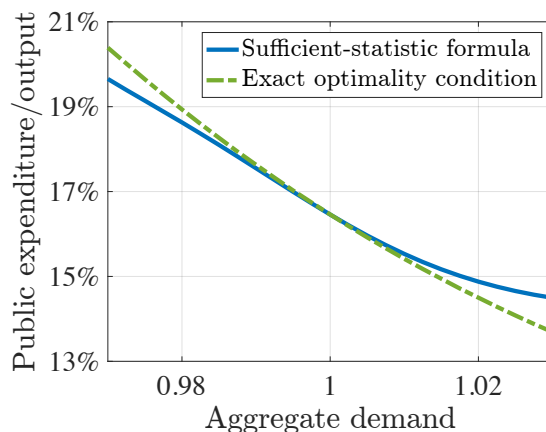


Figure 5: Accuracy of the Sufficient-Statistic Formula

Notes: This figure compares the level of public expenditure given by the sufficient-statistic formula (24) (solid, blue line) to that given by the exact optimality condition (18) (dashed, green line) over the business cycle. The business cycle is generated by changes in aggregate demand. The results are obtained by simulating the matching model with land of Section 2.4 under the calibration in Appendix B.

below $G/Y = 16.5\%$ so unemployment rises above its original level. For instance, at $\alpha = 1.03$ the unemployment rate increases from 4.4% to 5.2%. The unemployment multiplier heavily depends on the unemployment rate, so it adjusts accordingly.

5.3. Accuracy of the Sufficient-Statistic Formula

The simulations allow us to assess the accuracy of our sufficient-statistic formula. The formula is valid up to a second-order remainder. Since the unemployment rate displays large fluctuations, a potential concern is that the remainder could be large and the approximation given by the formula inaccurate. In our simulations, however, this does not happen.

Figure 5 shows that the sufficient-statistic formula is quite accurate. The figure compares the level of public expenditure given by our sufficient-statistic formula, which is approximate and given by (24), to the level given by the exact optimality condition (18). At $\alpha = 1$, the two formulas give the same public expenditure by construction. When α is further away from 1, the deviation between the levels of public expenditure remains below one percentage point: at $\alpha = 1.03$, the exact optimality condition gives $G/Y = 13.7\%$ while our formula gives $G/Y = 14.5\%$; at $\alpha = 0.97$, the exact condition gives $G/Y = 20.4\%$ while our formula gives $G/Y = 19.7\%$.

6. Conclusion

This paper has presented a theory of optimal public expenditure in the presence of unemployment. The theory shows that when unemployment is efficient, the Samuelson rule remains valid; but when unemployment is inefficient, optimal public expenditure deviates from the Samuelson rule to bring unemployment closer to its efficient level.

In the past few decades, monetary policy has been governments' preferred stabilization policy. Yet it has become clear that because of the zero lower bound on nominal interest rates, governments cannot rely on monetary policy alone to stabilize the economy—after the Great Recession, the zero lower bound was binding in Japan, the United States, and the eurozone. Our theory suggests that public expenditure could contribute to stabilization whenever monetary policy is constrained.

In addition, public expenditure could be helpful to members of monetary unions—for instance, countries in the eurozone or states in the United States. Indeed, these governments have no control over monetary policy, so they cannot use it to stabilize their economy. But they can adjust their public expenditure to tackle local unemployment. Furthermore, our theory focuses on budget-balanced spending, so it is particularly appropriate for US states, which cannot run budget deficits, and to eurozone countries, which face strict constraints on their budget deficits.

In this paper we have limited ourselves to static considerations and skirted around dynamic considerations. To gain a more complete understanding of optimal public expenditure when unemployment is inefficient, it would be useful to enrich our analysis with dynamic elements. Several such elements seem important: the political process leading to the design of stimulus packages (Battaglini and Coate 2016); the dynamic response of inflation to public spending, especially in a liquidity trap (Woodford 2011; Werning 2012); the intertemporal smoothing of distortionary taxes and the use of debt (Barro 1979); the distinction between temporary and permanent changes in public expenditure (Barro 1981); and public investment in infrastructure (Baxter and King 1993).

References

- Atkinson, A. B., and N. H. Stern. 1974. "Pigou, Taxation and Public Goods." *Review of Economic Studies*, 41(1): 119–128.
- Auerbach, Alan J., and Yuriy Gorodnichenko. 2012. "Measuring the Output Responses to Fiscal Policy." *American Economic Journal: Economic Policy*, 4(2): 1–27.

- Ballard, Charles L., and Don Fullerton.** 1992. "Distortionary Taxes and the Provision of Public Goods." *Journal of Economic Perspectives*, 6(3): 117–131.
- Barro, Robert J.** 1979. "On the Determination of the Public Debt." *Journal of Political Economy*, 87(5): 940–971.
- Barro, Robert J.** 1981. "Output Effects of Government Purchases." *Journal of Political Economy*, 89(6): 1086–1121.
- Barro, Robert J., and Charles J. Redlick.** 2011. "Macroeconomic Effects From Government Purchases and Taxes." *Quarterly Journal of Economics*, 126(1): 51–102.
- Battaglini, Marco, and Stephen Coate.** 2016. "A Political Economy Theory of Fiscal Policy and Unemployment." *Journal of the European Economic Association*, 14(2): 303–337.
- Baxter, Marianne, and Robert G. King.** 1993. "Fiscal Policy in General Equilibrium." *American Economic Review*, 83(3): 315–334.
- Blanchard, Olivier J., and Jordi Galí.** 2010. "Labor Markets and Monetary Policy: A New-Keynesian Model with Unemployment." *American Economic Journal: Macroeconomics*, 2(2): 1–30.
- Boone, Jan, and Lans Bovenberg.** 2002. "Optimal Labour Taxation and Search." *Journal of Public Economics*, 85(1): 53–97.
- Candelon, Bertrand, and Lenard Lieb.** 2013. "Fiscal Policy in Good and Bad Times." *Journal of Economic Dynamics and Control*, 37(12): 2679–2694.
- Chetty, Raj.** 2006. "A General Formula for the Optimal Level of Social Insurance." *Journal of Public Economics*, 90(10-11): 1879–1901.
- Chetty, Raj.** 2008. "Moral Hazard versus Liquidity and Optimal Unemployment Insurance." *Journal of Political Economy*, 116(2): 173–234.
- Chetty, Raj.** 2009. "Sufficient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form Methods." *Annual Review of Economics*, 1: 451–488.
- Christiansen, Vidar.** 1981. "Evaluation of Public Projects under Optimal Taxation." *Review of Economic Studies*, 48(3): 447–457.
- Coate, Stephen.** 2000. "An Efficiency Approach to the Evaluation of Policy Changes." *Economic Journal*, 110(463): 437–55.
- Cwik, Tobias, and Volker Wieland.** 2011. "Keynesian Government Spending Multipliers and Spillovers in the Euro Area." *Economic Policy*, 26(67): 493–549.
- Diamond, Peter A., and James A. Mirrlees.** 1971. "Optimal Taxation and Public Production II: Tax Rules." *American Economic Review*, 61(3): 261–278.
- Farhi, Emmanuel, and Xavier Gabaix.** 2015. "Optimal Taxation with Behavioral Agents." National Bureau of Economic Research (NBER) Working Paper 21524.
- Farmer, Roger E.A.** 2008. "Aggregate Demand and Supply." *International Journal of Economic Theory*, 4(1): 77–93.
- Fazzari, Steven M., James Morley, and Irina Panovska.** 2015. "State-Dependent Effects of Fiscal Policy." *Studies in Nonlinear Dynamics & Econometrics*, 19(3): 285–315.
- Hall, Robert E.** 2005. "Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market." *Review of Economics and Statistics*, 87(3): 397–407.
- Hendren, Nathaniel.** 2014. "The Inequality Deflator: Interpersonal Comparisons without a Social Welfare Function." National Bureau of Economic Research (NBER) Working Paper 20351.
- Hendren, Nathaniel.** 2016. "The Policy Elasticity." *Tax Policy and the Economy*, 30(1): 51–89.
- Hosios, Arthur J.** 1990. "On the Efficiency of Matching and Related Models of Search and Un-

- employment.” *Review of Economic Studies*, 57(2): 279–298.
- Hungerbühler, Mathias, and Etienne Lehmann.** 2009. “On the Optimality of a Minimum Wage: New Insights from Optimal Tax Theory.” *Journal of Public Economics*, 93(3-4): 464–481.
- Hylland, Aanund, and Richard Zeckhauser.** 1979. “Distributional Objectives Should Affect Taxes but Not Program Choice or Design.” *Scandinavian Journal of Economics*, 81(2): 264–284.
- Iacoviello, Matteo.** 2005. “House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle.” *American Economic Review*, 95(3): 739–764.
- Jacobs, Bas.** 2016. “The Marginal Cost of Public Funds is One at the Optimal Tax System.” <http://people.few.eur.nl/bjacobs/mcf.pdf>.
- Kaplow, Louis.** 1996. “The Optimal Supply of Public Goods and the Distortionary Cost of Taxation.” *National Tax Journal*, 49(4): 513–533.
- Kaplow, Louis.** 2004. “On the (Ir)Relevance of Distribution and Labor Supply Distortion to Government Policy.” *The Journal of Economic Perspectives*, 18(4): 159–175.
- Kleven, Henrik J., and Claus T. Kreiner.** 2006. “The Marginal Cost of Public Funds: Hours of Work Versus Labor Force Participation.” *Journal of Public Economics*, 90(10-11): 1955–1973.
- Kreiner, Claus T., and Nicolaj Verdelin.** 2012. “Optimal Provision of Public Goods: A Synthesis.” *Scandinavian Journal of Economics*, 114(2): 384–408.
- Landais, Camille, Pascal Michailat, and Emmanuel Saez.** 2016. “A Macroeconomic Approach to Optimal Unemployment Insurance: Theory.” *American Economic Journal: Economic Policy*, forthcoming.
- Landais, Camille, Pascal Michailat, and Emmanuel Saez.** 2017. “A Macroeconomic Approach to Optimal Unemployment Insurance: Applications.” <http://www.pascalnichailat.org/5.html>.
- Lehmann, Etienne, Alexis Parmentier, and Bruno Van Der Linden.** 2011. “Optimal Income Taxation with Endogenous Participation and Search Unemployment.” *Journal of Public Economics*, 95(11): 1523–1537.
- Liu, Zheng, Pengfei Wang, and Tao Zha.** 2013. “Land-Price Dynamics And Macroeconomic Fluctuations.” *Econometrica*, 81(3): 1147–1184.
- Mankiw, N. Gregory, and Matthew Weinzierl.** 2011. “An Exploration of Optimal Stabilization Policy.” *Brookings Papers on Economic Activity*, 42(1): 209–272.
- Michailat, Pascal.** 2012. “Do Matching Frictions Explain Unemployment? Not in Bad Times.” *American Economic Review*, 102(4): 1721–1750.
- Michailat, Pascal.** 2014. “A Theory of Countercyclical Government Multiplier.” *American Economic Journal: Macroeconomics*, 6(1): 190–217.
- Michailat, Pascal, and Emmanuel Saez.** 2015. “Aggregate Demand, Idle Time, and Unemployment.” *Quarterly Journal of Economics*, 130(2): 507–569.
- Monacelli, Tommaso, Roberto Perotti, and Antonella Trigari.** 2010. “Unemployment Fiscal Multipliers.” *Journal of Monetary Economics*, 57(5): 531–553.
- Okun, Arthur M.** 1963. “Potential GNP: Its Measurement and Significance.” Cowles Foundation Paper 190.
- Pissarides, Christopher A.** 2000. *Equilibrium Unemployment Theory*. 2nd ed., Cambridge, MA: MIT Press.
- Ramey, Valerie A.** 2011. “Can Government Purchases Stimulate the Economy?” *Journal of Economic Literature*, 49: 673–685.
- Ramey, Valerie A.** 2013. “Government Spending and Private Activity.” In *Fiscal Policy after the*

- Financial Crisis.* , ed. Alberto Alesina and Francesco Giavazzi, 19–55. Chicago, IL:University of Chicago Press.
- Romer, Christina D., and Jared Bernstein.** 2009. “The Job Impact of the American Recovery and Reinvestment Plan.” http://otrans.3cdn.net/45593e8ecbd339d074_13m6bt1te.pdf.
- Samuelson, Paul A.** 1954. “The Pure Theory of Public Expenditure.” *Review of Economics and Statistics*, 36(4): 387–389.
- Samuelson, Paul A.** 1955. “Diagrammatic Exposition of a Theory of Public Expenditure.” *Review of Economics and Statistics*, 37(4): 350–356.
- Stantcheva, Stefanie.** 2015. “Optimal Income, Education, and Bequest Taxes in an Intergenerational Model.” https://dl.dropboxusercontent.com/u/12222201/Education_Intergen.pdf.
- Stiglitz, J. E., and P. Dasgupta.** 1971. “Differential Taxation, Public Goods, and Economic Efficiency.” *Review of Economic Studies*, 38(2): 151–174.
- Werning, Iván.** 2012. “Managing a Liquidity Trap: Monetary and Fiscal Policy.” http://dl.dropboxusercontent.com/u/125966/zero_bound_2011.pdf.
- Werquin, Nicolas.** 2016. “Income Taxation with Frictional Labor Supply.” <https://www.dropbox.com/s/7n1ghyc9b2j0h4u/Werquin2015.pdf?>
- Woodford, Michael.** 2011. “Simple Analytics of the Government Expenditure Multiplier.” *American Economic Journal: Macroeconomics*, 3(1): 1–35.

Appendix A. Long Proofs

In this appendix we provide the proofs that are relatively long. We incorporate the shorter proofs directly in the main text.

Proof of Lemma 3

Since MRS_{gc} is a function of g/c , the first-order Taylor expansion of $MRS_{gc}(g/c)$ at $(g/c)^*$ is

$$MRS_{gc}(g/c) = MRS_{gc}((g/c)^*) + \frac{dMRS_{gc}}{dg/c} \cdot (g/c - (g/c)^*) + O([g/c - (g/c)^*]^2).$$

In addition, $MRS_{gc}((g/c)^*) = 1$ and $dMRS_{gc}/d(g/c) = -1/[\varepsilon \cdot (g/c)^*]$. Hence,

$$(A1) \quad 1 - MRS_{gc}(g/c) = \frac{1}{\varepsilon} \cdot \frac{g/c - (g/c)^*}{(g/c)^*} + O([g/c - (g/c)^*]^2).$$

The $1/\varepsilon$ in the Taylor expansion is evaluated at $(g/c)^*$. But we can replace it by the $1/\varepsilon$ evaluated at g/c because the difference between the two is proportional to $g/c - (g/c)^*$. So once the difference is multiplied by $g/c - (g/c)^*$ in (A1), it is absorbed in $O([g/c - (g/c)^*]^2)$. Thus, equation (A1) yields equation (20).

Next, we write $\partial \ln(y)/\partial \ln(x)$ as a function of u :

$$\frac{\partial \ln(y)}{\partial \ln(x)} = (1 - \eta) \cdot u - \eta \cdot \tau(u).$$

The function $\tau(u)$ is defined by $\tau(u) = \tau(x(u))$, where $\tau(x)$ is given by (3) and $x(u) = u^{-1}(u)$ is the inverse of the function $u(x)$ given by (2). We have

$$\tau'(u) = \tau'(x) \cdot x'(u) = \frac{\tau'(x)}{u'(x(u))} = \frac{\eta \cdot (1 + \tau) \cdot \tau/x}{-(1 - \eta) \cdot (1 - u) \cdot u/x} = -\frac{\eta \cdot (1 + \tau) \cdot \tau}{(1 - \eta) \cdot (1 - u) \cdot u}.$$

Since $(1 - \eta) \cdot u^* = \eta \cdot \tau(u^*)$, we have $\tau'(u^*) = -(1 + \tau(u^*))/(1 - u^*)$ and

$$-\eta \cdot \tau'(u^*) = \frac{\eta + \eta \cdot \tau(u^*)}{1 - u^*} = \frac{\eta + (1 - \eta) \cdot u^*}{1 - u^*} = \eta + \frac{u^*}{1 - u^*}.$$

Hence, the derivative of $\partial \ln(y)/\partial \ln(x)$ with respect to u at u^* is $(1 - \eta) - \eta \cdot \tau'(u^*) = 1/(1 - u^*)$. Furthermore, $\partial \ln(y)/\partial \ln(x) = 0$ at u^* . Thus, a first-order Taylor expansion of $\partial \ln(y)/\partial \ln(x)$ at u^* yields (21).

Finally, since the elasticity of $u(x)$ with respect to x is $-(1 - \eta) \cdot (1 - u)$, we find that

$$m = -y \cdot \frac{u}{g} \cdot \frac{d \ln(u)}{d \ln(g)} = \frac{y}{g} \cdot (1 - \eta) \cdot u \cdot (1 - u) \cdot \frac{d \ln(x)}{d \ln(g)} = \frac{y}{x} \cdot (1 - \eta) \cdot u \cdot (1 - u) \cdot \frac{dx}{dg}.$$

We obtain (22) by rearranging this equation.

Proof of Lemma 4

We start from (18). First, we approximate $1 - MRS_{gc}$ with (20). Next, we rewrite dx/dg with (22) and approximate $\partial y/\partial x$ with (21). This yields

$$(A2) \quad \frac{1}{\varepsilon} \cdot \frac{g/c - (g/c)^*}{(g/c)^*} = \frac{1}{1 - \eta} \cdot m \cdot \frac{(u - u^*)}{u \cdot (1 - u) \cdot (1 - u^*)} + O([g/c - (g/c)^*]^2 + [u - u^*]^2).$$

We can rewrite this as

$$\frac{1}{\varepsilon} \cdot \frac{g/c - (g/c)^*}{(g/c)^*} = \frac{1}{1 - \eta} \cdot m \cdot \frac{(u - u^*)}{u^* \cdot (1 - u^*)^2} + O([g/c - (g/c)^*]^2 + [u - u^*]^2).$$

This is because the difference between $1/[u \cdot (1 - u) \cdot (1 - u^*)]$ and $1/[u^* \cdot (1 - u^*)^2]$ is $O(u - u^*)$. Once this difference is multiplied by $u - u^*$ in (A2), it is absorbed by the term $O([g/c - (g/c)^*]^2 + [u - u^*]^2)$. We obtain (23) from this last equation.

Proof of Proposition 1

The economy starts at an equilibrium $[(g/c)^*, u_0]$, where the unemployment rate u_0 is inefficient. Since $u_0 \neq u^*$, the optimal g/c departs from $(g/c)^*$. In (23), the multiplier m and unemployment rate u are functions of g/c , so they respond as g/c moves away from $(g/c)^*$, and we cannot read the optimal g/c off the formula. In this proof, we derive a formula giving the optimal g/c as a function of fixed quantities.

First, we express the equilibrium values of all variables as functions of $[u, g/c]$. The proof of Lemma 3 showed that x and τ can be written as functions of u . Since $y = (1 - u) \cdot k/(1 + \tau)$, we can also write y as a function of u . Since $g = y \cdot (g/c)/[1 + g/c]$, g can be written as a function of u and g/c . As $c = y - g$, c can also be written as a function of u and g/c . Last, since $C = c \cdot (1 + \tau)$, $C = c \cdot (1 + \tau)$, and $G = c \cdot (1 + \tau)$, we can write C , G , and Y as functions of u and g/c .

Among all pairs $[u, g/c]$, the only pairs describing an equilibrium are those consistent with the equilibrium condition $u = u(x(g))$, where g is the function of u and g/c described above, $x(g)$ is the function defined by (7), and $u(x)$ is the function defined by (2). This equilibrium condition defines the unemployment rate as an implicit function of g/c , denoted $u(g/c)$. Then, the pairs $[u(g/c), g/c]$ for all $g/c > 0$ are the equilibria for all possible levels of public expenditure.

We start by linking u to u_0 and g/c . We write a first-order Taylor expansion of $u(g/c)$ around $u((g/c)^*) = u_0$, subtract u^* on both sides, and divide both sides by u^* :

$$(A3) \quad \frac{u - u^*}{u^*} = \frac{u_0 - u^*}{u^*} + \frac{1}{u^*} \cdot \frac{du}{d \ln(g/c)} \cdot \frac{g/c - (g/c)^*}{(g/c)^*} + O([g/c - (g/c)^*]^2).$$

We need to compute $du/d \ln(g/c)$. First, we decompose the derivative:

$$(A4) \quad \frac{du}{d \ln(g/c)} = \frac{du}{d \ln(g)} \cdot \frac{d \ln(g)}{d \ln(g/c)}.$$

Since all the derivatives in (A4) are evaluated at $[u_0, (g/c)^*]$, equation (8) implies that

$$\frac{du}{d\ln(g)} = m \cdot (g/y)^*,$$

where m is evaluated at $[u_0, (g/c)^*]$.

Next, we compute $d\ln(g)/d\ln(g/c)$ at $[u_0, (g/c)^*]$. We have

$$\ln(g/c) = \ln(g) - \ln(y(x(g/c), k) - g).$$

Differentiating with respect to $\ln(g/c)$ yields

$$(A5) \quad 1 = \frac{d\ln(g)}{d\ln(g/c)} - \frac{y}{c} \cdot \frac{\partial \ln(y)}{\partial \ln(x)} \frac{d\ln(x)}{d\ln(g/c)} + \frac{g}{c} \cdot \frac{d\ln(g)}{d\ln(g/c)}.$$

Reshuffling the terms, we obtain

$$\frac{d\ln(g)}{d\ln(g/c)} = \frac{c}{y} + \frac{\partial \ln(y)}{\partial \ln(x)} \frac{d\ln(x)}{d\ln(g/c)}.$$

At u^* , $\partial \ln(y)/\partial \ln(x) = 0$, so at u_0 , $\partial \ln(y)/\partial \ln(x)$ is $O([u_0 - u^*])$. Once this term is multiplied by $g/c - (g/c)^*$ in (A3), it creates a term that is $O([u_0 - u^*]^2 + [g/c - (g/c)^*]^2)$. Thus, we omit the term $(\partial \ln(y)/\partial \ln(x)) \cdot (d\ln(x)/d\ln(g/c))$ and set

$$\frac{d\ln(g)}{d\ln(g/c)} = (c/y)^*.$$

So far, we have showed that

$$(A6) \quad \frac{u - u^*}{u^*} = \frac{u_0 - u^*}{u^*} + m \cdot z_1 \cdot \frac{g/c - (g/c)^*}{(g/c)^*} + O([u_0 - u^*]^2 + [g/c - (g/c)^*]^2)$$

where $z_1 \equiv \frac{(g/y)^* \cdot (c/y)^*}{u^*}$.

Equation (23) includes a remainder that is $O([u - u^*]^2 + [g/c - (g/c)^*]^2)$. Equation (A6) implies that $(u - u^*)^2$ is $O([u_0 - u^*]^2 + [g/c - (g/c)^*]^2)$. Thus the remainder in formula (23) is $O([u_0 - u^*]^2 + [g/c - (g/c)^*]^2)$. Combining (23) and (A6), we therefore obtain

$$\frac{g/c - (g/c)^*}{(g/c)^*} = \left[\frac{\varepsilon \cdot m}{z_0} \right] \cdot \left[\frac{u_0 - u^*}{u^*} + m \cdot z_1 \cdot \frac{g/c - (g/c)^*}{(g/c)^*} \right] + O([u_0 - u^*]^2 + [g/c - (g/c)^*]^2),$$

where ε and m in the first bracket are evaluated at $[u, g/c]$. But instead we can use the values of ε and m evaluated at $[u_0, (g/c)^*]$ because the difference between the two values of each statistic is $O([u - u_0] + [g/c - (g/c)^*])$. So once the differences are multiplied by $g/c - (g/c)^*$ and $u_0 - u^*$ in the above equation, they are absorbed by the term $O([u_0 - u^*]^2 + [g/c - (g/c)^*]^2)$. Thus, this last equation yields equation (24).

To finish the proof, we derive (25). With the arguments that we have just used, (23) can be

written

$$\frac{g/c - (g/c)^*}{(g/c)^*} = \frac{\varepsilon \cdot m}{z_0} \cdot \frac{u - u^*}{u^*} + O([u_0 - u^*]^2 + [g/c - (g/c)^*]^2),$$

where ε and m are evaluated at $[u_0, (g/c)^*]$. Replacing the left-hand side of the last equation by the right-hand side in (24), and dividing everything by $\varepsilon \cdot m/z_0$, we obtain (25).

Derivation of Equation (27)

As $G = (1 + \tau(x(g))) \cdot g$ and the elasticity of $1 + \tau(x)$ with respect to x is $\eta \cdot \tau$, we find that

$$(A7) \quad \frac{d \ln(G)}{d \ln(g)} = 1 + \eta \cdot \tau \cdot \frac{d \ln(x)}{d \ln(g)} = 1 + \frac{g}{y} \cdot \frac{\eta}{1 - \eta} \cdot \frac{\tau}{u \cdot (1 - u)} \cdot m.$$

where the last equality is obtained using (22). Furthermore, the definitions of m and M imply that

$$m = -y \cdot \frac{du}{dg} = -\frac{Y}{1 + \tau(x)} \cdot \frac{du}{dG} \cdot \frac{dG}{dg} = \frac{g}{G} \cdot (1 - u) \cdot M \cdot \frac{dG}{dg} = (1 - u) \cdot M \cdot \frac{d \ln(G)}{d \ln(g)}.$$

We now plug into this equation the expression for $d \ln(G)/d \ln(g)$ obtained in (A7):

$$m = (1 - u) \cdot M + \frac{g}{y} \cdot \frac{\eta}{1 - \eta} \cdot \frac{\tau}{u} \cdot M \cdot m.$$

We obtain (27) by rearranging this equation.

Proof of Proposition 5

In the model of Section 4.1, Samuelson spending satisfies

$$MRS_{gc}(g/c^*) = 1 - \frac{d \ln(k)}{d \ln(g)},$$

so Lemma 6 implies that optimal public expenditure satisfies

$$MRS_{gc}((g/c)^*) - MRS_{gc}(g/c) = \frac{\partial y}{\partial x} \cdot \frac{dx}{dg}.$$

As in Lemma 3, we have

$$MRS_{gc}((g/c)^*) - MRS_{gc}(g/c) = \frac{1}{\varepsilon} \cdot \frac{g/c - (g/c)^*}{(g/c)^*}.$$

Moreover, (21) and (22) remain valid. Combining these results, we obtain (23).

Since formula (23) remains valid, the proof follows the same steps as the proof of Proposition 1. The only difference occurs once we reach equation (A5). With a supply-side response to taxation,

the equation becomes

$$1 = \frac{d \ln(g)}{d \ln(g/c)} - \frac{y}{c} \cdot \frac{\partial \ln(y)}{\partial \ln(x)} \frac{d \ln(x)}{d \ln(g/c)} - \frac{y}{c} \cdot \frac{\partial \ln(y)}{\partial \ln(k)} \cdot \frac{d \ln(k)}{d \ln(g)} \cdot \frac{d \ln(g)}{d \ln(g/c)} + \frac{g}{c} \cdot \frac{d \ln(g)}{d \ln(g/c)}.$$

Using the same argument as in the proof of Proposition 1, we can omit the term containing the factor $\partial \ln(y)/\partial \ln(x)$. Since $\partial \ln(y)/\partial \ln(k) = 1$, we therefore obtain $d \ln(g)/d \ln(g/c) = (c/y)^*/(1 - d \ln(k)/d \ln(g))$. Using the new expression for $d \ln(g)/d \ln(g/c)$, we conclude the proof like the proof of Proposition 1.

Appendix B. The Model with Land

We derive several results that are useful in the analysis and simulation of the matching model with land presented in Section 2.4. We also calibrate the model to US data.

Utility Function

We compute the derivatives of the utility function, given by (9):

$$\begin{aligned} \frac{\partial \ln(\mathcal{U})}{\partial \ln(c)} &= (1 - \gamma)^{\frac{1}{\varepsilon}} \cdot \left(\frac{c}{\mathcal{U}}\right)^{\frac{\varepsilon-1}{\varepsilon}}, & \mathcal{U}_c &\equiv \frac{\partial \mathcal{U}}{\partial c} = \left(\frac{(1 - \gamma) \cdot \mathcal{U}}{c}\right)^{\frac{1}{\varepsilon}} \\ \frac{\partial \ln(\mathcal{U})}{\partial \ln(g)} &= \gamma^{\frac{1}{\varepsilon}} \cdot \left(\frac{g}{\mathcal{U}}\right)^{\frac{\varepsilon-1}{\varepsilon}}, & \mathcal{U}_g &\equiv \frac{\partial \mathcal{U}}{\partial g} = \left(\frac{\gamma \cdot \mathcal{U}}{g}\right)^{\frac{1}{\varepsilon}} \\ \frac{\partial \ln(\mathcal{U}_c)}{\partial \ln(c)} &= \frac{1}{\varepsilon} \cdot \left(\frac{\partial \ln(\mathcal{U})}{\partial \ln(c)} - 1\right) \\ \frac{\partial \ln(\mathcal{U}_c)}{\partial \ln(g)} &= \frac{1}{\varepsilon} \cdot \frac{\partial \ln(\mathcal{U})}{\partial \ln(g)}. \end{aligned}$$

When the Samuelson rule holds, we have $MRS_{gc} = \mathcal{U}_g/\mathcal{U}_c = 1$, so

$$(g/c)^* = \frac{\gamma}{1 - \gamma}, \quad (g/y)^* = \gamma, \quad (c/y)^* = 1 - \gamma,$$

and the derivatives of the utility function simplify to

$$\begin{aligned} \frac{\partial \ln(\mathcal{U})}{\partial \ln(c)} &= 1 - \gamma, & \frac{\partial \ln(\mathcal{U})}{\partial \ln(g)} &= \gamma \\ \mathcal{U}_c &= 1, & \mathcal{U}_g &= 1 \\ \frac{\partial \ln(\mathcal{U}_c)}{\partial \ln(c)} &= -\frac{\gamma}{\varepsilon}, & \frac{\partial \ln(\mathcal{U}_c)}{\partial \ln(g)} &= \frac{\gamma}{\varepsilon}. \end{aligned}$$

Unemployment Multiplier

We now compute the unemployment multiplier. To compute the multiplier, we first express dx/dg as a function of the derivatives of the utility function. Then, we compute the theoretical unemploy-

ment multiplier m from dx/dg and the empirical unemployment multiplier M from m .

The price schedule is $p(g) = p_0 \cdot \mathcal{U}_c(y^* - g, g)^{1-r}$. The elasticity of the price schedule and its value at $g^* \equiv \gamma \cdot y^*$ are

$$\begin{aligned}\frac{d \ln(p)}{d \ln(g)} &= (1-r) \cdot \left[\frac{\partial \ln(\mathcal{U}_c)}{\partial \ln(g)} - \frac{g}{y^* - g} \cdot \frac{\partial \ln(\mathcal{U}_c)}{\partial \ln(c)} \right] \\ \frac{d \ln(p)}{d \ln(g)} &= (1-r) \cdot \frac{1}{\varepsilon} \cdot \frac{\gamma}{1-\gamma}.\end{aligned}$$

The private demand $c(x, g)$ is defined by $\mathcal{U}_c(c, g) = p(g) \cdot (1 + \tau(x)) \cdot \mathcal{V}'(l_0) / \delta$. The elasticities of the demand are

$$\begin{aligned}\frac{\partial \ln(c)}{\partial \ln(x)} &= \frac{\eta \cdot \tau(x)}{\partial \ln(\mathcal{U}_c) / \partial \ln(c)} \\ \frac{\partial \ln(c)}{\partial \ln(g)} &= - \frac{\partial \ln(\mathcal{U}_c) / \partial \ln(g) - \partial \ln(p) / \partial \ln(g)}{\partial \ln(\mathcal{U}_c) / \partial \ln(c)}.\end{aligned}$$

We calibrate the price level such that unemployment is efficient when public expenditure is at the Samuelson level, or equivalently $x(g^*) = x^*$. This means that $c(x^*, g^*) = c^* \equiv (1 - \gamma) \cdot y^*$ and $\eta \cdot \tau(x^*) = (1 - \eta) \cdot u^*$. Thus, the values of the elasticities at x^* and g^* are

$$\frac{\partial \ln(c)}{\partial \ln(x)} = -(1 - \eta) \cdot u^* \cdot \frac{\varepsilon}{\gamma} \quad \text{and} \quad \frac{\partial \ln(c)}{\partial \ln(g)} = \frac{r - \gamma}{1 - \gamma}.$$

The equilibrium condition determining tightness $x(g)$ is $y(x, k) = g + c(x, g)$. In the simulations, k is fixed. Differentiating this equation with respect to g , we obtain the elasticity of $x(g)$ with respect to g :

$$\begin{aligned}\frac{\partial \ln(y)}{\partial \ln(x)} \cdot \frac{d \ln(x)}{d \ln(g)} &= \frac{g}{y} + \frac{c}{y} \cdot \left[\frac{\partial \ln(c)}{\partial \ln(g)} + \frac{\partial \ln(c)}{\partial \ln(x)} \cdot \frac{d \ln(x)}{d \ln(g)} \right] \\ \text{so} \quad \frac{d \ln(x)}{d \ln(g)} &= \frac{(g/y) + (c/y) \cdot (\partial \ln(c) / \partial \ln(g))}{\partial \ln(y) / \partial \ln(x) - (c/y) \cdot (\partial \ln(c) / \partial \ln(x))}.\end{aligned}$$

As discussed above, we calibrate the model such that $x(g^*) = x^*$ and $c(x^*, g^*) = c^*$. In addition, $(c/y)^* = 1 - \gamma$ and $(g/y)^* = \gamma$. Hence, the value of the elasticity at g^* is

$$\frac{d \ln(x)}{d \ln(g)} = \frac{1}{(1 - \eta) \cdot u^*} \cdot \frac{r}{\varepsilon} \cdot \frac{\gamma}{1 - \gamma}.$$

From the expression for $d \ln(x) / d \ln(g)$, we obtain m and M using (22) and (27):

$$m = (1 - \eta) \cdot u \cdot (1 - u) \cdot \frac{y}{g} \cdot \frac{d \ln(x)}{d \ln(g)} \quad \text{and} \quad M = \frac{m}{1 - u + \frac{g}{y} \cdot \frac{\eta}{1 - \eta} \cdot \frac{\tau}{u} \cdot m}.$$

Table A1: Parameter Values in Simulations

	Description	Source
$\varepsilon = 1$	Elasticity of substitution between g and c	–
$\gamma = 0.16$	Parameter of utility function	Matches $(G/C)^* = 19.7\%$
$s = 2.8\%$	Monthly separation rate	Landais, Michailat and Saez (2017)
$\eta = 0.6$	Matching elasticity	Landais, Michailat and Saez (2017)
$\omega = 0.60$	Matching efficacy	Landais, Michailat and Saez (2017)
$\rho = 1.4$	Matching cost	Matches $u^* = 6.1\%$
$r = 0.46$	Price rigidity	Matches $M = 0.5$ at $\alpha = 1$
$p_0 = 0.78$	Price level	Matches $u = u^*$ at $\alpha = 1$

Since $(g/y)^* = \gamma$, the values of m and M at g^* are

$$m = \frac{r \cdot (1 - u^*)}{\varepsilon \cdot (1 - \gamma)} \quad \text{and} \quad M = \frac{r}{r \cdot \gamma + \varepsilon \cdot (1 - \gamma)}.$$

Calibration

Here we calibrate the matching model with land based on empirical evidence for the United States. The parameter values used in simulations are summarized in Table A1.

We begin by calibrating the utility function. We arbitrarily set the elasticity of substitution between public and private consumption to $\varepsilon = 1$. As showed above, the parameter γ determines Samuelson spending: $(G/C)^* = \gamma/(1 - \gamma)$. We assume that Samuelson spending is the average level of public expenditure in the United States for 1990–2014: $(G/C)^* = 19.7\%$ (Section 5). We therefore set $\gamma = 0.16$.

Then we calibrate matching parameters. Following Landais, Michailat and Saez (2017, Table A1), we use evidence from the US labor market data for 1990–2014 and set the separation rate to $s = 2.8\%$, the matching elasticity to $\eta = 0.6$, and the matching efficacy to $\omega = 0.60$. To calibrate the matching cost, we exploit the relationship $\tau = \rho \cdot s / [\omega \cdot x^{-\eta} - \rho \cdot s]$, which implies $\rho = \omega \cdot x^{-\eta} \cdot \tau / [s \cdot (1 + \tau)]$. This relationship holds for any τ and x , in particular when tightness is efficient so $x = x^*$ and $\tau = \tau^*$. We assume that the efficient unemployment rate and tightness are the average US unemployment rate and tightness for 1990–2014: $u^* = 6.1\%$ and $x^* = 0.43$ (Landais, Michailat and Saez 2017, Appendix G). When tightness is efficient, $\tau^* = u^* \cdot (1 - \eta)/\eta$ so $\tau^* = 4.1\%$. Plugging $x^* = 0.43$ and $\tau^* = 4.1\%$ in the expression for ρ , we obtain $\rho = 1.4$.

Last, we calibrate the price mechanism. The empirical evidence suggest that on average in the United States the unemployment multiplier is $M = 0.5$ (Section 5). Relying on (16), we set $r = 0.46$ to match $M = 0.5$. We calibrate the price level such that $u = u^* = 6.1\%$ when the aggregate demand parameter $\alpha \equiv \delta/\mathcal{V}'(l_0) = 1$ and $G/C = (G/C)^* = 19.7\%$. Using (15), we find that $p_0 = 0.78$.

Appendix C. Demand Side and Equilibrium: Other Examples

In Section 2.4 we describe a demand side with land for the matching model. Here we present other possible demand sides. These examples yield the same equilibrium as the example with land.

First of all, the variable $l(t)$ in the land model can be interpreted as a generic unproduced good, as in [Hart \(1982\)](#). It can also be interpreted as a specific unproduced good, such as gold. Of course the model would be unchanged under these alternative interpretations. We now turn to examples that are further from the land example.

Money in the Utility

The land model could be modified by replacing land by money and assuming that households derive utility from holding real money balances. Introducing money in the utility function is a classical way to generate an aggregate demand: following [Sidrauski \(1967\)](#), a large number of business-cycle models with money in the utility function have been developed (for example, [Barro and Grossman 1971](#); [Blanchard and Kiyotaki 1987](#)). Money is introduced in the utility function to capture the fact that money helps conducting transactions.

In the money model, a household holds $M(t)$ units of money and the supply of money is fixed at M_0 . In equilibrium, the money market clears and $M(t) = M_0$. The price of services in terms of money is $p(t)$. We specify a general price mechanism that determines the price of services: $p(t) = p(x(t), g(t))$. The household's instantaneous utility function is $\mathcal{U}(c(t), g(t)) + \mathcal{V}(M(t)/p(t))$. The law of motion of the household's real money balances $m(t) \equiv M(t)/p(t)$ is

$$\dot{m}(t) = (1 - u(x(t))) \cdot k - (1 + \tau(x(t))) \cdot c(t) - \pi(t) \cdot m(t) - T(t),$$

where $\pi(t) \equiv \dot{p}(t)/p(t)$ is the inflation rate. In steady state, g and thus p are fixed so inflation is zero. The equilibrium immediately converges to steady state. In steady state the desired private consumption $c(x, p, g)$ is given by

$$\frac{\partial \mathcal{U}}{\partial c} = (1 + \tau(x)) \cdot \frac{\mathcal{V}'(M_0/p)}{\delta}.$$

Equilibrium tightness $x(g)$ is implicitly defined by

$$c(x, p(x, g), g) + g = y(x, k).$$

Bonds in the Utility

The land model could also be modified by replacing land by nominal bonds and assuming that households derive utility from holding real bonds. Introducing bonds in the utility function is a simple way to generate an aggregate demand in a dynamic economy. It is especially adapted to modern, cashless macroeconomic models. The assumption that wealth enters the utility function has been used in growth models ([Kurz 1968](#); [Zou 1994](#)), microeconomic models ([Robson 1992](#); [Cole, Mailath and Postlewaite 1995](#)), life-cycle models ([Carroll 2000](#); [Francis 2009](#)), asset-pricing models ([Bakshi and Chen 1996](#); [Gong and Zou 2002](#)), business-cycle models ([Michaillat and Saez 2014](#); [Ono and Yamada 2014](#)), and public-economics models ([Saez and Stantcheva 2016](#)). Wealth is introduced in the utility function to capture the fact that people care about wealth not only as future consumption but for its own sake. Wealth could be valued for several reasons: high wealth provides high social status; high wealth provides political power; people value frugality and asceticism and thus dignify the accumulation of wealth; or people value bequeathing wealth.

In the bond model, bonds are issued and purchased by households, and they have a price of 1 in terms of money. Money only plays the role of a unit of account. A household holds $B(t)$ bonds and bonds are in zero net supply. In equilibrium, the bond market clears and $B(t) = 0$. The rate of return on bonds is the nominal interest rate $i(t)$. The nominal interest rate is determined by the central bank, which sets an interest rate $i(t) = i(x(t), g(t))$. The interest rate depends on public consumption so there is an interaction between monetary and fiscal policy.

The price of services in terms of money is $p(t)$. The inflation rate is $\pi(t) \equiv \dot{p}(t)/p(t)$. In the economy there are two goods—services and bonds—and hence one relative price (public and private services have the same price). The price of bonds relative to services is determined by the real interest rate, $i(t) - \pi(t)$. Since the nominal interest rate is determined by the central bank, it is the inflation rate that determines the real interest rate. The inflation rate is determined by a general price mechanism: $\pi(t) = \pi(x(t), g(t))$. Given the inflation rate, the price of services moves according to $\dot{p}(t) = \pi(t) \cdot p(t)$. The initial price $p(0)$ is given. Given the inflation rate and nominal interest rate, tightness adjusts such that supply equals demand on the market for services.

The household's instantaneous utility function is $\mathcal{U}(c(t), g(t)) + \mathcal{V}(B(t)/p(t))$. The law of motion of the household's real wealth $b(t) \equiv B(t)/p(t)$ is

$$\dot{b}(t) = (1 - u(x(t))) \cdot k - (1 + \tau(x(t))) \cdot c(t) + (i(t) - \pi(t)) \cdot b(t) - T(t).$$

As earlier, the equilibrium immediately converges to steady state. In steady state, the desired amount of private consumption $c(x, i, \pi, g)$ is given by

$$\frac{\partial \mathcal{U}}{\partial c} = (1 + \tau(x)) \cdot \frac{\mathcal{V}'(0)}{\delta - i + \pi}.$$

This equation is the usual consumption Euler equation modified by the utility of wealth and evaluated in steady state. The demand for saving arises in part from intertemporal consumption-smoothing considerations and in part from the utility provided by wealth. The equation implies that at the margin, the household is indifferent between consuming and holding real wealth. Equilibrium tightness $x(g)$ is implicitly defined by

$$c(x, i(x, g), \pi(x, g), g) + g = y(x, k).$$

References

- Bakshi, Gurdip S., and Zhiwu Chen.** 1996. "The Spirit of Capitalism and Stock-Market Prices." *American Economic Review*, 86(1): 133–157.
- Barro, Robert J., and Herschel I. Grossman.** 1971. "A General Disequilibrium Model of Income and Employment." *American Economic Review*, 61(1): 82–93.
- Blanchard, Olivier J., and Nobuhiro Kiyotaki.** 1987. "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review*, 77(4): 647–666.
- Carroll, Christopher D.** 2000. "Why Do the Rich Save So Much?" In *Does Atlas Shrug? The Economic Consequences of Taxing the Rich.*, ed. Joel B. Slemrod. Cambridge, MA:Harvard University Press.

- Cole, Harold L., George J. Mailath, and Andrew Postlewaite.** 1995. "Incorporating Concern for Relative Wealth Into Economic Models." *Quarterly Review*, 19(3): 12–21.
- Francis, Johanna.** 2009. "Wealth and the Capitalist Spirit." *Journal of Macroeconomics*, 31(3): 394–408.
- Gong, Liutang, and Heng-fu Zou.** 2002. "Direct preferences for wealth, the risk premium puzzle, growth, and policy effectiveness." *Journal of Economic Dynamics and Control*, 26(2): 247–270.
- Hart, Oliver.** 1982. "A Model of Imperfect Competition with Keynesian Features." *Quarterly Journal of Economics*, 97(1): 109–138.
- Kurz, Mordecai.** 1968. "Optimal Economic Growth and Wealth Effects." *International Economic Review*, 9(3): 348–357.
- Landais, Camille, Pascal Michailat, and Emmanuel Saez.** 2017. "A Macroeconomic Approach to Optimal Unemployment Insurance: Applications." <http://www.pascalnichailat.org/5.html>.
- Michailat, Pascal, and Emmanuel Saez.** 2014. "An Economical Business-Cycle Model." National Bureau of Economic Research (NBER) Working Paper 19777.
- Ono, Yoshiyasu, and Katsunori Yamada.** 2014. "Difference Or Ratio: Implications Of Status Preference On Stagnation." Institute of Social and Economic Research, Osaka University (ISER) Discussion Paper 0856.
- Robson, Arthur J.** 1992. "Status, the Distribution of Wealth, Private and Social Attitudes to Risk." *Econometrica*, 60(4): 837–857.
- Saez, Emmanuel, and Stefanie Stantcheva.** 2016. "A Simpler Theory of Capital Taxation." National Bureau of Economic Research (NBER) Working Paper 22664.
- Sidrauski, Miguel.** 1967. "Rational Choice and Patterns of Growth in a Monetary Economy." *American Economic Review*, 57(2): 534–544.
- Zou, Heng-fu.** 1994. "'The Spirit of Capitalism' and Long-Run Growth." *European Journal of Political Economy*, 10: 279–293.