The Optimal Use of Public-Good Spending for Macroeconomic Stabilization

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Abstract

This paper analyzes the optimal use of budget-balanced public-good spending for macroeconomic stabilization. We use a dynamic macroeconomic model with matching frictions to generate slack and with utility for wealth to generate an aggregate demand. We derive an optimal public-good spending formula expressed in terms of estimable sufficient statistics. The formula takes the form of the standard Samuelson formula plus a correction term. The correction term depends on the gap between actual tightness and optimal tightness (whether the economy is too slack or too tight) and the effect of public-good spending on tightness (which can be measured by the government-expenditure multiplier). When tightness is optimal, public-good spending should follow the standard Samuelson rule. If public-good spending increases tightness (positive multiplier), then public-good spending should be above the Samuelson rule when the economy is slack and below when it is tight. We use empirical estimates to calibrate our formula for the US economy. With a multiplier of 0.5, we find quantitatively large deviations of optimal public-good spending from the Samuelson rule during recessions—on the order of 4 points of GDP during the Great Recession.

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1. Introduction

There is a broad normative literature in public economics that analyzes optimal public-good spending. Optimal public-good spending should satisfy the famous Samuelson formula derived in Samuelson [1954] seminal contribution. The modern optimal public-good spending literature typically does not consider macroeconomic stabilization issues. Yet, fiscal policy is also traditionally considered as one potential tool for macroeconomic stabilization. In Keynesian models, more public spending translates into more overall output. Hence, increasing public spending can reduce the output gap in recessions. In neoclassical models however, there is no reason for public spending to increase output. Output may even decrease if distortionary taxes are increased to finance the extra public-good spending. There is a voluminous positive literature in macroeconomics that considers the effects of fiscal policy on macroeconomic stabilization with a particular emphasis in estimating the fiscal multiplier of government spending defined as the dollar increase in overall output for a \$1 increase in public-good spending [Ramey, 2011]. However, the modern macroeconomic literature typically does not derive optimal public-good spending formulas.[1] The goal of this paper is to bridge the gap between the normative public literature and the positive macroeconomic literature by deriving formulas for the optimal use of public-good spending for macroeconomic stabilization.[2]

Our model is designed to be as simple and tractable as possible so that we can obtain simple formulas, easily inspect the economic mechanisms, and hence understand the robustness of our results. We use the continuous time representative agent model of Michaillat and Saez [2014] but removing money for simplicity and adding public-good spending. Households derive utility from consumption, the public good, and real wealth. Utility for wealth is a simple way to capture aggregate demand effects in a real economy with no money.[3] Households are self-employed, producing and selling services on a market with matching frictions. There are no firms and hence

[1] See Woodford [2011] for a recent discussion of fiscal multipliers in New Keynesian models.

[2] A small older literature in public economics analyzes optimal policy in disequilibrium models. See for instance Roberts [1982], Drèze [1985], Guesnerie and Roberts [1987], and Marchand, Pestieau and Wibaut [1989]. Since our model of slack is much more tractable than the disequilibrium model (see Michaillat and Saez [2015] for a detailed discussion), our analysis is simpler than these earlier studies. Even though we work in a world with slack, we are able to obtain simple formulas that append a correction term to the classic Samuelson formula.

[3] Other possibilities would be to consider utility for money as in Michaillat and Saez [2015] or more generally utility for a nonproduced good. The advantage of using wealth is that it captures the intuition that shifts in preferences for "thrift" create aggregate demand shocks.

the single market for services is both a product market and a labor market. The services are purchased by other households, who consume the services, and by the government, which provides these services as a public good. Because of matching frictions, households can only sell a fraction of their potential labor services and hence are partly idle, which creates slack. Because of matching costs, buying services for private or public consumption requires extra matching costs spending. For example, consuming restaurant services requires taxi cab services to find a suitable restaurant. The state of the economy is determined by the market tightness. In a tight market, it is easy to sell services so that slack is small but buying services for consumption is difficult and matching costs are high. In a slack market, it is hard to sell services so that slack is high but buying services for consumption is easy and matching costs are low. As in standard matching models, there is an optimal tightness that maximizes output net of matching costs [Hosios, 1990].

Households also trade real bonds. They hold bonds to smooth consumption and also because they derive utility from wealth, which can only be stored in bonds. Bonds pay real interest which defines inter temporal prices. Matching frictions add market tightness as an additional variable in the model. Hence, an additional equation–the price mechanism–is needed to determine the economic equilibrium, a well known property of matching models.[4] In our economy, the price is the interest rate and we are agnostic on the price mechanism that determines interest. Our analysis applies for any price mechanism. If the interest rate is too high, aggregate demand is too low and the economy is slack (unemployment is too high and output is too low). If the interest is too low, aggregate demand is too high and the economy is tight (unemployment too low, output is too high, and too many resources are dissipated in matching frictions). Although the economy is dynamic, as there are no state variables, following a shock, the economy immediately jumps to the new steady-state making the analysis tractable and transparent.

We derive an optimal public-good spending formula expressed in terms of estimable sufficient statistics. It takes the form of the standard Samuelson formula plus a correction term. The Samuelson formula is simply that the the marginal rate of substitution between public good consumption and private consumption of the representative household should equal the marginal rate of transformation between the public and private goods (equal to one in our model). The correction term

[4]The traditional matching models assume that prices are determined by Nash bargaining between sellers and buyers.

depends on the gap between actual tightness and optimal tightness and the effect of public-good spending on tightness, which can be measured by the fiscal multiplier of public-good spending. When tightness is optimal, public-good spending has no additional welfare effect via changes in tightness (envelope theorem), and hence public-good spending should follow the standard Samuelson rule. Hence, our model nests the standard public economics case. However, if tightness is not optimal, the correction term is non zero, and optimal public-good spending should depart from the Samuelson rule as follows. If public-good spending increases tightness, which is the case when the fiscal multiplier is positive, then public-good spending should be above the Samuelson rule when the economy is slack and below when it is tight. Conversely, if public-good spending decreases tightness (the fiscal multiplier is negative), then public-good spending should be below the Samuelson rule when the economy is slack and above when it is tight. Optimal public policy formulas are typically expressed as implicit formulas as right-hand-side expressions are generally highly endogenous to the policy. In our model, with simple approximations, we can obtain a closed-form formula for public-good spending as a function of estimable parameters and the unemployment rate deviation from its initial optimal level. This makes our framework easy to apply.

The size of the deviation from the Samuelson rule depends on (1) the size of the initial unemployment shock, (2) the absolute size of the fiscal multiplier, (3) the elasticity of substitution between private consumption and public good consumption. With a small fiscal multiplier, the deviation of the Samuelson rule is small because public-good spending has only minimal effects on tightness. With a small elasticity of substitution between private consumption and public good consumption, extra public good beyond the Samuelson rule is an inefficient form of consumption, and the deviation from the Samuelson rule should be correspondingly small. Hence, in our theory, whether public goods are useful is a critical parameter. With a multiplier above one, increasing public-good spending also increases private good consumption and hence is always desirable in a recession, even if the public good is useless.[5] Interestingly, with a very large multiplier, the deviation from the Samuelson rule becomes smaller as public-good spending can quickly bring back tightness to its optimum.

If prices are partly rigid to shocks, then the fiscal multiplier in our model will be positive. In

[5] However, in our model, we would need unrealistic price mechanisms to obtain multipliers above one. We discuss the link with standard Keynesian theory at the end of the paper (in progress).

that case, starting from an optimal tightness, a negative aggregate demand shock reduces tightness and calls for an increase in public-good spending (relative to the Samuelson rule). In contrast, a negative aggregate supply shock increases tightness and calls for a decrease in public-good spending (relative to the Samuelson rule).

Finally, we use empirical estimates to calibrate our formula. First, we show that the ratio of private consumption to government consumption is counter-cyclical at business cycle frequency: In slumps, private consumption falls more than government consumption. With a standard assumption of a constant elasticity of substitution between private and public good consumption, this implies that actual governments deviate from the Samuelson rule in a counter-cyclical manner, consistent with our theory in the case of a positive fiscal multiplier. Second, we estimate deviations from optimal tightness in the US since 1951. Third, in order to provide quantitive guidance, we calibrate the model to the US economy using a range of realistic parameters for the fiscal multiplier and the elasticity of substitution between private and public good consumption. With a standard elasticity of substitution equal to one, with a fiscal multiplier of 0.5, we find large deviations of optimal public-good spending from the Samuelson rule during recessions. In particular, for a recession increasing unemployment from 6% (approximately the natural rate) to 9%, the optimal public good extra spending should be 4.2 GDP points. Even with a very modest multiplier of 0.2, the optimal deviation should still be 2.7 GDP points. Therefore, we find that public-good spending should be a key tool for macroeconomic stabilization as soon as the multiplier is positive and the elasticity of substitution between private and public consumption is positive.

2. A Model of Unemployment with Public-Good Spending

This section modifies the model of Michaillat and Saez [2014] to include public-good spending and remove money. The absence of money simplifies the analysis.

We work in continuous time. The model is dynamic and composed of a measure 1 of identical households. Households are self-employed, producing and selling services on a market with matching frictions. To simplify the analysis, we abstract from firms and assume that all production directly takes place within households.[6] The services are purchased by other households,

[6] Michaillat and Saez [2015] show how the model can be extended to feature a labor market and a product market, distinct but formally symmetric, and firms hiring workers on the labor market and selling their production on the

who consume the services, and by the government, who provides these services as a public good. Households also trade bonds; they hold bonds to smooth future consumption and because they derive utility from wealth, which can only be stored in bonds.

2.1. The Market for Labor Services

Households sell labor services on a market with matching frictions. A household has a productive capacity k. The productive capacity indicates the maximum amount of services that a household could deliver at any point in time; it is exogenous. At time t, a household sells $Y(t) < k$ units of services. All these services are sold through long-term relationships that separate at rate s. The idle capacity of the household at time t is $k - Y(t)$; since some of the capacity of the household is idle, some household members are unemployed. The rate of unemployment u is defined by

$$u(t) = \frac{k - Y(t)}{k},$$

where k is the aggregate productive capacity and $Y(t)$ is the aggregate output of services. All the workers are self-employed so $u(t)$ does not correspond to the usual definition of unemployment, which is the share of workers without a job; instead, $u(t)$ gives the share of workers who are idle and whose services are not employed by sellers.

Households also consume labor services, but they cannot consume their own services, so they purchase services from other households. The government also purchases services from households; the services purchased by the government constitute a public good. Hence, at time t households sell an amount $Y(t)$ of services on the product market; an amount $C(t)$ of these services are purchased by other households and an amount $G(t)$ is purchased by the government such that

$$Y(t) = C(t) + G(t).$$

To purchase labor services at time t, a household posts $v^h(t)$ help-wanted advertisements and the government posts $v^g(t)$ help-wanted advertisements. New long-term relationships are formed at a rate $h(k - Y(t), v(t))$, where h is the matching function, $k - Y(t)$ is the aggregate idle capacity,

product market.

and $v(t) = v^h(t) + v^g(t)$ is the aggregate number of help-wanted advertisements. The matching function has constant returns to scale, is differentiable, is increasing in both arguments, and has diminishing returns in both arguments.

The market tightness x is defined by

$$x(t) = \frac{v(t)}{k - Y(t)}.$$

The market tightness is the ratio of the two arguments in the matching function: aggregate help-wanted advertisements and aggregate idle capacity. Since it is an aggregate variable, the market tightness is taken as given by households. With constant returns to scale in matching, the market tightness determines the rates at which sellers and buyers enter into new long-term trading relationships. At time t, each of the $k - Y(t)$ units of available productive capacity is committed to a long-term relationship at rate

$$f(x(t)) = \frac{h(k - Y(t), v(t))}{k - Y(t)} = h(1, x(t))$$

and each of the $v(t)$ help-wanted advertisement is filled with a long-term relationship at rate

$$q(x(t)) = \frac{h(k - Y(t), v(t))}{v(t)} = h\left(\frac{1}{x(t)}, 1\right).$$

The function f is increasing and concave and the function q is decreasing. In other words, when the market tightness is higher, it is easier to sell services but harder to buy them. A useful property is that $q(x) = f(x)/x$. We denote by $1 - \eta$ and $-\eta$ the elasticities of f and q:

$$1 - \eta = x \cdot \frac{f'(x)}{f(x)} > 0$$
$$\eta = -x \cdot \frac{q'(x)}{q(x)} > 0.$$

According to the matching process, the number of relationships $Y(t)$ and the unemployment rate $1 - Y(t)/k$ are state variables. However, in practice, because the rate of outflow from unemployment is so large, the number of relationships and the unemployment rate are fast-moving state variables that adjust extremely rapidly to their steady-state levels.[7] Hence, the transitional dynamics of these variables are extremely fast the variables barely departs from their steady-state levels. Throughout the paper we therefore approximate output and the unemployment rate by the jump variables

$$Y(t) = \phi(x(t)) \cdot k \tag{1}$$

$$u(t) = 1 - \phi(x(t)) \tag{2}$$

where

$$\phi(x) = \frac{f(x)}{f(x) + s}. \tag{3}$$

Appendix A derives these expressions and demonstrates that transitional dynamics are unimportant. This approximation simplifies the analysis at virtually no cost.[8] In sum, because of the matching frictions, only a fraction $\phi(x(t))$ of the productive capacity in the economy is employed while a fraction $1 - \phi(x(t))$ remains idle. The employment rate in the economy is $\phi(x(t))$, and the unemployment rate is $1 - \phi(x(t))$.

Finally, posting help-wanted advertisements is costly. The cost of an advertisement is ρ units of labor services so that a total of $\rho \cdot v(t)$ services are spent at time t on filling help-wanted advertisements. These recruiting services represent the resources devoted to matching with an appropriate worker. Both households and government need to post costly help-wanted advertisements to purchase services; hence, expenditures on services $C(t)$ and $G(t)$ are divided between expenditures on consumption and expenditures on matching—by consumption we mean the amount of services that enters the household's utility function. We denote by $c(t) < C(t)$ personal consumption and by $g(t) < G(t)$ government consumption. Government consumption corresponds to the consumption of public good by the household. As we have just discussed, $C(t)$ and $G(t)$ are fast-moving state variables that can be well approximated by their stead-state levels. Hence, expenditure and

[7] Hall [2005*b*], Pissarides [2009], and Shimer [2012] make this point.

[8] This approximation is standard in the literature. See for instance Pissarides [2009] and Hall [2005*a*,*b*].

consumption are related by

$$C(t) = [1 + \tau(x(t))] \cdot c(t)$$
$$G(t) = [1 + \tau(x(t))] \cdot g(t)$$

where

$$\tau(x) = \frac{\rho \cdot s}{q(x) - \rho \cdot s}. \tag{4}$$

Appendix A derives these expressions. For households and the government, consuming one service requires to purchase $1 + \tau(x)$ services—one service for consumption plus $\tau(x)$ services to cover the cost of matching. From the buyer's perspective, it is as if it purchased 1 service at a unit price $1 + \tau(x)$. Effectively the matching frictions impose a wedge $\tau(x)$ on the price of services, which applies to both households and government.

The employment probability $\phi(x)$ and the matching wedge $\tau(x)$ play a central role in the analysis. The following proposition summarizes their properties:

PROPOSITION 1. *The employment probability $\phi(x)$ defined by (3) and the matching wedge $\tau(x)$ defined by (4) have the following properties:*

- *The function ϕ is $\in [0,1]$ and increasing on $[0,+\infty)$. The elasticity of ϕ is $x \cdot \phi'(x)/\phi(x) = (1-\eta) \cdot (1-\phi)$.*
- *The function τ is positive and increasing on $[0,x^m)$, where $x^m > 0$ is defined by $q(x^m) = \rho \cdot s$. The elasticity of $1+\tau$ is $x \cdot \tau'(x)/(1+\tau(x) = \eta \cdot \tau(x)$.*
- *The function $\phi/(1+\tau)$ is positive, increasing on $[0,x^*]$, and decreasing on $[x^*,x^m]$, where $x^* \in (0,x^m)$ is defined by*

$$\frac{1-\eta}{\eta} = \frac{\tau(x^*)}{1-\phi(x^*)}.$$

2.2. The Market for Bonds

Households can issue or buy riskless real bonds. Bonds are traded on a perfectly competitive market. At time t, a household holds $b(t)$ bonds, and the rate of return on bonds is the real interest rate $r(t)$. In equilibrium, the bond market clears and

$$b(t) = 0.$$

Household hold bonds to smooth future consumption and because they derive utility from real wealth, which can only be stored in bonds. Indeed, the real wealth of a household is the amount of bonds it owns, $b(t)$. The aggregate real wealth in the economy is zero.

2.3. The Government

The public good is financed by a lump-sum tax $T(t) = G(t)$. The government is only able to offer a fraction $1/(1+\tau(x(t)))$ of the services that it purchases for public-good consumption. The amount of consumption provided by the government as a public good is denoted by $g(t) = G(t)/(1+\tau(x(t)))$. The amount $g(t)$ is the amount that enters households' utility function.

The case of budget-balanced public-good spending is directly relevant for governments that cannot use debt such as the US states, which have budget-balance requirements, or for countries unable to issue new debt. When the government can issue debt, it could finance public-good spending with debt. If households are Ricardian—in the sense that they do not view government bonds as net wealth because such bonds need to be repaid with taxes later on—deficit finance and budget balance are economically equivalent in our model, exactly as in Barro [1974].[9]

[9] Naturally, households may not be Ricardian. In that case, the timing of taxes (keeping the present discounted value of taxes constant) can impact economic activity. Such non-Ricardian effects, however, are orthogonal to the effects of public-good spending studied here.

2.4. Households

The representative household spends part of its labor income on labor services and save part of it as bonds. The law of motion of the representative household's assets is

$$\dot{b}(t) = \phi(x(t)) \cdot k - (1 + \tau(x(t))) \cdot c(t) + r(t) \cdot b(t) - T(t). \tag{5}$$

Here, $b(t)$ are real bond holdings, $c(t)$ is the consumption of services, $\phi(x(t)) \cdot k$ is the quantity of services sold, $(1 + \tau(x(t))) \cdot c(t)$ is the quantity of services purchased, $r(t)$ is the real interest rate, and $T(t)$ is the lump-sum tax paid to the government. This budget constraint is standard but for two differences arising from the presence of matching frictions. First, income k is discounted by a factor $\phi(x(t)) \leq 1$ as only a fraction $\phi(x(t))$ of k is actually sold. Second, the price of consumption $c(t)$ is $1 + \tau(x(t)) \geq 1$ because some resources are dissipated in recruiting.

The representative household derives utility from consuming the $c(t)$ services that it purchases, consuming the $g(t)$ services purchased by the government and provided as a public good, and holding $b(t)$ units of real wealth. Its instantaneous utility function is $\mathscr{U}(c(t), g(t), b(t))$. The function $\mathscr{U}$ is twice differentiable, increasing in its three arguments, and concave. The utility function of a household at time 0 is the discounted sum of instantaneous utilities

$$\int_0^{+\infty} e^{-\delta \cdot t} \cdot \mathscr{U}(c(t), g(t), b(t))\, dt, \tag{6}$$

where $\delta > 0$ is the subjective discount rate.

Throughout, $[x(t)]_{t=0}^{+\infty}$ denotes the continuous-time path of variable $x(t)$. The representative household takes as given initial real wealth $b(0) = 0$ and the paths for market tightness, public-good provision, real interest rate, and tax $[x(t), g(t), r(t), T(t)]_{t=0}^{+\infty}$. Given these, the household chooses paths for consumption and real wealth $[c(t), b(t)]_{t=0}^{+\infty}$ to maximize (6) subject to (5).

To solve the household's problem, we set up the current-value Hamiltonian:

$$\mathscr{H}(t, c(t), b(t)) = \mathscr{U}(c(t), g(t), b(t)) + \lambda(t) \cdot [\phi(x(t)) \cdot k - (1 + \tau(x(t))) \cdot c(t) + r(t) \cdot b(t) - T(t)]$$

with control variable $c(t)$ and state variable $b(t)$, and current-value costate variable $\lambda(t)$. Throughout we use subscripts to denote partial derivatives. The necessary conditions for an interior solution to this maximization problem are $\mathscr{H}_c(t, c(t), b(t)) = 0$, $\mathscr{H}_b(t, c(t), b(t)) = \delta \cdot \lambda(t) - \dot{\lambda}(t)$, and the transversality condition $\lim_{t \to +\infty} e^{-\delta \cdot t} \cdot \lambda(t) \cdot b(t) = 0$. Given that $\mathscr{U}$ is concave and that $\mathscr{H}$ is the sum of $\mathscr{U}$ and a linear function of (c, b), $\mathscr{H}$ is concave in (c, b) and these conditions are also sufficient.

These two conditions imply that

$$\mathscr{U}_c(c(t), g(t), b(t)) = \lambda(t) \cdot (1 + \tau(x(t))) \tag{7}$$

$$\mathscr{U}_b(c(t), g(t), b(t)) = (\delta - r(t)) \cdot \lambda(t) - \dot{\lambda}(t). \tag{8}$$

Equations (7) and (8) imply that the marginal utilities from consumption and real wealth satisfy

$$(1 + \tau(x(t))) \cdot \frac{\mathscr{U}_b(c(t), g(t), b(t))}{\mathscr{U}_c(c(t), g(t), b(t))} + (r(t) - \delta) = -\frac{\dot{\lambda}(t)}{\lambda(t)}, \tag{9}$$

where $\dot{\lambda}(t)/\lambda(t)$ can be expressed as a function of $c(t)$, $g(t)$, $b(t)$, $x(t)$, and their derivatives using (7). This is the consumption Euler equation. It represents a demand for saving in part from intertemporal consumption-smoothing considerations and in part from the utility provided by wealth. This equation implies that at the margin, the household is indifferent between spending income on consumption and holding real wealth. This equation determines the level of aggregate demand and in steady state it defines the aggregate demand curve.

2.5. *Equilibrium*

We now define and characterize the equilibrium.[10] An equilibrium consists of paths for market tightness, personal consumption, public consumption, real wealth, and real interest rate, $[x(t), c(t), g(t), b(t), r(t)]_{t=0}^{+\infty}$, such that the following conditions hold:

(a) the representative household chooses personal consumption to maximize utility subject to its budget constraint;

(b) the government chooses public consumption;

[10] Michaillat and Saez [2015] provide more details about the equilibrium concept.

(c) the bond market clears;

(d) on the market for services, actual and posted tightnesses are equal;

(e) the real interest rate is given by a price mechanism.

In the economy there are two goods: labor services and bonds. Hence there is only one (relative) price. The price of bonds relative to services is determined by the real interest rate. Unlike on a Walrasian market, it is always required to specify a price mechanism on a matching market. Since the relative price of services is determined by the real interest rate, the price mechanism determines the real interest rate.

Condition (a) imposes that $[c(t)]_{t=0}^{+\infty}$ satisfies equation (9). Condition (b) says that $[g(t)]_{t=0}^{+\infty}$ is determined by the government. Condition (c) imposes that $b(t) = 0$. Condition (d) imposes that for all t,

$$c(t) + g(t) = \frac{\phi(x(t))}{1 + \tau(x(t))} \cdot k.$$

Finally, condition (e) says that $[r(t)]_{t=0}^{+\infty}$ is determined by a price mechanism, which we leave unspecified for the moment. The equilibrium consists of 5 endogenous variables determined by 5 conditions, so the equilibrium is well defined.

The equilibrium is a dynamical system. We now determine the steady state of the equilibrium before studying the transitional dynamics to the steady state.

2.6. *Steady State*

We determine the steady state for a public consumption g and a real interest rate r. We only need to determine tightness x and personal consumption c.

Setting $\dot{\lambda} = 0$ in equation (9), we find that c must satisfy

$$\frac{\mathcal{U}_b(c, g, 0)}{\mathcal{U}_c(c, g, 0)} = \frac{\delta - r}{1 + \tau(x)}. \tag{10}$$

Let $c^d(x, g, r)$ be the amount of personal consumption implicitly defined by this equation; then in steady state personal consumption satisfies

$$c = c^d(x, g, r).$$

The aggregate demand y^d is defined by

$$y^d(x, g, r) = g + c^d(x, g, r).$$

The aggregate demand gives the total amount of services that households and the government desire to consume. The aggregate supply y^s is defined by

$$y^s(x) = \frac{\phi(x)}{1 + \tau(x)} \cdot k. \tag{11}$$

This aggregate supply is strictly increasing on $[0, x^*]$ and strictly decreasing on $[x^*, x^m]$. Hence, x^* maximizes the aggregate supply. Figure 1 depicts the aggregate supply curve. In steady state, tightness satisfies

$$y^s(x) = y^d(x, g, r). \tag{12}$$

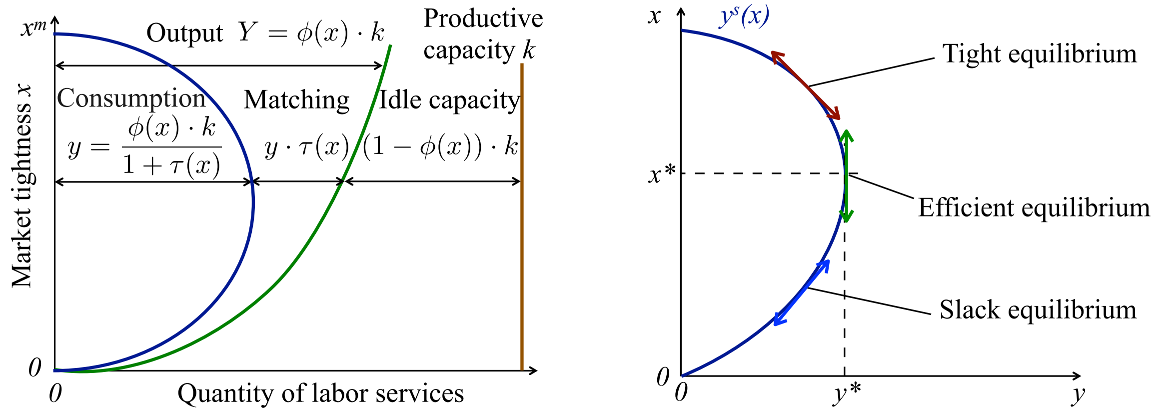
It is convenient to introduce an alternative aggregate supply Y^s, which gives the amount of services traded at a given tightness, and which is defined by

$$Y^s(x) = \phi(x) \cdot k.$$

The aggregate supply Y^s is strictly increasing on $[0, x^m]$.

The structure of the steady state is illustrated in Figure 2. Tightness and total consumption are given by the intersection of aggregate demand and supply in the (y, x) plane.

Depending on the price mechanism, the steady state can fall into three regimes. A steady state is efficient if it maximizes consumption. An inefficient steady state can be either slack, if an increase in tightness at the equilibrium point raises consumption, or tight, if an increase in tightness at the equilibrium point lowers consumption. Equivalently, a steady state is efficient if $x = x^*$, slack if $x < x^*$, and tight if $x > x^*$. The slack and tight equilibria are inefficient because their consumption



A. Description of matching frictions

B. The three regimes of the matching market

Figure 1: Aggregate Supply and Efficiency

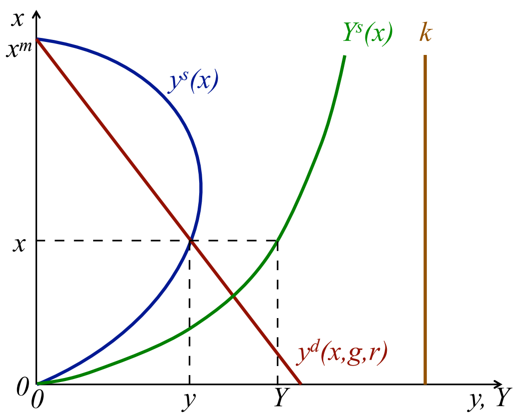


Figure 2: General Equilibrium

levels are below the efficient consumption level.

2.7. Transitional Dynamics

Here we describe the transitional dynamics toward the steady state. We make two simplifying assumptions: (a) $r < \delta$ is a function of the parameters so it is constant over time; (b) the utility for wealth enters separately in the utility function such that $\mathscr{U}_b(c(t), g(t), b(t) = 0)$ is a parameter and is constant over time; and (c) the government determines $g(t)$ as a function of the other endogenous variables at time t and the parameters.

Under these three assumptions, the dynamical system describing the equilibrium is described by one single endogenous variable: the costate variable $\lambda(t)$. All the variables can be recovered from $\lambda(t)$.

In equilibrium the law of motion for the costate variable is given by equation (8):

$$\dot{\lambda}(t) = (\delta - r) \cdot \lambda(t) - \mathscr{U}_b(c(t), g(t), 0) \equiv \phi(\lambda(t)).$$

The steady-state value of the costate variable satisfies $\phi(\lambda) = 0$ so

$$\lambda = \frac{\mathscr{U}_b(c(t), g(t), 0)}{\delta - r} > 0.$$

The nature of the dynamical system is given by the sign of $\phi'(\lambda)$. Since $\phi'(\lambda) = \delta - r > 0$, we infer that the system is a source. As there is no endogenous state variable, a source system jumps from one steady state to the other in response to an unexpected shock. Hence, comparative-statics analysis is sufficient to capture the full dynamics following unexpected shocks.

Below we assume that the property that the system is a source is always valid.

3. Optimal Public-Good Spending

In this section we study the optimal provision of public good. We compare the optimal provision of public good with the provision of public good given by the classical formula of Samuelson [1954]. When the economy is efficient, the provision of public good given by the Samuelson formula is optimal. But when the economy is inefficient, the optimal public-good formula departs from the Samuelson formula: it is optimal to provide more public good than suggested by the Samuelson formula when increasing the provision of public good brings the economy toward efficiency.

3.1. An Abstract Formula

When the government determines $g(t)$ as a function of the other endogenous variables at time t and the parameters, the dynamical system describing the equilibrium is a source so for any initial conditions, the system jumps to its steady state. In this steady state, public-good spending is constant. The question that we consider therefore is to determine the level g of public-good spending that maximizes steady-state welfare. This is equivalent to determining the level g of public-good spending that maximizes the instantaneous utility given by

$$\mathscr{W} = \mathscr{U}(c, g, 0).$$

Personal consumption is given by $c = c^d(x, g, r)$ because that is the consumption resulting from the household's problem in steady state. It is difficult to find the optimal policy with this expression for personal consumption because, without specifying the price mechanism, we do not have an expression for the equilibrium interest rate r. To simplify the analysis, we rewrite the equilibrium condition (12) as

$$c^d(x, g, r) = y^s(x) - g,$$

which allows us to eliminate r from the expression for social welfare. The social welfare can be written as a function of (x, g) only:

$$\mathscr{W}(x, g) = \mathscr{U}\left(y^s(x) - g, g, 0\right). \tag{13}$$

The government chooses g to maximize $\mathscr{W}(x, g)$, taking into account the fact that in equilibrium x is a function of g implicitly defined by the equilibrium system

$$\begin{cases} y^s(x) & = & y^d(x, g, r) \\ r & = & r(x, g) \end{cases}$$

where $r(x, g)$ is the most general price mechanism possible. In the government's problem, the government budget is balanced because the lump-sum tax T adjusts to cover the expenses incurred by the government on public good.

Taking the first-order condition of the government's problem, we find that the optimal provision of public good satisfies

$$0 = \left.\frac{\partial \mathscr{W}}{\partial g}\right|_x + \left.\frac{\partial \mathscr{W}}{\partial x}\right|_g \cdot \frac{dx}{dg}.$$

Table 1: Optimal Level of Public Good Compared to Samuelson Level

	$dx/dg < 0$	$dx/dg = 0$	$dx/dg > 0$
Slack equilibrium ($dy^s/dx > 0$)	lower	same	higher
Efficient equilibrium ($dy^s/dx = 0$)	same	same	same
Tight equilibrium ($dy^s/dx < 0$)	higher	same	lower

Notes: The Samuelson level of public good is given by $1 = (\partial\mathscr{U}/\partial g)/(\partial\mathscr{U}/\partial c)$. The optimal level of public good is higher than the Samuelson level if the stabilization term in (14) is positive, same as the Samuelson level if the stabilization term is zero, and lower than the Samuelson level if the stabilization term is negative.

Expression (13) shows that the effects of public good and tightness on welfare are simple:

$$\left.\frac{\partial\mathscr{W}}{\partial g}\right|_x = \frac{\partial\mathscr{U}}{\partial g} - \frac{\partial\mathscr{U}}{\partial c}$$
$$\left.\frac{\partial\mathscr{W}}{\partial x}\right|_g = \frac{\partial\mathscr{U}}{\partial c} \cdot \frac{dy^s}{dx}.$$

We can therefore rewrite the optimality condition as

$$\frac{\partial\mathscr{U}}{\partial c} = \frac{\partial\mathscr{U}}{\partial g} + \frac{\partial\mathscr{U}}{\partial c} \cdot \frac{dy^s}{dx} \cdot \frac{dx}{dg}.$$

Dividing the condition by $\partial\mathscr{U}/\partial c$ yields the formula for optimal public-good spending:

PROPOSITION 2. *The optimal provision of public good satisfies*

$$\underbrace{1 = \frac{\partial\mathscr{U}/\partial g}{\partial\mathscr{U}/\partial c}}_{\textit{Samuelson formula}} + \underbrace{\frac{dy^s}{dx} \cdot \frac{dx}{dg}}_{\textit{stabilization term}}. \tag{14}$$

The Samuelson formula is $1 = (\partial\mathscr{U}/\partial g)/(\partial\mathscr{U}/\partial c)$; it requires that the marginal utility from personal consumption equals the marginal utility from public consumption. Our optimal public-good formula is just the Samuelson formula plus a stabilization term equal to $(dy^s/dx) \cdot (dx/dg)$. Since $(\partial\mathscr{U}/\partial g)/(\partial\mathscr{U}/\partial c)$ is decreasing in g, our formula indicates that it is desirable to provide more public-good than in the Samuelson formula if the stabilization term is positive but less if the stabilization term is negative. If the stabilization term is zero, the Samuelson formula is valid.

The stabilization term is the product of the effect of public good on tightness and the effect of tightness on welfare. The stabilization term is positive if and only if providing more public good brings the economy toward efficiency. The structure of the optimal public-good formula—a standard public-finance term plus a stabilization term—is similar to the structure of the optimal unemployment-insurance formula derived by Landais, Michaillat and Saez [2010]. Furthermore, the response of tightness to the public good can be interpreted as a pecuniary externality. The reason is that tightness can be interpreted as a price, influencing welfare when the market is inefficient. Under this interpretation, the additive structure of the formula—a standard term plus a correction term—is similar to the structure of many optimal taxation formulas obtained in the presence of externalities.

There are two situations when the stabilization term is zero and the optimal provision of public good is given by the Samuelson formula. The first situation is when the market tightness is efficient such that $dy^s/dx = 0$. In that case, the marginal effect of the public good on tightness has no first-order effect on welfare; hence, the optimal provision of public good is governed by the same principles as in the Samuelson framework in which tightness is fixed. The second situation is when the public good has no effect on tightness such that $dx/dg = 0$. In that case, our model is isomorphic to the Samuelson framework, so optimal provision of public good is guided by the same principles.

In all other situations, the stabilization term is nonzero and the optimal provision of public good departs from the Samuelson formula. The main implication of our formula is that increasing the provision of public good above the Samuelson level is desirable if and only if increasing the provision of public good brings the market tightness closer to its efficient level. The provision of public good brings tightness closer to efficiency either if tightness is inefficiently low and the public good raises tightness or if tightness is inefficiently high and the public good lowers tightness. Table 1 summarizes all the possibilities.

As is standard in optimal tax formulas, the right-hand-side of (14) is endogenous to the policy. Even though the formula only characterizes the optimal provision of public good implicitly, it is useful. First, it transparently shows the economic forces at play. Second, it gives general conditions for the optimal level of public good to be above or below the Samuelson level.

3.2. An Exact Implicit Sufficient-Statistic Formula

To facilitate the application of the theory, we express our abstract formula for optimal public-good spending, given by (14), with estimable statistics. To simplify, we assume that households have a constant-elasticity-of-substitution utility given by

$$\mathscr{U}(c,g,b) = \frac{\mu_1}{1+\mu_1+\mu_2} \cdot c^{\frac{\varepsilon-1}{\varepsilon}} + \frac{\mu_2}{1+\mu_1+\mu_2} \cdot g^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+\mu_1+\mu_2} \cdot (b+\theta)^{\frac{\varepsilon-1}{\varepsilon}}, \tag{15}$$

where the parameter $\mu_1 > 0$ measures the taste for personal consumption relative to wealth, the parameter $\mu_2 > 0$ measures the taste for public consumption relative to wealth, the parameter $\varepsilon > 1$ is the elasticity of substitution between consumptions, and the parameter $\theta > 0$ governs the marginal utility of real wealth in general equilibrium.[11]

PROPOSITION 3. *The optimal provision of public good satisfies*

$$1 = \left(\frac{G/C}{G^*/C^*}\right)^{-\frac{1}{\varepsilon}} + \left(1 - \frac{\eta}{1-\eta} \cdot \frac{\tau(x)}{u}\right) \cdot \frac{dY}{dG} \cdot \left[1 - \frac{\eta}{1-\eta} \cdot \frac{\tau(x)}{u} \cdot \frac{1}{1+C/G} \cdot \frac{dY}{dG}\right]^{-1}. \tag{16}$$

The formula is expressed as a function of estimable statistics: multiplier dY/dG, elasticity of substitution across consumptions ε, ratio of personal to public consumption expenditures G/C, elasticity of the job-finding rate $1-\eta$, matching wedge $\tau(x)$, and unemployment rate u.

Proof. First, we express the ratio $(\partial\mathscr{U}/\partial g)/(\partial\mathscr{U}/\partial c)$ with estimable statistics under the specification (15). The term $(\partial\mathscr{U}/\partial g)/(\partial\mathscr{U}/\partial c)$ admits a simple expression:

$$\frac{\partial\mathscr{U}/\partial g}{\partial\mathscr{U}/\partial c} = \frac{\mu_2}{\mu_1} \cdot \left(\frac{G}{C}\right)^{-\frac{1}{\varepsilon}},$$

where $C = (1+\tau(x)) \cdot c$ is personal consumption expenditures and $G = (1+\tau(x)) \cdot g$ is public consumption expenditures.

We denote by G^*/C^* the ratio of personal consumption expenditures to public consumption expenditures that satisfy our formula when the equilibrium is efficient. This is also the ratio that

[11] It is conventional to assume that public consumption enters utility in a separable fashion. See for instance Hall [2009], Woodford [2011], and Ramey [2011].

satisfy the conventional Samuelson formula. This ratio determines μ_2/μ_1:

$$\left(\frac{G^*}{C^*}\right)^{\frac{1}{\varepsilon}} = \frac{\mu_2}{\mu_1}.$$

It is practical to express the ratio μ_2/μ_1 as a function of the ratio G^*/C^* because we will be able to measure G^*/C^* in the data. Indeed, under the assumption that the trend of the economy is efficient and that the government follows our optimal public-good formula, G^*/C^* can be measured in the data by the trend of the ratio G/C. Under the alternative assumption that the government determines the trend of government spending by following the well-known Samuelson formula, then G^*/C^* can also be measured in the data by the trend of the ratio G/C.

In sum, the term $(\partial\mathscr{U}/\partial g)/(\partial\mathscr{U}/\partial c)$ can be expressed with two estimable statistics:

$$\frac{\partial\mathscr{U}/\partial g}{\partial\mathscr{U}/\partial c} = \left(\frac{G/C}{G^*/C^*}\right)^{-\frac{1}{\varepsilon}}.$$

The first statistic is G^*/C^*, and the second statistic is ε, the elasticity of substitution between public and personal consumption.

Second, we express the stabilization term with estimable statistics. The stabilization term can be written as

$$\frac{dy^s}{dx} \cdot \frac{dx}{dg} = \frac{dy}{dg}$$

The term dy/dg is a multiplier: it gives the increase in total consumption of services when one more unit of publicly provided services is consumed. However, dy/dg is not directly estimable in macroeconomic data because researchers measure the amount of services that is purchased, and not the amount that is consumed. We therefore express dy/dg as a function of the multiplier dY/dG that researchers estimate in the data. The multiplier dY/dG gives the increase in output of services when one more service is purchased by the government.

Since $y = y^s(x) = Y^s(x)/(1+\tau(x))$, and given the elasticities of Y^s and τ, we find that

$$\frac{d\ln(y)}{d\ln(g)} = [(1-\eta)\cdot u - \eta\cdot\tau(x)]\cdot\frac{d\ln(x)}{d\ln(G)}\cdot\frac{d\ln(G)}{d\ln(g)}.$$

Next, given the elasticity of Y^s, we find that

$$\frac{d\ln(Y)}{d\ln(G)} = (1-\eta)\cdot u \cdot \frac{d\ln(x)}{d\ln(G)}.$$

Last, since $G = (1+\tau(x))\cdot g$, and given the elasticity τ, we find that

$$\frac{d\ln(G)}{d\ln(g)} = 1 + \eta \cdot \tau(x) \cdot \frac{d\ln(x)}{d\ln(G)} \cdot \frac{d\ln(G)}{d\ln(g)},$$

which implies that

$$\frac{d\ln(G)}{d\ln(g)} = \frac{1}{1-\eta\cdot\tau(x)\cdot\frac{d\ln(x)}{d\ln(G)}}.$$

Combining all these results, and using the fact that $g/y = G/Y$, we obtain

$$\frac{dy}{dg} = \left(1-\frac{\eta}{1-\eta}\cdot\frac{\tau(x)}{u}\right)\cdot\frac{dY}{dG}\cdot\left[1-\frac{\eta}{1-\eta}\cdot\frac{\tau(x)}{u}\cdot\left(\frac{1}{1+C/G}\right)\cdot\frac{dY}{dG}\right]^{-1}.$$

Bringing all the elements together, we obtain (16). □

3.3. An Approximate Explicit Sufficient-Statistic Formula

The formula (16) is a bit complex, and it can be greatly simplified with a few approximations. The approximate formula that we derive here is expressed with estimable statistics and is easy to apply to assess the change in public-good spending warranted by a given change in unemployment.

The formula (19) highlights the main empirical statistics that determine the optimal provision of public good G/Y. However, the right-hand side of (19) is endogenous to G/Y so we cannot use the formula to answer the following question: If the unemployment rate is 50% above its efficient level and the public good is at the Samuelson level, what should be the increase in public-good spending? Here we develop an *explicit sufficient-statistic formula* that we can use to address this question.

Assume that the economy is efficient and is hit by an unexpected permanent shock. The shock brings the economy to from an unemployment rate u^* to an unemployment rate u_0. As public-good spending G changes, the unemployment rate will endogenously respond. It is this endogenous response that we need to describe to obtain our explicit formula.

PROPOSITION 4. *Assume that the economy is efficient and is hit by an unexpected permanent shock. The shock brings the unemployment rate to u_0. Then the optimal response of public-good spending to this shock satisfies*

$$\frac{G/Y - G^*/Y^*}{G^*/Y^*} \approx \frac{\varepsilon \cdot \frac{dY}{dG} \cdot (1 - G/Y)}{1 - \eta + \varepsilon \cdot \left(\frac{dY}{dG}\right)^2 \cdot (1 - G/Y) \cdot \frac{G/Y}{u}} \cdot \frac{u_0 - u^*}{u^*}. \tag{17}$$

The formula is expressed as a function of the following sufficient statistics: multiplier dY/dG, elasticity of substitution across consumptions ε, ratio of public consumption expenditures to GDP G/Y, elasticity of the job-finding rate $1-\eta$, and unemployment rate u.

Proof. A first approximation is that

$$1 - \frac{\eta}{1-\eta} \cdot \frac{\tau(x)}{u} \cdot \frac{1}{1 + C/G} \cdot \frac{dY}{dG} \approx 1.$$

This approximation is valid as long as the ratio C/G is large enough and the multiplier dY/dG is not too large. In the US when the economy is efficient, $(1-\eta)/\eta = \tau(x)/u$, $C/G = 5$, and a reasonable estimate of the multiplier is $dY/dG = 0.5$. Hence the approximated term is 0.92, which is quite close to 1. The approximation becomes better when the economy becomes slack, and the slacker the economy the better the approximation.

The second approximation is that

$$\frac{\tau(x)}{u} = \frac{s \cdot \rho}{q(x) - s \cdot \rho} \cdot \frac{s + f(x)}{s} \approx \frac{s \cdot \rho}{q(x)} \cdot \frac{f(x)}{s} = \rho \cdot x.$$

This approximation is valid as long as $s \ll f(x)$ and $s \cdot \rho \ll q(x)$, which is always satisfied in practice. On average in the US, $s \approx s \cdot \rho \approx 4\%$ and $f(x) \approx 50\%$ and $q(x) \approx 100\%$. Let x^* and u^* be the efficient market tightness and unemployment rate. They satisfy

$$\frac{\tau(x^*)}{u^*} = \frac{1-\eta}{\eta} \approx \rho \cdot x^*.$$

Hence, we obtain the following approximation

$$1-\frac{\eta}{1-\eta}\cdot\frac{\tau(x)}{u}\approx 1-\frac{x}{x^*}.$$

The last approximation is a simple linear expansion:

$$1-\left(\frac{G/C}{G^*/C^*}\right)^{-\frac{1}{\varepsilon}}\approx\frac{1}{\varepsilon}\cdot\frac{G/C-G^*/C^*}{G^*/C^*}$$

Overall, the formula becomes

$$\frac{G/C-G^*/C^*}{G^*/C^*}\approx-\varepsilon\cdot\frac{x-x^*}{x^*}\cdot\frac{dY}{dG}. \tag{18}$$

This formula is extremely useful, in particular for the empirical analysis.

The last modification that we bring to the formula is to change variables and express it as a function of G/Y and u instead of G/C and x. This change of variable makes the formula easier to implement. To make the change of variables, we use the following linear approximations:

$$u=\frac{s}{s+f(x)}$$
$$\frac{u-u^*}{u^*}\approx d\ln(u)\approx-(1-u^*)\cdot(1-\eta)\cdot d\ln(x)\approx-(1-\eta)\cdot\frac{x-x^*}{x^*}$$

(where we also approximate $1-u^*\approx 1$, since the efficient unemployment rate is no doubt below 10%) and

$$\frac{G}{C}=\frac{1}{Y/G-1}$$
$$\frac{G/C-G^*/C^*}{G^*/C^*}\approx d\ln(G/C)\approx\frac{1}{1-G^*/Y^*}\cdot d\ln(G/Y)\approx\frac{1}{1-G^*/Y^*}\cdot\frac{G/Y-G^*/Y^*}{G^*/Y^*}.$$

The optimal public-good spending therefore satisfies the following approximate formula:

$$\frac{G/Y-G^*/Y^*}{G^*/Y^*}\approx\varepsilon\cdot\frac{dY}{dG}\cdot\frac{1-G^*/Y^*}{1-\eta}\cdot\frac{u-u^*}{u^*}. \tag{19}$$

Assume that the economy is efficient and is hit by an unexpected permanent shock. The shock brings the economy to from an unemployment rate u^* to an unemployment rate u_0. As public-good spending G changes, the unemployment rate will endogenously respond. The right-hand side of (19) is endogenous to G/Y. It is this endogenous response that we need to describe to obtain our explicit formula.

First, notice that

$$\frac{G/Y - G^*/Y^*}{G^*/Y^*} \approx d\ln\left(\frac{G}{Y}\right) \approx \left(1 - \frac{G^*}{Y^*} \cdot \frac{dY}{dG}\right) \cdot d\ln(G).$$

However, since $G^*/Y^* \ll 1$ and dY/dG is probably below 1, a good approximation is

$$\frac{G/Y - G^*/Y^*}{G^*/Y^*} \approx d\ln(G).$$

A linear expansion of $\ln(u)$ around $\ln(u_0)$ yields

$$\ln(u) - \ln(u^*) = \ln(u_0) - \ln(u^*) + \left(\frac{d\ln(u)}{d\ln(G)}\right) \cdot d\ln(G)$$

We have $(u-u^*)/u^* \approx \ln(u) - \ln(u^*)$ and $(u_0 - u^*)/u^* \approx \ln(u_0) - \ln(u^*)$. Furthermore, $u = 1 - Y/k$ and $u - 1 \approx -1$ so

$$\frac{d\ln(u)}{d\ln(G)} = \frac{u-1}{u} \cdot \frac{G}{Y} \cdot \frac{dY}{dG} \approx -\frac{G}{Y} \cdot \frac{1}{u} \cdot \frac{dY}{dG}.$$

Collecting these results yields

$$\frac{u-u^*}{u^*} = \frac{u_0 - u^*}{u^*} - \frac{G}{Y} \cdot \frac{1}{u} \cdot \frac{G/Y - G^*/Y^*}{G^*/Y^*}.$$

Plugging this expression in formula (19) yields formula (17). □

Formula (17) links the relative deviation of public-good spending (expressed as a share of GDP), G/Y, with the relative deviation of the unemployment rate, u. The formula involves three sufficient statistics: the multiplier dY/dG, the elasticity of substitution across consumptions ε, and the elasticity of the job-finding rate, $1-\eta$. The formula can be directly applied by policymakers to determine the optimal response of public-good spending to a shock that leads to an increase or decrease in the unemployment rate.

The formula confirms an intuition that many macroeconomists had but that had not been formalized before: optimal public-good spending depends positively on the multiplier dY/dG, at least for small values of the multiplier.

The formula also brings new insights. First, while public-good spending should be above the Samuelson level if unemployment is inefficiently high, public-good spending should be below the Samuelson level if unemployment is inefficiently low.

Second, while the multiplier plays an important role in determining the amplitude of deviation of public-good spending from the Samuelson level, the elasticity of substitution ε plays an equally important role. If the multiplier is zero, public-good spending is always given by the Samuelson level; the same is true of if the elasticity is zero (that is, if the utility function is Leontief). If the multiplier is large, public-good spending responds a lot to fluctuations of the unemployment rate; the same is true of if the elasticity is large (that is, if personal and public consumption are very substitutable).

Third, departures of unemployment from its efficient level have first-order effects on the optimal provision of public good, but fluctuations of the multiplier in response to a change in unemployment only have second-order effects.[12] To a first-order approximation, the average multiplier is sufficient to obtain the optimal response of public-good spending to a shock.

4. Empirical Applications

This section estimates the new statistics in the optimal public-good formulas (16) and (17). It then uses these estimates and existing estimates for the other statistics to address several policy questions. The empirical applications focus on the US for the 1951–2014 period.

[12] See for instance Auerbach and Gorodnichenko [2012, 2013] for empirical studies of the response of the multiplier to slack. See Michaillat [2014] for a model in which the multiplier is higher when the economy has more slack. See Parker [2011] for a discussion of the possible implications of state-dependent multipliers for optimal public-good spending.

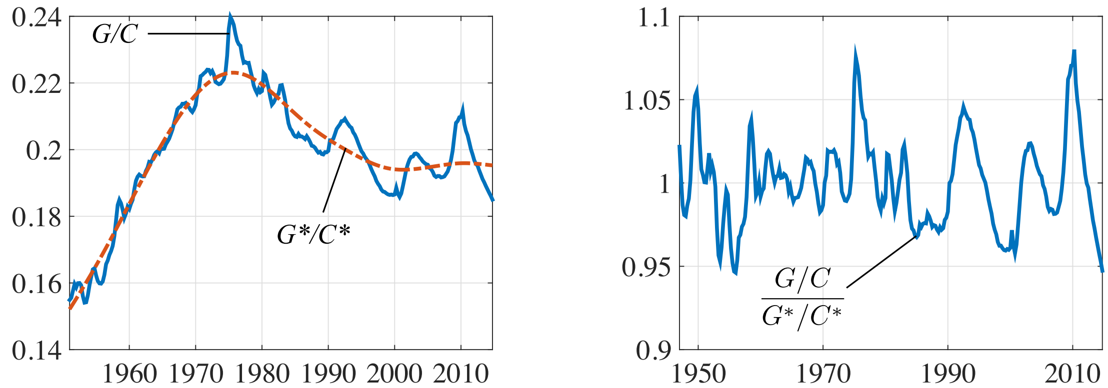


Figure 3: Public-Good Spending in the US, 1947–2014

Notes: The data are quarterly average of seasonally adjusted monthly data constructed by the BLS from the CES survey. The ratio G/C is the number of all employees in the government industry divided by the number of all employees in total private industry. The ratio G^*/C^* is a low-frequency trend of the ratio G/C, produced with a HP filter with smoothing parameter 10^5.

4.1. Estimates of the Sufficient Statistics

We estimate the ratios G/C and G^*/C^* and the elasticity $1-\eta$. in US data for the 1951–2014 period. Another critical statistic in the formula is the multiplier dY/dG. A vast literature aims to estimate the multiplier; therefore, we do not attempt to estimate our own multiplier but review the range of existing estimates instead. We also discuss possible estimates for the elasticity ε.

Estimates of the ratios G/C and G^*/C^*. The ratio G/C for the US can be measured using data constructed by the Bureau of Labor Statistics (BLS) from the Current Employment Statistics (CES) survey. We measure G/C as the ratio of employment in the public sector to employment in the private sector. Appendix C constructs an alternative measure of G/C using consumption expenditures data constructed by the Bureau of Economic Analysis (BEA) as part of the National Income and Product Accounts (NIPA). The cyclical behavior of the two series is very similar, and it is almost undistinguishable after 1990.

Figure 3 plots G/C. The ratio G/C started at 14.8% in 1947, peaked at 24.0% in 1975, fell back to 20.0% in 1990, and averages 20.5% since 1990. Using $G/Y = 1/(1+C/G)$, we also find that public employment constitutes 17.0% of total employment on average since 1990.

As discussed above, under the assumption that the trend of the economy is efficient and the government follows our optimal public-good formula, or under the assumption that the government determines the trend of government spending by following the well-known Samuelson formula, then the ratio G^*/C^* can be measured as the low-frequency trend of G/C. We produce the low-frequency trend of the quarterly series representing G/C with a Hodrick-Prescott (HP) filter with smoothing parameter 10^5. We also display G^*/C^* in Figure 3. We find that G^*/C^* has been relatively stable since 1990. We find that on average since 1990, $G^*/C^* = 19.6\%$.

Figure 3 also displays the ratio $(G/C)/(G^*/C^*)$. It is clear that G/C departs from G^*/C^*. Below we explore whether these deviations are optimal or not.

An Estimate of the Elasticity $1-\eta$. To estimate of $1-\eta$, we use measures of market tightness and job-finding rate in the US for the 1951–2014 period. The market tightness x_t is constructed by $x_t = v_t/u_t$, where u_t is the number of unemployed persons constructed by the BLS from the CPS and v_t is the number of vacancies that we construct by combining the vacancy index of Barnichon [2010] with the number of vacancies in JOLTS for 2001–2014. The job-finding rate f_t is constructed from CPS data following the methodology of Shimer [2012]. Appendix B details the construction of x_t and f_t.

The elasticity $1-\eta$ is defined by $1-\eta = d\ln(f(x))/d\ln(x)$. Since the relationship between $\ln(x_t)$ and $\ln(f_t)$ is nearly linear with a time trend in the intercept, we take $1-\eta$ to be a constant.[13] Furthermore, we estimate $1-\eta$ using a linear regression with a polynomial time trend. We run the regression

$$\ln(f_t) = (1-\hat{\eta})\cdot\ln(x_t) + \zeta_0 + \zeta_1\cdot t + \zeta_2\cdot t^2 + \zeta_3\cdot t^3 + \varepsilon_t.$$

The estimate that we obtain by ordinary least squares is $1-\hat{\eta} = 0.45$, with robust standard error 0.019 (this robust standard error accounts for possible heteroskedasticity and autocorrelation in the residuals). Hence, we estimate that $\eta = 0.55$. Our estimate of η falls in the middle of the range of estimates in the literature: in their survey of the literature estimating labor-market matching functions, Petrongolo and Pissarides [2001] conclude that most estimates of η fall in the 0.4–0.7 range. Our estimate for η is also close to the estimate of 0.72 obtained by Shimer [2005] using

[13] This finding is consistent with the result in Petrongolo and Pissarides [2001] that the matching function is well approximated by a Cobb-Douglas function.

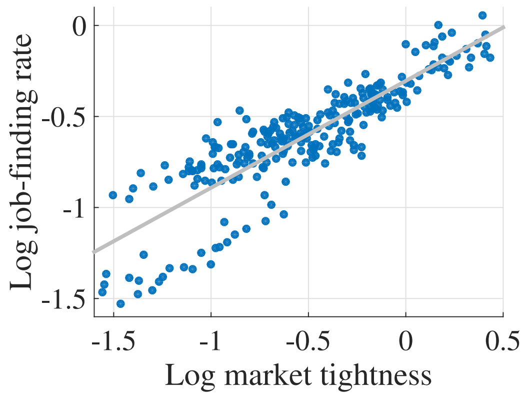


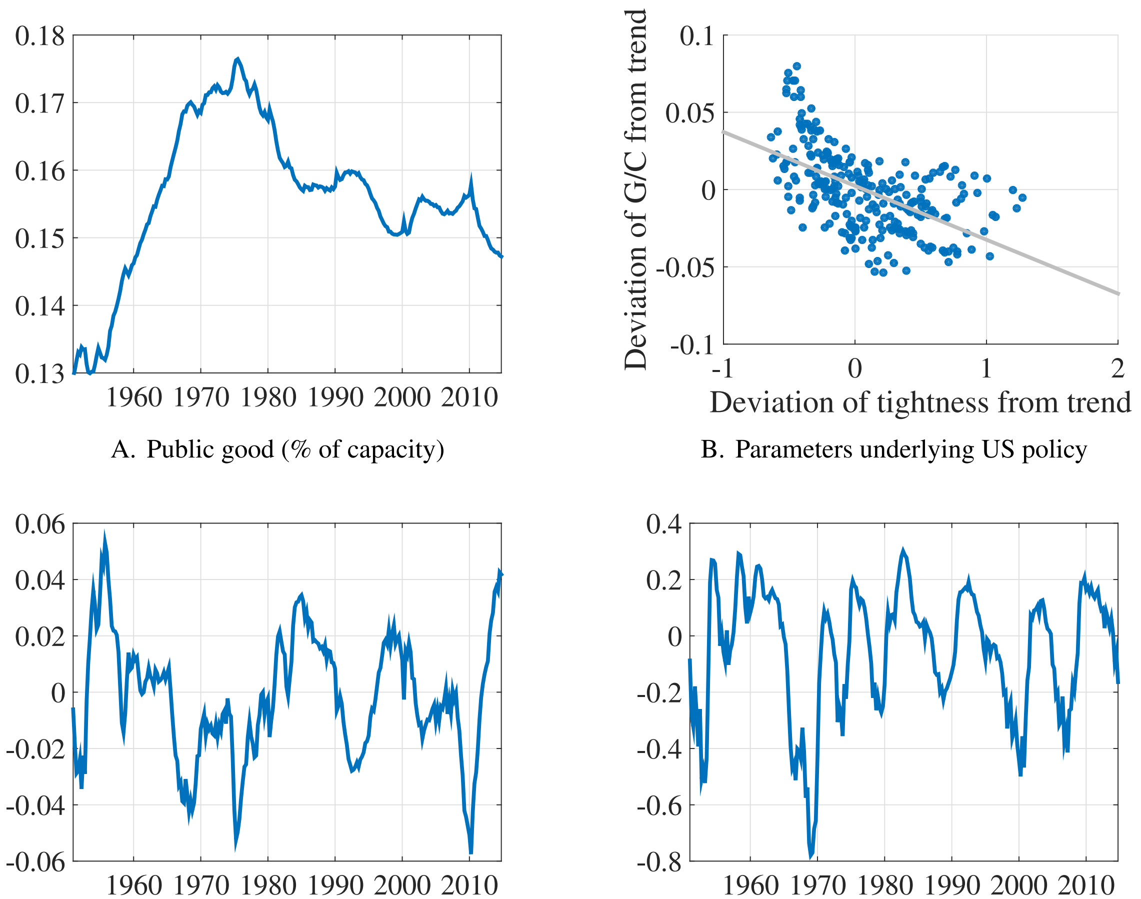
Figure 4: Estimating the Elasticity $1-\eta$ in the US, 1951–2014

Notes: This figure is a scatter plot of the logarithm of labor market tightness, $\ln(x_t)$, and the logarithm of the monthly job-finding rate, $\ln(f_t)$. The labor market tightness is constructed as the quarterly average of the monthly vacancy index constructed by Barnichon [2010], scaled to match the number of vacancies in JOLTS for 2001–2014, divided by the quarterly average of the seasonally adjusted monthly number of unemployed persons constructed by the BLS from the CPS. The monthly job-finding rate is constructed as $f_t = -\ln(1-F_t)$, where F_t is the monthly job-finding probability constructed following the methodology of Shimer [2012]. The series for F_t is available at quarterly frequency, so the series for f_t has a quarterly frequency. The figure also displays the least-squares regression line to the scatter plot.

similar data for the 1951–2003 period.[14]

Estimates of the Multiplier dY/dG. Next we discuss estimate of the multiplier dY/dG in the literature. The multiplier that enters the formula is a multiplier for budget-balanced and not deficit-financed government spending. Such a multiplier dY/dG has been measured in many studies. Ramey [2011] surveys the literature and concludes that the deficit-financed multiplier in the US is between 0.8 and 1.5. She adds that the data would not reject a multiplier of 0.5 or 2. According to Hall [2009] the deficit-financed multiplier in the US is between 0.7 and 1. Barro and Redlick [2011] find a deficit-financed multiplier of 0.8 and a tax multiplier (for distortionary taxation) of -1 so a balanced-budget multiplier of -0.2. The relevant multiplier in our model is a budget-balanced multiplier, but the balanced-budged and deficit-financed multiplier are the same if households are Ricardian in our model. Given the estimates in the literature, we use a multiplier of 0.5 as a baseline and consider lower and higher values.

[14] We use the same strategy as Shimer [2005] to estimate η, except that Shimer uses detrended data and a job-finding probability instead of a job-finding rate.



A. Public good (% of capacity)

B. Parameters underlying US policy

C. Evaluation of US policy with $dY/dG = 0.035$

D. Evaluation of US policy with $dY/dG = 0.5$

Figure 5: Assessment of Public-Good Spending in the US

Estimates of the Elasticity ε. The elasticity ε indicates the elasticity of substitution between personal consumption expenditures and government consumption expenditures. For instance, the utility function is Leontief and the consumptions are perfect complement if $\varepsilon = 0$, the utility function is Cobb-Douglas for $\varepsilon = 1$, and the utility function is linear and the consumptions are perfect substitute for $\varepsilon = +\infty$.

4.2. Assessment of Public-Good Spending in the US

The assessment of public-good spending in the US is presented in Figure 5. A first result, which appears in Panel B, is the US policy is optimal under statistics $\varepsilon \cdot (dY/dG) = 0.035$. This result is obtained by exploiting (18) and regressing $(G/C - G^*/C^*)/(G^*/C^*)$ on $(x - x^*)/x^*$. For an

Table 2: Optimal Response of Public-Good Spending (as Percentage of GDP) When the Unemployment Rate Increases from 6% to 9% for Several Values of Multiplier and Elasticity of Substitution

	$dY/dG = 0$	$dY/dG = 0.2$	$dY/dG = 0.5$	$dY/dG = 1$	$dY/dG = 2$
$\varepsilon = 0.1$	0%	+0.3%	+0.7%	+1.2%	+1.3%
$\varepsilon = 0.5$	0%	+1.4%	+2.7%	+2.8%	+2.0%
$\varepsilon = 1$	0%	+2.7%	+4.2%	+3.5%	+2.1%
$\varepsilon = 2$	0%	+4.8%	+5.7%	+3.9%	+2.2%

Notes: The table displays $\gamma - \gamma^*$ where $\gamma = G/Y$ is the optimal provision of public good in response to the shock and $\gamma^* = G^*/Y^*$ is the optimal provision of public good in the efficient equilibrium. The optimal response of public-good spending is given by formula (17). In (17), we use $\eta = 0.55$, $\gamma^* = 16.6\%$, $u_0 = 0.09$, and $\breve{u}_0 = 0.5$.

elasticity of substitution of $\varepsilon = 1$, the result implies that the US policy is optimal under a minuscule multiplier of $dY/dG = 0.035$.

Panel C evaluate the implicit sufficient-statistic formula for optimal public-good spending using the statistics measured in US data, an elasticity $\varepsilon = 1$, and a multiplier of $dY/dG = 0.5$. Consistent with the previous regression result, we find that if the multiplier is as large as $dY/dG = 0.5$, then public-good spending in the US is not countercyclical enough. Systematically, there is not enough spending in slumps and too much spending in booms.

4.3. Optimal Response of Public-Good Spending to an Unemployment Increase

Here we exploit the response formula, given by (17), and the estimates of the statistics described in the previous subsection, to compute the optimal response of public-good spending after a given increase in unemployment. As an illustration, we consider an increase in unemployment rate of 50%, from 6% to 9%. Table 2 displays the optimal increase in public-good spending for a range of possible estimates of the multiplier and the elasticity of substitution.

The first observation from the table is that if the multiplier is zero then public-good spending should remain at its initial level, given by the Samuelson formula.

The second observation is that the higher the elasticity of substitution between personal and public consumption, the higher the optimal increase in public-good spending. For instance with a multiplier of $dY/dG = 0.5$, public-good spending should increase from 16.6% to 17.3% of GDP with a low elasticity of 0.1, but it should increase from 16.6% to 22.3% of GDP with a high elasticity of 2. The elasticity of substitution therefore has a tremendous influence on the optimal response of public-good spending to a shock. The role of this elasticity has been neglected in previous work on the topic.

The third observation is that, unlike what conventional wisdom suggests, the optimal increase in public-good spending does not rise monotonically with the multiplier. It is true that the optimal increase in public-good spending rises for low values of the multiplier (in the table, $dY/dG \leq 0.5$). For instance with $\varepsilon = 1$, public-good spending should increase from 16.6% to 19.3% of GDP with a low multiplier of 0.2, but it should increase from 16.6% to 20.8% of GDP with a multiplier of 0.5. However, the optimal increase in public-good spending diminishes for high values of the multiplier (in the table, $dY/dG \geq 0.5$). The intuition is that if the multiplier is high, public-good spending is a very potent policy that can bring the economy close to the efficient tightness with only a modest amount of spending. As the multiplier rises, less public-good spending is required to bring the economy to the same tightness. For instance with $\varepsilon = 1$, public-good spending should increase from 16.6% to 20.8% of GDP with a low multiplier of 0.5, but it should only increase from 16.6% to 18.7% of GDP with a multiplier of 2. It seems that this property has not appeared previously in the literature.

4.4. Optimal Public-Good Spending in a Calibrated Model

To quantify more precisely the fluctuations of optimal public-good spending over the business cycle, we calibrate and simulate our model.

We showed in Section 2 that the economy jumps from one steady-state equilibrium to another in response to unexpected permanent shocks. Hence, we represent the different stages of the business cycle as a succession of steady states. Since we are interested in business cycles generated by aggregate demand shocks, we compute a collection of steady states parameterized by different values for the marginal utility of wealth, θ.[15] We perform two simulations: one in which public-good spending G/Y remains constant at its average value, and one in which public-good spending G/Y is at its optimal level, given by formula (16).

We calibrate the model to US data for the 1951–2014, as summarized in Table 3. Using the

[15]The empirical evidence presented in Michaillat and Saez [2015], based on the type of matching model used here, suggests that business cycles in the US are mostly caused by aggregate demand shocks.

Table 3: Parameter Values in the Simulation of the Model

	Description	Value	Source
	Panel A. Average values used for calibration		
$\overline{u}$	unemployment rate	5.9%	CPS, 1951–2014
$\overline{x}$	labor market tightness	0.65	Barnichon [2010], JOLTS, and CPS, 1951–2014
$\overline{G/Y}$	public employment / total employment	16.6%	CES, 1951–2014
$\overline{\tau}$	matching wedge	4.8%	efficiency on average
$\overline{dY/dG}$	multiplier	0.5	literature
$\overline{\theta}$	marginal utility of wealth	1	normalization
	Panel B. Calibrated parameters		
k	productive capacity	1	normalization
η	unemployment-elasticity of matching	0.55	CPS, 1951–2014
ε	elasticity of substitution	1	log utility
s	job-destruction rate (monthly)	3.3%	CPS, 1951–2014
ω	matching efficacy	0.64	matches average values
ρ	recruiting cost	1.12	matches average values
α	parameter of the demand shifter	1	rigidity with respect to θ
β	parameter of the demand shifter	0.5	matches $\overline{dY/dG}$

time series plotted in Figure A3, we find that over this period the average unemployment rate is $\overline{u} = 5.9\%$ and the average market tightness is $\overline{x} = 0.65$. Using the series plotted in Figure 3, we find that over this period the average public-good spending is $\overline{G/Y} = 16.6\%$. We assume that the government follows the Samuelson on average so that $\overline{G/Y} = 16.6\%$ satisfies the Samuelson formula when all the economic variables take their average values. In addition, we make the assumption that the economy is efficient on average.[16] This means that the average value of the matching wedge is $\overline{\tau} = \overline{u} \cdot (1-\eta)/\eta$. With $\overline{u} = 5.9\%$ and $\eta = 0.55$, we find that $\overline{\tau} = 4.8\%$. In the calibration, we target these average values.

First we make two normalizations. We will compute a collection of steady states parameterized by different values for the marginal utility of wealth θ, and we normalize the value of θ in the average state to $\overline{\theta} = 1$. We normalize the productive capacity to $k = 1$.

Next, we set the separation rate, s, to its average value over the 1951–2014 period, which is $s = 3.3\%$. We use a Cobb-Douglas matching function $h(u,v) = \omega \cdot u^{\eta} \cdot v^{1-\eta}$. We set $\eta = 0.55$ following

[16] If we had reliable estimates of the matching wedge $\tau(x)$, we would be able to test this assumption. However, we do not have sufficient evidence to establish whether the economy is efficient, slack, or tight on average. Landais, Michaillat and Saez [2010] explore two methods to test this assumption.

the estimate obtained above. To calibrate ω, we exploit the relationship $u \cdot f(x) = s \cdot (1-u)$, which implies $\omega = s \cdot x^{\eta-1} \cdot (1-u)/u$. With $\overline{u} = 5.9\%$ and $\overline{x} = 0.65$, we get $\omega = 0.64$.

To calibrate ρ we exploit the relationship $\tau = \rho \cdot s / [\omega \cdot x^{-\eta} - \rho \cdot s]$, which implies $\rho = \omega \cdot x^{-\eta} \cdot \tau / [s \cdot (1+\tau)]$. With $\omega = 0.64$, $s = 3.3\%$, $\overline{x} = 0.65$, and $\overline{\tau} = 4.8\%$, we obtain $\rho = 1.12$.

We use the utility function given by (15). Since we do not have good estimates for ε, we simply set $\varepsilon = 1$, which corresponds to log utility. In Appendix B, we perform addition simulations for $\varepsilon = xx$ to explore how the outcomes of the simulations depend on ε. With such a utility function the aggregate demand, defined by (10), satisfies

$$c^d(x, G, \theta) = [\mu_1 \cdot (\delta - r(G, \theta))]^{\varepsilon} \cdot (1 + \tau(x))^{-\varepsilon} \cdot \theta.$$

To simplify the exposition, we define the demand shifter R by

$$R(G, \theta) = [\mu_1 \cdot (\delta - r(G, \theta))]^{\varepsilon}.$$

The aggregate demand can be written as $c^d(x, G, \theta) = R(G, \theta) \cdot (1 + \tau(x))^{-\varepsilon} \cdot \theta$. The aggregate supply is given by

$$y^s(x) = \phi(x) \cdot k.$$

By calibrating η, ω, s, and k, we have calibrated the functions ϕ and y^s.

We need to calibrate the demand shifter $R(G, \theta)$, which is equivalent to calibrating the underlying real-interest-rate schedule $r(G, \theta)$. The calibration of $R(G, \theta)$ is critical because it determines the response of the economy to aggregate demand shocks as well as the value of the multiplier. For instance if $R(G, \theta) \propto 1/\theta$, then the parameter θ disappears from the aggregate demand c^d, and aggregate demand shocks have no effect. Or if $R(G, \theta)$ adjusts sufficiently to G that equilibrium tightness remains constant in response to changes in G, then the multiplier is necessarily zero.

We first derive the value $\overline{R}$ of the demand shifter in the average state. The value $\overline{R}$ must be such that for spending $\overline{G}$, output $\overline{Y} = Y^s(\overline{x}) = \phi(\overline{x}) \cdot k$, then $Y^d = \overline{Y}$. This condition requires

$$\begin{aligned}(1+\overline{\tau}) \cdot c^d + \overline{G} &= \overline{Y} \\ \overline{R} \cdot (1+\overline{\tau})^{1-\varepsilon} \cdot \overline{\theta} &= \overline{Y} - \overline{G} \\ \overline{R} &= \frac{\overline{Y} - \overline{G}}{\overline{\theta}} \cdot (1+\overline{\tau})^{\varepsilon-1}.\end{aligned}$$

We then derive the value R^* of the demand shifter that maintains the economy at efficiency. To simplify notations, let $Y^* = Y^s(x^*)$ and $\tau^* = \tau(x^*)$. At efficiency,

$$\begin{aligned}(1+\tau^*) \cdot c^d + G &= Y^* \\ R^* \cdot (1+\tau^*)^{1-\varepsilon} \cdot \theta &= Y^* - G \\ R^* &= \frac{Y^* - G}{\theta} \cdot (1+\tau^*)^{\varepsilon-1}.\end{aligned}$$

If the real-interest-rate schedule is such that $R = R^*$, then the economy is always at efficiency. However, if the real-interest-rate schedule is such that R responds less than R^* to fluctuations in government consumption expenditures G or marginal utility of wealth θ, then the economy responds to public-good spending shocks and aggregate demand shocks. To allow for this, we consider a general real-interest-rate schedule such that the demand shifter is

$$R(G,\theta) = \overline{R} \cdot \left(\frac{\overline{\theta}}{\theta}\right)^{1-\alpha} \cdot \left(\frac{Y^* - G}{Y^* - \overline{G}}\right)^{1-\beta}. \tag{20}$$

The parameter α determines the rigidity of the underlying real interest rate to shocks to the marginal utility of wealth, ω, and thus to aggregate demand shocks. If $\alpha = 1$, the real interest rate is completely rigid: it does not respond at all to aggregate demand shocks. If $\alpha = 0$, the real interest rate is completely flexible: it responds as much to aggregate demand shocks as the efficient real interest rate. When $\alpha = 0$, aggregate demand shocks are completely absorbed by the real interest rate such that the aggregate demands c^d and $y^d = c^d + g$ do not depend on θ.

The parameter β determines the rigidity of the underlying real interest rate to public-good spending shocks. If $\beta = 1$, the real interest rate is completely rigid: it does not respond at all to public-good spending shocks. If $\beta = 0$, the real interest rate is completely flexible: it responds as much to public-good spending shocks as the efficient real interest rate. When $\beta = 0$, public-good spending shocks are completely absorbed by the real interest rate such that the aggregate demand y^d does not depend on G and the multiplier dY/dG is zero.

The functional form for the demand shifter spans several interesting special cases. First, if $\alpha = \beta = 0$, the real interest rate r is always efficient: $R(G, \theta) = R^*$. This is because $\overline{Y} = Y^*$ and $\overline{\tau} = \tau^*$. Second, if $\alpha = \beta = 1$, the real interest rate r is constant at its average value: $R(G, \theta) = \overline{R}$. Last, the real interest rate r equals the average value $\overline{r}$ when $G = \overline{G}$ and $\theta = \overline{\theta}$: $R(\overline{G}, \overline{\theta}) = \overline{R}$.

For aggregate demand shocks to generate cyclical fluctuations, we need to set $\alpha > 0$. The value of α determines the elasticity of output and tightness to θ. Since we do not have estimates for the amplitude of the fluctuations of θ, the exact value of α is irrelevant. We therefore arbitrarily set $\alpha = 1$.

We calibrate β using available estimates of the multiplier dY/dG. Indeed, as established by the following lemma, β determines the amplitude of dY/dG.

LEMMA 1. *Assume that the utility function is given by (15) and the real interest rate is such that (20) holds. Then the multiplier satisfies*

$$\frac{dY}{dG} = \frac{1 - (1-\beta) \cdot \frac{Y-G}{Y^*-G}}{1 + \left(1 - \frac{G}{Y}\right) \cdot (\varepsilon - 1) \cdot \frac{\eta}{1-\eta} \cdot \frac{\tau(x)}{u}}. \tag{21}$$

When the economy is efficient, the multiplier simplifies to

$$\left(\frac{dY}{dG}\right)^* = \frac{\beta}{1 + \left(1 - \frac{G}{Y}\right) \cdot (\varepsilon - 1)}.$$

If in addition the elasticity of substitution $\varepsilon = 1$, then the multiplier is simply

$$\left(\frac{dY}{dG}\right)^* = \beta.$$

Proof. We differentiate the equilibrium condition $Y^s(x) = C^d(x, G) + G$ with respect to G. The condition can be written as

$$\phi(x) \cdot k = R(G, \theta) \cdot (1 + \tau(x))^{1-\varepsilon} \cdot \theta + G.$$

Log-differentiating with respect to G and using the expression (20) yields

$$(1-\eta)\cdot u\cdot\frac{d\ln(x)}{d\ln(G)}=\frac{G}{Y}+\frac{Y-G}{Y}\cdot\left[(1-\varepsilon)\cdot\eta\cdot\tau(x)\cdot\frac{d\ln(x)}{d\ln(G)}-(1-\beta)\cdot\frac{G}{Y^{*}-G}\right]$$
$$\frac{d\ln(x)}{d\ln(G)}=\frac{G}{Y}\cdot\frac{1-(1-\beta)\cdot\frac{Y-G}{Y^{*}-G}}{(1-\eta)\cdot u+\left(1-\frac{G}{Y}\right)\cdot(\varepsilon-1)\cdot\eta\cdot\tau(x)}.$$

Since $Y=Y^{s}(x)=\phi(x)\cdot k$, $d\ln(Y)/d\ln(G)=(1-\eta)\cdot u\cdot(d\ln(x)/d\ln(x))$. Of course, it is also true that $dY/dG=(Y/G)\cdot d\ln(Y)/d\ln(G)$. Combining these results, we obtain (21). □

An average value for the multiplier dY/dG found in the literature is $dY/dG=0.5$. As we assume that the economy is efficient on average, and since we set $\varepsilon=1$, we calibrate $\beta=0.5$ to match $dY/dG=0.5$ in the average state. Since $\tau(x)/u$ is very procyclical, the expression (21) for the multiplier dY/dG indicates that the multiplier is countercylical in the model, in line with the evidence provided by Auerbach and Gorodnichenko [2012, 2013]. The countercylicality of the multiplier arises from the sharp convexity of the aggregate supply Y^{s}, combined with the near-linearity of the downward-sloping aggregate demand Y^{d}. The mechanism behind the countercylicality of the multiplier is the same as that described in Michaillat [2014].

Figure 6 displays the simulations of the calibrated model. Each steady state is indexed by a marginal utility of wealth $\theta\in[0.94,1.02]$. Because of the rigidity of the real interest rate the steady states with low θ have a relatively high interest rate and therefore high unemployment: they represent slumps. Conversely, the steady states with high θ have a relatively low interest rate and low unemployment: they represent booms. As showed in Panel A, unemployment falls from 11% to 4% when θ increases from 0.94 to 1.02 and public-good spending remains constant. This numerical results implies that with some interest-rate rigidity, the model generates realistic fluctuations in unemployment. Relatedly, Panel B shows that when θ increases from 0.94 to 1.02 and public-good spending remains constant, the market tightness increases from 0.2 to 1.5.

It is not the unemployment rate that matters for optimal public-good spending but the efficiency term, depicted in Panel C. In a slump, the efficiency term is positive. The efficient term is 0 for $\theta=1$ since the model is calibrated such that the average state is efficient. In booms, the efficiency term is negative. The direct implication of this change of sign and the fact that the multiplier is positive is that public-good spending should be more generous than the Samuelson level in slumps

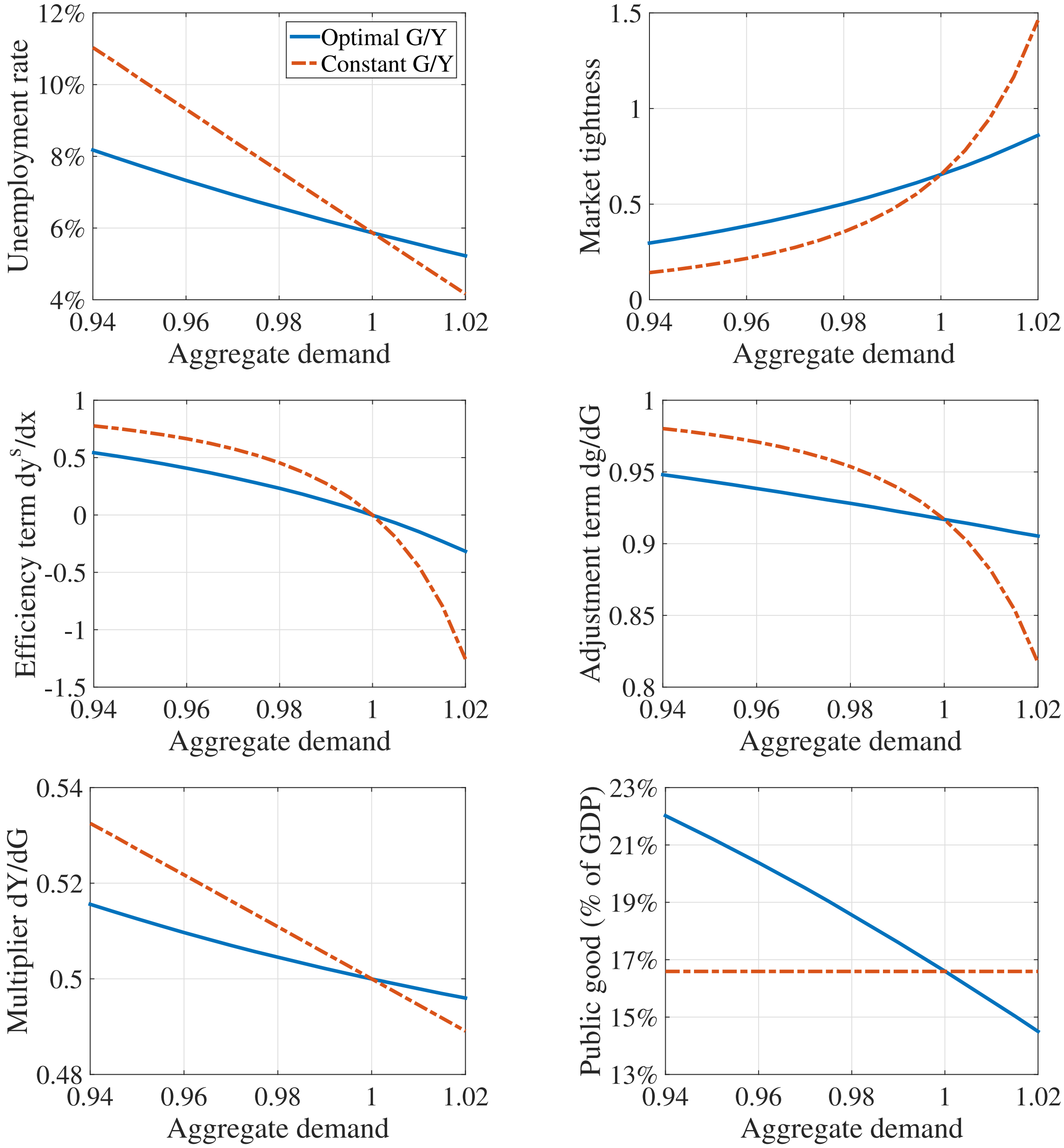


Figure 6: Optimal Public-Good Spending in a Calibrated Model

and less generous in booms.

Panel D depicts that term dg/dG that enters into the correction term in formula (16). This term is broadly constant around 0.9 for any value of θ, which justifies the assumption that we make when deriving the approximate formula (17).

Panel E displays the multiplier. The wedge is positive. In fact in an average situation with $\theta = 1$, the multiplier is 0.5 and matches exactly the empirical evidence. This is due to our calibration of the demand-shifter parameter, β. The multiplier is also slightly countercyclical.

Finally, Panel F shows that optimal public-good spending is countercyclical: optimal public-good spending falls from 22% of GDP when $\theta = 0.94$ to 15% of GDP when $\theta = 1.02$.

Of course, the unemployment rate responds to the adjustment of public-good spending from its original level of 16.6% to its optimal level. In slumps, optimal public-good spending is much higher than 16.6% so the unemployment rate decreases below its original level: at $\theta = 0.94$ the unemployment rate falls by 2.9 percentage point from 11% to 8.1%. In booms, optimal public-good spending is below 16.6% so the unemployment rate increases above its original level: at $\theta = 1.02$ the unemployment rate increases by 1.2 percentage point from 4.0% to 5.2%.

Last, the calibrated model allows us to evaluate the accuracy of the approximate explicit sufficient-statistic formula. We find that despite its simplicity, the approximate explicit sufficient-statistic formula is very accurate. Figure 7 compares the optimal public-good spendings obtained with the exact implicit sufficient-statistic formula, given by (16), and with the approximate explicit sufficient-statistic formula, given by (17). Despite the fact that the approximate explicit formula is an approximation of the exact implicit formula, the public-good spending suggested by the two formulas are almost identical.

5. Conclusion

In this paper we study the optimal use of public-good spending for macroeconomic stabilization. We derive an optimal public-good spending formula expressed in terms of estimable sufficient statistics. We use empirical estimates to calibrate our formula for the US economy. With a fiscal multiplier of 0.5, we find quantitatively large deviations of optimal public-good spending from the Samuelson rule during recessions, at around 4 points of GDP in the Great Recession. There are a

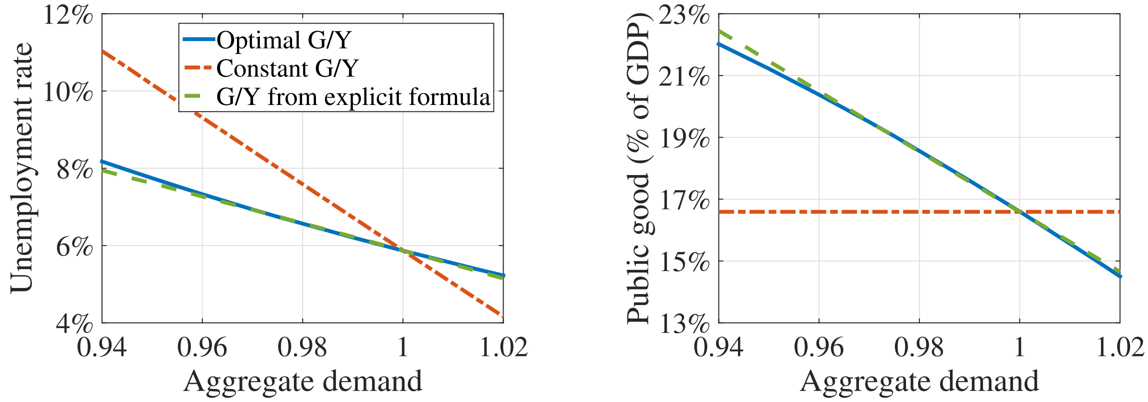


Figure 7: Comparison of the Public-Good Spendings Given by the Exact Implicit Formula and the Approximate Explicit Formula

number of extensions we would like to consider in future work.

Second, if there are supply-side responses and the public good is financed with distortionary taxation, changes in public-good spending can affect output through both supply-side effects (triggered by tax distortions) and through tightness effects. In that case, the correction term for optimal public-good spending still depends on how marginal public-good spending affects tightness. This parameter is no longer captured by the overall fiscal multiplier but by the net fiscal multiplier that parses out supply side responses. This net fiscal multiplier is defined at the overall macro level multiplier minus the micro level multiplier that captures the supply side responses of increasing public-good spending through distortionary taxation. These results are reminiscent of Landais, Michaillat and Saez [2010] who analyze optimal unemployment insurance over the business cycle and where the business cycle correction term (relative to the standard Baily-Chetty formula) depends on the difference between the macro and micro responses of unemployment to unemployment insurance.

Finally, our methodology could be applied to the optimal design of other public policies over the business cycle. We conjecture that a policy that maximizes welfare in an economy with inefficient fluctuations obeys the same general rule as the one derived in this paper and in the earlier study by Landais, Michaillat and Saez [2010]. The optimal policy is the sum of the optimal policy absent any inefficient cycle plus an externality-correction term when the economy is slack or tight. If a marginal increase of the policy increases tightness, the corrective term is positive in a slack economy and negative in a tight economy. For example, we conjecture that the rule could also be applied to income taxation. If high-income earners have a lower propensity to consume than low-income earners, then transfers from high incomes to low incomes stimulate aggregate demand and increase tightness. As a result, the top income tax rate should be higher than in the Mirrleesian optimal top income tax formula in a slack economy, and lower in a tight economy. This broad agenda in normative analysis could help bridge the gap between optimal policy analysis in public economics and business cycle analysis in macroeconomics.

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Appendix A: Employment Probability and Matching Wedge

In this appendix we derive the expressions for the employment probability $\phi(x)$ and matching wedge $\tau(x)$ given in the text. We also argue that transitional dynamics for output and unemployment are unimportant, which justifies the assumption made in the text.

We first derive the employment probability $\phi(x)$. The law of motion of the number of long-term relationships—this number is equal to output—is given by

$$\dot{Y}(t) = f(x(t)) \cdot (k - Y(t)) - s \cdot Y(t). \tag{A1}$$

In this law of motion, $f(x(t)) \cdot (k - Y(t))$ is the number of long-term relationships created at t and $s \cdot Y(t)$ is the number of long-term relationships separated at t.

According to the matching process summarized by the law of motion, the number of relationships $Y(t)$ and the unemployment rate $1 - Y(t)/k$ are state variables. However, in practice, because the rate of outflow from unemployment is so large, the number of relationships and the unemployment rate are fast-moving state variables that adjust extremely rapidly to their steady-state levels.[17] Hence, the transitional dynamics of the variable $Y(t)$ defined by the differential equation (A1) are extremely fast and $Y(t)$ barely departs from its steady-state level defined by

$$0 = f(x(t)) \cdot (k - Y(t)) - s \cdot Y(t),$$

or equivalently

$$Y(t) = \frac{f(x(t))}{f(x(t)) + s} \cdot k = \phi(x) \cdot k.$$

Similarly, the unemployment rate barely departs from its steady-state level defined by

$$u(t) = \frac{s}{f(x(t)) + s} = 1 - \phi(x).$$

Figure A1 demonstrates that transitional dynamics are unimportant. It compares the actual fluctuations of the unemployment rate with the fluctuations of the steady-state level of the unemployment rate, computed using (2) and current measures of the job-finding and separation rates.[18]

We now derive the matching wedge $\tau(x)$. The household posts costly help-wanted advertisements to purchase services. To consume $c(t)$ services, the household purchases $C(t) \geq c(t)$ services. The $C(t)$ services are divided between consumption and matching. At time t, the household adjusts its number of relationships by $\dot{C}(t)$, and it also replaces the $s \cdot C(t)$ relationships that have just separated. Making these $\dot{C}(t) + s \cdot C(t)$ new relationships requires $\left(\dot{C}(t) + s \cdot C(t)\right)/q(x(t))$ help-wanted advertisements, each costing ρ services.[19] Hence, purchases and consumption are

[17] Hall [2005*b*], Pissarides [2009], and Shimer [2012] make this point.

[18] This figure is similar to Figure 1 in Hall [2005*b*]. Even though we use different measures of the job-finding and separation rates and a longer time period, Hall's conclusion that transitional dynamics are irrelevant remains valid. We construct the job-finding and separation rates in Section 4 using the method developed by Shimer [2012].

[19] We abstract from randomness at the household level and assume that during each unit time, a household purchases exactly $q(x(t)) \cdot v(t)$ services out of $v(t)$ vacancies.

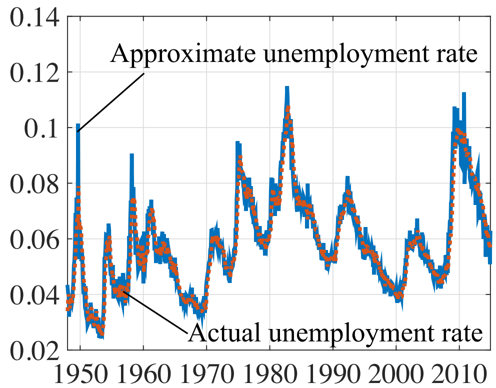


Figure A1: The Irrelevance of Transitional Dynamics For the Unemployment Rate

Notes: The actual unemployment rate (red dashed line) is the seasonally adjusted monthly unemployment rate constructed by the BLS from the CPS. The approximate unemployment rate (blue solid line) is computed using equation (2). This approximate rate abstracts from transitional dynamics and is computed using the job-finding and separation rates constructed in Section 4.

related by

$$C(t) = c(t) + \frac{\rho}{q(x(t))} \cdot \left(\dot{C}(t) + s \cdot C(t)\right)$$

which can be rewritten as the following differential equation:

$$\dot{C}(t) = -\frac{q(x(t))}{\rho} \cdot c(t) + \frac{q(x(t)) - s \cdot \rho}{\rho} \cdot C(t).$$

As we have just discussed above, the number of relationships $C(t)$ is a fast-moving state variable that can be well approximated by its stead-state level defined by

$$0 = -\left(q(x(t))/\rho\right) \cdot c(t) + \left[\left(q(x(t)/\rho\right) - s\right] \cdot C(t),$$

or equivalently

$$C(t) = \frac{q(x(t))}{q(x(t)) - s \cdot \rho} \cdot c(t) = (1 + \tau(x(t))) \cdot c(t).$$

The government purchases services on the market in the same way as households do. The government posts costly help-wanted advertisements to purchase services. To provide an amount $g(t)$ of public good, the government purchases $G(t) \geq g(t)$ services. The $G(t)$ services are divided between consumption and matching. At time t, the government adjusts its number of relationships by $\dot{G}(t)$, and it also replaces the $s \cdot G(t)$ relationships that have just separated. Making these $\dot{G}(t) + s \cdot G(t)$ new relationships requires $\left(\dot{G}(t) + s \cdot G(t)\right)/q(x(t))$ help-wanted advertisements,

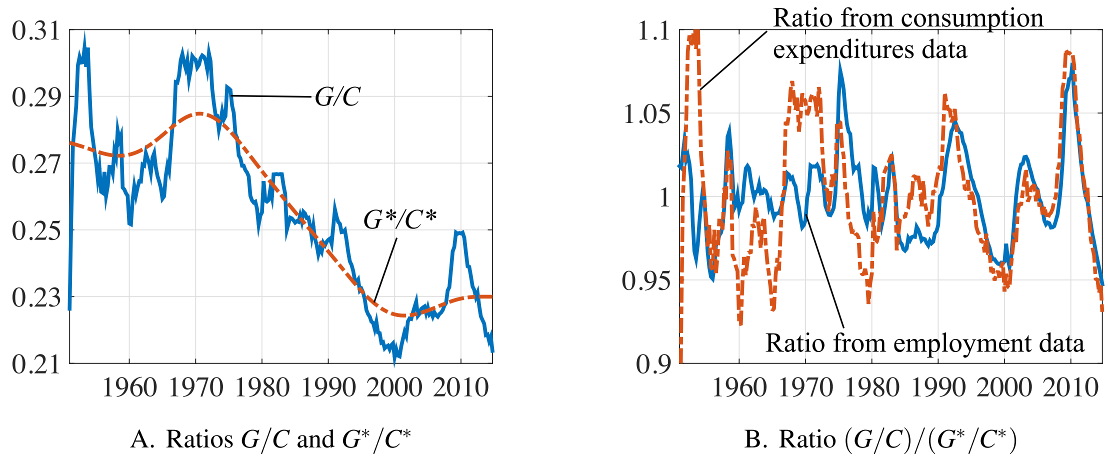


Figure A2: Public Consumption Expenditures in the US, 1951–2014

Notes: The data are seasonally adjusted quarterly data constructed by the BEA as part of the NIPA. The ratio G/C is government consumption expenditures in dollars divided by personal consumption expenditures in dollars. The ratio G^*/C^* is a low-frequency trend of the ratio G/C, produced with a HP filter with smoothing parameter 10^5. The ratio $(G/C)/(G^*/C^*)$ from employment data reproduces the times series in Figure 3.

each costing ρ services.[20] Hence, purchases and consumption are related by

$$G(t) = g(t) + \frac{\rho}{q(x(t))} \cdot \left(\dot{G}(t) + s \cdot G(t)\right).$$

The number of relationships $G(t)$ is a fast-moving state variable, and, following the same logic as above, it can be well approximated by its stead-state level given by

$$G(t) = \frac{q(x(t))}{q(x(t)) - s \cdot \rho} \cdot g(t) = (1 + \tau(x(t))) \cdot g(t).$$

Appendix B: Alternative Measure of Public-Good Spending Based on Consumption Expenditures Data

This appendix constructs the ratio G/C of personal consumption expenditures to government consumption expenditures. These data are constructed by the Bureau of Economic Analysis (BEA) as part of the National Income and Product Accounts (NIPA).

In levels, the two measures of G/C, plotted in Figures 3 and A2 are quite different. In terms of consumption expenditures, the ratio C/G started at 20.0% in 1947, peaked at 30.5% in 1953, hovered between 25% and 30.5% from 1953 to 1986, fell back to 24.7% in 1990, and averages 23.0% since 1990. We also find that public consumption expenditures constitute 18.7% of total consumption expenditures on average since 1990. We also find that on average since 1990, $G^*/C^* = 23.0\%$.

[20]We abstract from randomness at the government level and assume that during each unit time, a government purchases exactly $q(x(t)) \cdot v(t)$ services out of $v(t)$ vacancies.

The measure of $(G/C)/(G^*/C^*)$ based on consumption expenditures data is especially high in 1951–1953, which corresponds to the Korean war, and in 1967–1972, which corresponds to the Vietnam war. In part because military personnel is not counted as government employees in the BLS data, the government expenditures during wars do not appear in the measure of $(G/C)/(G^*/C^*)$ based on employment data.

As showed in Figure A2, after 1980, the two measures of $(G/C)/(G^*/C^*)$ nearly perfectly overlap. Changes in government expenditures compared to trend therefore track almost exactly changes in government employment compared to trend. It therefore does not matter how we measure public-good spending. For consistency, since we focus on labor market data in the empirical applications, we use the ratio $(G/C)/(G^*/C^*)$ obtained from employment data below.

Appendix C: Construction of the Labor Market Time Series Used For the Empirical Applications

To estimate the sufficient statistic $1-\eta$, we use time series for unemployment, vacancies, market tightness, and job-finding rate in the US for the 1951–2014 period. We construct and plot these data here.

The number of unemployed persons is constructed by the BLS from the Current Population Survey (CPS). Panel A in Figure A3 displays the number of unemployed persons, u_t.

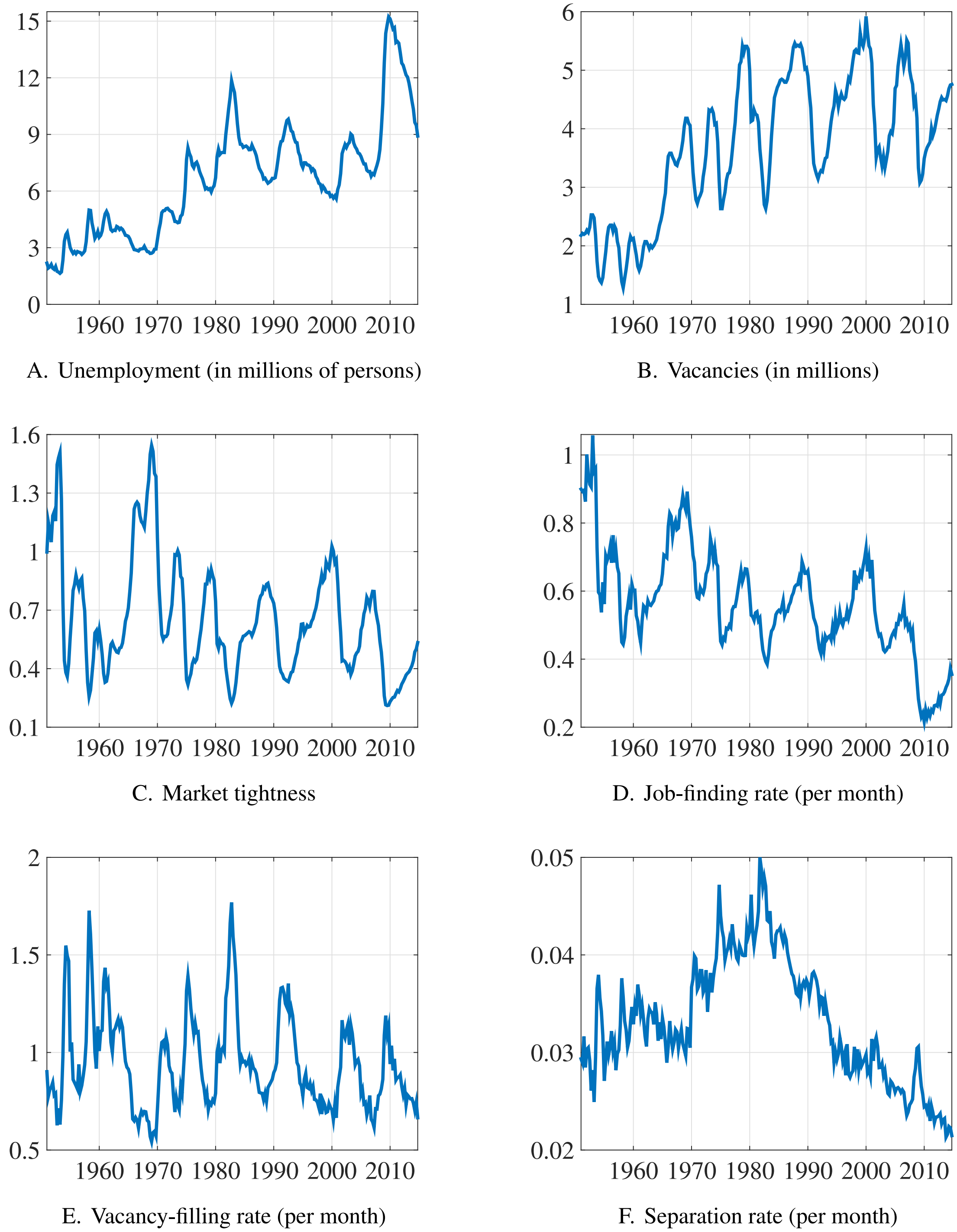
Since there are no long time series of vacancies in the US, we construct a proxy for vacancies. We start from the help-wanted advertising index constructed by Barnichon [2010]. This index combines the online and print help-wanted advertising indices constructed by the Conference Board (the print help-wanted advertising index is a standard proxy for vacancies).[21] Since December 2000, the BLS constructs the number of vacancies posted in the US using data collected in the Job Opening and Labor Turnover Survey (JOLTS). We then scale up the Barnichon index to transform it into a number of vacancies. The average value of the Barnichon index between December 2000 and December 2014 is 80.59. The average number of vacancies from JOLTS between December 2000 and December 2014 is 3.707 millions. Hence we multiply the Barnichon index by $3.707 \times 10^6/80.59 = 45,996$ to obtain a proxy for the number of vacancies since 1951. Panel B in Figure A3 displays the number of vacancies, v_t.

Dividing vacancies by unemployment, we construct a series of labor market tightness: $x_t = v_t/u_t$. Panel C in Figure A3 displays the market tightness, x_t.

The monthly job-finding rate is defined as $f_t = -\ln(1-F_t)$, where F_t is the monthly job-finding probability.[22] The job-finding probability F_t is constructed following the method developed by Shimer [2012] and using data constructed by the BLS from the CPS. Namely, we construct F_t using the relationship $F_t = 1-(u_{t+1}-u^s_{t+1})/u_t$, where u_t is the number of unemployed persons at time t and u^s_t is the number of short-term unemployed persons at time t. We measure u_t and u^s_t in the data constructed by the BLS from the CPS. The number of short-term unemployed persons is the number of unemployed persons with zero to four weeks duration, adjusted as in Shimer [2012] for the 1994–2014 period. Panel D in Figure A3 the monthly job-finding rate, f_t.

[21] Abraham and Wachter [1987] describes the print help-wanted advertising index in details, and argues that it provides an accurate proxy for vacancies.

[22] To obtain the relationship between f_t and F_t, we assume that unemployed workers find a job according to a Poisson process with arrival rate f_t.



A. Unemployment (in millions of persons)

B. Vacancies (in millions)

C. Market tightness

D. Job-finding rate (per month)

E. Vacancy-filling rate (per month)

F. Separation rate (per month)

Figure A3: Matching Process in the US, 1951–2014

Notes: Panel A displays the quarterly average of the seasonally adjusted monthly number of unemployed persons (in millions) constructed by the BLS from the CPS. We denote this series by u_t. Panel B displays the quarterly average of the monthly vacancy index constructed by Barnichon [2010], scaled to match the number of vacancies in JOLTS for 2001–2014. We denote this series by v_t. Panel C displays the labor market tightness $x_t = v_t/u_t$. Panel D displays the per-month job-finding rate f_t constructed following the methodology of Shimer [2012] and using CPS data. Panel E displays the per-month vacancy-filling rate $q_t = f_t/x_t$. Panel F displays the per-month separation rate constructed following the methodology of Shimer [2012] and using CPS data.

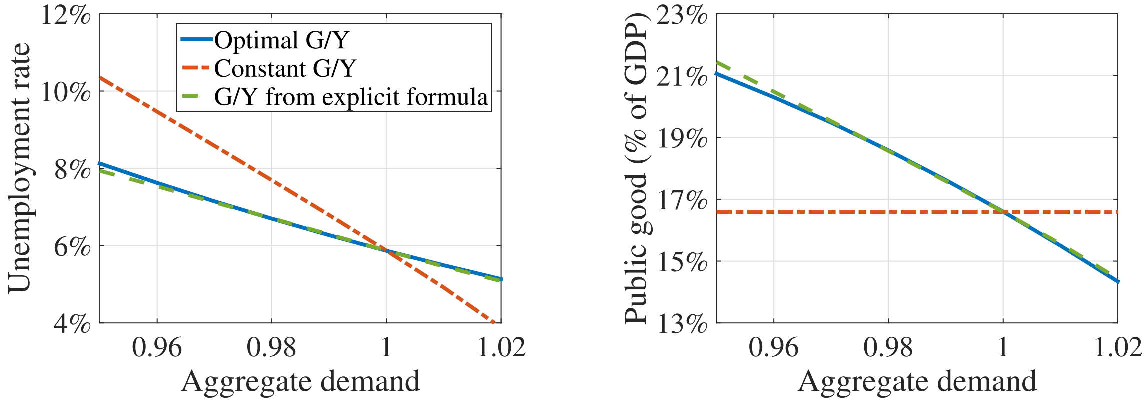
By displaying together unemployment, vacancies, market tightness, job-finding rate, vacancy-filling rate, and separation rate, Figure A3 also illustrates the matching process on the US labor market. The main facts about the matching process on the US labor market are well known.[23]

Appendix D: Robustness of Simulation Results to Alternative Calibrations

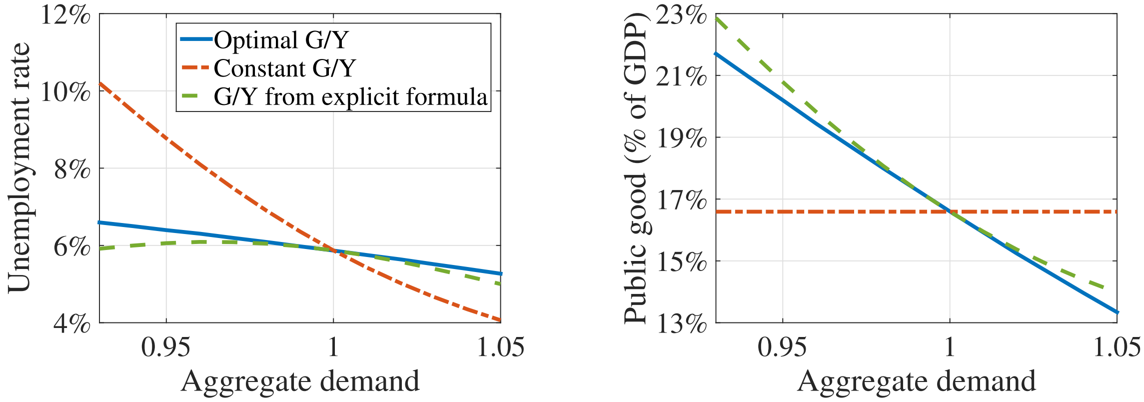
The simulation results in the text are obtained for an elasticity of substitution $\varepsilon = 1$ and a multiplier $dY/dG = 0.5$. This appendix describes the dependance of the results on the parameters ε and dY/dG.

We simulate the model for alternative calibrations and explore how the simulations outcomes are modified. Since the elasticity of substitution ε and the multiplier dY/dG are not estimated very accurately, we perform several simulations, each corresponding to a different calibration of ε and dY/dG. The insights from these additional simulations are consistent with the results obtained in Table 2.

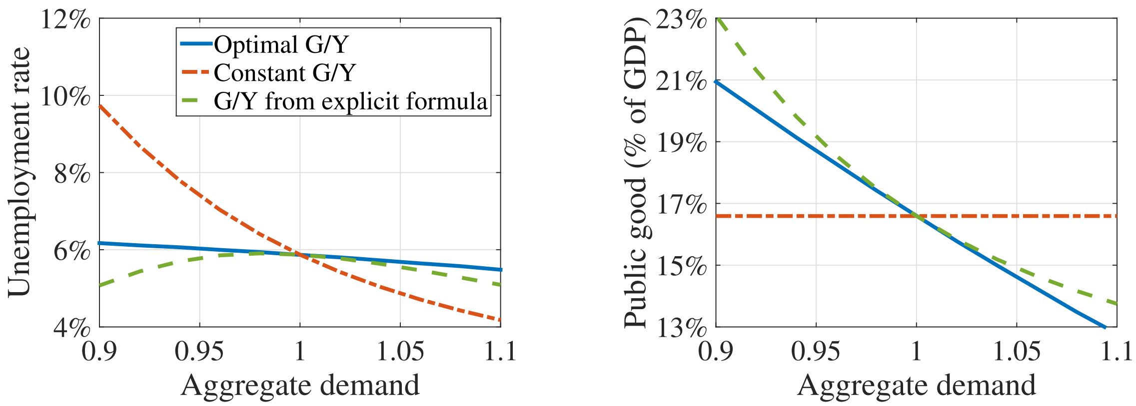
[23] See for instance Blanchard and Diamond [1989*a*,*b*, 1990] and Shimer [2005].



A. $\varepsilon = 0.9$ and $dY/dG = 0.5$

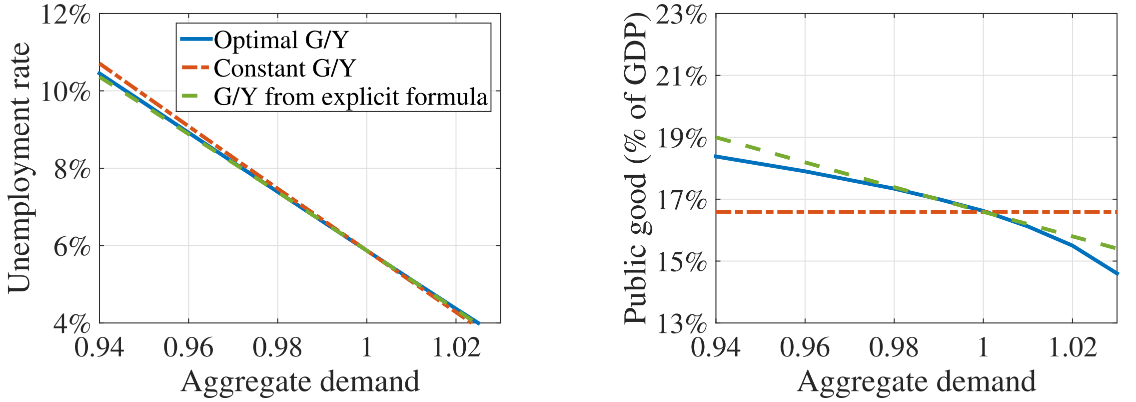


B. $\varepsilon = 2$ and $dY/dG = 0.5$

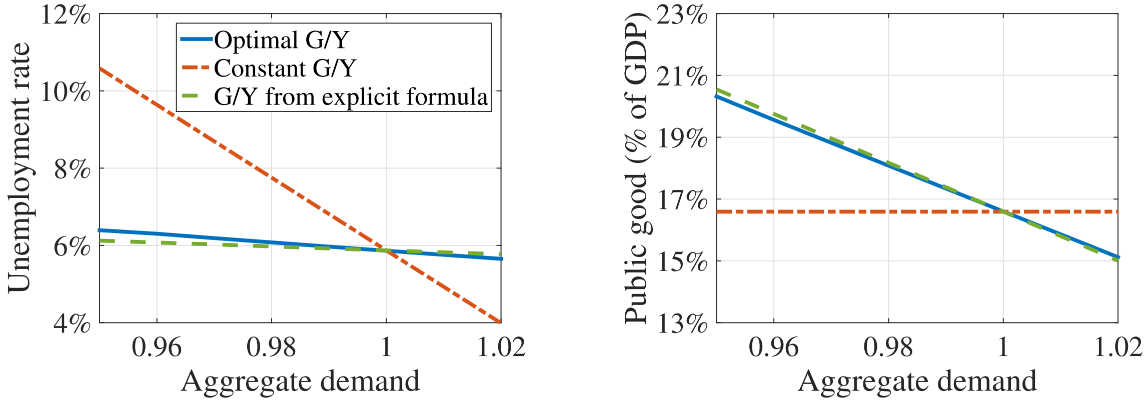


C. $\varepsilon = 4$ and $dY/dG = 0.5$

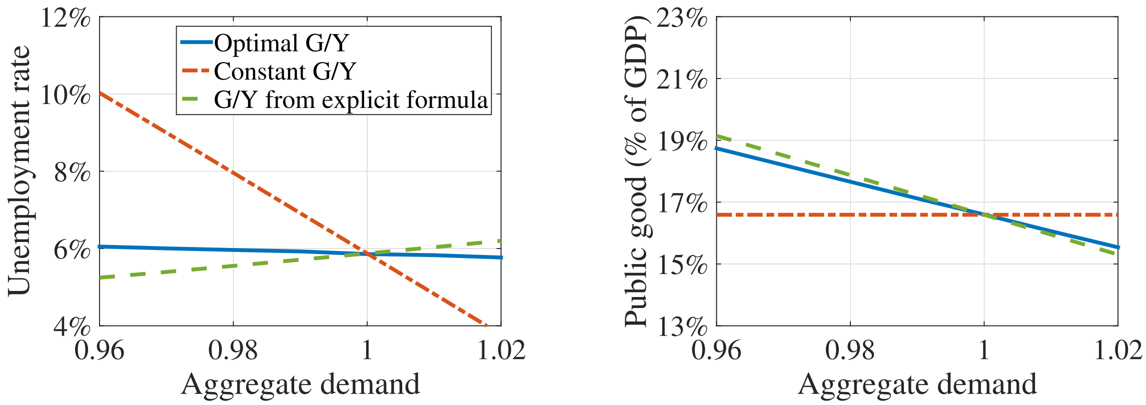
Figure A4: Optimal Public-Good Spending Under Alternative Calibrations of ε



A. $\varepsilon = 1$ and $dY/dG = 0.1$



B. $\varepsilon = 1$ and $dY/dG = 1$



C. $\varepsilon = 1$ and $dY/dG = 1.5$

Figure A5: Public-Good Spending Under Alternative Calibrations of dY/dG