The Optimal Use of Government Purchases for Macroeconomic Stabilization

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"You are Barack Obama. It is early 2009. The unemployment rate has reached 9% and the federal funds rate is 0%. By how much should you increase government purchases?"

we propose an answer based on 2 sufficient statistics:

- the government-purchases multiplier
- the elasticity of substitution between government and personal consumptions

A model of unemployment with government purchases

Overview

- dynamic continuous-time model
- measure 1 of identical self-employed households
- benevolent government
- a matching market where labor services are traded
    - not all services are sold in equilibrium $\Rightarrow$ unemployment
    - modeling follows Michaillat and Saez [QJE, 2015]
    - market represents both a labor and a product market

The matching market for labor services

- the productive capacity of each household is 1
- households buy $C(t)$ services
- government buys $G(t)$ services
- households sell $Y(t) = C(t) + G(t) < 1$ services
- **the unemployment rate is $u(t) = 1 - Y(t)$**
- services are traded through long-term relationships

Matching function and market tightness

- households and government advertise $v(t)$ vacancies
- matching function: $m = \omega \cdot (1 - Y(t))^{\eta} \cdot v(t)^{1-\eta}$
- **market tightness:** $x(t) \equiv v(t)/(1 - Y(t))$
- rates at which new relationships are formed:

$$f(\underset{+}{x(t)}) \equiv \frac{m}{1 - Y(t)} = \omega \cdot x(t)^{1-\eta}$$

$$q(\underset{-}{x(t)}) \equiv \frac{m}{v(t)} = \omega \cdot x(t)^{-\eta}$$

Market flows

- relationships separate at rate s
- output is a state variable: $\dot{Y} = f(x) \cdot (1 - Y) - s \cdot Y$
- assumption: flows are balanced, $f(x) \cdot (1 - Y) = s \cdot Y$
- output, unemployment become jump variables

$$Y(\underset{+}{x}) = \frac{f(x)}{s + f(x)}, \quad u(\underset{-}{x}) = \frac{s}{s + f(x)}$$

Matching cost: ρ services per vacancy

$$\underbrace{Y}_{\text{gross output}} = \underbrace{y}_{\text{net output}} + \underbrace{\rho \cdot v}_{\text{matching cost}}$$

market flows are balanced so $s \cdot Y = v \cdot q(x)$ and

$$Y = y + \rho \cdot \frac{s \cdot Y}{q(x)}$$

$$Y \cdot \left[1 - \frac{s \cdot \rho}{q(x)}\right] = y$$

$$Y = \left[1 + \frac{s \cdot \rho}{q(x) - s \cdot \rho}\right] \cdot y$$

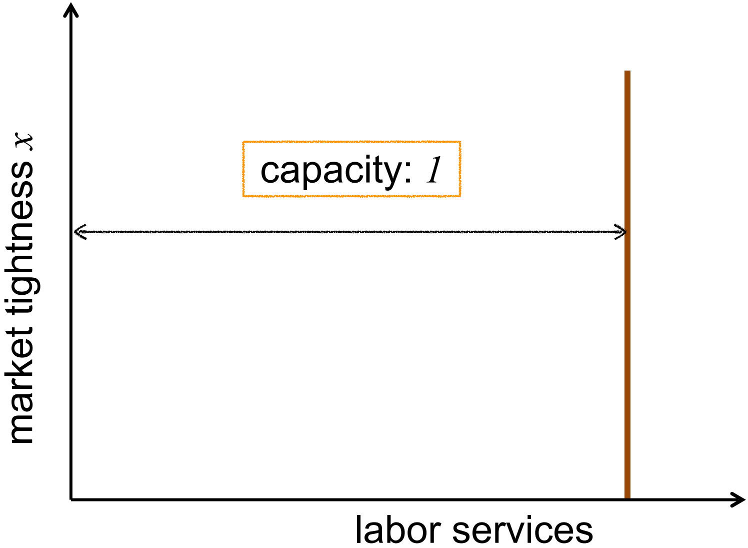
$$Y = [1 + \tau(\underset{+}{x})] \cdot y$$

Gross and net consumptions

net consumptions enter households' utility function

- C: gross personal consumption
- $c = C/(1+\tau(x))$: **net personal consumption**
- G: gross government consumption
- $g = G/(1+\tau(x))$: **net government consumption**

Gross output, unemployment, net output



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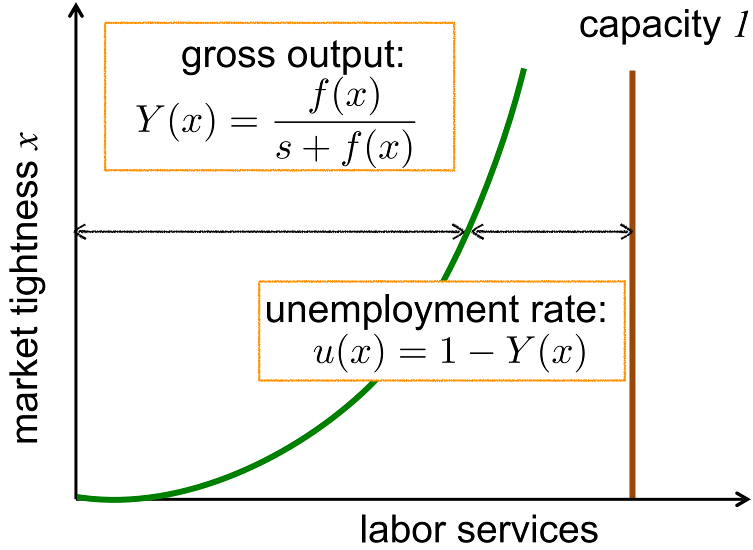

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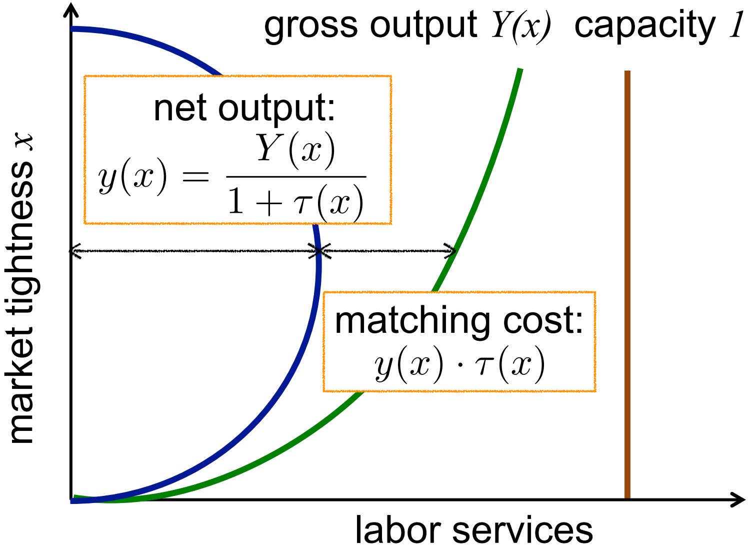
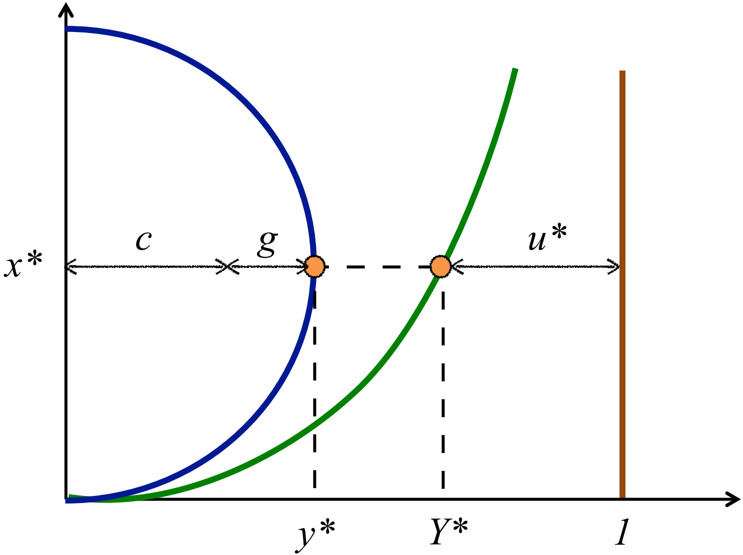


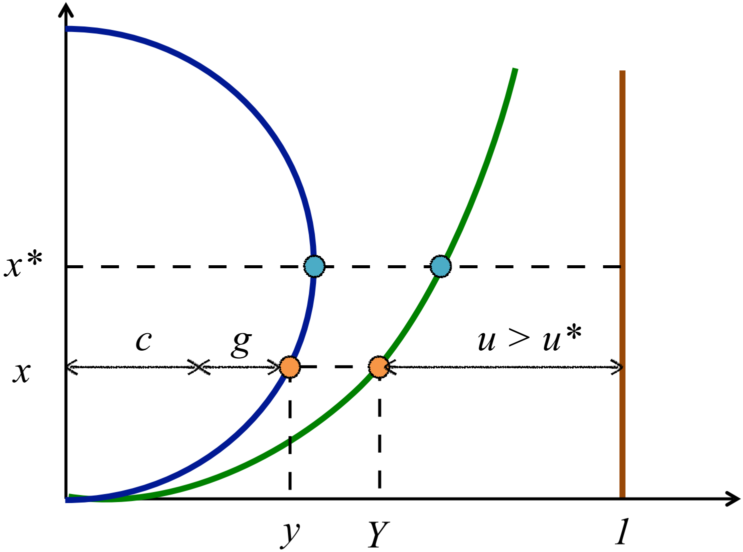
Feasible allocation and equilibrium

- a feasible allocation $[c, g, y, x]$ satisfies $y = y(x)$ and $y = c + g$
- an equilibrium function is $g \mapsto [c, g, y, x]$
- the equilibrium function reduces to $g \mapsto x(g)$

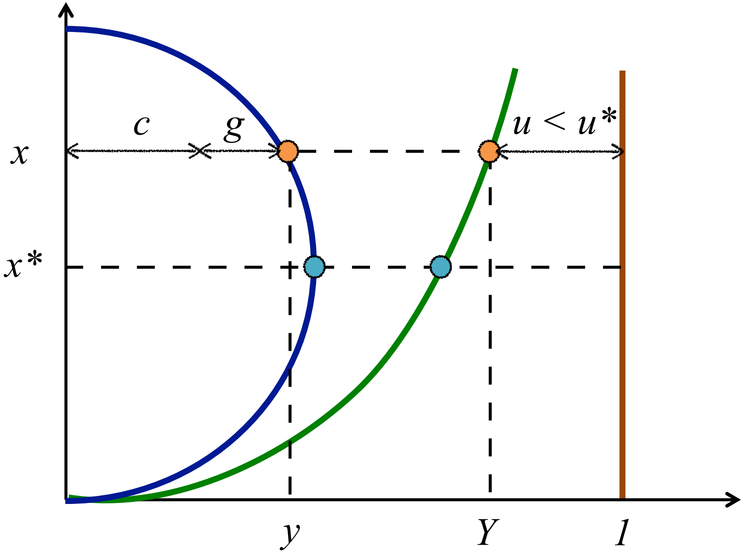
Efficient unemployment: $x(g) = x^*$



Inefficiently high unemployment: $x(g) < x^*$



Inefficiently low unemployment: $x(g) > x^*$



Sufficient-statistics formula for optimal government purchases

Value of government purchases

- we follow Samuelson [REStat, 1954]
- households' instantaneous utility is $\mathscr{U}(c, g)$
- $\mathscr{U}$ is homothetic so the marginal rate of substitution

$$MRS_{gc} \equiv \frac{\partial \mathscr{U} / \partial g}{\partial \mathscr{U} / \partial c}$$

  is a decreasing function of $g/c = G/C$

- MRS_{gc} is a decreasing function of G/Y

Government's problem

given an equilibrium function $x(g)$, the government's problem is to determine g to maximize welfare

$$\mathscr{U}\left(\underbrace{y(x(g)) - g}_{c}, g\right)$$

Formula in sufficient statistics

- the first-order condition is

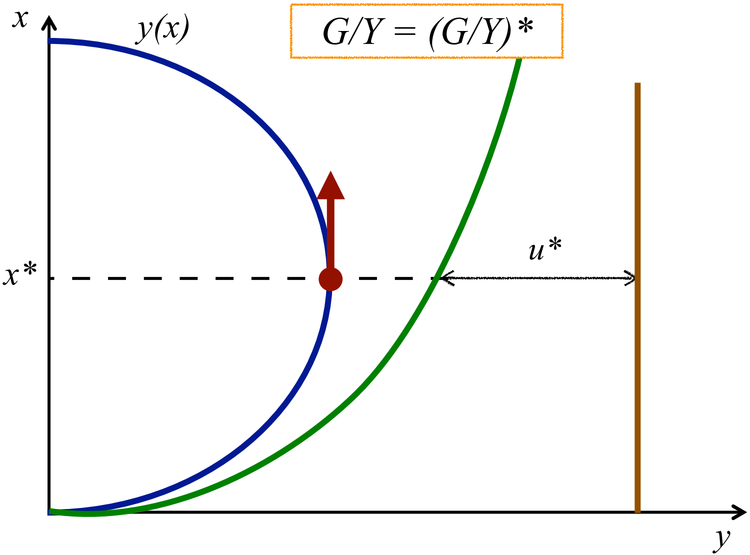
$$0 = \frac{\partial \mathscr{U}}{\partial g} - \frac{\partial \mathscr{U}}{\partial c} + \frac{\partial \mathscr{U}}{\partial c} \cdot \frac{dy}{dx} \cdot \frac{dx}{dg}$$

$$1 = \frac{\partial \mathscr{U} / \partial g}{\partial \mathscr{U} / \partial c} + \frac{dy}{dx} \cdot \frac{dx}{dg}$$

- optimal government purchases satisfy

$$\underbrace{1 = MRS_{gc}}_{\text{Samuelson formula}} + \underbrace{\frac{dy}{dx} \cdot \frac{dx}{dg}}_{\text{correction term}}$$

Optimal G/Y versus Samuelson's $(G/Y)^*$



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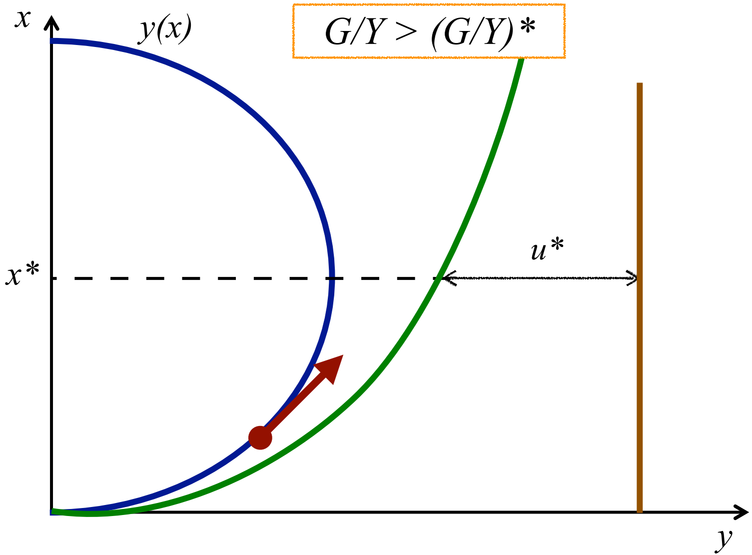

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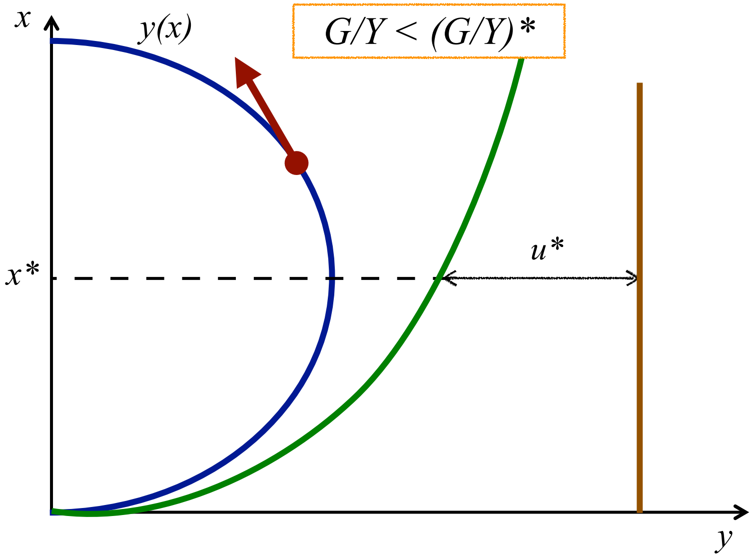

Assessment of actual US government purchases

Introducing estimable statistics

linearize abstract formula around efficient tightness x^* and Samuelson's ratio $(G/C)^*$:

$$1 - MRS_{gc} = \frac{dy}{dg}$$

$$\frac{1}{\epsilon} \cdot \frac{G/C - (G/C)^*}{(G/C)^*} \approx -\frac{x - x^*}{x^*} \cdot \frac{dY}{dG}$$

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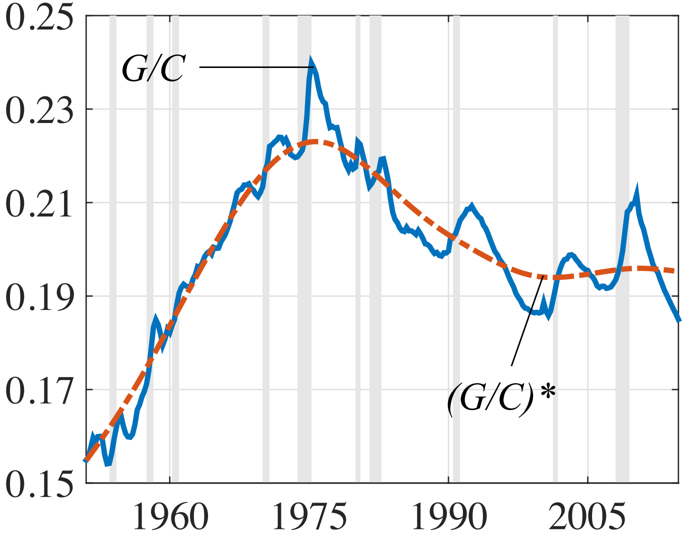
An implicit formula

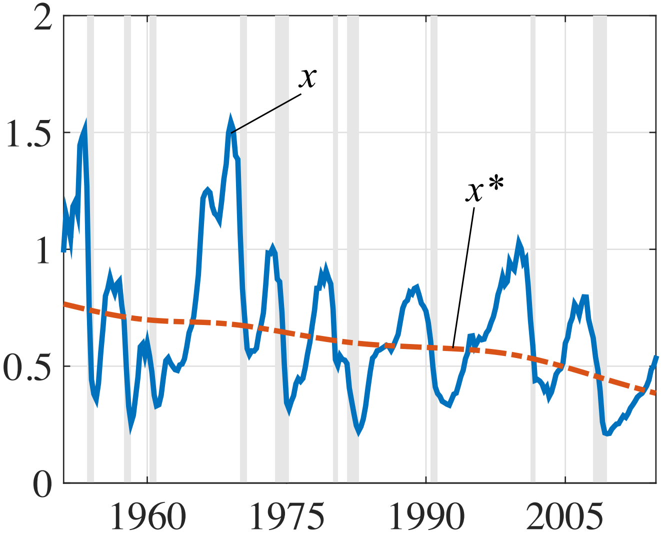
$$\frac{G/C-(G/C)^*}{(G/C)^*} \approx -\varepsilon \cdot \frac{dY}{dG} \cdot \frac{x-x^*}{x^*}$$

- dY/dG: government-purchases multiplier
- ε: elasticity of substitution between g and c
  - $\varepsilon = 0$: Leontief preferences, "bridges to nowhere"
  - $\varepsilon = 1$: Cobb-Douglas preferences, benchmark case
  - $\varepsilon \to +\infty$: linear preferences, perfect substitute

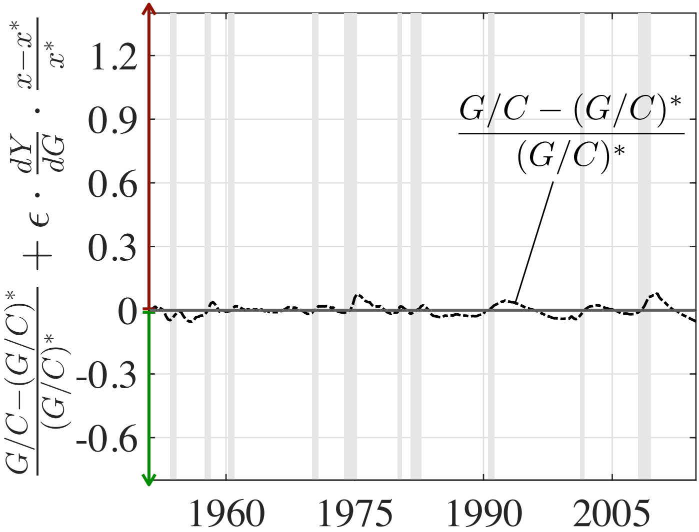
Public employment/private employment in the US



Labor market tightness in the US



Assessment of US government purchases



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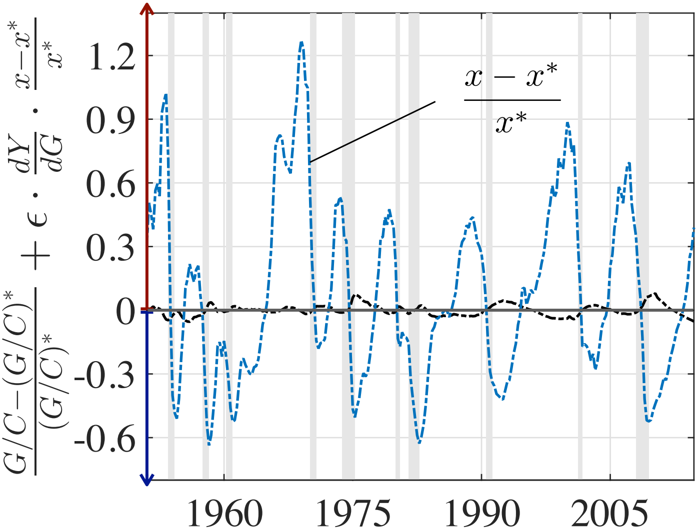

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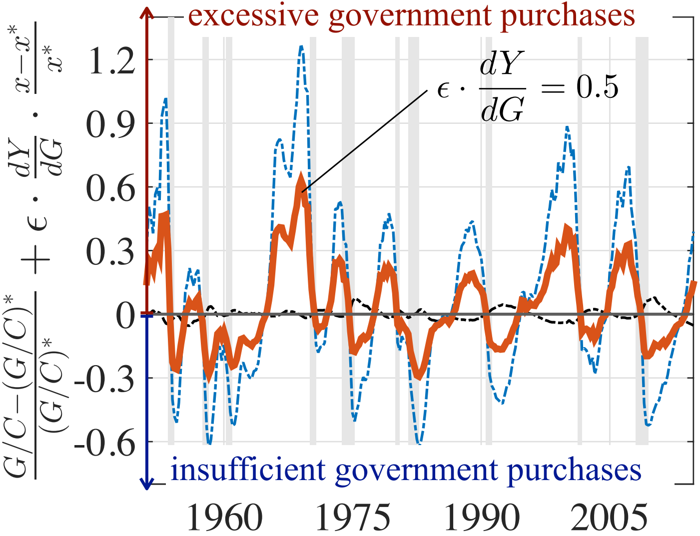

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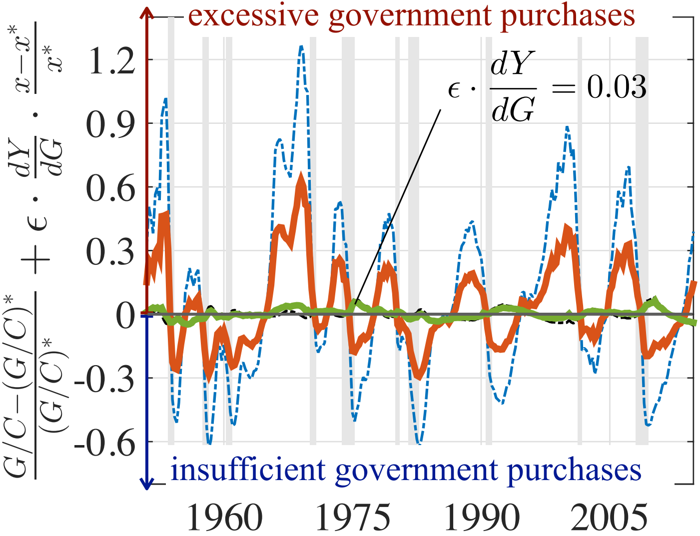

Optimal response to an increase in unemployment rate from 5.9% to 9%

An explicit formula

- consider a shock to $x(g)$ bringing the economy from $[x^*, (G/C)^*]$ to $[x_0, (G/C)^*]$
- the optimal response of government purchases is

$$\frac{G/C - (G/C)^*}{(G/C)^*} \approx -\varepsilon \cdot \frac{dY}{dG} \cdot \frac{x(G/C) - x^*}{x^*}$$

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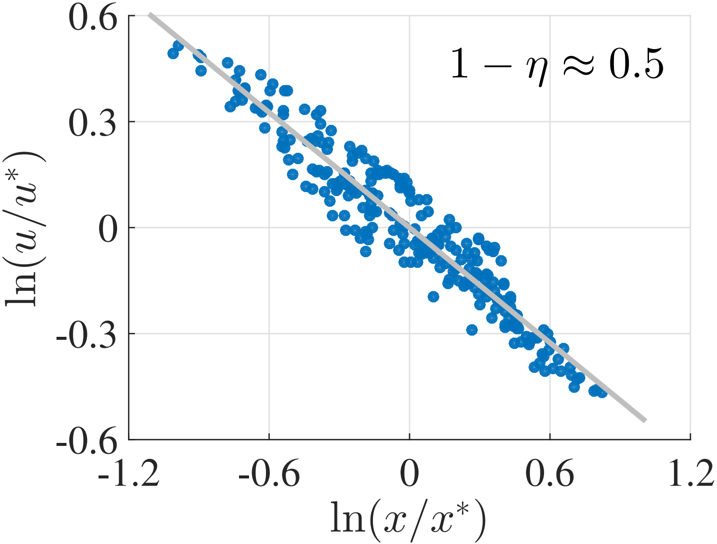
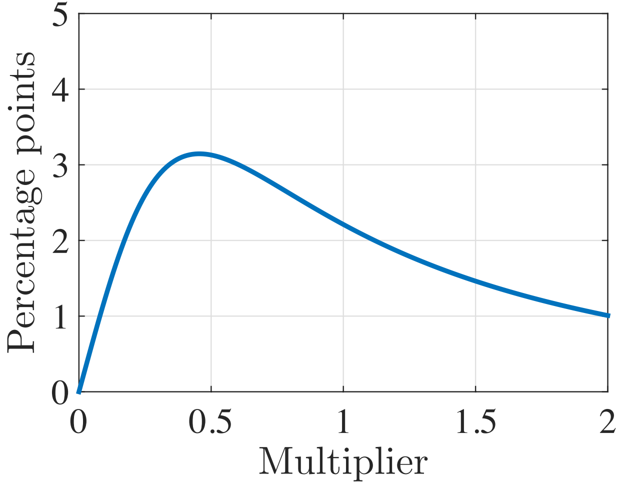
$$\frac{G/C - (G/C)^*}{(G/C)^*} \approx \frac{-\varepsilon \cdot \frac{dY}{dG}}{1 + \varepsilon \cdot \left(\frac{dY}{dG}\right)^2 \cdot \frac{(G/Y)^* \cdot (1-(G/Y)^*)}{(1-\eta) \cdot u^*}} \cdot \frac{x_0 - x^*}{x^*}$$

$$(u - u^*)/u^* \approx -(1-\eta)\cdot(x - x^*)/x^*$$



Optimal increase in G/Y when unemployment rises from 5.9% to 9%



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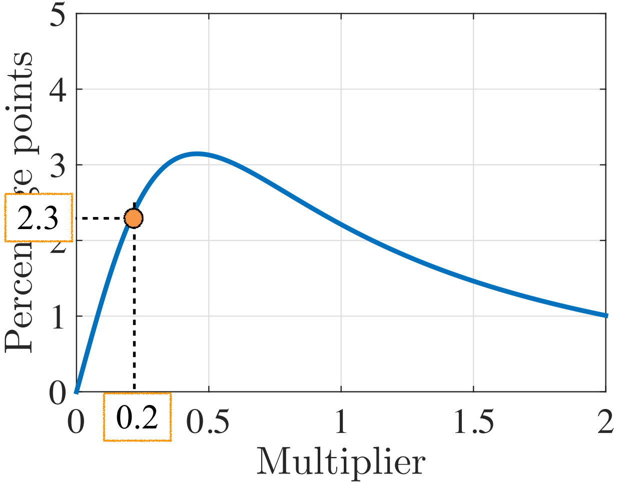

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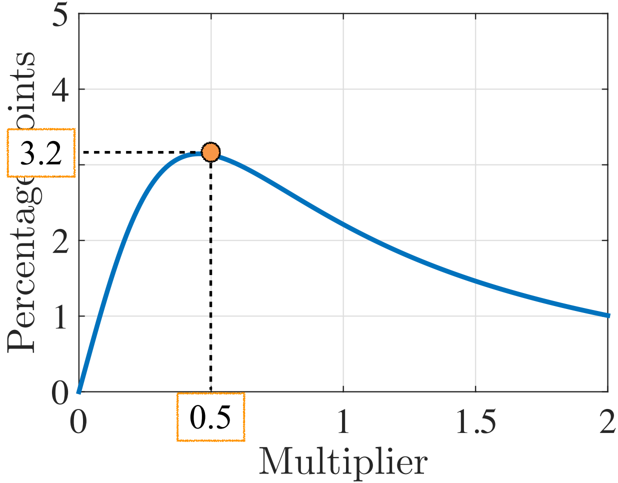

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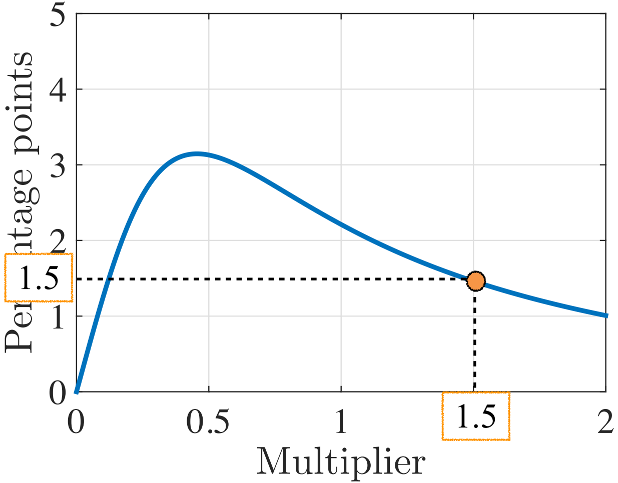

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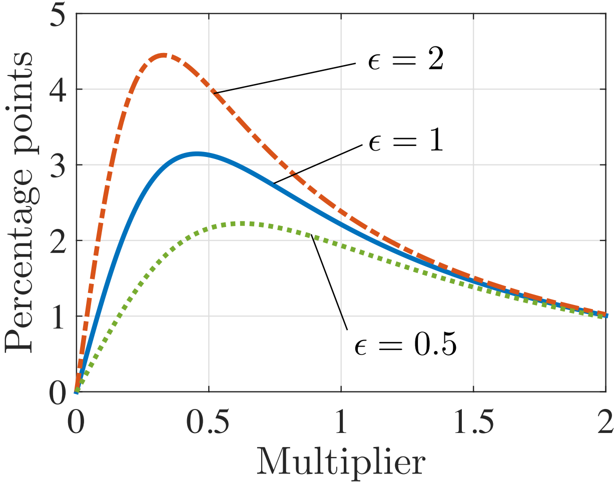

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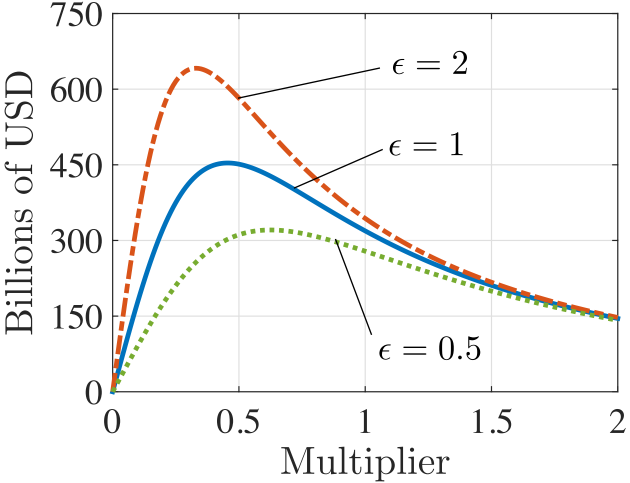

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